Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.5-Secant/121-4.5.2.1-a+b-sec-^m-c+d-sec-^n

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [241]. This is test number [121].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.59 (240)	0.41 (1)
Mathematica	97.10 (234)	2.90 (7)
Maple	90.46 (218)	9.54 (23)
Fricas	61.00 (147)	39.00 (94)
Maxima	40.66 (98)	59.34 (143)
Giac	31.54 (76)	68.46 (165)
Mupad	23.24 (56)	76.76 (185)
Sympy	2.49 (6)	97.51 (235)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

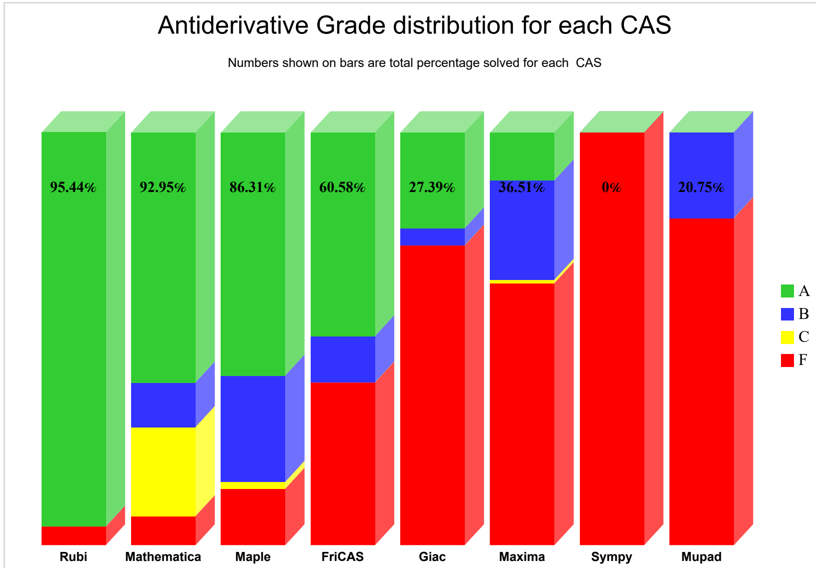
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

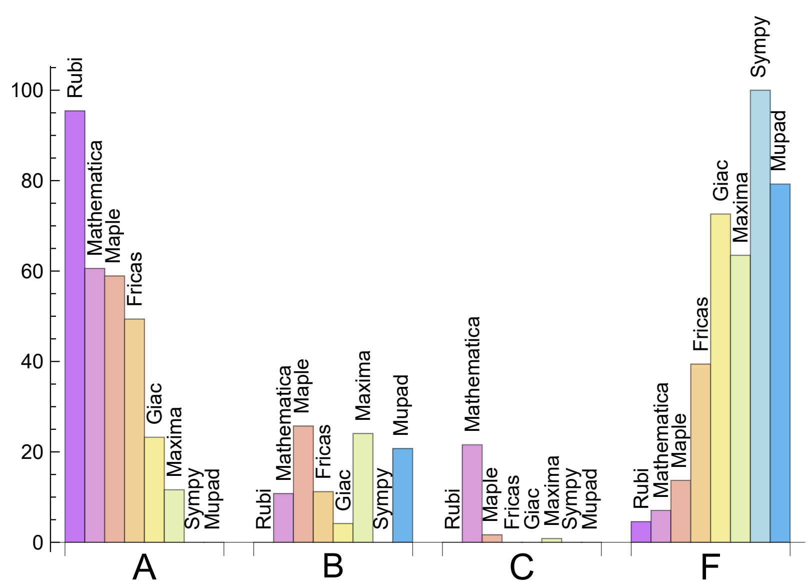
System	% A grade	% B grade	% C grade	% F grade
Rubi	95.436	0.000	0.000	4.564
Mathematica	60.581	10.788	21.577	7.054
Maple	58.921	25.726	1.660	13.693
Fricas	49.378	11.203	0.000	39.419
Giac	23.237	4.149	0.000	72.614
Maxima	11.618	24.066	0.830	63.485
Mupad	0.000	20.747	0.000	79.253
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	7	100.00	0.00	0.00
Maple	23	100.00	0.00	0.00
Fricas	94	68.09	31.91	0.00
Maxima	143	72.03	15.38	12.59
Giac	165	73.33	0.00	26.67
Mupad	185	0.00	100.00	0.00
Sympy	235	85.11	14.89	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.44
Giac	0.92
Maxima	1.37
Fricas	3.57
Mathematica	7.19
Maple	7.91
Mupad	24.53
Sympy	38.33

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	26.83	0.94	27.00	0.93
Giac	200.91	1.22	103.50	0.93
Rubi	212.70	1.00	153.00	1.00
Fricas	568.29	3.15	405.00	2.86
Maxima	770.86	4.75	255.50	2.45
Mathematica	813.40	2.38	122.00	0.90
Mupad	1614.84	6.20	122.00	1.09
Maple	5191.79	10.18	211.00	1.37

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

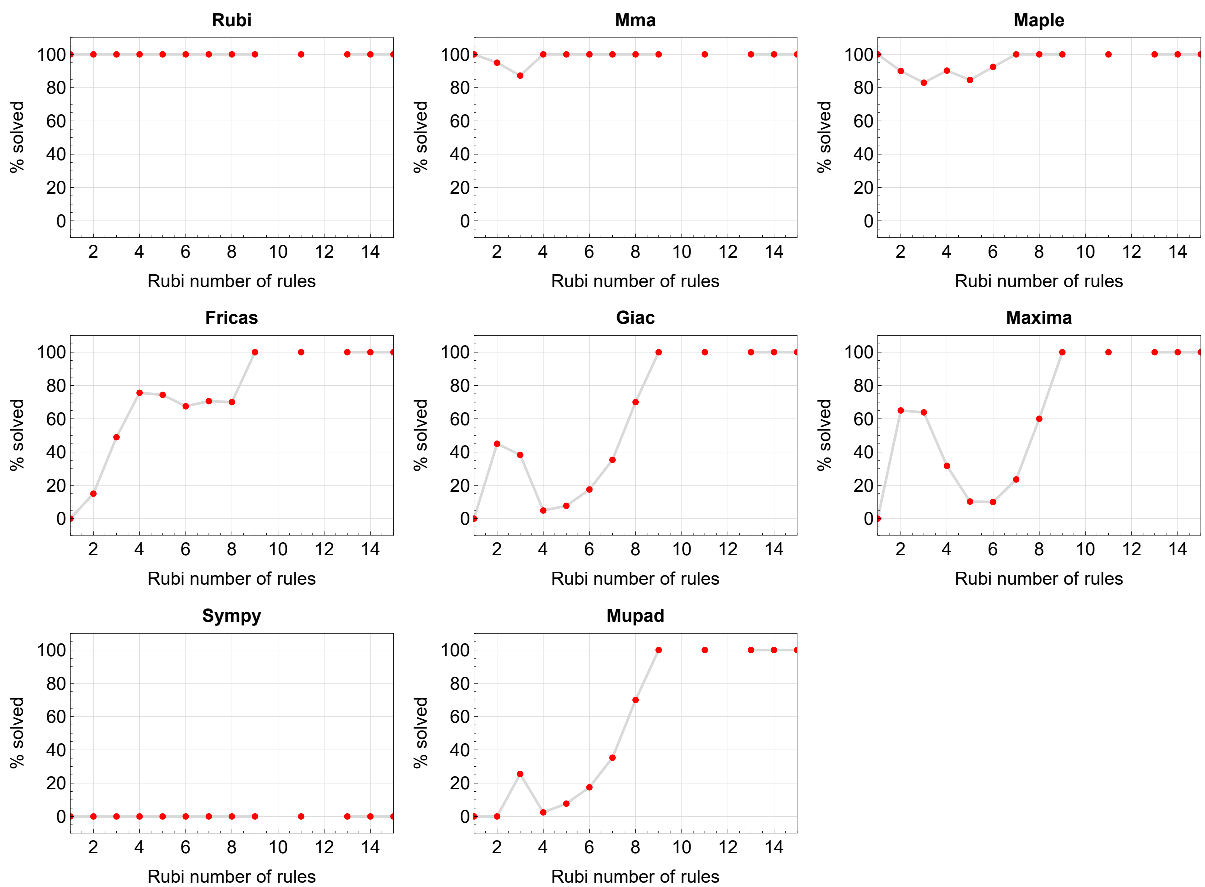


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

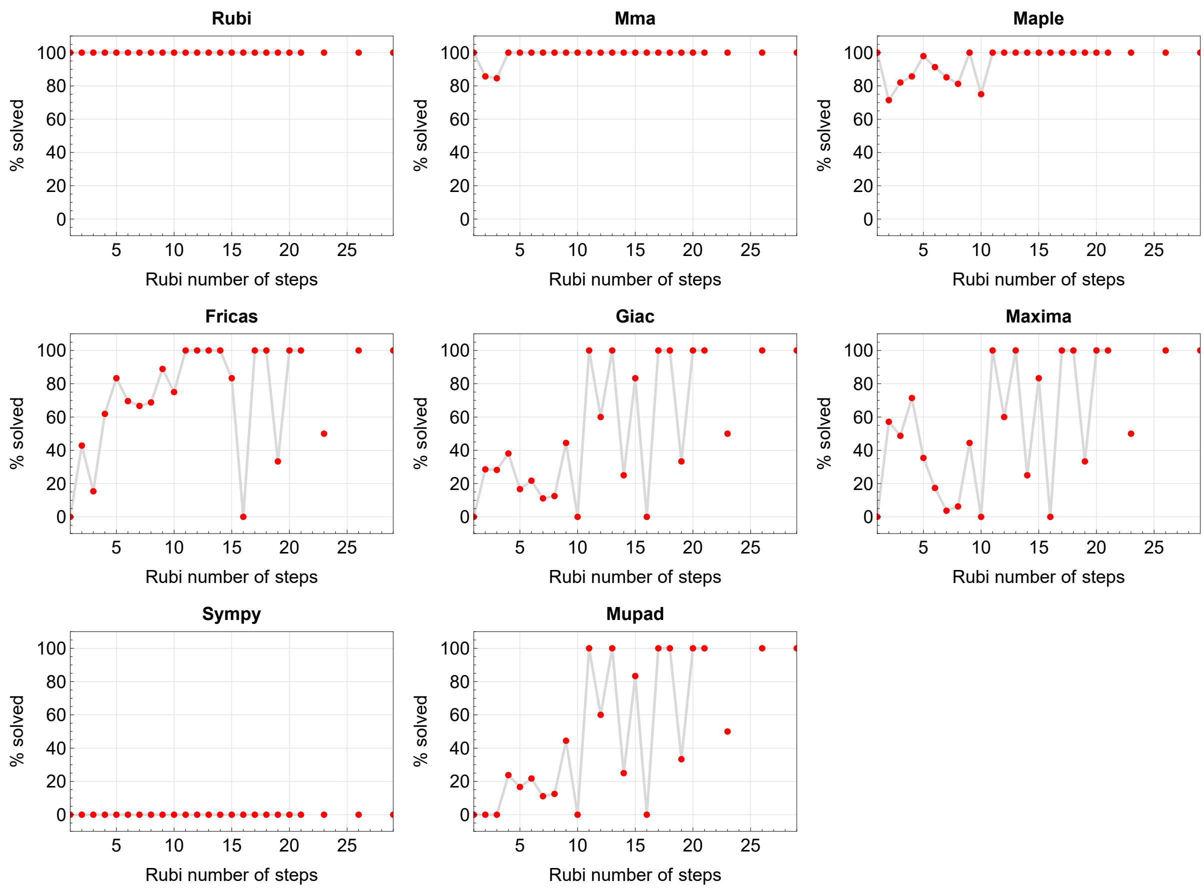


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

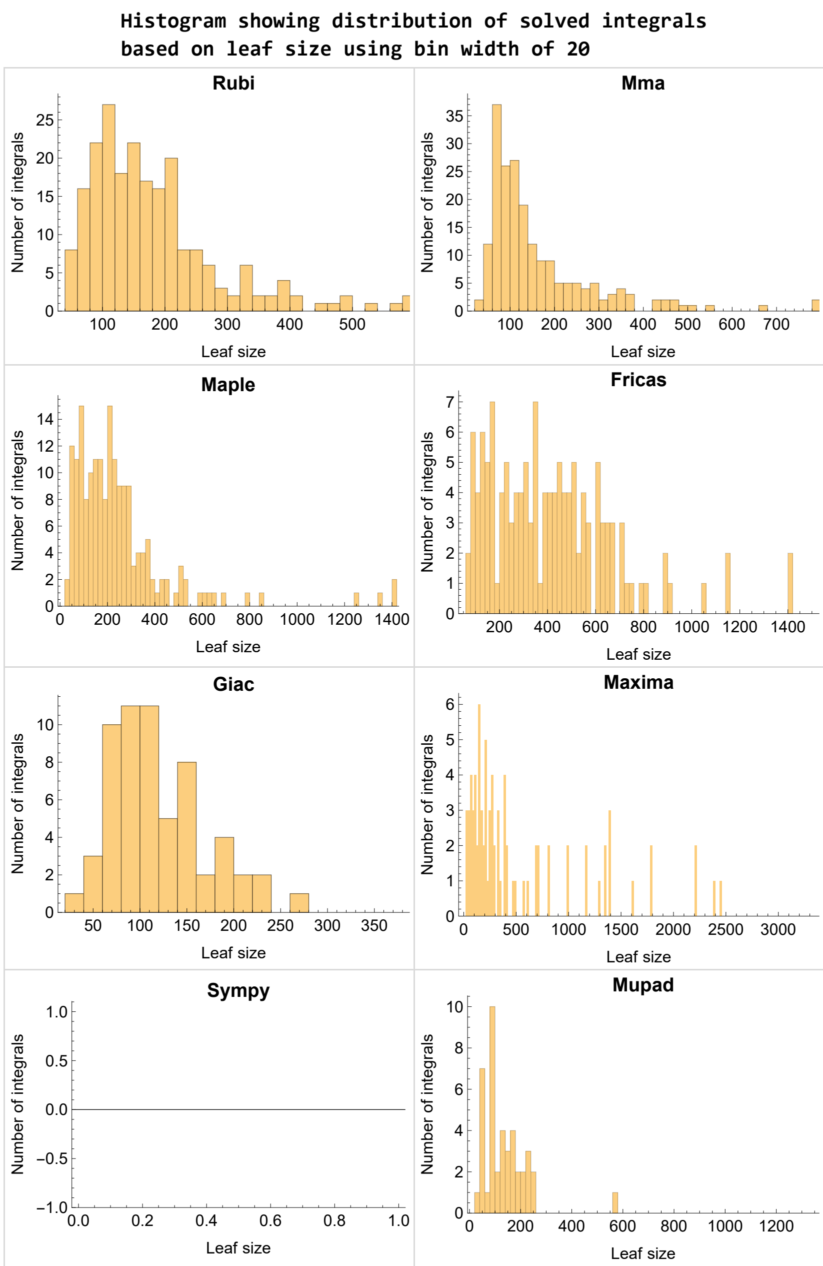


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

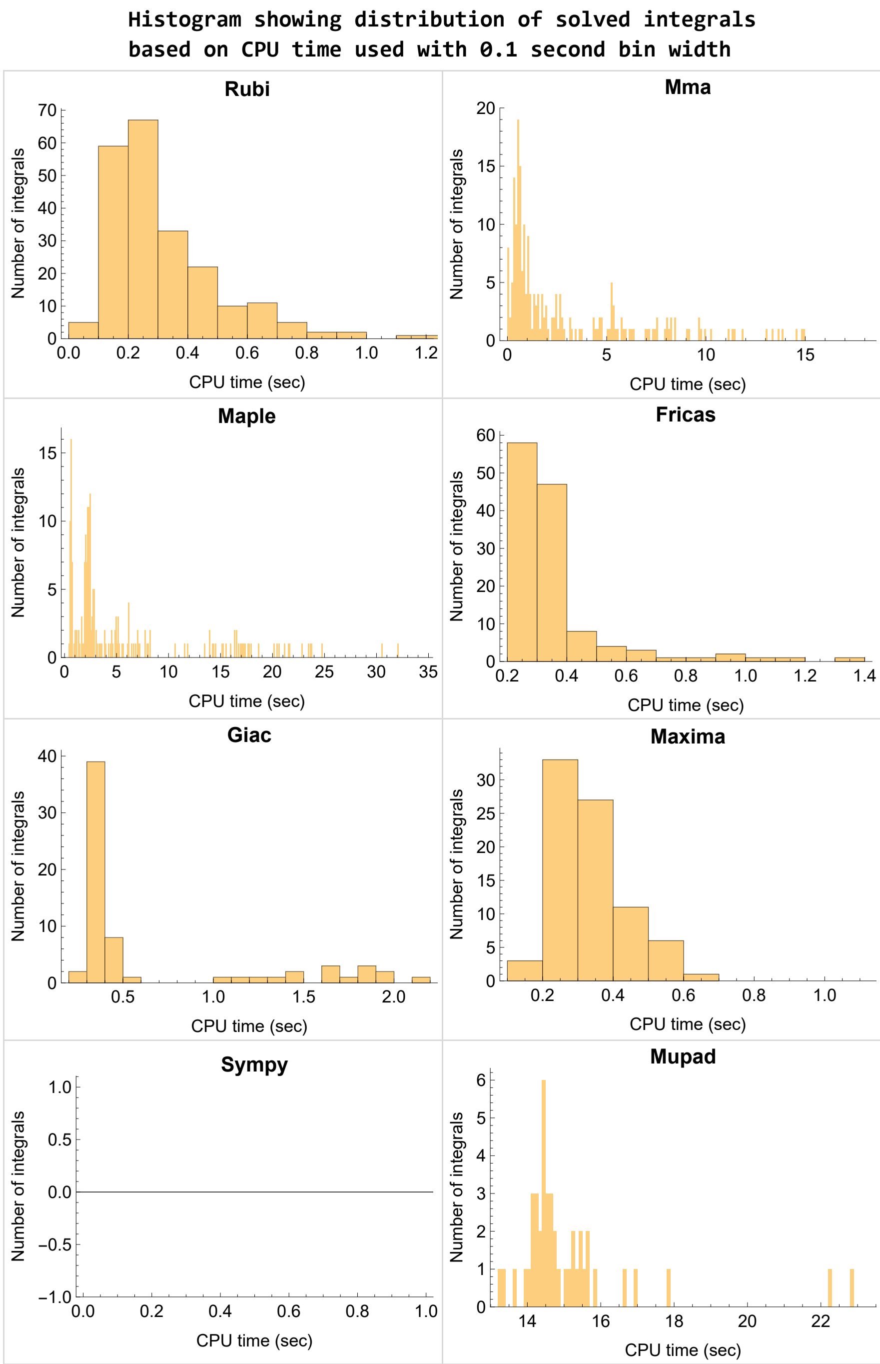


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

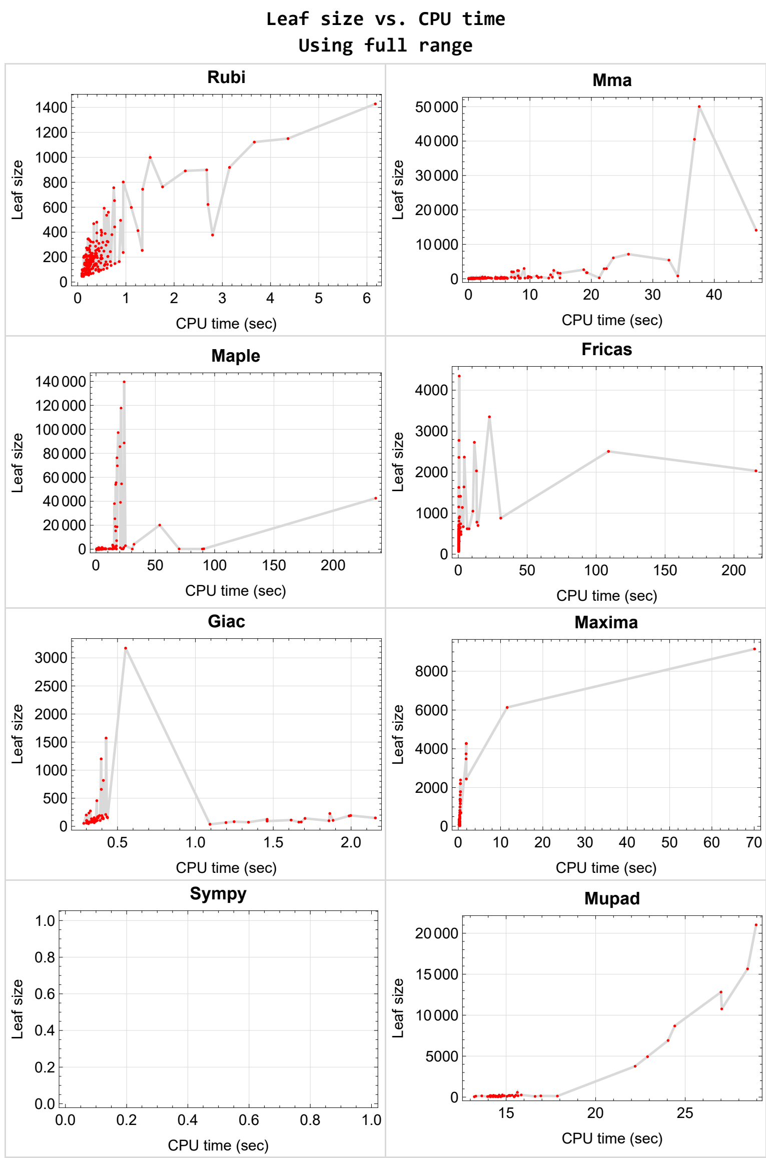


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{221, 222, 223, 224, 225, 226, 227, 228, 229, 236}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {216}

Mathematica {147, 149, 152, 153, 158, 159, 163, 165, 166, 167, 170, 171, 172, 175, 176, 177, 181, 182, 183, 200, 202, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 230, 240, 241}

Maple {42, 43, 50, 59, 65, 66, 72, 73, 79, 80, 81, 82, 83, 84, 100, 107, 108, 109, 123, 124, 129, 151, 152, 153, 157, 158, 159, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 187, 188, 205, 209, 210, 211, 212, 213, 214, 215, 216, 220}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima abs_integrate was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function LeafSize was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

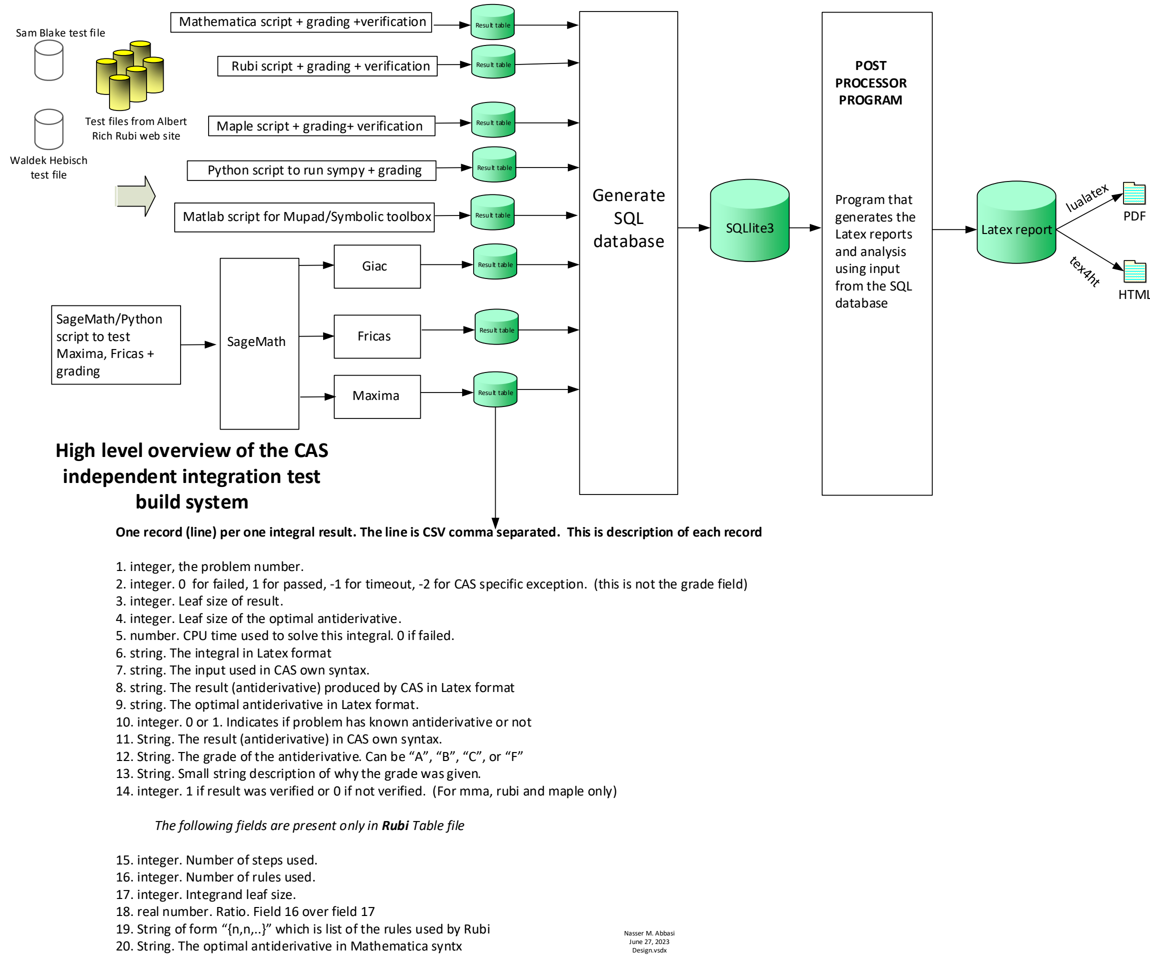
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
2.2 Detailed conclusion table per each integral for all CAS systems 26
2.3 Detailed conclusion table specific for Rubi results 75

2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 24
Giac 24
Mupad 25
Sympy 25

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

B grade { }

C grade { }

F normal fail { 217 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15, 19, 24, 33, 34, 42, 43, 44, 45, 50, 51, 52, 53, 57, 58, 59, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 141, 142, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 168, 169, 170, 173, 174, 175, 176, 178, 179, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 196, 197, 198, 199, 201, 203, 204, 208, 218, 231, 232, 233, 234, 235, 237, 238, 239 }

B grade { 6, 17, 25, 144, 171, 177, 180, 183, 193, 195, 200, 202, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 220, 230, 240, 241 }

C grade { 7, 9, 10, 16, 18, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 46, 47, 48, 49, 54, 55, 56, 60, 61, 62, 63, 64, 69, 70, 71, 76, 77, 78, 84, 85, 143, 149, 163, 166, 167, 172, 207, 217, 219 }

F normal fail { 131, 132, 133, 134, 135, 136, 137 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 53, 54, 57, 58, 59, 60, 65, 66, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 143, 144, 145, 146, 147, 148, 149, 150, 154, 155, 160, 161, 162, 173, 178, 179, 189, 190, 191, 192, 193, 194, 195, 196, 199, 201, 203, 204, 207, 208, 217, 218, 219 }

B grade { 48, 49, 52, 55, 56, 61, 62, 63, 64, 67, 71, 75, 76, 84, 105, 106, 118, 151, 152, 153, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 197, 198, 200, 202, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 220 }

C grade { 1, 11, 12, 13 }

F normal fail { 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 94, 95, 96, 101, 102, 103, 130, 143, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 175, 178, 179, 184, 185, 187, 188, 189 }

B grade { 5, 22, 23, 61, 62, 63, 74, 75, 83, 89, 105, 114, 118, 122, 153, 159, 174, 180, 186, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { 90, 91, 92, 93, 97, 98, 99, 100, 104, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 198, 200, 202, 205, 206, 208, 210, 211, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-1) timedout fail { 146, 171, 176, 177, 181, 182, 183, 199, 201, 203, 204, 207, 209, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 4, 5, 13, 15, 26, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 41, 89, 90, 97, 98, 106, 112, 113, 114, 119, 125 }

B grade { 3, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 31, 32, 33, 34, 45, 52, 59, 86, 87, 88, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 107, 108, 109, 115, 116, 117, 120, 121, 122, 123, 126, 127, 128, 129, 130, 150, 156, 162 }

C grade { 68, 168 }

F normal fail { 42, 43, 44, 46, 47, 48, 50, 51, 53, 58, 60, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 154, 155, 157, 161, 163, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 184, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-1) timedout fail { 49, 54, 55, 56, 57, 61, 62, 63, 64, 79, 80, 81, 153, 158, 159, 160, 164, 165, 177, 178, 182, 183 }

F(-2) exception fail { 104, 105, 110, 111, 118, 124, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197 }

Giac

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 190, 192 }

B grade { 5, 27, 38, 189, 191, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-1) timedout fail { }

F(-2) exception fail { 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 110, 111, 112, 117, 118, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 189, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { }

F(-1) timedout fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 226, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 112, 113, 114, 115, 116, 119, 120, 121, 122, 126, 127, 128, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

F(-1) timedout fail { 63, 64, 86, 87, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 117, 118, 123, 124, 125, 129, 130, 138, 139, 213, 214, 215, 216, 223, 226, 228, 229 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	165	206	334	179	0	191	228
N.S.	1	1.00	0.84	1.05	1.70	0.91	0.00	0.97	1.16
time (sec)	N/A	0.407	1.743	4.912	0.232	0.276	0.000	0.400	15.544

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	146	172	240	163	0	172	195
N.S.	1	1.00	1.04	1.23	1.71	1.16	0.00	1.23	1.39
time (sec)	N/A	0.258	1.160	3.126	0.214	0.269	0.000	0.373	15.150

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	122	133	203	147	0	153	163
N.S.	1	1.00	1.26	1.37	2.09	1.52	0.00	1.58	1.68
time (sec)	N/A	0.155	0.730	3.321	0.207	0.280	0.000	0.369	15.224

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	56	57	65	0	48	84
N.S.	1	1.00	0.96	1.19	1.21	1.38	0.00	1.02	1.79
time (sec)	N/A	0.086	0.025	1.699	0.205	0.248	0.000	0.313	16.617

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	97	84	95	103	0	103	91
N.S.	1	1.00	1.76	1.53	1.73	1.87	0.00	1.87	1.65
time (sec)	N/A	0.087	0.263	1.651	0.192	0.268	0.000	0.302	14.431

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	201	64	153	87	0	77	46
N.S.	1	1.00	3.59	1.14	2.73	1.55	0.00	1.38	0.82
time (sec)	N/A	0.229	1.609	0.490	0.279	0.260	0.000	0.313	14.136

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	53	41	174	88	0	57	40
N.S.	1	1.00	0.75	0.58	2.45	1.24	0.00	0.80	0.56
time (sec)	N/A	0.318	0.065	0.617	0.300	0.262	0.000	0.301	14.565

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	151	54	215	128	0	72	96
N.S.	1	1.00	1.48	0.53	2.11	1.25	0.00	0.71	0.94
time (sec)	N/A	0.435	1.356	0.598	0.296	0.253	0.000	0.324	14.427

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	119	67	294	172	0	88	124
N.S.	1	1.00	0.89	0.50	2.21	1.29	0.00	0.66	0.93
time (sec)	N/A	0.554	0.829	0.547	0.299	0.253	0.000	0.348	14.299

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	111	80	335	212	0	104	146
N.S.	1	1.00	0.68	0.49	2.04	1.29	0.00	0.63	0.89
time (sec)	N/A	0.676	0.867	0.686	0.296	0.255	0.000	0.370	14.414

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	189	217	356	195	0	210	259
N.S.	1	1.00	1.01	1.15	1.89	1.04	0.00	1.12	1.38
time (sec)	N/A	0.294	1.983	4.594	0.201	0.284	0.000	0.424	15.847

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	165	206	334	179	0	191	227
N.S.	1	1.00	1.25	1.56	2.53	1.36	0.00	1.45	1.72
time (sec)	N/A	0.172	1.162	3.828	0.201	0.282	0.000	0.384	15.336

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	81	94	81	0	65	122
N.S.	1	1.00	0.90	1.19	1.38	1.19	0.00	0.96	1.79
time (sec)	N/A	0.086	0.029	2.305	0.197	0.269	0.000	0.351	17.864

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	122	133	203	147	0	153	163
N.S.	1	1.00	1.26	1.37	2.09	1.52	0.00	1.58	1.68
time (sec)	N/A	0.141	0.464	2.742	0.197	0.266	0.000	0.350	15.664

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	101	77	107	118	0	104	104
N.S.	1	1.00	1.31	1.00	1.39	1.53	0.00	1.35	1.35
time (sec)	N/A	0.178	0.480	1.799	0.205	0.275	0.000	0.335	15.482

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	275	94	274	125	0	111	85
N.S.	1	1.00	3.53	1.21	3.51	1.60	0.00	1.42	1.09
time (sec)	N/A	0.250	4.392	0.650	0.285	0.253	0.000	0.343	14.380

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	226	66	274	156	0	80	45
N.S.	1	1.00	2.57	0.75	3.11	1.77	0.00	0.91	0.51
time (sec)	N/A	0.432	2.680	0.664	0.286	0.277	0.000	0.341	14.696

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	53	54	282	128	0	73	96
N.S.	1	1.00	0.52	0.53	2.76	1.25	0.00	0.72	0.94
time (sec)	N/A	0.536	0.062	0.548	0.294	0.269	0.000	0.334	14.503

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	161	67	383	172	0	88	122
N.S.	1	1.00	1.21	0.50	2.88	1.29	0.00	0.66	0.92
time (sec)	N/A	0.692	1.458	0.641	0.345	0.262	0.000	0.360	14.798

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	139	80	403	212	0	104	146
N.S.	1	1.00	0.85	0.49	2.46	1.29	0.00	0.63	0.89
time (sec)	N/A	0.862	1.452	0.718	0.358	0.261	0.000	0.389	16.938

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	153	351	137	603	242	0	153	145
N.S.	1	1.12	2.58	1.01	4.43	1.78	0.00	1.12	1.07
time (sec)	N/A	0.478	3.182	0.764	0.354	0.260	0.000	0.376	13.640

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	298	105	413	220	0	134	112
N.S.	1	1.00	2.92	1.03	4.05	2.16	0.00	1.31	1.10
time (sec)	N/A	0.378	2.443	0.682	0.321	0.296	0.000	0.333	13.307

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	189	58	268	173	0	80	46
N.S.	1	1.00	2.22	0.68	3.15	2.04	0.00	0.94	0.54
time (sec)	N/A	0.440	1.366	0.664	0.295	0.280	0.000	0.333	13.225

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	41	170	94	0	60	38
N.S.	1	1.00	1.00	0.61	2.54	1.40	0.00	0.90	0.57
time (sec)	N/A	0.275	0.045	0.543	0.289	0.247	0.000	0.308	14.535

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	139	37	119	86	0	53	41
N.S.	1	1.00	2.28	0.61	1.95	1.41	0.00	0.87	0.67
time (sec)	N/A	0.201	0.671	0.556	0.291	0.270	0.000	0.283	15.447

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	50	102	70	0	81	69
N.S.	1	1.00	0.84	0.72	1.48	1.01	0.00	1.17	1.00
time (sec)	N/A	0.149	0.759	0.554	0.293	0.246	0.000	0.304	13.963

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	39	38	46	81	0	95	58
N.S.	1	1.00	0.85	0.83	1.00	1.76	0.00	2.07	1.26
time (sec)	N/A	0.092	0.040	0.564	0.278	0.247	0.000	0.304	14.405

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	70	78	147	154	0	110	161
N.S.	1	1.00	0.71	0.80	1.50	1.57	0.00	1.12	1.64
time (sec)	N/A	0.190	1.032	0.575	0.278	0.250	0.000	0.340	14.175

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	81	91	167	166	0	122	185
N.S.	1	1.00	0.49	0.55	1.01	1.00	0.00	0.73	1.11
time (sec)	N/A	0.282	2.252	0.715	0.280	0.264	0.000	0.359	14.105

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	91	104	186	232	0	135	209
N.S.	1	1.00	0.43	0.50	0.89	1.10	0.00	0.64	1.00
time (sec)	N/A	0.371	5.911	0.737	0.288	0.261	0.000	0.409	14.298

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	372	118	562	289	0	154	134
N.S.	1	1.00	2.30	0.73	3.47	1.78	0.00	0.95	0.83
time (sec)	N/A	0.586	5.288	0.769	0.313	0.260	0.000	0.406	14.367

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	202	69	396	242	0	102	50
N.S.	1	1.00	1.36	0.47	2.68	1.64	0.00	0.69	0.34
time (sec)	N/A	0.772	2.657	0.620	0.299	0.265	0.000	0.352	14.267

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	90	54	277	138	0	80	93
N.S.	1	1.00	0.94	0.56	2.89	1.44	0.00	0.83	0.97
time (sec)	N/A	0.483	0.053	0.675	0.288	0.256	0.000	0.331	14.418

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	151	54	211	138	0	80	93
N.S.	1	1.00	1.57	0.56	2.20	1.44	0.00	0.83	0.97
time (sec)	N/A	0.374	1.272	0.624	0.294	0.261	0.000	0.323	14.468

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	79	50	159	124	0	71	85
N.S.	1	1.00	0.90	0.57	1.81	1.41	0.00	0.81	0.97
time (sec)	N/A	0.248	0.502	0.546	0.296	0.263	0.000	0.302	14.049

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	71	65	122	109	0	102	82
N.S.	1	1.00	0.56	0.52	0.97	0.87	0.00	0.81	0.65
time (sec)	N/A	0.249	0.623	0.618	0.303	0.243	0.000	0.302	14.700

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	70	78	146	154	0	116	161
N.S.	1	1.00	0.70	0.78	1.46	1.54	0.00	1.16	1.61
time (sec)	N/A	0.175	1.060	0.638	0.294	0.246	0.000	0.345	14.899

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	39	53	56	118	0	129	94
N.S.	1	1.00	0.58	0.79	0.84	1.76	0.00	1.93	1.40
time (sec)	N/A	0.103	0.067	0.648	0.282	0.254	0.000	0.349	15.053

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	81	104	187	232	0	142	209
N.S.	1	1.00	0.63	0.81	1.45	1.80	0.00	1.10	1.62
time (sec)	N/A	0.214	5.235	0.728	0.291	0.252	0.000	0.401	15.208

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	91	117	205	271	0	154	233
N.S.	1	1.00	0.43	0.56	0.98	1.29	0.00	0.73	1.11
time (sec)	N/A	0.295	6.333	0.738	0.291	0.259	0.000	0.436	14.610

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	787	130	227	310	0	169	257
N.S.	1	1.00	3.12	0.52	0.90	1.23	0.00	0.67	1.02
time (sec)	N/A	0.372	11.116	0.694	0.295	0.278	0.000	0.432	14.788

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	175	175	122	233	0	373	0	0	0
N.S.	1	1.00	0.70	1.33	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.231	5.732	7.131	0.000	0.291	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	140	140	112	211	0	347	0	0	0
N.S.	1	1.00	0.80	1.51	0.00	2.48	0.00	0.00	0.00
time (sec)	N/A	0.201	1.368	6.165	0.000	0.281	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	100	160	0	313	0	0	0
N.S.	1	1.00	0.95	1.52	0.00	2.98	0.00	0.00	0.00
time (sec)	N/A	0.198	0.821	4.938	0.000	0.278	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	91	93	147	234	0	0	0
N.S.	1	1.00	1.38	1.41	2.23	3.55	0.00	0.00	0.00
time (sec)	N/A	0.143	0.527	3.976	0.367	0.271	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	46	87	0	266	0	0	0
N.S.	1	1.00	0.67	1.26	0.00	3.86	0.00	0.00	0.00
time (sec)	N/A	0.195	0.365	2.256	0.000	0.322	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	59	180	0	339	0	0	0
N.S.	1	1.00	0.57	1.73	0.00	3.26	0.00	0.00	0.00
time (sec)	N/A	0.196	0.521	2.231	0.000	0.301	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	59	264	0	405	0	0	0
N.S.	1	1.00	0.42	1.90	0.00	2.91	0.00	0.00	0.00
time (sec)	N/A	0.203	0.654	2.653	0.000	0.336	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	59	348	0	475	0	0	0
N.S.	1	1.00	0.34	2.00	0.00	2.73	0.00	0.00	0.00
time (sec)	N/A	0.244	4.384	2.802	0.000	0.331	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	177	177	124	234	0	385	0	0	0
N.S.	1	1.00	0.70	1.32	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.247	1.704	8.283	0.000	0.264	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	114	189	0	355	0	0	0
N.S.	1	1.00	0.80	1.33	0.00	2.50	0.00	0.00	0.00
time (sec)	N/A	0.227	0.974	6.153	0.000	0.292	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	100	206	998	303	0	0	0
N.S.	1	1.00	0.99	2.04	9.88	3.00	0.00	0.00	0.00
time (sec)	N/A	0.164	0.588	1.109	0.436	0.271	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	78	90	0	269	0	0	0
N.S.	1	1.00	1.11	1.29	0.00	3.84	0.00	0.00	0.00
time (sec)	N/A	0.218	0.531	2.094	0.000	0.307	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	73	181	0	351	0	0	0
N.S.	1	1.00	0.72	1.77	0.00	3.44	0.00	0.00	0.00
time (sec)	N/A	0.220	0.644	2.067	0.000	0.303	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	73	265	0	417	0	0	0
N.S.	1	1.00	0.53	1.93	0.00	3.04	0.00	0.00	0.00
time (sec)	N/A	0.233	0.957	2.741	0.000	0.332	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	73	349	0	495	0	0	0
N.S.	1	1.00	0.42	2.03	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.238	4.602	2.461	0.000	0.349	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	134	223	0	441	0	0	0
N.S.	1	1.00	0.63	1.05	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.239	2.463	89.612	0.000	0.284	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	124	207	0	409	0	0	0
N.S.	1	1.00	0.70	1.17	0.00	2.31	0.00	0.00	0.00
time (sec)	N/A	0.215	1.096	22.837	0.000	0.295	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	132	132	110	190	1396	353	0	0	0
N.S.	1	1.00	0.83	1.44	10.58	2.67	0.00	0.00	0.00
time (sec)	N/A	0.184	0.622	7.990	0.418	0.280	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	66	100	0	291	0	0	0
N.S.	1	1.00	0.64	0.97	0.00	2.83	0.00	0.00	0.00
time (sec)	N/A	0.218	0.400	5.275	0.000	0.307	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	70	175	0	339	0	0	0
N.S.	1	1.00	0.95	2.36	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	0.203	0.607	13.920	0.000	0.323	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	73	267	0	441	0	0	0
N.S.	1	1.00	0.70	2.57	0.00	4.24	0.00	0.00	0.00
time (sec)	N/A	0.218	1.130	70.083	0.000	0.315	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	73	351	0	527	0	0	0
N.S.	1	1.00	0.52	2.51	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.231	4.699	1.636	0.000	0.345	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	73	435	0	601	0	0	0
N.S.	1	1.00	0.42	2.53	0.00	3.49	0.00	0.00	0.00
time (sec)	N/A	0.247	5.219	1.349	0.000	0.358	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	185	185	153	282	0	552	0	0	0
N.S.	1	1.00	0.83	1.52	0.00	2.98	0.00	0.00	0.00
time (sec)	N/A	0.322	5.428	6.491	0.000	0.701	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	152	152	166	260	0	518	0	0	0
N.S.	1	1.00	1.09	1.71	0.00	3.41	0.00	0.00	0.00
time (sec)	N/A	0.273	2.416	5.125	0.000	0.599	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	100	325	0	438	0	0	0
N.S.	1	1.00	0.84	2.73	0.00	3.68	0.00	0.00	0.00
time (sec)	N/A	0.224	0.685	5.185	0.000	0.382	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	102	137	699	298	0	0	0
N.S.	1	1.00	1.17	1.57	8.03	3.43	0.00	0.00	0.00
time (sec)	N/A	0.162	0.365	2.424	0.618	0.339	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	75	167	0	436	0	0	0
N.S.	1	1.00	0.62	1.38	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	0.229	0.433	2.476	0.000	0.354	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	83	202	0	520	0	0	0
N.S.	1	1.00	0.52	1.25	0.00	3.23	0.00	0.00	0.00
time (sec)	N/A	0.278	0.399	2.332	0.000	0.348	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	83	375	0	608	0	0	0
N.S.	1	1.00	0.42	1.91	0.00	3.10	0.00	0.00	0.00
time (sec)	N/A	0.349	0.533	2.847	0.000	0.365	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	203	203	196	282	0	634	0	0	0
N.S.	1	1.00	0.97	1.39	0.00	3.12	0.00	0.00	0.00
time (sec)	N/A	0.386	4.514	7.019	0.000	1.065	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	169	169	132	222	0	550	0	0	0
N.S.	1	1.00	0.78	1.31	0.00	3.25	0.00	0.00	0.00
time (sec)	N/A	0.285	2.757	5.666	0.000	0.560	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	134	119	200	0	542	0	0	0
N.S.	1	1.13	1.00	1.68	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.243	0.826	3.034	0.000	0.491	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	130	145	201	0	505	0	0	0
N.S.	1	1.15	1.28	1.78	0.00	4.47	0.00	0.00	0.00
time (sec)	N/A	0.208	0.640	2.430	0.000	0.387	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	102	339	0	514	0	0	0
N.S.	1	1.00	0.58	1.92	0.00	2.90	0.00	0.00	0.00
time (sec)	N/A	0.316	0.450	2.176	0.000	0.377	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	102	202	0	560	0	0	0
N.S.	1	1.00	0.48	0.94	0.00	2.62	0.00	0.00	0.00
time (sec)	N/A	0.489	0.470	2.693	0.000	0.372	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	102	232	0	714	0	0	0
N.S.	1	1.00	0.41	0.93	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.444	0.596	2.748	0.000	0.399	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	260	260	180	304	0	742	0	0	0
N.S.	1	1.00	0.69	1.17	0.00	2.85	0.00	0.00	0.00
time (sec)	N/A	0.407	8.452	7.706	0.000	1.695	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	229	164	246	0	655	0	0	0
N.S.	1	1.00	0.72	1.07	0.00	2.86	0.00	0.00	0.00
time (sec)	N/A	0.364	5.350	6.129	0.000	1.387	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	191	191	136	250	0	645	0	0	0
N.S.	1	1.00	0.71	1.31	0.00	3.38	0.00	0.00	0.00
time (sec)	N/A	0.302	2.652	3.888	0.000	1.122	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	189	189	145	245	0	645	0	0	0
N.S.	1	1.00	0.77	1.30	0.00	3.41	0.00	0.00	0.00
time (sec)	N/A	0.288	1.523	3.549	0.000	0.837	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	181	178	243	0	605	0	0	0
N.S.	1	1.22	1.20	1.64	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	0.248	1.490	2.662	0.000	0.564	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	230	230	117	500	0	608	0	0	0
N.S.	1	1.00	0.51	2.17	0.00	2.64	0.00	0.00	0.00
time (sec)	N/A	0.387	0.523	2.483	0.000	0.376	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	112	375	0	706	0	0	0
N.S.	1	1.00	0.42	1.39	0.00	2.62	0.00	0.00	0.00
time (sec)	N/A	0.433	0.530	2.705	0.000	0.393	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	84	167	1289	459	0	0	0
N.S.	1	1.00	0.45	0.90	6.97	2.48	0.00	0.00	0.00
time (sec)	N/A	0.484	1.825	2.492	0.463	0.334	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	72	157	710	425	0	0	0
N.S.	1	1.00	0.52	1.13	5.11	3.06	0.00	0.00	0.00
time (sec)	N/A	0.340	0.636	2.220	0.394	0.340	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	57	132	243	350	0	0	0
N.S.	1	1.00	0.61	1.42	2.61	3.76	0.00	0.00	0.00
time (sec)	N/A	0.222	0.502	2.338	0.398	0.350	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	87	39	200	0	0	0
N.S.	1	1.00	1.06	1.81	0.81	4.17	0.00	0.00	0.00
time (sec)	N/A	0.110	0.267	2.455	0.397	0.326	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	62	77	65	0	0	0	0
N.S.	1	1.00	1.22	1.51	1.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	0.356	2.112	0.308	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	72	148	399	0	0	0	0
N.S.	1	1.00	0.75	1.54	4.16	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.677	2.412	0.370	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	90	237	1173	0	0	0	0
N.S.	1	1.00	0.63	1.67	8.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	0.825	2.376	0.478	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	100	259	2444	0	0	0	0
N.S.	1	1.00	0.53	1.38	13.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.494	1.936	2.409	1.906	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	87	166	1356	467	0	0	0
N.S.	1	1.00	0.46	0.87	7.14	2.46	0.00	0.00	0.00
time (sec)	N/A	0.477	0.829	2.098	0.480	0.332	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	66	143	477	346	0	0	0
N.S.	1	1.00	0.64	1.39	4.63	3.36	0.00	0.00	0.00
time (sec)	N/A	0.147	0.578	2.298	0.403	0.321	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	68	118	243	347	0	0	0
N.S.	1	1.00	0.73	1.27	2.61	3.73	0.00	0.00	0.00
time (sec)	N/A	0.231	0.409	2.131	0.399	0.328	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	65	112	60	0	0	0	0
N.S.	1	1.00	0.62	1.08	0.58	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.370	1.990	0.385	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	76	145	95	0	0	0	0
N.S.	1	1.00	0.76	1.45	0.95	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.754	2.114	0.318	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	83	238	1786	0	0	0	0
N.S.	1	1.00	0.57	1.63	12.23	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	1.028	2.027	0.527	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	196	196	102	260	3480	0	0	0	0
N.S.	1	1.00	0.52	1.33	17.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.488	2.513	2.108	1.846	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	76	168	1619	405	0	0	0
N.S.	1	1.00	0.50	1.10	10.58	2.65	0.00	0.00	0.00
time (sec)	N/A	0.147	1.067	30.572	0.460	0.334	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	87	172	1356	467	0	0	0
N.S.	1	1.00	0.46	0.91	7.14	2.46	0.00	0.00	0.00
time (sec)	N/A	0.507	0.564	2.313	0.427	0.339	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	72	136	710	420	0	0	0
N.S.	1	1.00	0.52	0.98	5.11	3.02	0.00	0.00	0.00
time (sec)	N/A	0.365	0.309	2.237	0.398	0.341	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	69	155	0	0	0	0	0
N.S.	1	1.00	0.45	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.371	2.292	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	65	191	0	442	0	0	0
N.S.	1	1.00	0.68	1.99	0.00	4.60	0.00	0.00	0.00
time (sec)	N/A	0.239	0.613	2.231	0.000	0.335	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	76	240	139	0	0	0	0
N.S.	1	1.00	0.76	2.40	1.39	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	1.032	2.008	0.298	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	148	96	262	3738	0	0	0	0
N.S.	1	1.00	0.65	1.77	25.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	2.350	2.204	1.851	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	194	194	106	284	6134	0	0	0	0
N.S.	1	1.00	0.55	1.46	31.62	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	5.090	2.448	11.568	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	244	244	120	306	9150	0	0	0	0
N.S.	1	1.00	0.49	1.25	37.50	0.00	0.00	0.00	0.00
time (sec)	N/A	0.677	5.508	2.431	70.100	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	83	166	0	0	0	0	0
N.S.	1	1.00	0.41	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.166	1.906	2.118	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	69	146	0	0	0	0	0
N.S.	1	1.00	0.46	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.157	0.543	2.128	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	61	85	60	0	0	0	0
N.S.	1	1.00	0.60	0.83	0.59	0.00	0.00	0.00	0.00
time (sec)	N/A	0.137	0.364	2.021	0.369	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	57	52	34	0	0	36	0
N.S.	1	1.00	1.16	1.06	0.69	0.00	0.00	0.73	0.00
time (sec)	N/A	0.107	0.268	2.099	0.300	0.000	0.000	1.094	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	54	81	39	272	0	67	0
N.S.	1	1.00	1.17	1.76	0.85	5.91	0.00	1.46	0.00
time (sec)	N/A	0.121	0.325	1.981	0.360	0.432	0.000	1.196	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	217	89	151	818	0	0	106	0
N.S.	1	1.29	0.53	0.90	4.87	0.00	0.00	0.63	0.00
time (sec)	N/A	0.175	0.606	1.993	0.400	0.000	0.000	1.884	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	101	240	2206	0	0	141	0
N.S.	1	1.00	0.37	0.88	8.05	0.00	0.00	0.51	0.00
time (sec)	N/A	0.183	1.599	2.336	0.520	0.000	0.000	1.704	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	97	233	2393	0	0	0	0
N.S.	1	1.00	0.45	1.08	11.13	0.00	0.00	0.00	0.00
time (sec)	N/A	0.168	2.007	2.307	0.531	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	65	219	0	453	0	0	0
N.S.	1	1.00	0.68	2.28	0.00	4.72	0.00	0.00	0.00
time (sec)	N/A	0.235	0.555	1.958	0.000	0.342	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	80	97	70	0	0	73	0
N.S.	1	1.00	0.82	0.99	0.71	0.00	0.00	0.74	0.00
time (sec)	N/A	0.235	0.574	1.926	0.313	0.000	0.000	1.341	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	75	94	395	0	0	83	0
N.S.	1	1.00	0.80	1.00	4.20	0.00	0.00	0.88	0.00
time (sec)	N/A	0.236	0.262	2.344	0.371	0.000	0.000	1.249	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	108	136	818	0	0	99	0
N.S.	1	1.00	0.50	0.63	3.80	0.00	0.00	0.46	0.00
time (sec)	N/A	0.181	0.339	2.036	0.394	0.000	0.000	1.858	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	72	157	486	492	0	149	0
N.S.	1	1.00	0.71	1.55	4.81	4.87	0.00	1.48	0.00
time (sec)	N/A	0.133	0.463	2.263	0.379	0.489	0.000	2.156	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	347	347	118	262	4272	0	0	185	0
N.S.	1	1.00	0.34	0.76	12.31	0.00	0.00	0.53	0.00
time (sec)	N/A	0.222	0.991	1.999	1.882	0.000	0.000	1.987	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	220	220	85	216	0	0	0	93	0
N.S.	1	1.00	0.39	0.98	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.168	1.780	2.362	0.000	0.000	0.000	1.461	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	79	142	102	0	0	75	0
N.S.	1	1.00	0.81	1.45	1.04	0.00	0.00	0.77	0.00
time (sec)	N/A	0.231	0.725	1.956	0.313	0.000	0.000	1.664	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	86	154	1786	0	0	111	0
N.S.	1	1.00	0.60	1.07	12.40	0.00	0.00	0.77	0.00
time (sec)	N/A	0.389	0.591	2.242	0.509	0.000	0.000	1.615	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	89	129	1165	0	0	124	0
N.S.	1	1.00	0.64	0.92	8.32	0.00	0.00	0.89	0.00
time (sec)	N/A	0.384	0.416	2.077	0.486	0.000	0.000	1.461	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	101	195	2206	0	0	77	0
N.S.	1	1.00	0.37	0.72	8.17	0.00	0.00	0.29	0.00
time (sec)	N/A	0.195	1.016	2.516	0.507	0.000	0.000	1.679	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	345	345	118	262	4272	0	0	193	0
N.S.	1	1.00	0.34	0.76	12.38	0.00	0.00	0.56	0.00
time (sec)	N/A	0.212	0.894	2.262	1.898	0.000	0.000	1.997	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	84	222	1386	564	0	228	0
N.S.	1	1.00	0.56	1.47	9.18	3.74	0.00	1.51	0.00
time (sec)	N/A	0.171	0.749	2.317	0.499	0.635	0.000	1.863	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.115	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.122	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	172	101	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.601	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	96	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.539	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	69	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.350	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	94	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	205	122	0	0	0	0	0	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	0.399	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	133	136	0	293	0	0	0
N.S.	1	1.00	1.46	1.49	0.00	3.22	0.00	0.00	0.00
time (sec)	N/A	0.100	1.001	1.822	0.000	0.346	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	810	257	0	0	0	0	0
N.S.	1	1.00	3.51	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	34.066	8.246	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	178	247	0	0	0	0	0
N.S.	1	1.00	0.79	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	7.529	5.050	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	182	287	0	0	0	0	0
N.S.	1	1.00	0.57	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	8.235	7.295	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	229	300	0	472	0	0	0
N.S.	1	1.00	0.85	1.11	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.231	8.411	6.826	0.000	0.302	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	192	223	0	392	0	0	0
N.S.	1	1.00	0.94	1.09	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.175	9.191	4.842	0.000	0.283	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	444	159	0	320	0	0	0
N.S.	1	1.00	3.08	1.10	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.143	6.983	4.575	0.000	0.277	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	76	96	147	235	0	0	0
N.S.	1	1.00	1.15	1.45	2.23	3.56	0.00	0.00	0.00
time (sec)	N/A	0.105	0.419	4.244	0.341	0.280	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	105	144	499	0	669	0	0	0
N.S.	1	1.00	1.37	4.75	0.00	6.37	0.00	0.00	0.00
time (sec)	N/A	0.283	2.893	15.146	0.000	0.497	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	219	219	244	38052	0	1413	0	0	0
N.S.	1	1.00	1.11	173.75	0.00	6.45	0.00	0.00	0.00
time (sec)	N/A	0.274	5.179	15.518	0.000	1.574	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	287	287	311	76269	0	2368	0	0	0
N.S.	1	1.00	1.08	265.75	0.00	8.25	0.00	0.00	0.00
time (sec)	N/A	0.370	7.574	17.656	0.000	4.365	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	219	297	0	482	0	0	0
N.S.	1	1.00	0.91	1.23	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.252	3.630	6.620	0.000	0.309	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	145	232	0	398	0	0	0
N.S.	1	1.00	0.82	1.32	0.00	2.26	0.00	0.00	0.00
time (sec)	N/A	0.175	1.515	4.699	0.000	0.278	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	102	206	998	316	0	0	0
N.S.	1	1.00	0.97	1.96	9.50	3.01	0.00	0.00	0.00
time (sec)	N/A	0.190	0.597	1.460	0.394	0.273	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	110	135	840	0	731	0	0	0
N.S.	1	1.00	1.23	7.64	0.00	6.65	0.00	0.00	0.00
time (sec)	N/A	0.312	0.865	14.277	0.000	0.908	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	229	229	293	25448	0	1640	0	0	0
N.S.	1	1.00	1.28	111.13	0.00	7.16	0.00	0.00	0.00
time (sec)	N/A	0.287	4.742	15.931	0.000	4.126	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	310	310	359	55536	0	2729	0	0	0
N.S.	1	1.00	1.16	179.15	0.00	8.80	0.00	0.00	0.00
time (sec)	N/A	0.439	5.862	16.912	0.000	11.665	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	286	367	0	620	0	0	0
N.S.	1	1.00	0.85	1.09	0.00	1.85	0.00	0.00	0.00
time (sec)	N/A	0.247	4.731	90.860	0.000	0.314	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	191	280	0	500	0	0	0
N.S.	1	1.00	0.74	1.09	0.00	1.94	0.00	0.00	0.00
time (sec)	N/A	0.199	2.398	21.676	0.000	0.295	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	128	211	1396	390	0	0	0
N.S.	1	1.00	0.90	1.49	9.83	2.75	0.00	0.00	0.00
time (sec)	N/A	0.272	0.905	7.703	0.429	0.306	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	203	203	343	1402	0	1140	0	0	0
N.S.	1	1.00	1.69	6.91	0.00	5.62	0.00	0.00	0.00
time (sec)	N/A	0.252	5.708	23.447	0.000	2.927	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	329	329	280	20138	0	2031	0	0	0
N.S.	1	1.00	0.85	61.21	0.00	6.17	0.00	0.00	0.00
time (sec)	N/A	0.396	3.260	53.694	0.000	13.167	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	536	536	551	42563	0	3351	0	0	0
N.S.	1	1.00	1.03	79.41	0.00	6.25	0.00	0.00	0.00
time (sec)	N/A	0.601	9.973	235.817	0.000	22.582	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	258	258	787	509	0	619	0	0	0
N.S.	1	1.00	3.05	1.97	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.246	8.070	5.597	0.000	7.795	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	183	183	295	348	0	481	0	0	0
N.S.	1	1.00	1.61	1.90	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.190	2.729	5.114	0.000	1.961	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	92	203	699	314	0	0	0
N.S.	1	1.00	1.01	2.23	7.68	3.45	0.00	0.00	0.00
time (sec)	N/A	0.125	0.537	2.401	0.598	0.685	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	166	166	243	659	0	1050	0	0	0
N.S.	1	1.00	1.46	3.97	0.00	6.33	0.00	0.00	0.00
time (sec)	N/A	0.437	3.439	15.288	0.000	10.595	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	416	416	337	54030	0	2508	0	0	0
N.S.	1	1.00	0.81	129.88	0.00	6.03	0.00	0.00	0.00
time (sec)	N/A	0.485	11.361	16.575	0.000	108.960	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	653	653	2940	97277	0	0	0	0	0
N.S.	1	1.00	4.50	148.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.763	22.452	18.669	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	324	324	856	587	0	701	0	0	0
N.S.	1	1.00	2.64	1.81	0.00	2.16	0.00	0.00	0.00
time (sec)	N/A	0.272	8.094	6.055	0.000	14.300	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	290	290	177	389	0	620	0	0	0
N.S.	1	1.00	0.61	1.34	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.256	5.217	3.462	0.000	6.467	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	230	289	0	548	0	0	0
N.S.	1	1.00	1.81	2.28	0.00	4.31	0.00	0.00	0.00
time (sec)	N/A	0.215	2.454	3.022	0.000	1.879	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	394	394	302	1419	0	2033	0	0	0
N.S.	1	1.00	0.77	3.60	0.00	5.16	0.00	0.00	0.00
time (sec)	N/A	0.397	6.105	16.332	0.000	215.922	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	560	560	470	69595	0	0	0	0	0
N.S.	1	1.00	0.84	124.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.635	11.849	17.956	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	802	802	2632	117747	0	0	0	0	0
N.S.	1	1.00	3.28	146.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.945	18.761	21.104	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	480	480	436	688	0	880	0	0	0
N.S.	1	1.00	0.91	1.43	0.00	1.83	0.00	0.00	0.00
time (sec)	N/A	0.395	9.677	4.859	0.000	30.821	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	468	468	262	523	0	782	0	0	0
N.S.	1	1.00	0.56	1.12	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.332	6.250	4.484	0.000	13.489	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	164	164	343	376	0	670	0	0	0
N.S.	1	1.00	2.09	2.29	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	0.321	7.472	2.893	0.000	3.610	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	592	592	360	2345	0	0	0	0	0
N.S.	1	1.00	0.61	3.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.549	11.406	16.762	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	756	756	465	85544	0	0	0	0	0
N.S.	1	1.00	0.62	113.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.748	13.651	20.180	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	999	999	2904	139613	0	0	0	0	0
N.S.	1	1.00	2.91	139.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.503	22.075	23.723	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	123	240	1496	0	806	0	0	0
N.S.	1	1.00	1.95	12.16	0.00	6.55	0.00	0.00	0.00
time (sec)	N/A	0.415	14.890	4.998	0.000	0.499	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	102	166	0	206	0	0	0
N.S.	1	1.00	1.67	2.72	0.00	3.38	0.00	0.00	0.00
time (sec)	N/A	0.165	0.687	2.899	0.000	0.338	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	135	609	0	517	0	0	0
N.S.	1	1.00	1.22	5.49	0.00	4.66	0.00	0.00	0.00
time (sec)	N/A	0.608	1.161	2.847	0.000	0.358	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	184	423	0	883	0	0	0
N.S.	1	1.00	1.30	3.00	0.00	6.26	0.00	0.00	0.00
time (sec)	N/A	0.617	13.066	2.717	0.000	0.575	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	171	367	0	913	0	0	0
N.S.	1	1.00	1.21	2.60	0.00	6.48	0.00	0.00	0.00
time (sec)	N/A	0.565	0.820	2.501	0.000	0.915	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	73	0	250	0	274	573
N.S.	1	1.00	1.01	1.09	0.00	3.73	0.00	4.09	8.55
time (sec)	N/A	0.189	0.201	0.541	0.000	0.280	0.000	0.326	15.629

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	155	168	0	561	0	201	3763
N.S.	1	1.00	1.26	1.37	0.00	4.56	0.00	1.63	30.59
time (sec)	N/A	0.318	1.065	0.634	0.000	0.306	0.000	0.299	22.207

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	267	287	0	1152	0	457	6909
N.S.	1	1.00	1.31	1.41	0.00	5.65	0.00	2.24	33.87
time (sec)	N/A	0.623	1.818	0.921	0.000	0.343	0.000	0.367	24.046

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	136	195	0	671	0	237	4934
N.S.	1	1.00	1.02	1.47	0.00	5.05	0.00	1.78	37.10
time (sec)	N/A	0.356	1.321	0.623	0.000	0.316	0.000	0.317	22.898

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	493	386	0	1409	0	658	8682
N.S.	1	1.00	2.08	1.63	0.00	5.95	0.00	2.78	36.63
time (sec)	N/A	0.944	2.694	1.008	0.000	0.360	0.000	0.396	24.417

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	438	635	0	2362	0	1201	12818
N.S.	1	1.00	1.16	1.68	0.00	6.27	0.00	3.19	34.00
time (sec)	N/A	2.798	4.424	1.095	0.000	0.450	0.000	0.395	27.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	517	458	0	1629	0	818	10759
N.S.	1	1.00	2.04	1.80	0.00	6.41	0.00	3.22	42.36
time (sec)	N/A	1.338	2.247	1.186	0.000	0.404	0.000	0.409	27.042

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	459	785	0	2776	0	1572	15647
N.S.	1	1.00	1.11	1.91	0.00	6.74	0.00	3.82	37.98
time (sec)	N/A	1.250	5.386	1.335	0.000	0.479	0.000	0.427	28.485

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	622	622	668	1345	0	4346	0	3173	21021
N.S.	1	1.00	1.07	2.16	0.00	6.99	0.00	5.10	33.80
time (sec)	N/A	2.703	9.725	2.379	0.000	0.688	0.000	0.552	28.964

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	379	1249	0	0	0	0	0
N.S.	1	1.00	1.18	3.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	10.202	20.479	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	225	400	0	0	0	0	0
N.S.	1	1.00	1.02	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	8.253	6.164	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	6063	2984	0	0	0	0	0
N.S.	1	1.00	15.96	7.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.710	23.573	24.793	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	230	532	0	0	0	0	0
N.S.	1	1.00	0.71	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.611	9.678	8.067	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	7138	4161	0	0	0	0	0
N.S.	1	1.00	16.15	9.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.766	26.039	32.018	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	145	182	0	0	0	0	0
N.S.	1	1.00	0.70	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	3.778	17.243	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	251	281	0	0	0	0	0
N.S.	1	1.00	1.16	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	21.285	7.082	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	376	376	1138	2883	0	0	0	0	0
N.S.	1	1.00	3.03	7.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	13.360	14.500	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	1589	7173	0	0	0	0	0
N.S.	1	1.00	3.21	14.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.889	14.926	17.337	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	40517	510	0	0	0	0	0
N.S.	1	1.00	104.16	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.569	36.774	11.872	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	336	322	0	0	0	0	0
N.S.	1	1.00	1.70	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	5.315	10.626	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	598	598	1708	2444	0	0	0	0	0
N.S.	1	1.00	2.86	4.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.112	14.547	14.380	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	899	899	1990	18680	0	0	0	0	0
N.S.	1	1.00	2.21	20.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.676	7.333	17.709	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	744	744	1750	3569	0	0	0	0	0
N.S.	1	1.00	2.35	4.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.347	19.215	13.973	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	919	919	1960	15314	0	0	0	0	0
N.S.	1	1.00	2.13	16.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.149	7.166	16.550	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1122	1122	2385	54551	0	0	0	0	0
N.S.	1	1.00	2.13	48.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.664	7.984	21.529	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	891	891	2026	18991	0	0	0	0	0
N.S.	1	1.00	2.27	21.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.232	7.027	16.321	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1150	1150	2344	39100	0	0	0	0	0
N.S.	1	1.00	2.04	34.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.363	8.150	20.623	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	1428	1428	2979	88656	0	0	0	0	0
N.S.	1	1.00	2.09	62.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.175	9.058	23.698	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	652	0	50041	457	0	0	0	0	0
N.S.	1	0.00	76.75	0.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	37.554	17.069	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	325	322	0	0	0	0	0
N.S.	1	1.00	1.64	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	5.201	11.562	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	249	263	0	0	0	0	0
N.S.	1	1.00	0.63	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.497	3.141	13.405	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	622	763	1761	2867	0	0	0	0	0
N.S.	1	1.23	2.83	4.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.760	19.317	17.183	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.07
time (sec)	N/A	0.215	14.574	0.667	0.731	0.000	1.101	0.912	99.948

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.07
time (sec)	N/A	0.100	87.531	0.527	1.028	0.000	10.451	1.231	105.691

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	0.00
time (sec)	N/A	0.098	84.795	0.602	1.422	0.000	0.000	1.428	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.07
time (sec)	N/A	0.243	14.913	0.646	0.777	0.000	3.792	1.683	107.171

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.07
time (sec)	N/A	0.105	89.115	0.536	1.062	0.000	61.552	2.937	107.375

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	0.00
time (sec)	N/A	0.101	87.715	0.593	1.396	0.000	0.000	5.147	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.07
time (sec)	N/A	0.245	75.841	0.597	0.755	0.000	147.970	2.906	110.554

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	0.00
time (sec)	N/A	0.110	98.957	0.546	1.148	0.000	0.000	3.674	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	0.00
time (sec)	N/A	0.103	130.902	0.634	2.376	0.000	0.000	4.900	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	2425	0	0	0	0	0	0
N.S.	1	1.00	22.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	13.876	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	191	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	1.789	0.000	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	205	152	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	0.753	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	124	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	208	159	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	0.632	0.000	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	248	248	215	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.544	1.516	0.000	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	26	29	33
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.96	1.07	1.22
time (sec)	N/A	0.141	6.636	1.290	4.191	0.313	5.086	0.878	15.968

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	296	278	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.587	0.816	0.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	200	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	125	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	206	206	5411	0	0	0	0	0	0
N.S.	1	1.00	26.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.627	32.619	0.000	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	322	322	14108	0	0	0	0	0	0
N.S.	1	1.00	43.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.628	46.813	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [31] had the largest ratio of [.5768999999999999968]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	15	9	1.00	26	0.346
2	A	11	8	1.00	26	0.308
3	A	5	3	1.00	26	0.115
4	A	4	3	1.00	26	0.115
5	A	4	3	1.00	24	0.125
6	A	8	6	1.00	26	0.231
7	A	9	6	1.00	26	0.231
8	A	12	7	1.00	26	0.269
9	A	15	7	1.00	26	0.269
10	A	18	7	1.00	26	0.269
11	A	13	8	1.00	26	0.308
12	A	6	3	1.00	26	0.115
13	A	5	3	1.00	26	0.115
14	A	5	3	1.00	26	0.115
15	A	9	8	1.00	24	0.333
16	A	15	11	1.00	26	0.423
17	A	13	9	1.00	26	0.346
18	A	15	9	1.00	26	0.346
19	A	19	9	1.00	26	0.346
20	A	23	9	1.00	26	0.346
21	A	26	14	1.12	26	0.538
22	A	21	13	1.00	26	0.500
23	A	13	9	1.00	26	0.346
24	A	9	6	1.00	26	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	7	5	1.00	24	0.208
26	A	4	3	1.00	26	0.115
27	A	4	3	1.00	26	0.115
28	A	5	3	1.00	26	0.115
29	A	13	8	1.00	26	0.308
30	A	17	9	1.00	26	0.346
31	A	29	15	1.00	26	0.577
32	A	20	13	1.00	26	0.500
33	A	15	9	1.00	26	0.346
34	A	12	7	1.00	26	0.269
35	A	9	6	1.00	24	0.250
36	A	12	8	1.00	26	0.308
37	A	5	3	1.00	26	0.115
38	A	5	3	1.00	26	0.115
39	A	6	3	1.00	26	0.115
40	A	14	8	1.00	26	0.308
41	A	18	9	1.00	26	0.346
42	A	5	4	1.00	28	0.143
43	A	5	4	1.00	28	0.143
44	A	5	4	1.00	28	0.143
45	A	4	4	1.00	26	0.154
46	A	4	4	1.00	28	0.143
47	A	5	4	1.00	28	0.143
48	A	6	4	1.00	28	0.143
49	A	7	4	1.00	28	0.143
50	A	6	5	1.00	28	0.179
51	A	6	5	1.00	28	0.179
52	A	5	5	1.00	26	0.192
53	A	4	4	1.00	28	0.143
54	A	5	5	1.00	28	0.179
55	A	6	5	1.00	28	0.179
56	A	7	5	1.00	28	0.179
57	A	5	4	1.00	28	0.143
58	A	5	4	1.00	28	0.143
59	A	5	4	1.00	26	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	5	4	1.00	28	0.143
61	A	5	4	1.00	28	0.143
62	A	5	4	1.00	28	0.143
63	A	5	4	1.00	28	0.143
64	A	5	4	1.00	28	0.143
65	A	8	6	1.00	28	0.214
66	A	7	6	1.00	28	0.214
67	A	6	5	1.00	28	0.179
68	A	5	4	1.00	26	0.154
69	A	6	5	1.00	28	0.179
70	A	7	6	1.00	28	0.214
71	A	8	6	1.00	28	0.214
72	A	8	6	1.00	28	0.214
73	A	7	6	1.00	28	0.214
74	A	7	6	1.13	28	0.214
75	A	6	5	1.15	26	0.192
76	A	7	6	1.00	28	0.214
77	A	8	6	1.00	28	0.214
78	A	9	6	1.00	28	0.214
79	A	9	7	1.00	28	0.250
80	A	8	7	1.00	28	0.250
81	A	7	6	1.00	28	0.214
82	A	7	6	1.00	28	0.214
83	A	7	6	1.22	26	0.231
84	A	8	7	1.00	28	0.250
85	A	9	7	1.00	28	0.250
86	A	5	3	1.00	30	0.100
87	A	4	3	1.00	30	0.100
88	A	3	3	1.00	30	0.100
89	A	2	2	1.00	30	0.067
90	A	2	2	1.00	30	0.067
91	A	3	3	1.00	30	0.100
92	A	4	3	1.00	30	0.100
93	A	5	3	1.00	30	0.100
94	A	5	4	1.00	30	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	3	3	1.00	30	0.100
96	A	3	3	1.00	30	0.100
97	A	3	2	1.00	30	0.067
98	A	3	3	1.00	30	0.100
99	A	4	4	1.00	30	0.133
100	A	5	4	1.00	30	0.133
101	A	4	3	1.00	30	0.100
102	A	5	4	1.00	30	0.133
103	A	4	3	1.00	30	0.100
104	A	3	2	1.00	30	0.067
105	A	3	3	1.00	30	0.100
106	A	3	3	1.00	30	0.100
107	A	4	4	1.00	30	0.133
108	A	5	4	1.00	30	0.133
109	A	6	4	1.00	30	0.133
110	A	3	2	1.00	30	0.067
111	A	3	2	1.00	30	0.067
112	A	3	2	1.00	30	0.067
113	A	2	2	1.00	30	0.067
114	A	2	2	1.00	30	0.067
115	A	3	2	1.29	30	0.067
116	A	3	2	1.00	30	0.067
117	A	3	2	1.00	30	0.067
118	A	3	3	1.00	30	0.100
119	A	3	3	1.00	30	0.100
120	A	3	3	1.00	30	0.100
121	A	3	2	1.00	30	0.067
122	A	3	3	1.00	30	0.100
123	A	3	2	1.00	30	0.067
124	A	3	2	1.00	30	0.067
125	A	3	3	1.00	30	0.100
126	A	4	4	1.00	30	0.133
127	A	4	3	1.00	30	0.100
128	A	3	2	1.00	30	0.067
129	A	3	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	4	3	1.00	30	0.100
131	A	2	2	1.00	24	0.083
132	A	3	3	1.00	26	0.115
133	A	3	3	1.00	26	0.115
134	A	3	3	1.00	26	0.115
135	A	3	3	1.00	24	0.125
136	A	3	3	1.00	26	0.115
137	A	3	3	1.00	26	0.115
138	A	4	3	1.00	28	0.107
139	A	3	3	1.00	28	0.107
140	A	2	2	1.00	28	0.071
141	A	4	4	1.00	28	0.143
142	A	5	5	1.00	28	0.179
143	A	6	5	1.00	27	0.185
144	A	3	3	1.00	27	0.111
145	A	3	3	1.00	27	0.111
146	A	5	5	1.00	27	0.185
147	A	5	4	1.00	27	0.148
148	A	5	4	1.00	27	0.148
149	A	5	4	1.00	27	0.148
150	A	4	4	1.00	25	0.160
151	A	5	5	1.00	27	0.185
152	A	7	6	1.00	27	0.222
153	A	8	7	1.00	27	0.259
154	A	6	5	1.00	27	0.185
155	A	5	5	1.00	27	0.185
156	A	5	5	1.00	25	0.200
157	A	5	5	1.00	27	0.185
158	A	7	6	1.00	27	0.222
159	A	8	6	1.00	27	0.222
160	A	5	4	1.00	27	0.148
161	A	5	4	1.00	27	0.148
162	A	6	5	1.00	25	0.200
163	A	7	5	1.00	27	0.185
164	A	10	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	14	6	1.00	27	0.222
166	A	9	5	1.00	27	0.185
167	A	7	4	1.00	27	0.148
168	A	5	4	1.00	25	0.160
169	A	8	7	1.00	27	0.259
170	A	12	6	1.00	27	0.222
171	A	16	6	1.00	27	0.222
172	A	10	5	1.00	27	0.185
173	A	10	5	1.00	27	0.185
174	A	6	5	1.00	25	0.200
175	A	12	6	1.00	27	0.222
176	A	15	6	1.00	27	0.222
177	A	19	6	1.00	27	0.222
178	A	14	5	1.00	27	0.185
179	A	14	5	1.00	27	0.185
180	A	7	5	1.00	25	0.200
181	A	16	6	1.00	27	0.222
182	A	19	6	1.00	27	0.222
183	A	23	6	1.00	27	0.222
184	A	5	5	1.00	29	0.172
185	A	2	2	1.00	29	0.069
186	A	5	5	1.00	29	0.172
187	A	5	4	1.00	29	0.138
188	A	5	4	1.00	29	0.138
189	A	4	4	1.00	23	0.174
190	A	5	5	1.00	23	0.217
191	A	6	6	1.00	23	0.261
192	A	5	5	1.00	25	0.200
193	A	6	6	1.00	25	0.240
194	A	7	7	1.00	25	0.280
195	A	6	6	1.00	25	0.240
196	A	7	7	1.00	25	0.280
197	A	8	8	1.00	25	0.320
198	A	5	5	1.00	25	0.200
199	A	3	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	6	6	1.00	25	0.240
201	A	5	5	1.00	27	0.185
202	A	7	7	1.00	25	0.280
203	A	3	3	1.00	25	0.120
204	A	3	3	1.00	27	0.111
205	A	6	6	1.00	25	0.240
206	A	7	7	1.00	25	0.280
207	A	3	3	1.00	29	0.103
208	A	1	1	1.00	29	0.034
209	A	5	5	1.00	29	0.172
210	A	7	7	1.00	29	0.241
211	A	6	6	1.00	29	0.207
212	A	7	7	1.00	29	0.241
213	A	8	8	1.00	29	0.276
214	A	7	7	1.00	29	0.241
215	A	8	8	1.00	29	0.276
216	A	9	8	1.00	29	0.276
217	F	0	0	N/A	0.000	N/A
218	A	1	1	1.00	29	0.034
219	A	3	3	1.00	29	0.103
220	A	6	6	1.23	29	0.207
221	N/A	0	0	1.00	29	0.000
222	N/A	0	0	1.00	29	0.000
223	N/A	0	0	1.00	29	0.000
224	N/A	0	0	1.00	29	0.000
225	N/A	0	0	1.00	29	0.000
226	N/A	0	0	1.00	29	0.000
227	N/A	0	0	1.00	29	0.000
228	N/A	0	0	1.00	29	0.000
229	N/A	0	0	1.00	29	0.000
230	A	4	4	1.00	27	0.148
231	A	8	6	1.00	27	0.222
232	A	7	5	1.00	27	0.185
233	A	6	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
234	A	7	5	1.00	27	0.185
235	A	8	6	1.00	27	0.222
236	N/A	0	0	1.00	27	0.000
237	A	8	6	1.00	27	0.222
238	A	7	5	1.00	27	0.185
239	A	6	4	1.00	25	0.160
240	A	7	5	1.00	27	0.185
241	A	10	5	1.00	27	0.185

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$ 91
3.2 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$ 98
3.3 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$ 105
3.4 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$ 110
3.5 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$ 115
3.6 $\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$ 120
3.7 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$ 125
3.8 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$ 130
3.9 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$ 136
3.10 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$ 142
3.11 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$ 149
3.12 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$ 156
3.13 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$ 162
3.14 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$ 167
3.15 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$ 172
3.16 $\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$ 178
3.17 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$ 185
3.18 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$ 191
3.19 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$ 197
3.20 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$ 204
3.21 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$ 211
3.22 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$ 219
3.23 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$ 226

3.24 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$ . . . 232
3.25 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx$ . . . 237
3.26 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$ . . . 242
3.27 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$ . . . 247
3.28 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$ . . . 251
3.29 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$ . . . 256
3.30 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$ . . . 262
3.31 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$ . . . 269
3.32 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$ . . . 278
3.33 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$ . . . 286
3.34 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$ . . . 292
3.35 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$ . . . 298
3.36 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$ . . . 304
3.37 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$ . . . 310
3.38 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$ . . . 315
3.39 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$ . . . 320
3.40 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$ . . . 326
3.41 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$ . . . 333
3.42 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$ . . . 342
3.43 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$ . . . 353
3.44 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx$ . . . 362
3.45 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$ . . . 369
3.46 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$ . . . 374
3.47 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx$ . . . 379
3.48 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx$ . . . 384
3.49 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx$ . . . 389
3.50 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx$ . . . 395
3.51 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx$ . . . 408
3.52 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx$ . . . 418
3.53 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx$ . . . 424
3.54 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx$ . . . 428
3.55 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx$ . . . 433
3.56 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx$ . . . 438
3.57 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx$ . . . 444
3.58 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx$ . . . 450
3.59 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx$ . . . 461

3.60 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx$ . . . 467
3.61 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx$ . . . 472
3.62 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx$ . . . 477
3.63 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx$ . . . 482
3.64 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx$ . . . 487
3.65 $\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 492
3.66 $\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 499
3.67 $\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 506
3.68 $\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 512
3.69 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$ . . . 518
3.70 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx$ . . . 524
3.71 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx$ . . . 530
3.72 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 537
3.73 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 544
3.74 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 551
3.75 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 557
3.76 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx$ . . . 563
3.77 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx$ . . . 569
3.78 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx$ . . . 575
3.79 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 582
3.80 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 590
3.81 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 597
3.82 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 604
3.83 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 611
3.84 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx$ . . . 617
3.85 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx$ . . . 624
3.86 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx$ . . . 632
3.87 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx$ . . . 638
3.88 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx$ . . . 644
3.89 $\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$ . . . 649
3.90 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 653
3.91 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 657
3.92 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 662
3.93 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx$ . . . 667

3.94 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$ . . . 674
3.95 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$ . . . 680
3.96 $\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$ . . . 685
3.97 $\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 690
3.98 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 694
3.99 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 698
3.100 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$ . . . 704
3.101 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$ . . . 712
3.102 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$ . . . 718
3.103 $\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$ . . . 724
3.104 $\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 729
3.105 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 733
3.106 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 737
3.107 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$ . . . 741
3.108 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$ . . . 748
3.109 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$ . . . 757
3.110 $\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 769
3.111 $\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 773
3.112 $\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 777
3.113 $\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 781
3.114 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 785
3.115 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 789
3.116 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 794
3.117 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 800
3.118 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 806
3.119 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 811
3.120 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 815
3.121 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 820
3.122 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 825
3.123 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 830
3.124 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 838
3.125 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 843
3.126 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 848

3.127 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$ 854
3.128 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$ 860
3.129 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$ 866
3.130 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$ 874
3.131 $\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$ 880
3.132 $\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$ 884
3.133 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$ 888
3.134 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$ 892
3.135 $\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$ 896
3.136 $\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx$ 900
3.137 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$ 904
3.138 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx$ 908
3.139 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx$ 912
3.140 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$ 916
3.141 $\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$ 920
3.142 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$ 924
3.143 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$ 929
3.144 $\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$ 934
3.145 $\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$ 939
3.146 $\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$ 944
3.147 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$ 950
3.148 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$ 963
3.149 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$ 972
3.150 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$ 980
3.151 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$ 985
3.152 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$ 990
3.153 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$ 997
3.154 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx$ 1005
3.155 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx$ 1021
3.156 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$ 1032
3.157 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$ 1038
3.158 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx$ 1043
3.159 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx$ 1050
3.160 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx$ 1058
3.161 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx$ 1065
3.162 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$ 1077
3.163 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx$ 1083

3.164 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx$ 1089
3.165 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx$ 1096
3.166 $\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$ 1106
3.167 $\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$ 1113
3.168 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$ 1119
3.169 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$ 1124
3.170 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx$ 1130
3.171 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx$ 1139
3.172 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$ 1149
3.173 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$ 1157
3.174 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$ 1164
3.175 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx$ 1169
3.176 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx$ 1178
3.177 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx$ 1186
3.178 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$ 1198
3.179 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$ 1207
3.180 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$ 1216
3.181 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx$ 1222
3.182 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx$ 1232
3.183 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx$ 1243
3.184 $\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$ 1257
3.185 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$ 1263
3.186 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$ 1267
3.187 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$ 1273
3.188 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$ 1279
3.189 $\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$ 1285
3.190 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$ 1290
3.191 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$ 1297
3.192 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$ 1307
3.193 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$ 1315
3.194 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$ 1326
3.195 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$ 1341
3.196 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$ 1354
3.197 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$ 1371

3.198 $\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$ 1395
3.199 $\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$ 1401
3.200 $\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$ 1406
3.201 $\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$ 1414
3.202 $\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$ 1420
3.203 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$ 1428
3.204 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$ 1433
3.205 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$ 1438
3.206 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$ 1446
3.207 $\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$ 1453
3.208 $\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$ 1458
3.209 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$ 1462
3.210 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx$ 1470
3.211 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$ 1478
3.212 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$ 1487
3.213 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$ 1495
3.214 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$ 1504
3.215 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$ 1512
3.216 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$ 1521
3.217 $\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$ 1532
3.218 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx$ 1536
3.219 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$ 1540
3.220 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx$ 1545
3.221 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$ 1554
3.222 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$ 1558
3.223 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$ 1561
3.224 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$ 1564
3.225 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$ 1567
3.226 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$ 1570
3.227 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$ 1573
3.228 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$ 1576

3.229 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$ 1579
3.230 $\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))^m\,dx$ 1582
3.231 $\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))^3\,dx$ 1587
3.232 $\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))^2\,dx$ 1593
3.233 $\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))\,dx$ 1599
3.234 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$ 1603
3.235 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$ 1608
3.236 $\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))^m\,dx$ 1614
3.237 $\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))^3\,dx$ 1618
3.238 $\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))^2\,dx$ 1624
3.239 $\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))\,dx$ 1629
3.240 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$ 1633
3.241 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx$ 1638

3.1 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$

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Mathematica [A] (verified) 94
Maple [C] (verified) 95
Fricas [A] (verification not implemented) 95
Sympy [F] 96
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Giac [A] (verification not implemented) 97
Mupad [B] (verification not implemented) 97

Optimal result

Integrand size = 26, antiderivative size = 196

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = a^2c^5x-\frac{19a^2c^5\text{arctanh}(\sin(e+fx))}{16f}-\frac{a^2c^5\tan(e+fx)}{f}+\frac{17a^2c^5\sec(e+fx)\tan(e+fx)}{16f}+\frac{a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^5\tan^3(e+fx)}{3f}-\frac{3a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f}-\frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{6f}+\frac{3a^2c^5\tan^5(e+fx)}{5f}$$

```
[Out] a^2*c^5*x-19/16*a^2*c^5*arctanh(sin(f*x+e))/f-a^2*c^5*tan(f*x+e)/f+17/16*a^
2*c^5*sec(f*x+e)*tan(f*x+e)/f+1/8*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f+1/3*a^2
*c^5*tan(f*x+e)^3/f-3/4*a^2*c^5*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^2*c^5*sec(f
*x+e)^3*tan(f*x+e)^3/f+3/5*a^2*c^5*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3989, 3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = -\frac{19a^2c^5\text{arctanh}(\sin(e+fx))}{16f}+\frac{3a^2c^5\tan^5(e+fx)}{5f}+\frac{a^2c^5\tan^3(e+fx)}{3f}-\frac{a^2c^5\tan(e+fx)}{f}-\frac{a^2c^5\tan^3(e+fx)\sec^3(e+fx)}{6f}+\frac{a^2c^5\tan(e+fx)\sec^3(e+fx)}{8f}-\frac{3a^2c^5\tan^3(e+fx)\sec(e+fx)}{4f}+\frac{17a^2c^5\tan(e+fx)\sec(e+fx)}{16f}+a^2c^5x$$

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] a^2*c^5*x - (19*a^2*c^5*ArcTanh[Sin[e + f*x]])/(16*f) - (a^2*c^5*Tan[e + f*
x])/f + (17*a^2*c^5*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (a^2*c^5*Sec[e + f*
x]^3*Tan[e + f*x])/(8*f) + (a^2*c^5*Tan[e + f*x]^3)/(3*f) - (3*a^2*c^5*Sec[
e + f*x]*Tan[e + f*x]^3)/(4*f) - (a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6
*f) + (3*a^2*c^5*Tan[e + f*x]^5)/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int (c-c\sec(e+fx))^3\tan^4(e+fx)\,dx\\
&= \left(a^2c^2\right)\int \left(c^3\tan^4(e+fx)-3c^3\sec(e+fx)\tan^4(e+fx)\right.\\
&\qquad\left.+3c^3\sec^2(e+fx)\tan^4(e+fx)-c^3\sec^3(e+fx)\tan^4(e+fx)\right)\,dx\\
&= \left(a^2c^5\right)\int \tan^4(e+fx)\,dx-\left(a^2c^5\right)\int \sec^3(e+fx)\tan^4(e+fx)\,dx\\
&\quad-\left(3a^2c^5\right)\int \sec(e+fx)\tan^4(e+fx)\,dx+\left(3a^2c^5\right)\int \sec^2(e+fx)\tan^4(e+fx)\,dx\\
&= \frac{a^2c^5\tan^3(e+fx)}{3f}-\frac{3a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f}\\
&\quad-\frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{6f}+\frac{1}{2}\left(a^2c^5\right)\int \sec^3(e+fx)\tan^2(e\\
&\qquad+fx)\,dx-\left(a^2c^5\right)\int \tan^2(e+fx)\,dx\\
&\quad+\frac{1}{4}\left(9a^2c^5\right)\int \sec(e+fx)\tan^2(e+fx)\,dx+\frac{\left(3a^2c^5\right)\text{Subst}\left(\int x^4\,dx,x,\tan(e+fx)\right)}{f}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a^2c^5\tan(e+fx)}{f} + \frac{9a^2c^5\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{a^2c^5\tan^3(e+fx)}{3f} - \frac{3a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f} \\
&\quad - \frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{6f} + \frac{3a^2c^5\tan^5(e+fx)}{5f} \\
&\quad - \frac{1}{8}\left(a^2c^5\right)\int \sec^3(e+fx)\,dx + \left(a^2c^5\right)\int 1\,dx - \frac{1}{8}\left(9a^2c^5\right)\int \sec(e+fx)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= a^2c^5x - \frac{9a^2c^5\operatorname{arctanh}(\sin(e+fx))}{8f} - \frac{a^2c^5\tan(e+fx)}{f} + \frac{17a^2c^5\sec(e+fx)\tan(e+fx)}{16f} \\
&\quad + \frac{a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^5\tan^3(e+fx)}{3f} - \frac{3a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f} \\
&\quad - \frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{6f} + \frac{3a^2c^5\tan^5(e+fx)}{5f} - \frac{1}{16}\left(a^2c^5\right)\int \sec(e+fx)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= a^2c^5x - \frac{19a^2c^5\operatorname{arctanh}(\sin(e+fx))}{16f} - \frac{a^2c^5\tan(e+fx)}{f} \\
&\quad + \frac{17a^2c^5\sec(e+fx)\tan(e+fx)}{16f} + \frac{a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{a^2c^5\tan^3(e+fx)}{3f} - \frac{3a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f} \\
&\quad - \frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{6f} + \frac{3a^2c^5\tan^5(e+fx)}{5f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

$$
\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx
$$
$$
= \frac{a^2c^5\sec^6(e+fx)\left(1200e+1200fx-4560\operatorname{arctanh}(\sin(e+fx))\cos^6(e+fx)+1800(e+fx)\cos(2(e+fx)\right.}{}
$$

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*c^5*Sec[e + f*x]^6*(1200*e + 1200*f*x - 4560*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] + 72
0*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x)]
- 210*Sin[e + f*x] - 120*Sin[2*(e + f*x)] + 865*Sin[3*(e + f*x)] - 768*Sin[
4*(e + f*x)] + 435*Sin[5*(e + f*x)] - 88*Sin[6*(e + f*x)]))/(3840*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05

method	result
risch	$a^{2} c^{5} x-\frac{i c^{5} a^{2} \left(435 \,{\mathrm e}^{11 i \left(f x +e \right)}-240 \,{\mathrm e}^{10 i \left(f x +e \right)}+865 \,{\mathrm e}^{9 i \left(f x +e \right)}+1200 \,{\mathrm e}^{8 i \left(f x +e \right)}-210 \,{\mathrm e}^{7 i \left(f x +e \right)}+1760 \,{\mathrm e}^{6 i \left(f x +e \right)}+210 \,{\mathrm e}^{\cdots}\right.}{120 f \left(1+{\mathrm e}^{2 i \left(f x +e \right)}\right)^{6}}$
parallelrisch	$-\frac{a^{2} c^{5} \left(\frac{19 \left(-5-\frac{15 \cos \left(2 f x +2 e \right)}{2}-3 \cos \left(4 f x +4 e \right)-\frac{\cos \left(6 f x +6 e \right)}{2}\right) \ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{8}+\frac{19 \left(5+\frac{\cos \left(6 f x +6 e \right)}{2}+3 \cos \left(4 f x +4 e \right)+\frac{15 \cos\cdots}{\cdots}\right.}{8}\right.}{\cdots}$
derivativedivides	$\frac{-c^{5} a^{2} \left(-\left(-\frac{\sec \left(f x +e \right)^{5}}{6}-\frac{5 \sec \left(f x +e \right)^{3}}{24}-\frac{5 \sec \left(f x +e \right)}{16}\right) \tan \left(f x +e \right)+\frac{5 \ln \left(\sec \left(f x +e \right)+\tan \left(f x +e \right)\right)}{16}\right)-3 c^{5} a^{2} \left(-\frac{8}{15}-\frac{\sec \left(f x +e \right)^{\cdots}}{5}\cdots\right.}{\cdots}$
default	$\frac{-c^{5} a^{2} \left(-\left(-\frac{\sec \left(f x +e \right)^{5}}{6}-\frac{5 \sec \left(f x +e \right)^{3}}{24}-\frac{5 \sec \left(f x +e \right)}{16}\right) \tan \left(f x +e \right)+\frac{5 \ln \left(\sec \left(f x +e \right)+\tan \left(f x +e \right)\right)}{16}\right)-3 c^{5} a^{2} \left(-\frac{8}{15}-\frac{\sec \left(f x +e \right)^{\cdots}}{5}\cdots\right.}{\cdots}$
parts	$a^{2} c^{5} x +\frac{a^{2} c^{5} \tan \left(f x +e \right)}{f}-\frac{3 c^{5} a^{2} \ln \left(\sec \left(f x +e \right)+\tan \left(f x +e \right)\right)}{f}+\frac{5 c^{5} a^{2} \left(\frac{\sec \left(f x +e \right) \tan \left(f x +e \right)}{2}+\frac{\ln \left(\sec \left(f x +e \right)+\tan \left(f x\cdots\right)\right)}{2}\right.}{f}$
norman	$\frac{a^{2} c^{5} x +a^{2} c^{5} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{12}-6 a^{2} c^{5} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+15 a^{2} c^{5} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-20 a^{2} c^{5} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}+15 a^{2} c^{5} x \tan \left(\frac{f x}{2}\cdots\right.}{\cdots}$

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c^5*x-1/120*I*c^5*a^2*(435*exp(11*I*(f*x+e))-240*exp(10*I*(f*x+e))+865*
exp(9*I*(f*x+e))+1200*exp(8*I*(f*x+e))-210*exp(7*I*(f*x+e))+1760*exp(6*I*(f
*x+e))+210*exp(5*I*(f*x+e))+1440*exp(4*I*(f*x+e))-865*exp(3*I*(f*x+e))+1296
*exp(2*I*(f*x+e))-435*exp(I*(f*x+e))+176)/f/(1+exp(2*I*(f*x+e)))^6+19/16*c^
5*a^2/f*ln(exp(I*(f*x+e))-I)-19/16*c^5*a^2/f*ln(exp(I*(f*x+e))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.91

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$=\frac{480\,a^{2}c^{5}fx\cos\left(fx+e\right)^{6}-285\,a^{2}c^{5}\cos\left(fx+e\right)^{6}\log\left(\sin\left(fx+e\right)+1\right)+285\,a^{2}c^{5}\cos\left(fx+e\right)^{6}\log\left(-\sin\cdots\right)}{\cdots}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/480*(480*a^2*c^5*f*x*cos(f*x + e)^6 - 285*a^2*c^5*cos(f*x + e)^6*log(sin(
f*x + e) + 1) + 285*a^2*c^5*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(176*
```

```
a^2*c^5*cos(f*x + e)^5 - 435*a^2*c^5*cos(f*x + e)^4 + 208*a^2*c^5*cos(f*x +
 e)^3 + 110*a^2*c^5*cos(f*x + e)^2 - 144*a^2*c^5*cos(f*x + e) + 40*a^2*c^5)
*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = -a^2c^5\left(\int(-1)\,dx+\int 3\sec(e+fx)\,dx+\int\left(-\sec^2(e+fx)\right)\,dx + \int\left(-5\sec^3(e+fx)\right)\,dx+\int 5\sec^4(e+fx)\,dx+\int\sec^5(e+fx)\,dx + \int\left(-3\sec^6(e+fx)\right)\,dx+\int\sec^7(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*c**5*(Integral(-1, x) + Integral(3*sec(e + f*x), x) + Integral(-sec(e
 + f*x)**2, x) + Integral(-5*sec(e + f*x)**3, x) + Integral(5*sec(e + f*x)*
*4, x) + Integral(sec(e + f*x)**5, x) + Integral(-3*sec(e + f*x)**6, x) + I
ntegral(sec(e + f*x)**7, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.70

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = \frac{96\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^2c^5-800\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2c^5}{}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/480*(96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^5
- 800*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^5 + 480*(f*x + e)*a^2*c^5 + 5
*a^2*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(
f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e)
 + 1) + 15*log(sin(f*x + e) - 1)) + 30*a^2*c^5*(2*(3*sin(f*x + e)^3 - 5*sin
(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1
) + 3*log(sin(f*x + e) - 1)) - 600*a^2*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2
- 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 1440*a^2*c^5*log(se
c(f*x + e) + tan(f*x + e)) + 480*a^2*c^5*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.97

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$= \frac{240\,(fx+e)a^2c^5 - 285\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 285\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(525\,a^2\right.}{}}{}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)*a^2*c^5 - 285*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1
)) + 285*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(525*a^2*c^5*tan(1/
2*f*x + 1/2*e)^11 - 3135*a^2*c^5*tan(1/2*f*x + 1/2*e)^9 + 1746*a^2*c^5*tan(
1/2*f*x + 1/2*e)^7 - 366*a^2*c^5*tan(1/2*f*x + 1/2*e)^5 - 95*a^2*c^5*tan(1/
2*f*x + 1/2*e)^3 + 45*a^2*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2
 - 1)^6)/f
```

Mupad [B] (verification not implemented)

Time = 15.54 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.16

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = a^2\,c^5\,x$$
$$-\frac{-\frac{35\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}+\frac{209\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{8}-\frac{291\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{20}+\frac{61\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{20}+\frac{19\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24}-}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-6\,\mathrm{ta}\right.}$$
$$-\frac{19\,a^2\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5,x)
```

```
[Out] a^2*c^5*x - ((19*a^2*c^5*tan(e/2 + (f*x)/2)^3)/24 + (61*a^2*c^5*tan(e/2 + (
f*x)/2)^5)/20 - (291*a^2*c^5*tan(e/2 + (f*x)/2)^7)/20 + (209*a^2*c^5*tan(e/
2 + (f*x)/2)^9)/8 - (35*a^2*c^5*tan(e/2 + (f*x)/2)^11)/8 - (3*a^2*c^5*tan(e
/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20
*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 +
 tan(e/2 + (f*x)/2)^12 + 1)) - (19*a^2*c^5*atanh(tan(e/2 + (f*x)/2)))/(8*f)
```

3.2 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 140

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx = a^2c^4x-\frac{3a^2c^4\text{arctanh}(\sin(e+fx))}{4f}-\frac{a^2c^4\tan(e+fx)}{f}+\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{4f}+\frac{a^2c^4\tan^3(e+fx)}{3f}-\frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{2f}+\frac{a^2c^4\tan^5(e+fx)}{5f}$$

```
[Out] a^2*c^4*x-3/4*a^2*c^4*arctanh(sin(f*x+e))/f-a^2*c^4*tan(f*x+e)/f+3/4*a^2*c^
4*sec(f*x+e)*tan(f*x+e)/f+1/3*a^2*c^4*tan(f*x+e)^3/f-1/2*a^2*c^4*sec(f*x+e)
*tan(f*x+e)^3/f+1/5*a^2*c^4*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3989, 3971, 3554, 8, 2691, 3855, 2687, 30}

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx = -\frac{3a^2c^4\text{arctanh}(\sin(e+fx))}{4f}+\frac{a^2c^4\tan^5(e+fx)}{5f}+\frac{a^2c^4\tan^3(e+fx)}{3f}-\frac{a^2c^4\tan(e+fx)}{f}-\frac{a^2c^4\tan^3(e+fx)\sec(e+fx)}{2f}+\frac{3a^2c^4\tan(e+fx)\sec(e+fx)}{4f}+a^2c^4x$$

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] a^2*c^4*x - (3*a^2*c^4*ArcTanh[Sin[e + f*x]])/(4*f) - (a^2*c^4*Tan[e + f*x]
)/f + (3*a^2*c^4*Sec[e + f*x]*Tan[e + f*x])/(4*f) + (a^2*c^4*Tan[e + f*x]^3
)/(3*f) - (a^2*c^4*Sec[e + f*x]*Tan[e + f*x]^3)/(2*f) + (a^2*c^4*Tan[e + f*
x]^5)/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int (c-c\sec(e+fx))^2\tan^4(e+fx)\,dx\\
&= \left(a^2c^2\right)\int \left(c^2\tan^4(e+fx)-2c^2\sec(e+fx)\tan^4(e+fx)+c^2\sec^2(e+fx)\tan^4(e+fx)\right)\,dx\\
&= \left(a^2c^4\right)\int \tan^4(e+fx)\,dx+\left(a^2c^4\right)\int \sec^2(e+fx)\tan^4(e+fx)\,dx\\
&\quad -\left(2a^2c^4\right)\int \sec(e+fx)\tan^4(e+fx)\,dx\\
&= \frac{a^2c^4\tan^3(e+fx)}{3f}-\frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{2f}-\left(a^2c^4\right)\int \tan^2(e+fx)\,dx\\
&\quad +\frac{1}{2}\left(3a^2c^4\right)\int \sec(e+fx)\tan^2(e+fx)\,dx+\frac{\left(a^2c^4\right)\text{Subst}\left(\int x^4\,dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{a^2c^4\tan(e+fx)}{f}+\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{4f}\\
&\quad +\frac{a^2c^4\tan^3(e+fx)}{3f}-\frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{2f}\\
&\quad +\frac{a^2c^4\tan^5(e+fx)}{5f}-\frac{1}{4}\left(3a^2c^4\right)\int \sec(e+fx)\,dx+\left(a^2c^4\right)\int 1\,dx\\
&= a^2c^4x-\frac{3a^2c^4\text{arctanh}(\sin(e+fx))}{4f}-\frac{a^2c^4\tan(e+fx)}{f}\\
&\quad +\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{4f}+\frac{a^2c^4\tan^3(e+fx)}{3f}\\
&\quad -\frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{2f}+\frac{a^2c^4\tan^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= \frac{a^2c^4\sec^5(e+fx)\,(600(e+fx)\cos(e+fx)-720\text{arctanh}(\sin(e+fx))\cos^5(e+fx)+300e\cos(3(e+fx)}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*Sec[e + f*x]^5*(600*(e + f*x)*Cos[e + f*x] - 720*ArcTanh[Sin[e + f
*x]]*Cos[e + f*x]^5 + 300*e*Cos[3*(e + f*x)] + 300*f*x*Cos[3*(e + f*x)] + 6
0*e*Cos[5*(e + f*x)] + 60*f*x*Cos[5*(e + f*x)] + 40*Sin[e + f*x] + 60*Sin[2
*(e + f*x)] - 220*Sin[3*(e + f*x)] + 150*Sin[4*(e + f*x)] - 68*Sin[5*(e + f
*x)]))/(960*f)
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

method	result
parts	$a^2c^4x-\frac{c^4a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}-\frac{a^2c^4\tan(fx+e)}{f}+\frac{2a^2c^4\sec(fx+e)\tan(fx+e)}{f}+$
risch	$a^2c^4x-\frac{ic^4a^2\left(75\,e^{9i(fx+e)}+60\,e^{8i(fx+e)}+30\,e^{7i(fx+e)}+360\,e^{6i(fx+e)}+320\,e^{4i(fx+e)}-30\,e^{3i(fx+e)}+280\,e^{2i(fx+e)}-\right.}{30f\left(1+e^{2i(fx+e)}\right)^5}$
derivativedivides	$\frac{-c^4a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2c^4a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)}{8}\right.}{}$
default	$\frac{-c^4a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2c^4a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)}{8}\right.}{}$
parallelrisch	$\frac{a^2c^4\left(\left(\frac{15\cos(fx+e)}{2}+\frac{15\cos(3fx+3e)}{4}+\frac{3\cos(5fx+5e)}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{15\cos(fx+e)}{2}-\frac{15\cos(3fx+3e)}{4}-\frac{3\cos(5fx+}{4}\right.\right.}{f(\cos}$
norman	$\frac{a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-a^2c^4x+5a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-10a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+10a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-5a^2c^4x\tan\left(\frac{fx}{2}+\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c^4*x-c^4*a^2/f*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-a
^2*c^4*tan(f*x+e)/f+2*a^2*c^4*sec(f*x+e)*tan(f*x+e)/f+c^4*a^2/f*(-2/3-1/3*s
ec(f*x+e)^2)*tan(f*x+e)-2*c^4*a^2/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*ta
n(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$=\frac{120\,a^2c^4fx\cos{(fx+e)}^5-45\,a^2c^4\cos{(fx+e)}^5\log{(\sin{(fx+e)}+1)}+45\,a^2c^4\cos{(fx+e)}^5\log{(-\sin{(fx}}{}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*c^4*f*x*cos(f*x + e)^5 - 45*a^2*c^4*cos(f*x + e)^5*log(sin(f
*x + e) + 1) + 45*a^2*c^4*cos(f*x + e)^5*log(-sin(f*x + e) + 1) - 2*(68*a^2
*c^4*cos(f*x + e)^4 - 75*a^2*c^4*cos(f*x + e)^3 + 4*a^2*c^4*cos(f*x + e)^2
+ 30*a^2*c^4*cos(f*x + e) - 12*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$=a^2c^4\left(\int 1\,dx+\int\left(-2\sec{(e+fx)}\right)\,dx+\int\left(-\sec^2{(e+fx)}\right)\,dx+\int 4\sec^3{(e+fx)}\,dx\right.$$
$$\left.+\int\left(-\sec^4{(e+fx)}\right)\,dx+\int\left(-2\sec^5{(e+fx)}\right)\,dx+\int\sec^6{(e+fx)}\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*c**4*(Integral(1, x) + Integral(-2*sec(e + f*x), x) + Integral(-sec(e
+ f*x)**2, x) + Integral(4*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)**4,
 x) + Integral(-2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.71

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$=\frac{8\left(3\,\tan{(fx+e)}^5+10\,\tan{(fx+e)}^3+15\,\tan{(fx+e)}\right)a^2c^4-40\left(\tan{(fx+e)}^3+3\,\tan{(fx+e)}\right)a^2c^4+}{}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^4 -
 40*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 120*(f*x + e)*a^2*c^4 + 15*
a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x
+ e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*
c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) - 240*a^2*c^4*log(sec(f*x + e) + tan(f*x + e)) - 120*a^2*c^4
*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^4\,dx = \frac{60\,(fx + e)a^2c^4 - 45\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 1\right|\right) + 45\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right|\right) + \frac{2\left(105\,a^2c^4\tan\right.}{}}{60\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/60*(60*(f*x + e)*a^2*c^4 - 45*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))
+ 45*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(105*a^2*c^4*tan(1/2*f*
x + 1/2*e)^9 - 530*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 328*a^2*c^4*tan(1/2*f*x
 + 1/2*e)^5 - 110*a^2*c^4*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c^4*tan(1/2*f*x +
 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^4\,dx = a^2\,c^4\,x + \frac{\frac{7\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{2} - \frac{53\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3} + \frac{164\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15} - \frac{11\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3} + \frac{a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10} - 5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8 + 10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6 - 10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 + 5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 - 1\right)} - \frac{3\,a^2\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}$$

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4,x)
```

```
[Out] a^2*c^4*x + ((164*a^2*c^4*tan(e/2 + (f*x)/2)^5)/15 - (11*a^2*c^4*tan(e/2 +
(f*x)/2)^3)/3 - (53*a^2*c^4*tan(e/2 + (f*x)/2)^7)/3 + (7*a^2*c^4*tan(e/2 +
```

```
(f*x)/2)^9)/2 + (a^2*c^4*tan(e/2 + (f*x)/2))/2)/(f*(5*tan(e/2 + (f*x)/2)^2
- 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^
8 + tan(e/2 + (f*x)/2)^10 - 1)) - (3*a^2*c^4*atanh(tan(e/2 + (f*x)/2)))/(2*
f)
```

3.3 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 97

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = a^2c^3x-\frac{3a^2c^3\text{arctanh}(\sin(e+fx))}{8f}-\frac{a^2(8c^3-3c^3\sec(e+fx))\tan(e+fx)}{8f}+\frac{a^2(4c^3-3c^3\sec(e+fx))\tan^3(e+fx)}{12f}$$

```
[Out] a^2*c^3*x-3/8*a^2*c^3*arctanh(sin(f*x+e))/f-1/8*a^2*(8*c^3-3*c^3*sec(f*x+e)
)*tan(f*x+e)/f+1/12*a^2*(4*c^3-3*c^3*sec(f*x+e))*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3966, 3855}

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = -\frac{3a^2c^3\text{arctanh}(\sin(e+fx))}{8f}+\frac{a^2\tan^3(e+fx)\,(4c^3-3c^3\sec(e+fx))}{12f}-\frac{a^2\tan(e+fx)\,(8c^3-3c^3\sec(e+fx))}{8f}+a^2c^3x$$

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] a^2*c^3*x - (3*a^2*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (a^2*(8*c^3 - 3*c^3*S
ec[e + f*x])*Tan[e + f*x])/(8*f) + (a^2*(4*c^3 - 3*c^3*Sec[e + f*x])*Tan[e
+ f*x]^3)/(12*f)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int (c-c\sec(e+fx))\tan^4(e+fx)\,dx\\
&= \frac{a^2(4c^3-3c^3\sec(e+fx))\tan^3(e+fx)}{12f}-\frac{1}{4}\left(a^2c^2\right)\int (4c-3c\sec(e+fx))\tan^2(e+fx)\,dx\\
&= -\frac{a^2(8c^3-3c^3\sec(e+fx))\tan(e+fx)}{8f}\\
&\quad+\frac{a^2(4c^3-3c^3\sec(e+fx))\tan^3(e+fx)}{12f}+\frac{1}{8}\left(a^2c^2\right)\int (8c-3c\sec(e+fx))\,dx\\
&= a^2c^3x-\frac{a^2(8c^3-3c^3\sec(e+fx))\tan(e+fx)}{8f}\\
&\quad+\frac{a^2(4c^3-3c^3\sec(e+fx))\tan^3(e+fx)}{12f}-\frac{1}{8}\left(3a^2c^3\right)\int \sec(e+fx)\,dx\\
&= a^2c^3x-\frac{3a^2c^3\operatorname{arctanh}(\sin(e+fx))}{8f}-\frac{a^2(8c^3-3c^3\sec(e+fx))\tan(e+fx)}{8f}\\
&\quad+\frac{a^2(4c^3-3c^3\sec(e+fx))\tan^3(e+fx)}{12f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$= \frac{a^2c^3\sec^4(e+fx)\,(72e+72fx-72\text{arctanh}(\sin(e+fx))\cos^4(e+fx)+96(e+fx)\cos(2(e+fx))+24e}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*Sec[e + f*x]^4*(72*e + 72*f*x - 72*ArcTanh[Sin[e + f*x]]*Cos[e + f
*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Cos[
4*(e + f*x)] - 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] + 30*Sin[3*(e + f*x)]
- 32*Sin[4*(e + f*x)]))/(192*f)
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37

method	result
parts	$a^2c^3x-\frac{a^2c^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{2a^2c^3\tan(fx+e)}{f}+\frac{a^2c^3\sec(fx+e)\tan(fx+e)}{f}-\frac{a^2c^3\left(-\left(-\frac{\text{se}}{}\right.\right.}{}$
risch	$a^2c^3x-\frac{ia^2c^3\left(15\,\mathrm{e}^{7i(fx+e)}+48\,\mathrm{e}^{6i(fx+e)}-9\,\mathrm{e}^{5i(fx+e)}+96\,\mathrm{e}^{4i(fx+e)}+9\,\mathrm{e}^{3i(fx+e)}+80\,\mathrm{e}^{2i(fx+e)}-15\,\mathrm{e}^{i(fx+e)}+32\right)}{12f\left(1+\mathrm{e}^{2i(fx+e)}\right)^4}+$
derivativedivides	$\frac{-a^2c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-a^2c^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+}{}$
default	$\frac{-a^2c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-a^2c^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+}{}$
parallelrisch	$\frac{a^2c^3\left(9(3+\cos(4fx+4e)+4\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+9(-\cos(4fx+4e)-4\cos(2fx+2e)-3)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.\right.}{24f(3+\cos(4fx+4e}$
norman	$\frac{a^2c^3x+a^2c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-\frac{5a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{71a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{12f}-\frac{137a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{12f}+\frac{11a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}-4a}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c^3*x-a^2*c^3/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a^2*c^3/f*tan(f*x+
e)+a^2*c^3/f*sec(f*x+e)*tan(f*x+e)-a^2*c^3/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f
*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^3\,dx = \frac{48\,a^2c^3fx\cos(fx+e)^4 - 9\,a^2c^3\cos(fx+e)^4\log(\sin(fx+e)+1) + 9\,a^2c^3\cos(fx+e)^4\log(-\sin(fx+e$$
$$48\,f\cos(fx$$

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] 1/48*(48*a^2*c^3*f*x*cos(f*x + e)^4 - 9*a^2*c^3*cos(f*x + e)^4*log(sin(f*x + e) + 1) + 9*a^2*c^3*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^2*c^3*cos(f*x + e)^3 - 15*a^2*c^3*cos(f*x + e)^2 - 8*a^2*c^3*cos(f*x + e) + 6*a^2*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)

Sympy [F]

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^3\,dx = -a^2c^3\left(\int (-1)\,dx + \int \sec(e+fx)\,dx + \int 2\sec^2(e+fx)\,dx + \int \left(-2\sec^3(e+fx)\right)dx + \int \left(-\sec^4(e+fx)\right)dx + \int \sec^5(e+fx)\,dx\right)$$

[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)

[Out] -a**2*c**3*(Integral(-1, x) + Integral(sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(91) = 182.

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.09

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^3\,dx = \frac{16\left(\tan(fx+e)^3 + 3\tan(fx+e)\right)a^2c^3 + 48\,(fx+e)a^2c^3 + 3\,a^2c^3\left(\frac{2\left(3\sin(fx+e)^3 - 5\sin(fx+e)\right)}{\sin(fx+e)^4 - 2\sin(fx+e)^2 + 1} - 3\log(\sin(\right.}{}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 48*(f*x + e)*a^2*c^3 +
 3*a^2*c^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f
*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 24*a^
2*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(si
n(f*x + e) - 1)) - 48*a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) - 96*a^2*c^3
*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$=\frac{24\,(fx+e)a^2c^3-9\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)+9\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(33\,a^2c^3\tan\left(\frac{1}{2}\right.\right.}{}}{24\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="giac")
[Out] 1/24*(24*(f*x + e)*a^2*c^3 - 9*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) +
 9*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(33*a^2*c^3*tan(1/2*f*x +
 1/2*e)^7 - 137*a^2*c^3*tan(1/2*f*x + 1/2*e)^5 + 71*a^2*c^3*tan(1/2*f*x + 1
/2*e)^3 - 15*a^2*c^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/
f
```

Mupad [B] (verification not implemented)

Time = 15.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$=\frac{\frac{11\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{137\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{71\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}-\frac{5\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$
$$+a^2\,c^3\,x-\frac{3\,a^2\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3,x)
[Out] ((71*a^2*c^3*tan(e/2 + (f*x)/2)^3)/12 - (137*a^2*c^3*tan(e/2 + (f*x)/2)^5)/
12 + (11*a^2*c^3*tan(e/2 + (f*x)/2)^7)/4 - (5*a^2*c^3*tan(e/2 + (f*x)/2))/4
)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2
)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + a^2*c^3*x - (3*a^2*c^3*atanh(tan(e/2 + (
f*x)/2)))/(4*f)
```

3.4 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 47

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2x-\frac{a^2c^2\tan(e+fx)}{f}+\frac{a^2c^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*c^2*x-a^2*c^2*tan(f*x+e)/f+1/3*a^2*c^2*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3554, 8}

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{a^2c^2\tan^3(e+fx)}{3f}-\frac{a^2c^2\tan(e+fx)}{f}+a^2c^2x$$

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] a^2*c^2*x - (a^2*c^2*Tan[e + f*x])/f + (a^2*c^2*Tan[e + f*x]^3)/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \tan^4(e+fx)\,dx\\
&= \frac{a^2c^2\tan^3(e+fx)}{3f} - \left(a^2c^2\right)\int \tan^2(e+fx)\,dx\\
&= -\frac{a^2c^2\tan(e+fx)}{f} + \frac{a^2c^2\tan^3(e+fx)}{3f} + \left(a^2c^2\right)\int 1\,dx\\
&= a^2c^2x - \frac{a^2c^2\tan(e+fx)}{f} + \frac{a^2c^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2\left(\frac{\arctan(\tan(e+fx))}{f} - \frac{\tan(e+fx)}{f} + \frac{\tan^3(e+fx)}{3f}\right)$$

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
[Out] a^2*c^2*(ArcTan[Tan[e + f*x]]/f - Tan[e + f*x]/f + Tan[e + f*x]^3/(3*f))
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

method	result
parts	$a^{2} c^{2} x-\frac{a^{2} c^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x+e\right)^{2}}{3}\right) \tan \left(f x+e\right)}{f}-\frac{2 a^{2} c^{2} \tan \left(f x+e\right)}{f}$
derivativedivides	$\frac{-a^{2} c^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x+e\right)^{2}}{3}\right) \tan \left(f x+e\right)-2 a^{2} c^{2} \tan \left(f x+e\right)+a^{2} c^{2} \left(f x+e\right)}{f}$
default	$\frac{-a^{2} c^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x+e\right)^{2}}{3}\right) \tan \left(f x+e\right)-2 a^{2} c^{2} \tan \left(f x+e\right)+a^{2} c^{2} \left(f x+e\right)}{f}$
risch	$a^{2} c^{2} x-\frac{4 i a^{2} c^{2} \left(3 \,{\mathrm e}^{4 i \left(f x+e\right)}+3 \,{\mathrm e}^{2 i \left(f x+e\right)}+2\right)}{3 f \left(1+{\mathrm e}^{2 i \left(f x+e\right)}\right)^{3}}$
parallelrisch	$\frac{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6} x f-3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4} x f+2 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}+3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2} x f-\frac{20 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-f x+2 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right) a^{2} c^{2}}{f \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{3} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}$
norman	$\frac{a^{2} c^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-a^{2} c^{2} x+\frac{2 a^{2} c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{f}-\frac{20 a^{2} c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 f}+\frac{2 a^{2} c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{f}+3 a^{2} c^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-3 a^{2} c^{2}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c^2*x-a^2*c^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a^2*c^2*tan(f*x+e)
/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{3\,a^{2} c^{2} f x \cos\left(f x + e\right)^{3} - {\left(4\,a^{2} c^{2} \cos\left(f x + e\right)^{2} - a^{2} c^{2}\right)} \sin\left(f x + e\right)}{3\,f \cos\left(f x + e\right)^{3}}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*c^2*f*x*cos(f*x + e)^3 - (4*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*si
n(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2\left(\int 1\,dx + \int \left(-2\sec^2{(e+fx)}\right)\,dx + \int \sec^4{(e+fx)}\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
[Out] a**2*c**2*(Integral(1, x) + Integral(-2*sec(e + f*x)**2, x) + Integral(sec(
e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{\left(\tan{(fx+e)}^3+3\tan{(fx+e)}\right)a^2c^2+3\,(fx+e)a^2c^2-6\,a^2c^2\tan{(fx+e)}}{3\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^2 + 3*(f*x + e)*a^2*c^2 - 6*a^
2*c^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{a^2c^2\tan{(fx+e)}^3+3\,(fx+e)a^2c^2-3\,a^2c^2\tan{(fx+e)}}{3\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="giac")
[Out] 1/3*(a^2*c^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2*c^2 - 3*a^2*c^2*tan(f*x + e))
/f
```

Mupad [B] (verification not implemented)

Time = 16.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^2 \, dx$$
$$= a^2 c^2 x + \frac{2 a^2 c^2 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^5 - \frac{20 a^2 c^2 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^3}{3} + 2 a^2 c^2 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)}{f \left(\tan\left(\frac{e}{2} + \frac{f x}{2}\right)^2 - 1\right)^3}$$

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2,x)
```

```
[Out] a^2*c^2*x + (2*a^2*c^2*tan(e/2 + (f*x)/2)^5 - (20*a^2*c^2*tan(e/2 + (f*x)/2
)^3)/3 + 2*a^2*c^2*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^2 - 1)^3)
```

3.5 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 55

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = a^2cx+\frac{a^2c\,\text{arctanh}(\sin(e+fx))}{2f}-\frac{c(2a^2+a^2\sec(e+fx))\tan(e+fx)}{2f}$$

```
[Out] a^2*c*x+1/2*a^2*c*arctanh(sin(f*x+e))/f-1/2*c*(2*a^2+a^2*sec(f*x+e))*tan(f*
x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3989, 3966, 3855}

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{a^2c\,\text{arctanh}(\sin(e+fx))}{2f}-\frac{c\tan(e+fx)\left(a^2\sec(e+fx)+2a^2\right)}{2f}+a^2cx$$

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] a^2*c*x + (a^2*c*ArcTanh[Sin[e + f*x]])/(2*f) - (c*(2*a^2 + a^2*Sec[e + f*x
])*Tan[e + f*x])/(2*f)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left((ac)\int (a+a\sec(e+fx))\tan^2(e+fx)\,dx\right)\\
&= -\frac{c(2a^2+a^2\sec(e+fx))\tan(e+fx)}{2f}+\frac{1}{2}(ac)\int (2a+a\sec(e+fx))\,dx\\
&= a^2cx-\frac{c(2a^2+a^2\sec(e+fx))\tan(e+fx)}{2f}+\frac{1}{2}(a^2c)\int \sec(e+fx)\,dx\\
&= a^2cx+\frac{a^2c\,\mathrm{arctanh}(\sin(e+fx))}{2f}-\frac{c(2a^2+a^2\sec(e+fx))\tan(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.76

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=\frac{a^2c\sec^2(e+fx)\,(e+fx+6\arctan(\tan(e+fx))\cos^2(e+fx)+4\mathrm{arctanh}(\sin(e+fx))\cos^2(e+fx)+e\,c}{8f}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*Sec[e + f*x]^2*(e + f*x + 6*ArcTan[Tan[e + f*x]]*Cos[e + f*x]^2 + 4*
ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^2 + e*Cos[2*(e + f*x)] + f*x*Cos[2*(e +
f*x)] - 4*Sin[e + f*x] - 4*Sin[2*(e + f*x)]))/(8*f)
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2c\tan(fx+e)+a^2c\ln(\sec(fx+e)+\tan(fx+e))+a^2c(fx+e)}{f}$
default	$\frac{-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2c\tan(fx+e)+a^2c\ln(\sec(fx+e)+\tan(fx+e))+a^2c(fx+e)}{f}$
parts	$a^2cx+\frac{a^2c\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a^2c\tan(fx+e)}{f}-\frac{a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
risch	$a^2cx+\frac{ia^2c(e^{3i(fx+e)}-2e^{2i(fx+e)}-e^{i(fx+e)}-2)}{f(1+e^{2i(fx+e)})^2}+\frac{a^2c\ln(e^{i(fx+e)}+i)}{2f}-\frac{a^2c\ln(e^{i(fx+e)}-i)}{2f}$
parallelrisch	$-\frac{\left((1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+(-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-2fx\cos(2fx+2e)-2fx+2\sin(f}{2f(1+\cos(2fx+2e))}$
norman	$\frac{a^2cx+a^2cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-2a^2cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{3a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}-\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+$

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-a^2*c
*tan(f*x+e)+a^2*c*ln(sec(f*x+e)+tan(f*x+e))+a^2*c*(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(51) = 102.

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{4\,a^2cfx\cos(fx+e)^2+a^2c\cos(fx+e)^2\log(\sin(fx+e)+1)-a^2c\cos(fx+e)^2\log(-\sin(fx+e)+1}{4\,f\cos(fx+e)^2}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*c*f*x*cos(f*x + e)^2 + a^2*c*cos(f*x + e)^2*log(sin(f*x + e) + 1
) - a^2*c*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*(2*a^2*c*cos(f*x + e) +
 a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -a^2c\left(\int (-1)\,dx+\int (-\sec(e+fx))\,dx + \int \sec^2(e+fx)\,dx+\int \sec^3(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
[Out] -a**2*c*(Integral(-1, x) + Integral(-sec(e + f*x), x) + Integral(sec(e + f*
x)**2, x) + Integral(sec(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{4\,(fx+e)a^2c+a^2c\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+4\,a^2c\log(\sec(fx+e)}{4\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="maxima")
[Out] 1/4*(4*(f*x + e)*a^2*c + a^2*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(s
in(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 4*a^2*c*log(sec(f*x + e) + tan(
f*x + e)) - 4*a^2*c*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(51) = 102.

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{2\,(fx+e)a^2c+a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^2c}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}}{2\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] 1/2*(2*(f*x + e)*a^2*c + a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - a^2*c*l
og(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(a^2*c*tan(1/2*f*x + 1/2*e)^3 - 3*a^2
*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=a^2\,c\,x-\frac{3\,a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}+\frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)),x)
```

```
[Out] a^2*c*x - (3*a^2*c*tan(e/2 + (f*x)/2) - a^2*c*tan(e/2 + (f*x)/2)^3)/(f*(tan
(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1)) + (a^2*c*atanh(tan(e/2 + (
f*x)/2)))/f
```

3.6 $\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 56

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx=\frac{a^2x}{c}-\frac{a^2\text{arctanh}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}$$

```
[Out] a^2*x/c-a^2*arctanh(sin(f*x+e))/c/f-4*a^2*tan(f*x+e)/c/f/(1-sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3988, 3862, 8, 3879, 3874, 3855}

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx=-\frac{a^2\text{arctanh}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}+\frac{a^2x}{c}$$

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]
[Out] (a^2*x)/c - (a^2*ArcTanh[Sin[e + f*x]])/(c*f) - (4*a^2*Tan[e + f*x])/(c*f*(
1 - Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{a^2}{1-\sec(e+fx)}+\frac{2a^2\sec(e+fx)}{1-\sec(e+fx)}+\frac{a^2\sec^2(e+fx)}{1-\sec(e+fx)}\right)dx}{c}\\
&= \frac{a^2\int \frac{1}{1-\sec(e+fx)}\,dx}{c}+\frac{a^2\int \frac{\sec^2(e+fx)}{1-\sec(e+fx)}\,dx}{c}+\frac{(2a^2)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{c}\\
&= -\frac{3a^2\tan(e+fx)}{cf(1-\sec(e+fx))}-\frac{a^2\int -1\,dx}{c}-\frac{a^2\int \sec(e+fx)\,dx}{c}+\frac{a^2\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{c}\\
&= \frac{a^2x}{c}-\frac{a^2\text{arctanh}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. $2(56) = 112$.

Time = 1.61 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.59

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{a^{3/2}\tan(e+fx)\left(4\sqrt{c}\left(\sqrt{a}\sqrt{1-\sec(e+fx)}(1+\sec(e+fx))+\arcsin\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)\sec(e+fx)\sqrt{\cdots}\right.\right.}{c^{3/2}f(1-\sec(e+fx\cdots}$$

[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]

[Out] -((a^(3/2)*Tan[e + f*x]*(4*Sqrt[c]*(Sqrt[a]*Sqrt[1 - Sec[e + f*x]]*(1 + Sec[e + f*x]) + ArcSin[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^2) - ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[1 - Sec[e + f*x]]*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(c^(3/2)*f*(1 - Sec[e + f*x])^(3/2)*(1 + Sec[e + f*x])))

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

method	result	si
derivativedivides	$\frac{4a^2\left(\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fc}$	6
default	$\frac{4a^2\left(\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fc}$	6
parallelrisch	$\frac{a^2\left(4+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)fx+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	8
risch	$\frac{a^2x}{c}+\frac{8ia^2}{fc\left(e^{i(fx+e)}-1\right)}+\frac{a^2\ln\left(e^{i(fx+e)}-i\right)}{cf}-\frac{a^2\ln\left(e^{i(fx+e)}+i\right)}{cf}$	8
norman	$\frac{\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{c}-\frac{4a^2}{cf}+\frac{4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}-\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{cf}-\frac{a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{cf}$	1

[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 4/f*a^2/c*(1/tan(1/2*f*x+1/2*e)+1/4*ln(tan(1/2*f*x+1/2*e)-1)+1/2*arctan(tan(1/2*f*x+1/2*e))-1/4*ln(tan(1/2*f*x+1/2*e)+1))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = \frac{2\,a^2fx\sin\left(fx+e\right)-a^2\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+a^2\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+8\,a}{2\,cf\sin\left(fx+e\right)}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*f*x*sin(f*x + e) - a^2*log(sin(f*x + e) + 1)*sin(f*x + e) + a^2*
log(-sin(f*x + e) + 1)*sin(f*x + e) + 8*a^2*cos(f*x + e) + 8*a^2)/(c*f*sin(
f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{a^2\left(\int \frac{2\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{1}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x) - 1), x))/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.73

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = \frac{a^2\left(\frac{2\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}+\frac{\cos(fx+e)+1}{c\sin(fx+e)}\right)-a^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}-\frac{\cos(fx+e)+1}{c\sin(fx+e)}\right)+\frac{2\,a^2(\cos(fx}{c\sin(fx}}{f}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] (a^2*(2*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c + (cos(f*x + e) + 1)/(c*s
in(f*x + e))) - a^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) +
2*a^2*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = \frac{\frac{(fx+e)a^2}{c} - \frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c} + \frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c} + \frac{4\,a^2}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{f}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^2/c - a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c + a^2*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/c + 4*a^2/(c*tan(1/2*f*x + 1/2*e)))/f
```

Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = \frac{a^2\,x}{c} - \frac{a^2\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right) - \frac{4}{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{c\,f}$$

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x)),x)
```

```
[Out] (a^2*x)/c - (a^2*(2*atanh(tan(e/2 + (f*x)/2)) - 4/tan(e/2 + (f*x)/2)))/(c*f
)
```

3.7 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$

Optimal result 125
Rubi [A] (verified) 125
Mathematica [C] (verified) 127
Maple [A] (verified) 127
Fricas [A] (verification not implemented) 128
Sympy [F] 128
Maxima [B] (verification not implemented) 128
Giac [A] (verification not implemented) 129
Mupad [B] (verification not implemented) 129

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx=\frac{a^2x}{c^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

```
[Out] a^2*x/c^2-4/3*a^2*tan(f*x+e)/c^2/f/(1-sec(f*x+e))^2-4/3*a^2*tan(f*x+e)/c^2/
f/(1-sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3988, 3862, 4004, 3879, 3881, 3882}

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx=-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^2x}{c^2}$$

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*x)/c^2 - (4*a^2*Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f*x])^2) - (4*a^2*
Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^2}+\frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^2}+\frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^2}\right)dx}{c^2}\\
&= \frac{a^2\int \frac{1}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{a^2\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{(2a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}\\
&= -\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{a^2\int \frac{-3-\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}
\end{aligned}$$

$$= \frac{a^2x}{c^2} - \frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2} + \frac{(4a^2)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}dx}{3c^2}$$
$$= \frac{a^2x}{c^2} - \frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2} - \frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$$
$$= -\frac{2a^2\cot^3\left(\frac{e}{2}+\frac{fx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3c^2f}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]
[Out] (-2*a^2*Cot[e/2 + (f*x)/2]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e/2 + (f
*x)/2]^2])/(3*c^2*f)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$-\frac{a^2\left(2\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3-3fx-6\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3c^2f}$	41
derivativedivides	$\frac{2a^2\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^2}$	47
default	$\frac{2a^2\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^2}$	47
risch	$\frac{a^2x}{c^2}+\frac{8ia^2\left(3\,e^{2i(fx+e)}-3\,e^{i(fx+e)}+2\right)}{3f\,c^2\left(e^{i(fx+e)}-1\right)^3}$	59
norman	$\frac{\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{c}+\frac{2a^2}{3cf}-\frac{8a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3cf}+\frac{2a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}-\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{c}}{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	126

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] -1/3*a^2*(2*cot(1/2*f*x+1/2*e)^3-3*f*x-6*cot(1/2*f*x+1/2*e))/c^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^2}\,dx = \frac{8\,a^2\cos\left(fx + e\right)^2 + 4\,a^2\cos\left(fx + e\right) - 4\,a^2 + 3\,\left(a^2 fx\cos\left(fx + e\right) - a^2 fx\right)\sin\left(fx + e\right)}{3\,\left(c^2 f\cos\left(fx + e\right) - c^2 f\right)\sin\left(fx + e\right)}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(8*a^2*cos(f*x + e)^2 + 4*a^2*cos(f*x + e) - 4*a^2 + 3*(a^2*f*x*cos(f*x
 + e) - a^2*f*x)*sin(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^2}\,dx = \frac{a^2\left(\int \frac{2\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx + \int \frac{\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx + \int \frac{1}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx\right)}{c^2}$$

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(63) = 126.

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.45

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^2}\,dx = \frac{a^2\left(\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2} + \frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right) - \frac{a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3} + \frac{2\,a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right.}{c^2\sin(}}{6\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*(12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2 + (9*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - a^
2*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin
(f*x + e)^3) + 2*a^2*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x +
 e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx=\frac{\frac{3\,(fx+e)a^2}{c^2}+\frac{2\,\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^2\right)}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^2/c^2 + 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 - a^2)/(c^2*tan(
1/2*f*x + 1/2*e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx=\frac{a^2\,\left(-2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+6\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)+3\,f\,x\right)}{3\,c^2\,f}$$

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^2,x)
```

```
[Out] (a^2*(6*cot(e/2 + (f*x)/2) - 2*cot(e/2 + (f*x)/2)^3 + 3*f*x))/(3*c^2*f)
```

3.8 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$

Optimal result 130
Rubi [A] (verified) 130
Mathematica [A] (verified) 132
Maple [A] (verified) 133
Fricas [A] (verification not implemented) 133
Sympy [F] 134
Maxima [B] (verification not implemented) 134
Giac [A] (verification not implemented) 135
Mupad [B] (verification not implemented) 135

Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2x}{c^3} - \frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} - \frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} - \frac{23a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

```
[Out] a^2*x/c^3-4/5*a^2*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^3-8/15*a^2*tan(f*x+e)/c^3
/f/(1-sec(f*x+e))^2-23/15*a^2*tan(f*x+e)/c^3/f/(1-sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882}

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = -\frac{23a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))} - \frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} - \frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} + \frac{a^2x}{c^3}$$

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*x)/c^3 - (4*a^2*Tan[e + f*x])/(5*c^3*f*(1 - Sec[e + f*x])^3) - (8*a^2*
Tan[e + f*x])/(15*c^3*f*(1 - Sec[e + f*x])^2) - (23*a^2*Tan[e + f*x])/(15*c
^3*f*(1 - Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
```

```
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^3}+\frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^3}+\frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^3}\right)dx}{c^3}\\
&= \frac{a^2\int \frac{1}{(1-\sec(e+fx))^3}\,dx}{c^3}+\frac{a^2\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3}+\frac{(2a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3}\\
&= -\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{a^2\int \frac{-5-2\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3}\\
&\quad -\frac{(3a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3}+\frac{(4a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3}\\
&= -\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}\\
&\quad +\frac{a^2\int \frac{15+7\sec(e+fx)}{1-\sec(e+fx)}\,dx}{15c^3}-\frac{a^2\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{5c^3}+\frac{(4a^2)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{15c^3}\\
&= \frac{a^2x}{c^3}-\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}\\
&\quad -\frac{a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))}+\frac{(22a^2)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{15c^3}\\
&= \frac{a^2x}{c^3}-\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{23a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.48

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{a^{3/2}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(43-11\sec(e+fx)-31\sec^2(e+fx)+23\sec^3(e+fx))-60\text{arctanh}\left(\frac{\sqrt{-ac\tan^2(\cdots}}{\sqrt{a}\sqrt{c}\cdots}\right.\right.}{15c^{7/2}f(-1+\sec(e+fx))^3(1+\sec(e+fx))}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^(3/2)*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(43 - 11*Sec[e + f*x] - 31*Sec[e + f
*x]^2 + 23*Sec[e + f*x]^3) - 60*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a
]*Sqrt[c])]*Sec[e + f*x]^2*Sin[(e + f*x)/2]^4*Sqrt[-(a*c*Tan[e + f*x]^2)]))
/(15*c^(7/2)*f*(-1 + Sec[e + f*x])^3*(1 + Sec[e + f*x]))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.53

method	result	size
parallelrisch	$\frac{a^{2}\left(3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-10\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+15fx+30\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15c^{3}f}$	54
derivativedivides	$\frac{a^{2}\left(\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,c^{3}}$	63
default	$\frac{a^{2}\left(\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,c^{3}}$	63
risch	$\frac{a^{2}x}{c^{3}}+\frac{2ia^{2}\left(75\,\mathrm{e}^{4i(fx+e)}-180\,\mathrm{e}^{3i(fx+e)}+250\,\mathrm{e}^{2i(fx+e)}-140\,\mathrm{e}^{i(fx+e)}+43\right)}{15f\,c^{3}\left(\mathrm{e}^{i(fx+e)}-1\right)^{5}}$	81
norman	$\frac{\frac{a^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{c}-\frac{a^{2}}{5cf}+\frac{13a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{15cf}-\frac{8a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{3cf}+\frac{2a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{cf}-\frac{a^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$	148

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*a^2*(3*cot(1/2*f*x+1/2*e)^5-10*cot(1/2*f*x+1/2*e)^3+15*f*x+30*cot(1/2*
f*x+1/2*e))/c^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{43\,a^2\cos\left(fx+e\right)^3-11\,a^2\cos\left(fx+e\right)^2-31\,a^2\cos\left(fx+e\right)+23\,a^2+15\,\left(a^2fx\cos\left(fx+e\right)^2-2\,a^2fx\,\mathrm{co}\right.}{15\,\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(43*a^2*cos(f*x + e)^3 - 11*a^2*cos(f*x + e)^2 - 31*a^2*cos(f*x + e) +
 23*a^2 + 15*(a^2*f*x*cos(f*x + e)^2 - 2*a^2*f*x*cos(f*x + e) + a^2*f*x)*si
n(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x
+ e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx =$$
$$-\frac{a^2\left(\int \frac{2\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{1}{\sec^3(e+fx)-3\sec^2(e-}\right.}{c^3}$$

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec
(e + f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**3 - 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(90) = 180.

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.11

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$$
$$=\frac{a^2\left(\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right)-\frac{2a^2\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(}{c^3\sin(fx+e)^5}}{60f}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(a^2*(120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3 - (20*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(c
os(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5)) - 2*a^2*(10*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) +
 1)^5/(c^3*sin(f*x + e)^5) - 3*a^2*(5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)a^2}{c^3}+\frac{30\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-10\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^2}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*a^2/c^3 + (30*a^2*tan(1/2*f*x + 1/2*e)^4 - 10*a^2*tan(1/
2*f*x + 1/2*e)^2 + 3*a^2)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2\,x}{c^3}+\frac{\frac{a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{2\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{c^3\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^3,x)
```

```
[Out] (a^2*x)/c^3 + ((a^2*cos(e/2 + (f*x)/2)^5)/5 + 2*a^2*cos(e/2 + (f*x)/2)*sin(
e/2 + (f*x)/2)^4 - (2*a^2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^2)/3)/(c^
3*f*sin(e/2 + (f*x)/2)^5)
```

3.9 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4} dx$

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Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4} dx = \frac{a^2x}{c^4} - \frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4} - \frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3} - \frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2} - \frac{164a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))}$$

```
[Out] a^2*x/c^4-4/7*a^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^4-12/35*a^2*tan(f*x+e)/c^
4/f/(1-sec(f*x+e))^3-59/105*a^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^2-164/105*a
^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882}

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4} dx = -\frac{164a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))} - \frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2} - \frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3} - \frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4} + \frac{a^2x}{c^4}$$

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*x)/c^4 - (4*a^2*Tan[e + f*x])/(7*c^4*f*(1 - Sec[e + f*x])^4) - (12*a^2
*Tan[e + f*x])/(35*c^4*f*(1 - Sec[e + f*x])^3) - (59*a^2*Tan[e + f*x])/(105
*c^4*f*(1 - Sec[e + f*x])^2) - (164*a^2*Tan[e + f*x])/(105*c^4*f*(1 - Sec[e
 + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^4}+\frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^4}+\frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^4}\right)dx}{c^4}\\
&= \frac{a^2\int \frac{1}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{a^2\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{(2a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}\\
&= -\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{a^2\int \frac{-7-3\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}\\
&\quad -\frac{(4a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}+\frac{(6a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}\\
&= -\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}+\frac{a^2\int \frac{35+20\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^4}\\
&\quad -\frac{(8a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^4}+\frac{(12a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^4}\\
&= -\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}\\
&\quad -\frac{a^2\int \frac{-105-55\sec(e+fx)}{1-\sec(e+fx)}\,dx}{105c^4}-\frac{(8a^2)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{105c^4}+\frac{(4a^2)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{35c^4}\\
&= \frac{a^2x}{c^4}-\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}\\
&\quad -\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))}+\frac{(32a^2)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{21c^4}\\
&= \frac{a^2x}{c^4}-\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}\\
&\quad -\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{164a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.83 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^4}\,dx = -\frac{a^2\csc^7(e + fx)\left(7032 + 18165\cos(e + fx) + 19348\cos(2(e + fx)) + 9303\cos(3(e + fx)) + 3080\cos(4\right.}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/6720*(a^2*Csc[e + f*x]^7*(7032 + 18165*Cos[e + f*x] + 19348*Cos[2*(e + f
*x)] + 9303*Cos[3*(e + f*x)] + 3080*Cos[4*(e + f*x)] + 2149*Cos[5*(e + f*x)
] + 1260*Cos[6*(e + f*x)] + 143*Cos[7*(e + f*x)] + 960*Cos[e + f*x]^7*Hyper
geometric2F1[-7/2, 1, -5/2, -Tan[e + f*x]^2]))/(c^4*f)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

method	result
parallelrisch	$-\frac{a^2\left(15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7-63\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5+140\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3-210fx-420\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{210c^4f}$
derivativedivides	$\frac{a^2\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2f\,c^4}$
default	$\frac{a^2\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2f\,c^4}$
risch	$\frac{a^2x}{c^4}+\frac{2ia^2\left(630\,\mathrm{e}^{6i(fx+e)}-2415\,\mathrm{e}^{5i(fx+e)}+5215\,\mathrm{e}^{4i(fx+e)}-5950\,\mathrm{e}^{3i(fx+e)}+4284\,\mathrm{e}^{2i(fx+e)}-1603\,\mathrm{e}^{i(fx+e)}+319\right)}{105f\,c^4\left(\mathrm{e}^{i(fx+e)}-1\right)^7}$
norman	$\frac{\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{c}+\frac{a^2}{14cf}-\frac{13a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{35cf}+\frac{29a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{30cf}-\frac{8a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3cf}+\frac{2a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{cf}-\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/210*a^2*(15*cot(1/2*f*x+1/2*e)^7-63*cot(1/2*f*x+1/2*e)^5+140*cot(1/2*f*x
+1/2*e)^3-210*f*x-420*cot(1/2*f*x+1/2*e))/c^4/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{319\,a^2\cos\left(fx+e\right)^4-327\,a^2\cos\left(fx+e\right)^3-95\,a^2\cos\left(fx+e\right)^2+387\,a^2\cos\left(fx+e\right)-164\,a^2+105\,(a^2 f\cdots}{105\left(c^4 f\cos\left(fx+e\right)^3-3\,c^4 f\cos\left(fx+e\right)^2+3\,c^4 f\cos\cdots}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(319*a^2*cos(f*x + e)^4 - 327*a^2*cos(f*x + e)^3 - 95*a^2*cos(f*x + e
)^2 + 387*a^2*cos(f*x + e) - 164*a^2 + 105*(a^2*f*x*cos(f*x + e)^3 - 3*a^2*
f*x*cos(f*x + e)^2 + 3*a^2*f*x*cos(f*x + e) - a^2*f*x)*sin(f*x + e))/((c^4*
f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*s
in(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2\left(\int \frac{2\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\cdots\right.}{c^4}$$

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(
e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(1/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec
(e + f*x) + 1), x))/c**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(117) = 234.

Time = 0.30 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.21

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{5\,a^2\left(\frac{336\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4}+\frac{\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{77\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{315\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-3\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}\right)+\frac{a^2\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\,\sin(\cdots}{(\cos(fx\cdots}\right.}{\cdots}}{840\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(5*a^2*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x
 + e)^7)) + a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(
f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 6*a^2*(21*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{\frac{210\,(fx+e)a^2}{c^4}+\frac{420\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-140\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+63\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,a^2}{c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{210\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/210*(210*(f*x + e)*a^2/c^4 + (420*a^2*tan(1/2*f*x + 1/2*e)^6 - 140*a^2*ta
n(1/2*f*x + 1/2*e)^4 + 63*a^2*tan(1/2*f*x + 1/2*e)^2 - 15*a^2)/(c^4*tan(1/2
*f*x + 1/2*e)^7))/f
```

Mupad [B] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2\,x}{c^4} - \frac{\frac{a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{14}-\frac{3\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{10}+\frac{2\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3}-2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{c^4\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}$$

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^4,x)
```

```
[Out] (a^2*x)/c^4 - ((a^2*cos(e/2 + (f*x)/2)^7)/14 - 2*a^2*cos(e/2 + (f*x)/2)*sin
(e/2 + (f*x)/2)^6 + (2*a^2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^4)/3 - (
3*a^2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2)/10)/(c^4*f*sin(e/2 + (f*x)
/2)^7)
```

3.10 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5} dx$

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Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5} dx = \frac{a^2x}{c^5} - \frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} - \frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{179a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} - \frac{494a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))}$$

```
[Out] a^2*x/c^5-4/9*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5-16/63*a^2*tan(f*x+e)/c^
5/f/(1-sec(f*x+e))^4-37/105*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^3-179/315*a
^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^2-494/315*a^2*tan(f*x+e)/c^5/f/(1-sec(f*
x+e))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882}

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5} dx = -\frac{494a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))} - \frac{179a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} - \frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} + \frac{a^2x}{c^5}$$

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*x)/c^5 - (4*a^2*Tan[e + f*x])/(9*c^5*f*(1 - Sec[e + f*x])^5) - (16*a^2
*Tan[e + f*x])/(63*c^5*f*(1 - Sec[e + f*x])^4) - (37*a^2*Tan[e + f*x])/(105
*c^5*f*(1 - Sec[e + f*x])^3) - (179*a^2*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e
 + f*x])^2) - (494*a^2*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^5} + \frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^5} + \frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^5}\right) dx}{c^5}$$

$$= \frac{a^2 \int \frac{1}{(1-\sec(e+fx))^5} dx}{c^5} + \frac{a^2 \int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^5} dx}{c^5} + \frac{(2a^2) \int \frac{\sec(e+fx)}{(1-\sec(e+fx))^5} dx}{c^5}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} - \frac{a^2 \int \frac{-9-4\sec(e+fx)}{(1-\sec(e+fx))^4} dx}{9c^5} - \frac{(5a^2) \int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4} dx}{9c^5} + \frac{(8a^2) \int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4} dx}{9c^5}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} - \frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} + \frac{a^2 \int \frac{63+39\sec(e+fx)}{(1-\sec(e+fx))^3} dx}{63c^5} - \frac{(5a^2) \int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3} dx}{21c^5} + \frac{(8a^2) \int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3} dx}{21c^5}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} - \frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{a^2 \int \frac{-315-204\sec(e+fx)}{(1-\sec(e+fx))^2} dx}{315c^5} - \frac{(2a^2) \int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2} dx}{21c^5} + \frac{(16a^2) \int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2} dx}{105c^5}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} - \frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{179a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} + \frac{a^2 \int \frac{945+519\sec(e+fx)}{1-\sec(e+fx)} dx}{945c^5} - \frac{(2a^2) \int \frac{\sec(e+fx)}{1-\sec(e+fx)} dx}{63c^5} + \frac{(16a^2) \int \frac{\sec(e+fx)}{1-\sec(e+fx)} dx}{315c^5}$$

$$= \frac{a^2x}{c^5} - \frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} - \frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{179a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} - \frac{2a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))} + \frac{(488a^2) \int \frac{\sec(e+fx)}{1-\sec(e+fx)} dx}{315c^5}$$

$$
= \frac{a^2 x}{c^5} - \frac{4a^2 \tan(e+fx)}{9c^5 f (1-\sec(e+fx))^5} - \frac{16a^2 \tan(e+fx)}{63c^5 f (1-\sec(e+fx))^4} - \frac{37a^2 \tan(e+fx)}{105c^5 f (1-\sec(e+fx))^3} - \frac{179a^2 \tan(e+fx)}{315c^5 f (1-\sec(e+fx))^2} - \frac{494a^2 \tan(e+fx)}{315c^5 f (1-\sec(e+fx))}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.87 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.68

$$
\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5} \, dx = \frac{a^2 \cot^9(e+fx) \left(441 + 35 \, \text{Hypergeometric2F1}\left(-\frac{9}{2}, 1, -\frac{7}{2}, -\tan^2(e+fx)\right) + 2205 \sec(e+fx) + 1323 \sec
$$

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]
[Out] (a^2*Cot[e + f*x]^9*(441 + 35*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x
]^2] + 2205*Sec[e + f*x] + 1323*Sec[e + f*x]^2 - 2205*Sec[e + f*x]^3 + 441*
Sec[e + f*x]^4 + 3969*Sec[e + f*x]^5 - 2223*Sec[e + f*x]^7 + 494*Sec[e + f*
x]^9))/(315*c^5*f)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

method	result
parallelrisch	$\frac{a^2 \left(35 \cot\left(\frac{fx}{2}+\frac{e}{2}\right)^9 - 180 \cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7 + 441 \cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5 - 840 \cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3 + 1260 fx + 2520 \cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{1260 c^5 f}$
derivativedivides	$\frac{a^2 \left(8 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right) + \frac{1}{9 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9} - \frac{4}{7 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7} + \frac{7}{5 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5} - \frac{8}{3 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3} + \frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4 f c^5}$
default	$\frac{a^2 \left(8 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right) + \frac{1}{9 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9} - \frac{4}{7 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7} + \frac{7}{5 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5} - \frac{8}{3 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3} + \frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4 f c^5}$
risch	$\frac{a^2 x}{c^5} + \frac{2 i a^2 \left(2205 \, e^{8 i (fx+e)} - 11655 \, e^{7 i (fx+e)} + 34335 \, e^{6 i (fx+e)} - 58905 \, e^{5 i (fx+e)} + 67599 \, e^{4 i (fx+e)} - 50001 \, e^{3 i (fx+e)} + 2}{315 f c^5 \left(e^{i (fx+e)} - 1\right)^9}$
norman	$\frac{\frac{a^2 x \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{c} - \frac{a^2}{36 c f} + \frac{43 a^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{252 c f} - \frac{69 a^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{140 c f} + \frac{61 a^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{60 c f} - \frac{8 a^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{3 c f} + \frac{2 a^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{c f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 - 1\right) c^4 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1260*a^2*(35*cot(1/2*f*x+1/2*e)^9-180*cot(1/2*f*x+1/2*e)^7+441*cot(1/2*f*
x+1/2*e)^5-840*cot(1/2*f*x+1/2*e)^3+1260*f*x+2520*cot(1/2*f*x+1/2*e))/c^5/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{1004\,a^2\cos(fx+e)^5-1811\,a^2\cos(fx+e)^4+797\,a^2\cos(fx+e)^3+1457\,a^2\cos(fx+e)^2-1661\,a^2\cos(\ldots}{315\,\left(c^5f\cos(fx+e)^4-4\,c^5f\cos(fx+\ldots\right.}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(1004*a^2*cos(f*x + e)^5 - 1811*a^2*cos(f*x + e)^4 + 797*a^2*cos(f*x
+ e)^3 + 1457*a^2*cos(f*x + e)^2 - 1661*a^2*cos(f*x + e) + 494*a^2 + 315*(a
^2*f*x*cos(f*x + e)^4 - 4*a^2*f*x*cos(f*x + e)^3 + 6*a^2*f*x*cos(f*x + e)^2
 - 4*a^2*f*x*cos(f*x + e) + a^2*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)^4 -
 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c
^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = -\frac{a^2\left(\int \frac{2\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx+\int \frac{\sec^2(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-\ldots}\right.}{c^5}$$

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*se
c(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**5 -
5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x
) - 1), x))/c**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(144) = 288.

Time = 0.30 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.04

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{a^2\left(\frac{10080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5} - \frac{\left(\frac{270\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1008\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2730\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{9765\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}\right) - \frac{2a^2\left(\cdots\right.}{}}{}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(a^2*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 2*a^2*(180*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f*x + e
) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^2*(18*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x
 + e)^9))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{1260\,(fx+e)a^2}{c^5} + \frac{2520\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-840\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+441\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-180\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^2}{c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}}{1260\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/1260*(1260*(f*x + e)*a^2/c^5 + (2520*a^2*tan(1/2*f*x + 1/2*e)^8 - 840*a^2
*tan(1/2*f*x + 1/2*e)^6 + 441*a^2*tan(1/2*f*x + 1/2*e)^4 - 180*a^2*tan(1/2*
f*x + 1/2*e)^2 + 35*a^2)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f
```

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2\left(\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^9}{36}-\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^7\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{7}+\frac{7\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{20}-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6}{3}+2\,\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right.}{c^5\,f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^5,x)
```

```
[Out] (a^2*(cos(e/2 + (f*x)/2)^9/36 + 2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^8 +
 sin(e/2 + (f*x)/2)^9*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2
)^6)/3 + (7*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^4)/20 - (cos(e/2 + (f*x
)/2)^7*sin(e/2 + (f*x)/2)^2)/7))/(c^5*f*sin(e/2 + (f*x)/2)^9)
```

3.11 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 188

$$\begin{aligned}\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx &= a^3c^5x-\frac{5a^3c^5\text{arctanh}(\sin(e+fx))}{8f}-\frac{a^3c^5\tan(e+fx)}{f}\\&\quad+\frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^3c^5\tan^3(e+fx)}{3f}-\frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{12f}\\&\quad-\frac{a^3c^5\tan^5(e+fx)}{5f}+\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{3f}-\frac{a^3c^5\tan^7(e+fx)}{7f}\end{aligned}$$

```
[Out] a^3*c^5*x-5/8*a^3*c^5*arctanh(sin(f*x+e))/f-a^3*c^5*tan(f*x+e)/f+5/8*a^3*c^
5*sec(f*x+e)*tan(f*x+e)/f+1/3*a^3*c^5*tan(f*x+e)^3/f-5/12*a^3*c^5*sec(f*x+e
)*tan(f*x+e)^3/f-1/5*a^3*c^5*tan(f*x+e)^5/f+1/3*a^3*c^5*sec(f*x+e)*tan(f*x+
e)^5/f-1/7*a^3*c^5*tan(f*x+e)^7/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3989, 3971, 3554, 8, 2691, 3855, 2687, 30}

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx=-\frac{5a^3c^5\text{arctanh}(\sin(e+fx))}{8f}-\frac{a^3c^5\tan^7(e+fx)}{7f}-\frac{a^3c^5\tan^5(e+fx)}{5f}+\frac{a^3c^5\tan^3(e+fx)}{3f}-\frac{a^3c^5\tan(e+fx)}{f}+\frac{a^3c^5\tan^5(e+fx)\sec(e+fx)}{3f}-\frac{5a^3c^5\tan^3(e+fx)\sec(e+fx)}{12f}+\frac{5a^3c^5\tan(e+fx)\sec(e+fx)}{8f}+a^3c^5x$$

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] a^3*c^5*x - (5*a^3*c^5*ArcTanh[Sin[e + f*x]])/(8*f) - (a^3*c^5*Tan[e + f*x]
)/f + (5*a^3*c^5*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (a^3*c^5*Tan[e + f*x]^3
)/(3*f) - (5*a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^3)/(12*f) - (a^3*c^5*Tan[e +
 f*x]^5)/(5*f) + (a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^5)/(3*f) - (a^3*c^5*Tan
[e + f*x]^7)/(7*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(a^3c^3\right)\int (c-c\sec(e+fx))^2\tan^6(e+fx)\,dx\right)\\
&= -\left(\left(a^3c^3\right)\int \left(c^2\tan^6(e+fx)-2c^2\sec(e+fx)\tan^6(e+fx)\right.\right.\\
&\qquad\left.\left.+c^2\sec^2(e+fx)\tan^6(e+fx)\right)\,dx\right)\\
&= -\left(\left(a^3c^5\right)\int \tan^6(e+fx)\,dx\right)-\left(a^3c^5\right)\int \sec^2(e+fx)\tan^6(e+fx)\,dx\\
&\quad+\left(2a^3c^5\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\\
&= -\frac{a^3c^5\tan^5(e+fx)}{5f}+\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{3f}+\left(a^3c^5\right)\int \tan^4(e+fx)\,dx\\
&\quad-\frac{1}{3}\left(5a^3c^5\right)\int \sec(e+fx)\tan^4(e+fx)\,dx-\frac{\left(a^3c^5\right)\operatorname{Subst}\left(\int x^6\,dx,x,\tan(e+fx)\right)}{f}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a^3c^5\tan^3(e+fx)}{3f} - \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{12f} - \frac{a^3c^5\tan^5(e+fx)}{5f} \\
&\quad + \frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{3f} - \frac{a^3c^5\tan^7(e+fx)}{7f} \\
&\quad - (a^3c^5)\int \tan^2(e+fx)\,dx + \frac{1}{4}(5a^3c^5)\int \sec(e+fx)\tan^2(e+fx)\,dx \\
&= -\frac{a^3c^5\tan(e+fx)}{f} + \frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{a^3c^5\tan^3(e+fx)}{3f} - \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{12f} \\
&\quad - \frac{a^3c^5\tan^5(e+fx)}{5f} + \frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{3f} \\
&\quad - \frac{a^3c^5\tan^7(e+fx)}{7f} - \frac{1}{8}(5a^3c^5)\int \sec(e+fx)\,dx + (a^3c^5)\int 1\,dx \\
&= a^3c^5x - \frac{5a^3c^5\text{arctanh}(\sin(e+fx))}{8f} - \frac{a^3c^5\tan(e+fx)}{f} \\
&\quad + \frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^3c^5\tan^3(e+fx)}{3f} \\
&\quad - \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{12f} - \frac{a^3c^5\tan^5(e+fx)}{5f} \\
&\quad + \frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{3f} - \frac{a^3c^5\tan^7(e+fx)}{7f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01

$$
\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx
$$
$$
= \frac{a^3c^5\sec^7(e+fx)\,(14700(e+fx)\cos(e+fx) - 16800\text{arctanh}(\sin(e+fx))\cos^7(e+fx) + 8820e\cos(3(e+
$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*c^5*Sec[e + f*x]^7*(14700*(e + f*x)*Cos[e + f*x] - 16800*ArcTanh[Sin[e
 + f*x]]*Cos[e + f*x]^7 + 8820*e*Cos[3*(e + f*x)] + 8820*f*x*Cos[3*(e + f*x
)] + 2940*e*Cos[5*(e + f*x)] + 2940*f*x*Cos[5*(e + f*x)] + 420*e*Cos[7*(e +
 f*x)] + 420*f*x*Cos[7*(e + f*x)] - 4200*Sin[e + f*x] + 2975*Sin[2*(e + f*x
)] - 2184*Sin[3*(e + f*x)] + 980*Sin[4*(e + f*x)] - 2408*Sin[5*(e + f*x)] +
 1155*Sin[6*(e + f*x)] - 584*Sin[7*(e + f*x)]))/(26880*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.15

method	result
risch	$a^3c^5x - \frac{ic^5a^3(1155\,\mathrm{e}^{13i(fx+e)}+1680\,\mathrm{e}^{12i(fx+e)}+980\,\mathrm{e}^{11i(fx+e)}+10080\,\mathrm{e}^{10i(fx+e)}+2975\,\mathrm{e}^{9i(fx+e)}+16240\,\mathrm{e}^{8i(fx+e)}}{420f(1+\mathrm{e}^{2}}$
parts	$a^3c^5x + \frac{c^5a^3\ln(\sec(fx+e)+\tan(fx+e))}{f} - \frac{2a^3c^5\tan(fx+e)}{f} + \frac{3a^3c^5\sec(fx+e)\tan(fx+e)}{f} - \frac{6c^5a^3\left(-\left(-\frac{\sec(}{}\right.\right.}{}$
parallelrisch	$-\frac{10a^3c^5\left(\frac{(-\cos(7fx+7e)-7\cos(5fx+5e)-21\cos(3fx+3e)-35\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{(\cos(7fx+7e)+7\cos(5fx+5e)+2}{}\right.}{}$
derivativedivides	$\frac{c^5a^3\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)+2c^5a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\right.}{}$
default	$\frac{c^5a^3\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)+2c^5a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\right.}{}$
norman	$\frac{a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}-a^3c^5x+7a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-21a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+35a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-35a^3c^5x\tan\left(\frac{fx}{2}\right.}{}$

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c^5*x-1/420*I*c^5*a^3*(1155*exp(13*I*(f*x+e))+1680*exp(12*I*(f*x+e))+98
0*exp(11*I*(f*x+e))+10080*exp(10*I*(f*x+e))+2975*exp(9*I*(f*x+e))+16240*exp
(8*I*(f*x+e))+24640*exp(6*I*(f*x+e))-2975*exp(5*I*(f*x+e))+14448*exp(4*I*(f
*x+e))-980*exp(3*I*(f*x+e))+6496*exp(2*I*(f*x+e))-1155*exp(I*(f*x+e))+1168)
/f/(1+exp(2*I*(f*x+e)))^7+5/8*c^5*a^3/f*ln(exp(I*(f*x+e))-I)-5/8*c^5*a^3/f*
ln(exp(I*(f*x+e))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$= \frac{1680\,a^3c^5fx\cos(fx+e)^7 - 525\,a^3c^5\cos(fx+e)^7\log(\sin(fx+e)+1) + 525\,a^3c^5\cos(fx+e)^7\log(-\mathrm{si}}{}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/1680*(1680*a^3*c^5*f*x*cos(f*x + e)^7 - 525*a^3*c^5*cos(f*x + e)^7*log(si
n(f*x + e) + 1) + 525*a^3*c^5*cos(f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(11
```

```
68*a^3*c^5*cos(f*x + e)^6 - 1155*a^3*c^5*cos(f*x + e)^5 - 256*a^3*c^5*cos(f
*x + e)^4 + 910*a^3*c^5*cos(f*x + e)^3 - 192*a^3*c^5*cos(f*x + e)^2 - 280*a
^3*c^5*cos(f*x + e) + 120*a^3*c^5)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$= -a^3c^5\bigg(\int(-1)\;dx+\int 2\sec{(e+fx)}\,dx+\int 2\sec^2{(e+fx)}\,dx + \int\left(-6\sec^3{(e+fx)}\right)\;dx+\int 6\sec^5{(e+fx)}\,dx+\int\left(-2\sec^6{(e+fx)}\right)\;dx + \int\left(-2\sec^7{(e+fx)}\right)\;dx+\int\sec^8{(e+fx)}\,dx\bigg)$$

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)
[Out] -a**3*c**5*(Integral(-1, x) + Integral(2*sec(e + f*x), x) + Integral(2*sec(
e + f*x)**2, x) + Integral(-6*sec(e + f*x)**3, x) + Integral(6*sec(e + f*x)
**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7, x)
 + Integral(sec(e + f*x)**8, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(174) = 348.

Time = 0.20 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.89

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx =$$
$$-\frac{48\left(5\tan{(fx+e)}^7+21\tan{(fx+e)}^5+35\tan{(fx+e)}^3+35\tan{(fx+e)}\right)a^3c^5-224\left(3\tan{(fx+e)}\right.}{}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="maxima")
[Out] -1/1680*(48*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*
tan(f*x + e))*a^3*c^5 - 224*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(
f*x + e))*a^3*c^5 - 1680*(f*x + e)*a^3*c^5 + 35*a^3*c^5*(2*(15*sin(f*x + e)
^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^
4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e)
- 1)) - 630*a^3*c^5*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4
- 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)
) + 2520*a^3*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) +
1) + log(sin(f*x + e) - 1)) + 3360*a^3*c^5*log(sec(f*x + e) + tan(f*x + e))
 + 3360*a^3*c^5*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$=\frac{840\,(fx+e)a^3c^5-525\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)+525\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(1365\,a^3\right.}{}}{}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/840*(840*(f*x + e)*a^3*c^5 - 525*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1
)) + 525*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(1365*a^3*c^5*tan(1
/2*f*x + 1/2*e)^13 - 9660*a^3*c^5*tan(1/2*f*x + 1/2*e)^11 + 29673*a^3*c^5*t
an(1/2*f*x + 1/2*e)^9 - 21216*a^3*c^5*tan(1/2*f*x + 1/2*e)^7 + 9863*a^3*c^5
*tan(1/2*f*x + 1/2*e)^5 - 2660*a^3*c^5*tan(1/2*f*x + 1/2*e)^3 + 315*a^3*c^5
*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^7)/f
```

Mupad [B] (verification not implemented)

Time = 15.85 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.38

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$=\frac{\frac{13\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{4}-23\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}+\frac{1413\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{20}-\frac{1768\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{35}+\frac{1409\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.}{60}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-21\right.}$$
$$+a^3\,c^5\,x-\frac{5\,a^3\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5,x)
```

```
[Out] ((1409*a^3*c^5*tan(e/2 + (f*x)/2)^5)/60 - (19*a^3*c^5*tan(e/2 + (f*x)/2)^3)
/3 - (1768*a^3*c^5*tan(e/2 + (f*x)/2)^7)/35 + (1413*a^3*c^5*tan(e/2 + (f*x)
/2)^9)/20 - 23*a^3*c^5*tan(e/2 + (f*x)/2)^11 + (13*a^3*c^5*tan(e/2 + (f*x)/
2)^13)/4 + (3*a^3*c^5*tan(e/2 + (f*x)/2))/4)/(f*(7*tan(e/2 + (f*x)/2)^2 - 2
1*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^8
+ 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^1
4 - 1)) + a^3*c^5*x - (5*a^3*c^5*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

3.12 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 132

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = a^3c^4x-\frac{5a^3c^4\text{arctanh}(\sin(e+fx))}{16f}-\frac{a^3(16c^4-5c^4\sec(e+fx))\tan(e+fx)}{16f}+\frac{a^3(8c^4-5c^4\sec(e+fx))\tan^3(e+fx)}{24f}-\frac{a^3(6c^4-5c^4\sec(e+fx))\tan^5(e+fx)}{30f}$$

```
[Out] a^3*c^4*x-5/16*a^3*c^4*arctanh(sin(f*x+e))/f-1/16*a^3*(16*c^4-5*c^4*sec(f*x
+e))*tan(f*x+e)/f+1/24*a^3*(8*c^4-5*c^4*sec(f*x+e))*tan(f*x+e)^3/f-1/30*a^3
*(6*c^4-5*c^4*sec(f*x+e))*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3989, 3966, 3855}

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = -\frac{5a^3c^4\text{arctanh}(\sin(e+fx))}{16f} - \frac{a^3\tan^5(e+fx)\left(6c^4-5c^4\sec(e+fx)\right)}{30f} + \frac{a^3\tan^3(e+fx)\left(8c^4-5c^4\sec(e+fx)\right)}{24f} - \frac{a^3\tan(e+fx)\left(16c^4-5c^4\sec(e+fx)\right)}{16f} + a^3c^4x$$

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] a^3*c^4*x - (5*a^3*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (a^3*(16*c^4 - 5*c^4
*Sec[e + f*x])*Tan[e + f*x])/(16*f) + (a^3*(8*c^4 - 5*c^4*Sec[e + f*x])*Tan
[e + f*x]^3)/(24*f) - (a^3*(6*c^4 - 5*c^4*Sec[e + f*x])*Tan[e + f*x]^5)/(30
*f)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\left(\left(a^3c^3\right)\int (c - c\sec(e+fx))\tan^6(e+fx)\,dx\right)$$
$$= -\frac{a^3(6c^4 - 5c^4\sec(e+fx))\tan^5(e+fx)}{30f} + \frac{1}{6}\left(a^3c^3\right)\int (6c - 5c\sec(e+fx))\tan^4(e+fx)\,dx$$
$$= \frac{a^3(8c^4 - 5c^4\sec(e+fx))\tan^3(e+fx)}{24f} - \frac{a^3(6c^4 - 5c^4\sec(e+fx))\tan^5(e+fx)}{30f} - \frac{1}{24}\left(a^3c^3\right)\int (24c - 15c\sec(e+fx))\tan^2(e+fx)\,dx$$
$$= -\frac{a^3(16c^4 - 5c^4\sec(e+fx))\tan(e+fx)}{16f} + \frac{a^3(8c^4 - 5c^4\sec(e+fx))\tan^3(e+fx)}{24f} - \frac{a^3(6c^4 - 5c^4\sec(e+fx))\tan^5(e+fx)}{30f} + \frac{1}{48}\left(a^3c^3\right)\int (48c - 15c\sec(e+fx))\,dx$$
$$= a^3c^4x - \frac{a^3(16c^4 - 5c^4\sec(e+fx))\tan(e+fx)}{16f} + \frac{a^3(8c^4 - 5c^4\sec(e+fx))\tan^3(e+fx)}{24f} - \frac{a^3(6c^4 - 5c^4\sec(e+fx))\tan^5(e+fx)}{30f} - \frac{1}{16}\left(5a^3c^4\right)\int \sec(e+fx)\,dx$$
$$= a^3c^4x - \frac{5a^3c^4\text{arctanh}(\sin(e+fx))}{16f} - \frac{a^3(16c^4 - 5c^4\sec(e+fx))\tan(e+fx)}{16f} + \frac{a^3(8c^4 - 5c^4\sec(e+fx))\tan^3(e+fx)}{24f} - \frac{a^3(6c^4 - 5c^4\sec(e+fx))\tan^5(e+fx)}{30f}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.25

$$\int (a + a\sec(e+fx))^3(c - c\sec(e+fx))^4\,dx$$
$$= \frac{a^3c^4\sec^6(e+fx)\left(1200e + 1200fx - 1200\text{arctanh}(\sin(e+fx))\cos^6(e+fx) + 1800(e+fx)\cos(2(e+fx)\right.}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*c^4*Sec[e + f*x]^6*(1200*e + 1200*f*x - 1200*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] + 72
0*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x)]
+ 450*Sin[e + f*x] - 600*Sin[2*(e + f*x)] - 25*Sin[3*(e + f*x)] - 384*Sin[4
*(e + f*x)] + 165*Sin[5*(e + f*x)] - 184*Sin[6*(e + f*x)]))/(3840*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.83 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.56

method	result
risch	$a^{3} c^{4} x - \frac{i c^{4} a^{3} \left(165 \,{\mathrm e}^{11 i (f x+e)}+720 \,{\mathrm e}^{10 i (f x+e)}-25 \,{\mathrm e}^{9 i (f x+e)}+2160 \,{\mathrm e}^{8 i (f x+e)}+450 \,{\mathrm e}^{7 i (f x+e)}+3680 \,{\mathrm e}^{6 i (f x+e)}-450 \,{\mathrm e}^{5 i}\right.}{120 f \left(1+{\mathrm e}^{2 i (f x+e)}\right)^{6}}$
parallelrisch	$-\frac{5 a^{3} \left(\frac{\left(-5-\frac{15 \cos(2 f x+2 e)}{2}-3 \cos(4 f x+4 e)-\frac{\cos(6 f x+6 e)}{2}\right) \ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{8}+\frac{\left(5+\frac{\cos(6 f x+6 e)}{2}+3 \cos(4 f x+4 e)+\frac{15 \cos(2 f}{2}\right.}{8}\right.}{}$
derivativedivides	$\frac{c^{4} a^{3} \left(-\left(-\frac{\sec(f x+e)^{5}}{6}-\frac{5 \sec(f x+e)^{3}}{24}-\frac{5 \sec(f x+e)}{16}\right) \tan(f x+e)+\frac{5 \ln(\sec(f x+e)+\tan(f x+e))}{16}\right)+c^{4} a^{3} \left(-\frac{8}{15}-\frac{\sec(f x+e)^{4}}{5}-\right.}{}$
default	$\frac{c^{4} a^{3} \left(-\left(-\frac{\sec(f x+e)^{5}}{6}-\frac{5 \sec(f x+e)^{3}}{24}-\frac{5 \sec(f x+e)}{16}\right) \tan(f x+e)+\frac{5 \ln(\sec(f x+e)+\tan(f x+e))}{16}\right)+c^{4} a^{3} \left(-\frac{8}{15}-\frac{\sec(f x+e)^{4}}{5}-\right.}{}$
parts	$a^{3} c^{4} x + \frac{c^{4} a^{3} \left(-\left(-\frac{\sec(f x+e)^{5}}{6}-\frac{5 \sec(f x+e)^{3}}{24}-\frac{5 \sec(f x+e)}{16}\right) \tan(f x+e)+\frac{5 \ln(\sec(f x+e)+\tan(f x+e))}{16}\right)}{f} - \frac{c^{4} a^{3} \ln(\sec}{}$
norman	$\frac{a^{3} c^{4} x+a^{3} c^{4} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{12}-6 a^{3} c^{4} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+15 a^{3} c^{4} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-20 a^{3} c^{4} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}+15 a^{3} c^{4} x \tan\left(\frac{f x}{2}\right.}{}$

[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)

[Out] a^3*c^4*x-1/120*I*c^4*a^3*(165*exp(11*I*(f*x+e))+720*exp(10*I*(f*x+e))-25*exp(9*I*(f*x+e))+2160*exp(8*I*(f*x+e))+450*exp(7*I*(f*x+e))+3680*exp(6*I*(f*x+e))-450*exp(5*I*(f*x+e))+3360*exp(4*I*(f*x+e))+25*exp(3*I*(f*x+e))+1488*exp(2*I*(f*x+e))-165*exp(I*(f*x+e))+368)/f/(1+exp(2*I*(f*x+e)))^6+5/16*c^4*a^3/f*ln(exp(I*(f*x+e))-I)-5/16*c^4*a^3/f*ln(exp(I*(f*x+e))+I)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

$$\int (a + a\sec(e + fx))^3 (c - c\sec(e + fx))^4 \, dx = \frac{480 \, a^{3} c^{4} f x \cos\left(f x + e\right)^{6} - 75 \, a^{3} c^{4} \cos\left(f x + e\right)^{6} \log\left(\sin\left(f x + e\right) + 1\right) + 75 \, a^{3} c^{4} \cos\left(f x + e\right)^{6} \log\left(-\sin\left(f\right.\right.}{}$$

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="fricas")

[Out] 1/480*(480*a^3*c^4*f*x*cos(f*x + e)^6 - 75*a^3*c^4*cos(f*x + e)^6*log(sin(f*x + e) + 1) + 75*a^3*c^4*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(368*a^

```
3*c^4*cos(f*x + e)^5 - 165*a^3*c^4*cos(f*x + e)^4 - 176*a^3*c^4*cos(f*x + e
)^3 + 130*a^3*c^4*cos(f*x + e)^2 + 48*a^3*c^4*cos(f*x + e) - 40*a^3*c^4)*si
n(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = a^3c^4\left(\int 1\,dx+\int(-\sec(e+fx))\,dx+\int\left(-3\sec^2(e+fx)\right)dx+\int 3\sec^3(e+fx)\,dx+\int 3\sec^4(e+fx)\,dx+\int\left(-3\sec^5(e+fx)\right)dx+\int\left(-\sec^6(e+fx)\right)dx+\int\sec^7(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*c**4*(Integral(1, x) + Integral(-sec(e + f*x), x) + Integral(-3*sec(e
+ f*x)**2, x) + Integral(3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)**4
, x) + Integral(-3*sec(e + f*x)**5, x) + Integral(-sec(e + f*x)**6, x) + In
tegral(sec(e + f*x)**7, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(124) = 248$.

Time = 0.20 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.53

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = -\frac{32\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^3c^4-480\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3}{}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(32*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^4
 - 480*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^4 - 480*(f*x + e)*a^3*c^4 +
5*a^3*c^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin
(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e
) + 1) + 15*log(sin(f*x + e) - 1)) - 90*a^3*c^4*(2*(3*sin(f*x + e)^3 - 5*si
n(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) +
1) + 3*log(sin(f*x + e) - 1)) + 360*a^3*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2
 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 480*a^3*c^4*log(se
c(f*x + e) + tan(f*x + e)) + 1440*a^3*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$= \frac{240\,(fx+e)a^3c^4 - 75\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 75\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(315\,a^3c^4\,\mathrm{t}\right.}{}}{}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)*a^3*c^4 - 75*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)
) + 75*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(315*a^3*c^4*tan(1/2*
f*x + 1/2*e)^11 - 1945*a^3*c^4*tan(1/2*f*x + 1/2*e)^9 + 5118*a^3*c^4*tan(1/
2*f*x + 1/2*e)^7 - 3138*a^3*c^4*tan(1/2*f*x + 1/2*e)^5 + 1095*a^3*c^4*tan(1
/2*f*x + 1/2*e)^3 - 165*a^3*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)
^2 - 1)^6)/f
```

Mupad [B] (verification not implemented)

Time = 15.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.72

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = a^3\,c^4\,x$$
$$+ \frac{\frac{21\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8} - \frac{389\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24} + \frac{853\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{20} - \frac{523\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{20} + \frac{73\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{8} -}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12} - 6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10} + 15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8} - 20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6} + 15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4} - 6\,\mathrm{ta}\right.}$$
$$- \frac{5\,a^3\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4,x)
```

```
[Out] a^3*c^4*x + ((73*a^3*c^4*tan(e/2 + (f*x)/2)^3)/8 - (523*a^3*c^4*tan(e/2 + (
f*x)/2)^5)/20 + (853*a^3*c^4*tan(e/2 + (f*x)/2)^7)/20 - (389*a^3*c^4*tan(e/
2 + (f*x)/2)^9)/24 + (21*a^3*c^4*tan(e/2 + (f*x)/2)^11)/8 - (11*a^3*c^4*tan
(e/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 -
20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10
 + tan(e/2 + (f*x)/2)^12 + 1)) - (5*a^3*c^4*atanh(tan(e/2 + (f*x)/2)))/(8*f
)
```

3.13 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 68

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx=a^3c^3x-\frac{a^3c^3\tan(e+fx)}{f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan^5(e+fx)}{5f}$$

```
[Out] a^3*c^3*x-a^3*c^3*tan(f*x+e)/f+1/3*a^3*c^3*tan(f*x+e)^3/f-1/5*a^3*c^3*tan(f
*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3554, 8}

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx=-\frac{a^3c^3\tan^5(e+fx)}{5f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan(e+fx)}{f}+a^3c^3x$$

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] a^3*c^3*x - (a^3*c^3*Tan[e + f*x])/f + (a^3*c^3*Tan[e + f*x]^3)/(3*f) - (a^
3*c^3*Tan[e + f*x]^5)/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(a^3c^3\right)\int \tan^6(e+fx)\,dx\right)\\
&= -\frac{a^3c^3\tan^5(e+fx)}{5f}+\left(a^3c^3\right)\int \tan^4(e+fx)\,dx\\
&= \frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan^5(e+fx)}{5f}-\left(a^3c^3\right)\int \tan^2(e+fx)\,dx\\
&= -\frac{a^3c^3\tan(e+fx)}{f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan^5(e+fx)}{5f}+\left(a^3c^3\right)\int 1\,dx\\
&= a^3c^3x-\frac{a^3c^3\tan(e+fx)}{f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -a^3c^3\left(-\frac{\arctan(\tan(e+fx))}{f}+\frac{\tan(e+fx)}{f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan^5(e+fx)}{5f}\right)$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -(a^3*c^3*(-(ArcTan[Tan[e + f*x]]/f) + Tan[e + f*x]/f - Tan[e + f*x]^3/(3*f
) + Tan[e + f*x]^5/(5*f)))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

method	result
risch	$a^3c^3x-\frac{2ic^3a^3\left(45\,\mathrm{e}^{8i(fx+e)}+90\,\mathrm{e}^{6i(fx+e)}+140\,\mathrm{e}^{4i(fx+e)}+70\,\mathrm{e}^{2i(fx+e)}+23\right)}{15f\left(1+\mathrm{e}^{2i(fx+e)}\right)^5}$
derivativedivides	$\frac{c^3a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-3c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-3c^3a^3\tan(fx+e)+c^3a^3(fx+e}{f}$
default	$\frac{c^3a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-3c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-3c^3a^3\tan(fx+e)+c^3a^3(fx+e}{f}$
parts	$a^3c^3x-\frac{3a^3c^3\tan(fx+e)}{f}-\frac{3c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{c^3a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx}{f}$
parallelrisch	$\frac{a^3c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}xf-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}xf+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}xf-\frac{32\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{3}-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}xf\right.}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
norman	$\frac{a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-a^3c^3x+5a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-10a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+10a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-5a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right.}$

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c^3*x-2/15*I*c^3*a^3*(45*exp(8*I*(f*x+e))+90*exp(6*I*(f*x+e))+140*exp(4
*I*(f*x+e))+70*exp(2*I*(f*x+e))+23)/f/(1+exp(2*I*(f*x+e)))^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$
$$=\frac{15\,a^3c^3fx\cos\left(fx+e\right)^5-\left(23\,a^3c^3\cos\left(fx+e\right)^4-11\,a^3c^3\cos\left(fx+e\right)^2+3\,a^3c^3\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^3*c^3*f*x*cos(f*x + e)^5 - (23*a^3*c^3*cos(f*x + e)^4 - 11*a^3*c
^3*cos(f*x + e)^2 + 3*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -a^3c^3\bigg(\int(-1)\;dx + \int 3\sec^2(e+fx)\,dx + \int\left(-3\sec^4(e+fx)\right)\;dx + \int \sec^6(e+fx)\,dx\bigg)$$

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*c**3*(Integral(-1, x) + Integral(3*sec(e + f*x)**2, x) + Integral(-3*
sec(e + f*x)**4, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -\frac{\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^3c^3-15\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c^3}{15\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^3 - 1
5*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 - 15*(f*x + e)*a^3*c^3 + 45*a^3
*c^3*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -\frac{3\,a^3c^3\tan(fx+e)^5-5\,a^3c^3\tan(fx+e)^3-15\,(fx+e)a^3c^3+15\,a^3c^3\tan(fx+e)}{15\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(3*a^3*c^3*tan(f*x + e)^5 - 5*a^3*c^3*tan(f*x + e)^3 - 15*(f*x + e)*a
^3*c^3 + 15*a^3*c^3*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 17.86 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.79

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = a^3\,c^3\,x + \frac{2\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9-\frac{32\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3}+\frac{356\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}-\frac{32\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+2\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^5}$$

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3,x)
```

```
[Out] a^3*c^3*x + ((356*a^3*c^3*tan(e/2 + (f*x)/2)^5)/15 - (32*a^3*c^3*tan(e/2 +
(f*x)/2)^3)/3 - (32*a^3*c^3*tan(e/2 + (f*x)/2)^7)/3 + 2*a^3*c^3*tan(e/2 + (
f*x)/2)^9 + 2*a^3*c^3*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^2 - 1)^5)
```

3.14 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 97

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = a^3c^2x+\frac{3a^3c^2\text{arctanh}(\sin(e+fx))}{8f} - \frac{c^2(8a^3+3a^3\sec(e+fx))\tan(e+fx)}{8f} + \frac{c^2(4a^3+3a^3\sec(e+fx))\tan^3(e+fx)}{12f}$$

```
[Out] a^3*c^2*x+3/8*a^3*c^2*arctanh(sin(f*x+e))/f-1/8*c^2*(8*a^3+3*a^3*sec(f*x+e)
)*tan(f*x+e)/f+1/12*c^2*(4*a^3+3*a^3*sec(f*x+e))*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3966, 3855}

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \frac{3a^3c^2\text{arctanh}(\sin(e+fx))}{8f} + \frac{c^2\tan^3(e+fx)\,(3a^3\sec(e+fx)+4a^3)}{12f} - \frac{c^2\tan(e+fx)\,(3a^3\sec(e+fx)+8a^3)}{8f} + a^3c^2x$$

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] a^3*c^2*x + (3*a^3*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (c^2*(8*a^3 + 3*a^3*S
ec[e + f*x])*Tan[e + f*x])/(8*f) + (c^2*(4*a^3 + 3*a^3*Sec[e + f*x])*Tan[e
+ f*x]^3)/(12*f)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int (a+a\sec(e+fx))\tan^4(e+fx)\,dx\\
&= \frac{c^2(4a^3+3a^3\sec(e+fx))\tan^3(e+fx)}{12f}-\frac{1}{4}\left(a^2c^2\right)\int (4a+3a\sec(e+fx))\tan^2(e+fx)\,dx\\
&= -\frac{c^2(8a^3+3a^3\sec(e+fx))\tan(e+fx)}{8f}\\
&\quad+\frac{c^2(4a^3+3a^3\sec(e+fx))\tan^3(e+fx)}{12f}+\frac{1}{8}\left(a^2c^2\right)\int (8a+3a\sec(e+fx))\,dx\\
&= a^3c^2x-\frac{c^2(8a^3+3a^3\sec(e+fx))\tan(e+fx)}{8f}\\
&\quad+\frac{c^2(4a^3+3a^3\sec(e+fx))\tan^3(e+fx)}{12f}+\frac{1}{8}\left(3a^3c^2\right)\int \sec(e+fx)\,dx\\
&= a^3c^2x+\frac{3a^3c^2\operatorname{arctanh}(\sin(e+fx))}{8f}-\frac{c^2(8a^3+3a^3\sec(e+fx))\tan(e+fx)}{8f}\\
&\quad+\frac{c^2(4a^3+3a^3\sec(e+fx))\tan^3(e+fx)}{12f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$=\frac{a^3c^2\sec^4(e+fx)\,(72e+72fx+72\mathrm{arctanh}(\sin(e+fx))\cos^4(e+fx)+96(e+fx)\cos(2(e+fx))+24e\cdots}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*Sec[e + f*x]^4*(72*e + 72*f*x + 72*ArcTanh[Sin[e + f*x]]*Cos[e + f
*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Cos[
4*(e + f*x)] + 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] - 30*Sin[3*(e + f*x)]
- 32*Sin[4*(e + f*x)]))/(192*f)
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37

method	result
parts	$a^3c^2x-\frac{c^2a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{c^2a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\mathrm{ta}\cdots}{8}\right.}{f}$
risch	$a^3c^2x+\frac{ic^2a^3\left(15\,\mathrm{e}^{7i(fx+e)}-48\,\mathrm{e}^{6i(fx+e)}-9\,\mathrm{e}^{5i(fx+e)}-96\,\mathrm{e}^{4i(fx+e)}+9\,\mathrm{e}^{3i(fx+e)}-80\,\mathrm{e}^{2i(fx+e)}-15\,\mathrm{e}^{i(fx+e)}-32\right)}{12f\left(1+\mathrm{e}^{2i(fx+e)}\right)^4}+\cdots$
derivativedivides	$\frac{c^2a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-c^2a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2c\cdots}{f}$
default	$\frac{c^2a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-c^2a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2c\cdots}{f}$
parallelrisch	$\frac{a^3c^2\left(-9(3+\cos(4fx+4e)+4\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-9(-\cos(4fx+4e)-4\cos(2fx+2e)-3)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.\cdots}{24f(3+\cos(4fx+\cdots}$
norman	$\frac{a^3c^2x+a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-4a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+6a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-4a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-\frac{11c^2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}\cdots}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c^2*x-c^2*a^3/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+c^2*a^3/f*(-(-1/4*se
c(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-2*c^2*
a^3/f*tan(f*x+e)-c^2*a^3/f*tan(f*x+e)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= \frac{48\,a^3c^2fx\cos\left(fx+e\right)^4+9\,a^3c^2\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-9\,a^3c^2\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+\right.\right.}{48\,f\cos\left(fx\right.}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a^3*c^2*f*x*cos(f*x + e)^4 + 9*a^3*c^2*cos(f*x + e)^4*log(sin(f*x
+ e) + 1) - 9*a^3*c^2*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^3*c^2
*cos(f*x + e)^3 + 15*a^3*c^2*cos(f*x + e)^2 - 8*a^3*c^2*cos(f*x + e) - 6*a^
3*c^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = a^3c^2\left(\int 1\,dx+\int \sec\left(e+fx\right)dx + \int \left(-2\sec^2\left(e+fx\right)\right)\,dx + \int \left(-2\sec^3\left(e+fx\right)\right)\,dx + \int \sec^4\left(e+fx\right)dx+\int \sec^5\left(e+fx\right)dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*c**2*(Integral(1, x) + Integral(sec(e + f*x), x) + Integral(-2*sec(e +
 f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4,
x) + Integral(sec(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(91) = 182.

Time = 0.20 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.09

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= \frac{16\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^3c^2+48\left(fx+e\right)a^3c^2-3\,a^3c^2\left(\frac{2\left(3\,\sin(fx+e)^3-5\,\sin(fx+e)\right)}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}-3\,\log\left(\sin\left(\right.\right.\right.}{}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 + 48*(f*x + e)*a^3*c^2 -
 3*a^3*c^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f
*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 24*a^
3*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(si
n(f*x + e) - 1)) + 48*a^3*c^2*log(sec(f*x + e) + tan(f*x + e)) - 96*a^3*c^2
*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= \frac{24\,(fx+e)a^3c^2 + 9\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) - 9\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(15\,a^3c^2\tan\left(\frac{1}{2}\right.\right.}{}}{24\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(f*x + e)*a^3*c^2 + 9*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) -
 9*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(15*a^3*c^2*tan(1/2*f*x +
 1/2*e)^7 - 71*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 137*a^3*c^2*tan(1/2*f*x + 1
/2*e)^3 - 33*a^3*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/
f
```

Mupad [B] (verification not implemented)

Time = 15.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= \frac{\frac{5\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4} - \frac{71\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12} + \frac{137\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12} - \frac{11\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8 - 4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6 + 6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 - 4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + 1\right)}$$
$$+ a^3\,c^2\,x + \frac{3\,a^3\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2,x)
```

```
[Out] ((137*a^3*c^2*tan(e/2 + (f*x)/2)^3)/12 - (71*a^3*c^2*tan(e/2 + (f*x)/2)^5)/
12 + (5*a^3*c^2*tan(e/2 + (f*x)/2)^7)/4 - (11*a^3*c^2*tan(e/2 + (f*x)/2))/4
)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2
)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + a^3*c^2*x + (3*a^3*c^2*atanh(tan(e/2 + (
f*x)/2)))/(4*f)
```

3.15 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 77

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = a^3cx+\frac{a^3c\operatorname{arctanh}(\sin(e+fx))}{f}-\frac{a^3c\tan(e+fx)}{f}-\frac{a^3c\sec(e+fx)\tan(e+fx)}{f}-\frac{a^3c\tan^3(e+fx)}{3f}$$

```
[Out] a^3*c*x+a^3*c*arctanh(sin(f*x+e))/f-a^3*c*tan(f*x+e)/f-a^3*c*sec(f*x+e)*tan
(f*x+e)/f-1/3*a^3*c*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3989, 3971, 3554, 8, 2691, 3855, 2687, 30}

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = \frac{a^3c\operatorname{arctanh}(\sin(e+fx))}{f}-\frac{a^3c\tan^3(e+fx)}{3f}-\frac{a^3c\tan(e+fx)}{f}-\frac{a^3c\tan(e+fx)\sec(e+fx)}{f}+a^3cx$$

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

[Out] a^3*c*x + (a^3*c*ArcTanh[Sin[e + f*x]])/f - (a^3*c*Tan[e + f*x])/f - (a^3*c*Sec[e + f*x]*Tan[e + f*x])/f - (a^3*c*Tan[e + f*x]^3)/(3*f)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
```

```
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left((ac)\int (a+a\sec(e+fx))^2\tan^2(e+fx)\,dx\right)\\
&= -\left((ac)\int \left(a^2\tan^2(e+fx)+2a^2\sec(e+fx)\tan^2(e+fx)+a^2\sec^2(e+fx)\tan^2(e+fx)\right)\,dx\right)\\
&= -\left((a^3c)\int \tan^2(e+fx)\,dx\right)-(a^3c)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\\
&\quad-(2a^3c)\int \sec(e+fx)\tan^2(e+fx)\,dx\\
&= -\frac{a^3c\tan(e+fx)}{f}-\frac{a^3c\sec(e+fx)\tan(e+fx)}{f}+(a^3c)\int 1\,dx\\
&\quad+(a^3c)\int \sec(e+fx)\,dx-\frac{(a^3c)\,\text{Subst}\left(\int x^2\,dx,x,\tan(e+fx)\right)}{f}\\
&= a^3cx+\frac{a^3c\,\text{arctanh}(\sin(e+fx))}{f}-\frac{a^3c\tan(e+fx)}{f}\\
&\quad-\frac{a^3c\sec(e+fx)\tan(e+fx)}{f}-\frac{a^3c\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$= \frac{a^3c\sec^3(e+fx)\left(9(e+fx)\cos(e+fx)+12\,\text{arctanh}(\sin(e+fx))\cos^3(e+fx)+3e\cos(3(e+fx))+3fx\right.}{12f}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*Sec[e + f*x]^3*(9*(e + f*x)*Cos[e + f*x] + 12*ArcTanh[Sin[e + f*x]]*
Cos[e + f*x]^3 + 3*e*Cos[3*(e + f*x)] + 3*f*x*Cos[3*(e + f*x)] - 6*Sin[e +
f*x] - 6*Sin[2*(e + f*x)] - 2*Sin[3*(e + f*x)]))/(12*f)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
parts	$a^{3}cx+\frac{a^{3}c\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a^{3}c\sec(fx+e)\tan(fx+e)}{f}+\frac{a^{3}c\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}$
derivativedivides	$\frac{a^{3}c\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)-2a^{3}c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2a^{3}c\ln(\sec(fx+e)+\tan(fx-}{f}$
default	$\frac{a^{3}c\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)-2a^{3}c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2a^{3}c\ln(\sec(fx+e)+\tan(fx-}{f}$
risch	$a^{3}cx+\frac{2ia^{3}c(3\,e^{5i(fx+e)}-6\,e^{2i(fx+e)}-3\,e^{i(fx+e)}-2)}{3f\left(1+e^{2i(fx+e)}\right)^{3}}+\frac{a^{3}c\ln(e^{i(fx+e)}+i)}{f}-\frac{a^{3}c\ln(e^{i(fx+e)}-i)}{f}$
parallelrisch	$-\frac{2\left(\frac{3(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}-\frac{9fx\cos(fx+e)}{2}-\frac{3j}{}\right.}{3f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{a^{3}cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-a^{3}cx+\frac{4a^{3}c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{4a^{3}c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3f}+3a^{3}cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-3a^{3}cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}+\frac{a^{3}c\ln(}{}$

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] a^3*c*x+a^3*c/f*ln(sec(f*x+e)+tan(f*x+e))-a^3*c*sec(f*x+e)*tan(f*x+e)/f+a^3
*c/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{6\,a^{3}cfx\cos\left(fx+e\right)^{3}+3\,a^{3}c\cos\left(fx+e\right)^{3}\log\left(\sin\left(fx+e\right)+1\right)-3\,a^{3}c\cos\left(fx+e\right)^{3}\log\left(-\sin\left(fx+e\right)\right.}{6\,f\cos\left(fx+e\right)^{3}}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c*f*x*cos(f*x + e)^3 + 3*a^3*c*cos(f*x + e)^3*log(sin(f*x + e) +
 1) - 3*a^3*c*cos(f*x + e)^3*log(-sin(f*x + e) + 1) - 2*(2*a^3*c*cos(f*x +
e)^2 + 3*a^3*c*cos(f*x + e) + a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -a^3c\left(\int(-1)\,dx+\int(-2\sec(e+fx))\,dx+\int 2\sec^3(e+fx)\,dx+\int\sec^4(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
[Out] -a**3*c*(Integral(-1, x) + Integral(-2*sec(e + f*x), x) + Integral(2*sec(e
+ f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx =$$
$$-\frac{2\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c-6(fx+e)a^3c-3a^3c\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log\cdots\right.}{6f}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="maxima")
[Out] -1/6*(2*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 6*(f*x + e)*a^3*c - 3*a^3
*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f
*x + e) - 1)) - 12*a^3*c*log(sec(f*x + e) + tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{3(fx+e)a^3c+3a^3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-3a^3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{4\left(a^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-\cdots\right.}{\left(\tan\left(\frac{1}{2}fx+\cdots\right.\right.}}{3f}$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] 1/3*(3*(f*x + e)*a^3*c + 3*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^3
*c*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 4*(a^3*c*tan(1/2*f*x + 1/2*e)^3 - 3
*a^3*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 15.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int (a + a\sec(e + fx))^3 (c - c\sec(e + fx))\,dx$$
$$= \frac{4\,a^3\,c\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right) - \frac{4\,a^3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}}{f\,\left(\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^6 - 3\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^4 + 3\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^2 - 1\right)} + a^3\,c\,x + \frac{2\,a^3\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)\right)}{f}$$

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)),x)
```

```
[Out] (4*a^3*c*tan(e/2 + (f*x)/2) - (4*a^3*c*tan(e/2 + (f*x)/2)^3)/3)/(f*(3*tan(e
/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1)) + a^3
*c*x + (2*a^3*c*atanh(tan(e/2 + (f*x)/2)))/f
```

3.16 $\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = \frac{a^3x}{c} - \frac{4a^3\text{arctanh}(\sin(e+fx))}{cf} + \frac{8a^3\cot(e+fx)}{cf} + \frac{8a^3\csc(e+fx)}{cf} - \frac{a^3\tan(e+fx)}{cf}$$

```
[Out] a^3*x/c-4*a^3*arctanh(sin(f*x+e))/c/f+8*a^3*cot(f*x+e)/c/f+8*a^3*csc(f*x+e)
/c/f-a^3*tan(f*x+e)/c/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {3989, 3971, 3554, 8, 2686, 3852, 2701, 327, 213, 2700, 14}

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{4a^3\text{arctanh}(\sin(e+fx))}{cf} - \frac{a^3\tan(e+fx)}{cf} + \frac{8a^3\cot(e+fx)}{cf} + \frac{8a^3\csc(e+fx)}{cf} + \frac{a^3x}{c}$$

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*x)/c - (4*a^3*ArcTanh[Sin[e + f*x]])/(c*f) + (8*a^3*Cot[e + f*x])/(c*f
) + (8*a^3*Csc[e + f*x])/(c*f) - (a^3*Tan[e + f*x])/(c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^4\,dx}{ac}$$

$$= -\frac{\int \left(a^4\cot^2(e+fx)+4a^4\cot(e+fx)\csc(e+fx)+6a^4\csc^2(e+fx)+4a^4\csc^2(e+fx)\sec(e+ ...$$

$$= -\frac{a^3\int \cot^2(e+fx)\,dx}{c}-\frac{a^3\int \csc^2(e+fx)\sec^2(e+fx)\,dx}{c} -\frac{(4a^3)\int \cot(e+fx)\csc(e+fx)\,dx}{c} -\frac{(4a^3)\int \csc^2(e+fx)\sec(e+fx)\,dx}{c}-\frac{(6a^3)\int \csc^2(e+fx)\,dx}{c}$$

$$= \frac{a^3\cot(e+fx)}{cf}+\frac{a^3\int 1\,dx}{c}-\frac{a^3\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(e+fx)\right)}{cf} +\frac{(4a^3)\,\text{Subst}(\int 1\,dx,x,\csc(e+fx))}{cf}+\frac{(4a^3)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(e+fx)\right)}{cf} +\frac{(6a^3)\,\text{Subst}(\int 1\,dx,x,\cot(e+fx))}{cf}$$

$$= \frac{a^3x}{c}+\frac{7a^3\cot(e+fx)}{cf}+\frac{8a^3\csc(e+fx)}{cf}-\frac{a^3\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx,x,\tan(e+fx)\right)}{cf} +\frac{(4a^3)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(e+fx)\right)}{cf}$$

$$= \frac{a^3x}{c} - \frac{4a^3\operatorname{arctanh}(\sin(e+fx))}{cf} + \frac{8a^3\cot(e+fx)}{cf} + \frac{8a^3\csc(e+fx)}{cf} - \frac{a^3\tan(e+fx)}{cf}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.39 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.53

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx =$$
$$-\frac{\tan(e+fx)\left(-8\sqrt{2}a^3\sqrt{c}\cos^6\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sec^4(e+f\ldots\right.}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/5*(Tan[e + f*x]*(-8*Sqrt[2]*a^3*Sqrt[c]*Cos[(e + f*x)/2]^6*Hypergeometri
c2F1[3/2, 5/2, 7/2, (1 + Sec[e + f*x])/2]*Sec[e + f*x]^4*Sin[(e + f*x)/2]^2
 + 5*a^(5/2)*(4*Sqrt[c]*(Sqrt[a]*Sqrt[1 - Sec[e + f*x]]*(1 + Sec[e + f*x])
+ ArcSin[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*Sec[e + f*x]*Sqrt[a*
(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^2) - ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2
)]/(Sqrt[a]*Sqrt[c])]*Sqrt[1 - Sec[e + f*x]]*Sqrt[-(a*c*Tan[e + f*x]^2)])))
/(c^(3/2)*f*(1 - Sec[e + f*x])^(3/2)*(1 + Sec[e + f*x]))
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{8a^3\left(\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}\right)}{fc}$
default	$\frac{8a^3\left(\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}\right)}{fc}$
parallelrisch	$\frac{a^3\left(fx\cos(fx+e)+4\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-4\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+9\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\cos(fx+e)-\cot\ldots\right.}{cf\cos(fx+e)}$
risch	$\frac{a^3x}{c}+\frac{2ia^3\left(8\,e^{2i(fx+e)}-e^{i(fx+e)}+9\right)}{fc\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}-1\right)}-\frac{4a^3\ln\left(e^{i(fx+e)}+i\right)}{cf}+\frac{4a^3\ln\left(e^{i(fx+e)}-i\right)}{cf}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{c}+\frac{8a^3}{cf}-\frac{18a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}+\frac{10a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}-\frac{2a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{4a^3\ln\left(\tan\left(\ldots\right.\right.}{cf}$

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

[Out] 8/f*a^3/c*(1/8/(tan(1/2*f*x+1/2*e)+1)-1/2*ln(tan(1/2*f*x+1/2*e)+1)+1/8/(tan(1/2*f*x+1/2*e)-1)+1/2*ln(tan(1/2*f*x+1/2*e)-1)+1/tan(1/2*f*x+1/2*e)+1/4*arctan(tan(1/2*f*x+1/2*e)))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.60

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = \frac{a^3fx\cos(fx+e)\sin(fx+e)-2\,a^3\cos(fx+e)\log(\sin(fx+e)+1)\sin(fx+e)+2\,a^3\cos(fx+e)\log}{cf\cos(fx+e)\sin(fx+e)}$$

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] (a^3*f*x*cos(f*x + e)*sin(f*x + e) - 2*a^3*cos(f*x + e)*log(sin(f*x + e) + 1)*sin(f*x + e) + 2*a^3*cos(f*x + e)*log(-sin(f*x + e) + 1)*sin(f*x + e) + 9*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x + e) - a^3)/(c*f*cos(f*x + e)*sin(f*x + e))

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{a^3\left(\int \frac{3\sec(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{3\sec^2(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{\sec^3(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{1}{\sec(e+fx)-1}\,dx\right)}{c}$$

[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)

[Out] -a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x) - 1), x))/c

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.51

$$\int \frac{(a + a\sec(e + fx))^3}{c - c\sec(e + fx)}\,dx = -\frac{a^3\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right) - a^3\left(\frac{2\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} + \frac{\cos(fx+e)+}{c\sin(fx+e)}\right.}{f}$$

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")

[Out] -(a^3*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x + e)/(cos(f*x + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c) - a^3*(2*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c + (cos(f*x + e) + 1)/(c*sin(f*x + e))) + 3*a^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - 3*a^3*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int \frac{(a + a\sec(e + fx))^3}{c - c\sec(e + fx)}\,dx = \frac{\frac{(fx+e)a^3}{c} - \frac{4a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{c} + \frac{4a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{c} + \frac{2\left(5a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-4a^3\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)c}}{f}$$

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")

[Out] ((f*x + e)*a^3/c - 4*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c + 4*a^3*log(abs(tan(1/2*f*x + 1/2*e) - 1))/c + 2*(5*a^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^3)/((tan(1/2*f*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e))*c))/f

Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = \frac{a^3\,x}{c} - \frac{10\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-8\,a^3}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)} - \frac{8\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

```
[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x)),x)
```

```
[Out] (a^3*x)/c - (10*a^3*tan(e/2 + (f*x)/2)^2 - 8*a^3)/(f*(c*tan(e/2 + (f*x)/2)
- c*tan(e/2 + (f*x)/2)^3)) - (8*a^3*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

3.17 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3x}{c^2}+\frac{a^3\text{arctanh}(\sin(e+fx))}{c^2f} - \frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

```
[Out] a^3*x/c^2+a^3*arctanh(sin(f*x+e))/c^2/f-8/3*a^3*tan(f*x+e)/c^2/f/(1-sec(f*x
+e))^2+4/3*a^3*tan(f*x+e)/c^2/f/(1-sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3988, 3862, 4004, 3879, 3881, 3882, 3884, 4083, 3855}

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3\text{arctanh}(\sin(e+fx))}{c^2f}+\frac{4a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))} - \frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^3x}{c^2}$$

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*x)/c^2 + (a^3*ArcTanh[Sin[e + f*x]])/(c^2*f) - (8*a^3*Tan[e + f*x])/(3
*c^2*f*(1 - Sec[e + f*x])^2) + (4*a^3*Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f
*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

```
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^2}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^2}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^2}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^2}\right)dx}{c^2}$$

$$= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2} + \frac{(3a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{(3a^3)\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}$$

$$= -\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{a^3\int \frac{-3-\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2} + \frac{a^3\int \frac{(-2-3\sec(e+fx))\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}+\frac{a^3\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{c^2}-\frac{(2a^3)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{c^2}$$

$$= \frac{a^3x}{c^2}-\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^3\tan(e+fx)}{c^2f(1-\sec(e+fx))} + \frac{a^3\int \sec(e+fx)\,dx}{c^2}+\frac{(4a^3)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}-\frac{(5a^3)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}$$

$$= \frac{a^3x}{c^2}+\frac{a^3\operatorname{arctanh}(\sin(e+fx))}{c^2f}-\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(88) = 176.

Time = 2.68 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.57

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{a^{5/2}\tan(e+fx)\left(4\sqrt{c}\left(\sqrt{a}\sqrt{1-\sec(e+fx)}(1+\sec(e+fx))^2+6\arcsin\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)\sec^2(e+fx\right.\right.}{3c^{5/2}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(a^(5/2)*Tan[e + f*x]*(4*Sqrt[c]*(Sqrt[a]*Sqrt[1 - Sec[e + f*x]]*(1 +
Sec[e + f*x])^2 + 6*ArcSin[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*Se
c[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^4) + 6*ArcTanh[Sqr
t[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[1 - Sec[e + f*x]]*Sec[e +
f*x]*Sin[(e + f*x)/2]^2*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(c^(5/2)*f*(1 - Sec[e
 + f*x])^(5/2)*(1 + Sec[e + f*x]))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{4a^3\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f\,c^2}$
default	$\frac{4a^3\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f\,c^2}$
parallelrisch	$\frac{a^3\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 fx+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-4\right)}{3f\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$
risch	$\frac{a^3x}{c^2}+\frac{8ia^3\left(3\,e^{2i(fx+e)}+1\right)}{3f\,c^2\left(e^{i(fx+e)}-1\right)^3}+\frac{a^3\ln\left(e^{i(fx+e)}+i\right)}{c^2f}-\frac{a^3\ln\left(e^{i(fx+e)}-i\right)}{c^2f}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{c}-\frac{4a^3}{3cf}+\frac{8a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3cf}-\frac{4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{3cf}-\frac{2a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{a^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{c^2f}$

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*a^3/c^2*(-1/3/tan(1/2*f*x+1/2*e)^3+1/2*arctan(tan(1/2*f*x+1/2*e))-1/4*l
n(tan(1/2*f*x+1/2*e)-1)+1/4*ln(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{8\,a^3\cos\left(fx+e\right)^2+16\,a^3\cos\left(fx+e\right)+8\,a^3+3\,(a^3\cos\left(fx+e\right)-a^3)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{6\,(c^2f\cos\left(fx+e\right)}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(8*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 8*a^3 + 3*(a^3*cos(f*x +
e) - a^3)*log(sin(f*x + e) + 1)*sin(f*x + e) - 3*(a^3*cos(f*x + e) - a^3)*l
og(-sin(f*x + e) + 1)*sin(f*x + e) + 6*(a^3*f*x*cos(f*x + e) - a^3*f*x)*sin
(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{}{\sec^2\left(e+fx\right)-}\right.}{c^2}$$

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Int
egral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral
(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(80) = 160.

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.11

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3\left(\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}+\frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right)+a^3\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2}-\right.}{6\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^3*(12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2 + (9*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) + a^
3*(6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(
f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - 3*a^3*(3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) + 3*a^3*(3*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)
)/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{\frac{3\,(fx+e)a^3}{c^2}+\frac{3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^2}-\frac{3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^2}-\frac{4\,a^3}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^3/c^2 + 3*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 3
*a^3*log(abs(tan(1/2*f*x + 1/2*e) - 1))/c^2 - 4*a^3/(c^2*tan(1/2*f*x + 1/2*
e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3\,x}{c^2}+\frac{a^3\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{4\,\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}\right)}{c^2\,f}$$

```
[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^2,x)
```

```
[Out] (a^3*x)/c^2 + (a^3*(2*atanh(tan(e/2 + (f*x)/2)) - (4*cot(e/2 + (f*x)/2)^3)/
3))/(c^2*f)
```

3.18 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$

Optimal result 191
Rubi [A] (verified) 191
Mathematica [C] (verified) 194
Maple [A] (verified) 194
Fricas [A] (verification not implemented) 195
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Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx=\frac{a^3x}{c^3}-\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{26a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

```
[Out] a^3*x/c^3-8/5*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^3+4/15*a^3*tan(f*x+e)/c^3
/f/(1-sec(f*x+e))^2-26/15*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882, 3884, 4085}

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx=-\frac{26a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{a^3x}{c^3}$$

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*x)/c^3 - (8*a^3*Tan[e + f*x])/(5*c^3*f*(1 - Sec[e + f*x])^3) + (4*a^3*
Tan[e + f*x])/(15*c^3*f*(1 - Sec[e + f*x])^2) - (26*a^3*Tan[e + f*x])/(15*c
^3*f*(1 - Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
```

```
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^3} + \frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^3} + \frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^3} + \frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^3}\right) dx}{c^3}$$

$$= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^3}\,dx}{c^3} + \frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3} + \frac{(3a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3} + \frac{(3a^3)\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3}$$

$$= -\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} - \frac{a^3\int \frac{-5-2\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3} + \frac{a^3\int \frac{(-3-5\sec(e+fx))\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3} + \frac{(6a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3} - \frac{(9a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3}$$

$$= -\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} + \frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} + \frac{a^3\int \frac{15+7\sec(e+fx)}{1-\sec(e+fx)}\,dx}{15c^3} + \frac{(2a^3)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{5c^3} + \frac{(7a^3)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{15c^3} - \frac{(3a^3)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{5c^3}$$

$$= \frac{a^3x}{c^3} - \frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} + \frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} - \frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))} + \frac{(22a^3)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{15c^3}$$

$$= \frac{a^3x}{c^3} - \frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} + \frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} - \frac{26a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{2a^3\cot^5\left(\frac{e}{2}+\frac{fx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5c^3f}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]
[Out] (2*a^3*Cot[e/2 + (f*x)/2]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e/2 + (f*
x)/2]^2])/(5*c^3*f)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.53

method	result
parallelrisch	$\frac{a^3\left(6\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5-10\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+15fx+30\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15c^3f}$
derivativedivides	$\frac{2a^3\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^3}$
default	$\frac{2a^3\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^3}$
risch	$\frac{a^3x}{c^3}+\frac{4ia^3\left(45\,e^{4i(fx+e)}-90\,e^{3i(fx+e)}+140\,e^{2i(fx+e)}-70\,e^{i(fx+e)}+23\right)}{15f\,c^3\left(e^{i(fx+e)}-1\right)^5}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{c}+\frac{2a^3}{5cf}-\frac{22a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{15cf}+\frac{56a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{15cf}-\frac{14a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3cf}+\frac{2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{cf}-}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 1/15*a^3*(6*cot(1/2*f*x+1/2*e)^5-10*cot(1/2*f*x+1/2*e)^3+15*f*x+30*cot(1/2*
f*x+1/2*e))/c^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{46\,a^3\cos\left(fx+e\right)^3-2\,a^3\cos\left(fx+e\right)^2-22\,a^3\cos\left(fx+e\right)+26\,a^3+15\,\left(a^3fx\cos\left(fx+e\right)^2-2\,a^3fx\cos\right.}{15\,\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(46*a^3*cos(f*x + e)^3 - 2*a^3*cos(f*x + e)^2 - 22*a^3*cos(f*x + e) +
26*a^3 + 15*(a^3*f*x*cos(f*x + e)^2 - 2*a^3*f*x*cos(f*x + e) + a^3*f*x)*sin
(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x +
 e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{a^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^3}{\sec^3\left(e+fx\right)-3\sec^2\left(\right.}\right.}{c^3}$$

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec
(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x
)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*
x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(90) = 180$.

Time = 0.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.76

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{a^3\left(\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right)+\frac{a^3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos}{c^3\sin(fx+e)^5}}{60\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(a^3*(120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3 - (20*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(c
os(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5)) + a^3*(10*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1
)^5/(c^3*sin(f*x + e)^5) - 3*a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f
*x + e)^5) - 9*a^3*(5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e
) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)a^3}{c^3}+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^3\right)}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*a^3/c^3 + 2*(15*a^3*tan(1/2*f*x + 1/2*e)^4 - 5*a^3*tan(1
/2*f*x + 1/2*e)^2 + 3*a^3)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 14.50 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{a^3\,x}{c^3}+\frac{\frac{2\,a^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{2\,a^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{c^3\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

```
[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^3,x)
```

```
[Out] (a^3*x)/c^3 + ((2*a^3*cos(e/2 + (f*x)/2)^5)/5 + 2*a^3*cos(e/2 + (f*x)/2)*si
n(e/2 + (f*x)/2)^4 - (2*a^3*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^2)/3)/(
c^3*f*sin(e/2 + (f*x)/2)^5)
```

3.19 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$

Optimal result 197
Rubi [A] (verified) 197
Mathematica [A] (verified) 200
Maple [A] (verified) 200
Fricas [A] (verification not implemented) 201
Sympy [F] 202
Maxima [B] (verification not implemented) 202
Giac [A] (verification not implemented) 203
Mupad [B] (verification not implemented) 203

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx=\frac{a^3x}{c^4}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{167a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))}$$

```
[Out] a^3*x/c^4-8/7*a^3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^4+4/35*a^3*tan(f*x+e)/c^4
/f/(1-sec(f*x+e))^3-62/105*a^3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^2-167/105*a^
3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882, 3884, 4085}

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx=-\frac{167a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{a^3x}{c^4}$$

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*x)/c^4 - (8*a^3*Tan[e + f*x])/(7*c^4*f*(1 - Sec[e + f*x])^4) + (4*a^3*
Tan[e + f*x])/(35*c^4*f*(1 - Sec[e + f*x])^3) - (62*a^3*Tan[e + f*x])/(105*
c^4*f*(1 - Sec[e + f*x])^2) - (167*a^3*Tan[e + f*x])/(105*c^4*f*(1 - Sec[e
+ f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^4}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^4}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^4}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^4}\right)dx}{c^4}\\
&= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}\\
&\quad+\frac{(3a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{(3a^3)\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}\\
&= -\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{a^3\int \frac{-7-3\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}+\frac{a^3\int \frac{(-4-7\sec(e+fx))\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}\\
&\quad+\frac{(9a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}-\frac{(12a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}\\
&= -\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}+\frac{a^3\int \frac{35+20\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^4}\\
&\quad+\frac{(13a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^4}+\frac{(18a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^4}-\frac{(24a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4} + \frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3} \\
&\quad - \frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2} - \frac{a^3\int\frac{-105-55\sec(e+fx)}{1-\sec(e+fx)}dx}{105c^4} \\
&\quad + \frac{(13a^3)\int\frac{\sec(e+fx)}{1-\sec(e+fx)}dx}{105c^4} + \frac{(6a^3)\int\frac{\sec(e+fx)}{1-\sec(e+fx)}dx}{35c^4} - \frac{(8a^3)\int\frac{\sec(e+fx)}{1-\sec(e+fx)}dx}{35c^4} \\
&= \frac{a^3x}{c^4} - \frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4} + \frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3} \\
&\quad - \frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2} - \frac{a^3\tan(e+fx)}{15c^4f(1-\sec(e+fx))} + \frac{(32a^3)\int\frac{\sec(e+fx)}{1-\sec(e+fx)}dx}{21c^4} \\
&= \frac{a^3x}{c^4} - \frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4} + \frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3} \\
&\quad - \frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2} - \frac{167a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$
\int\frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^{5/2}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(-337+276\sec(e+fx)+50\sec^2(e+fx)-396\sec^3(e+fx)+167\sec^4(e+fx)\right.}{105c^{9/2}f(-1+\sec(e+fx))^4(1+\mathrm{s}}
$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^(5/2)*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(-337 + 276*Sec[e + f*x] + 50*Sec[e
+ f*x]^2 - 396*Sec[e + f*x]^3 + 167*Sec[e + f*x]^4) - 840*ArcTanh[Sqrt[-(a*
c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f*x]^3*Sin[(e + f*x)/2]^6*Sqr
t[-(a*c*Tan[e + f*x]^2)]))/(105*c^(9/2)*f*(-1 + Sec[e + f*x])^4*(1 + Sec[e
+ f*x]))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

method	result
parallelrisch	$-\frac{a^3\left(15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7-42\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5+70\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3-105fx-210\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{105c^4f}$
derivativedivides	$\frac{a^3\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,c^4}$
default	$\frac{a^3\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,c^4}$
risch	$\frac{a^3x}{c^4}+\frac{2ia^3\left(735\,e^{6i(fx+e)}-2520\,e^{5i(fx+e)}+5635\,e^{4i(fx+e)}-6160\,e^{3i(fx+e)}+4557\,e^{2i(fx+e)}-1624\,e^{i(fx+e)}+337\right)}{105f\,c^4\left(e^{i(fx+e)}-1\right)^7}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{c}-\frac{a^3}{7cf}+\frac{24a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{35cf}-\frac{169a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{105cf}+\frac{56a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{15cf}-\frac{14a^3\tan\left(\frac{fx}{2}+\right.}{3cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*a^3*(15*cot(1/2*f*x+1/2*e)^7-42*cot(1/2*f*x+1/2*e)^5+70*cot(1/2*f*x+
1/2*e)^3-105*f*x-210*cot(1/2*f*x+1/2*e))/c^4/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{337\,a^3\cos(fx+e)^4-276\,a^3\cos(fx+e)^3-50\,a^3\cos(fx+e)^2+396\,a^3\cos(fx+e)-167\,a^3+105\,(a^3}{105\,(c^4f\cos(fx+e)^3-3\,c^4f\cos(fx+e)^2+3\,c^4f\,\mathrm{co}}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(337*a^3*cos(f*x + e)^4 - 276*a^3*cos(f*x + e)^3 - 50*a^3*cos(f*x + e
)^2 + 396*a^3*cos(f*x + e) - 167*a^3 + 105*(a^3*f*x*cos(f*x + e)^3 - 3*a^3*
f*x*cos(f*x + e)^2 + 3*a^3*f*x*cos(f*x + e) - a^3*f*x)*sin(f*x + e))/((c^4*
f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*s
in(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{a^3\left(\int \frac{3\sec(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{3\sec^2(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \dots\right.}{c^4}$$

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(
e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e +
 f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)
+ Integral(sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e +
 f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**4 - 4*sec(e
+ f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(117) = 234$.

Time = 0.34 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.88

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{5\,a^3\left(\frac{336\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4} + \frac{\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{77\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{315\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 3\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}\right) + \frac{3\,a^3\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{35\sin\dots}{(\cos(f\dots}\right.}{\dots}}{\dots}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(5*a^3*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x
 + e)^7)) + 3*a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(co
s(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 9*a^3*(21*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e) + 1)^7/(c^
4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{\frac{105\,(fx+e)a^3}{c^4}+\frac{210\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-70\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+42\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,a^3}{c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{105\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*(f*x + e)*a^3/c^4 + (210*a^3*tan(1/2*f*x + 1/2*e)^6 - 70*a^3*tan
(1/2*f*x + 1/2*e)^4 + 42*a^3*tan(1/2*f*x + 1/2*e)^2 - 15*a^3)/(c^4*tan(1/2*
f*x + 1/2*e)^7))/f
```

Mupad [B] (verification not implemented)

Time = 14.80 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^3\left(-\frac{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{7}+\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{5}-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3}+2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+(e-\right.}{c^4\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}$$

```
[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^4,x)
```

```
[Out] (a^3*(2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^6 - cos(e/2 + (f*x)/2)^7/7 +
sin(e/2 + (f*x)/2)^7*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)
^4)/3 + (2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2)/5))/(c^4*f*sin(e/2 +
(f*x)/2)^7)
```

3.20 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3x}{c^5} - \frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} + \frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} - \frac{496a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))}$$

```
[Out] a^3*x/c^5-8/9*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5+4/63*a^3*tan(f*x+e)/c^5
/f/(1-sec(f*x+e))^4-38/105*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^3-181/315*a^
3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^2-496/315*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x
+e))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882, 3884, 4085}

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{496a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))} - \frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} - \frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} + \frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} + \frac{a^3x}{c^5}$$

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*x)/c^5 - (8*a^3*Tan[e + f*x])/(9*c^5*f*(1 - Sec[e + f*x])^5) + (4*a^3*
Tan[e + f*x])/(63*c^5*f*(1 - Sec[e + f*x])^4) - (38*a^3*Tan[e + f*x])/(105*
c^5*f*(1 - Sec[e + f*x])^3) - (181*a^3*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e
+ f*x])^2) - (496*a^3*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^5}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^5}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^5}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^5}\right)dx}{c^5}\\
&= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}\\
&\quad+\frac{(3a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{(3a^3)\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}\\
&= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{a^3\int \frac{-9-4\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{9c^5}+\frac{a^3\int \frac{(-5-9\sec(e+fx))\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{9c^5}\\
&\quad+\frac{(4a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{3c^5}-\frac{(5a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{3c^5}
\end{aligned}$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} + \frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} + \frac{a^3\int\frac{63+39\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{63c^5} + \frac{a^3\int\frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{3c^5} + \frac{(4a^3)\int\frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^5} - \frac{(5a^3)\int\frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^5}$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} + \frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{a^3\int\frac{-315-204\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{315c^5} + \frac{(2a^3)\int\frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{15c^5} + \frac{(8a^3)\int\frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^5} - \frac{(2a^3)\int\frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{7c^5}$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} + \frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} + \frac{a^3\int\frac{945+519\sec(e+fx)}{1-\sec(e+fx)}\,dx}{945c^5} + \frac{(2a^3)\int\frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{45c^5} + \frac{(8a^3)\int\frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{105c^5} - \frac{(2a^3)\int\frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{21c^5}$$

$$= \frac{a^3x}{c^5} - \frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} + \frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} - \frac{8a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))} + \frac{(488a^3)\int\frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{315c^5}$$

$$= \frac{a^3x}{c^5} - \frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} + \frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} - \frac{496a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.85

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3\csc^9(e+fx)\left(48242+81711\cos(e+fx)+59544\cos(2(e+fx))+45591\cos(3(e+fx))+30744\cos(\ldots\right.}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]
```

[Out] (a^3*Csc[e + f*x]^9*(48242 + 81711*Cos[e + f*x] + 59544*Cos[2*(e + f*x)] + 45591*Cos[3*(e + f*x)] + 30744*Cos[4*(e + f*x)] + 13221*Cos[5*(e + f*x)] + 4200*Cos[6*(e + f*x)] + 1656*Cos[7*(e + f*x)] + 630*Cos[8*(e + f*x)] + 61*Cos[9*(e + f*x)] + 1120*Cos[e + f*x]^9*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2]))/(10080*c^5*f)

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

method	result
parallelrisch	$\frac{a^3\left(35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^9-135\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7+252\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5-420\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+630fx+1260\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{630c^5f}$
derivativedivides	$\frac{a^3\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2f\,c^5}$
default	$\frac{a^3\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2f\,c^5}$
risch	$\frac{a^3x}{c^5}+\frac{2ia^3\left(2520\,e^{8i(fx+e)}-12285\,e^{7i(fx+e)}+36645\,e^{6i(fx+e)}-61425\,e^{5i(fx+e)}+71001\,e^{4i(fx+e)}-51639\,e^{3i(fx+e)}+255\right.}{315f\,c^5\left(e^{i(fx+e)}-1\right)^9}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{c}+\frac{a^3}{18cf}-\frac{41a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{126cf}+\frac{557a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{630cf}-\frac{353a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{210cf}+\frac{56a^3\tan\left(\frac{fx}{2}+\right.}{15cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)

[Out] 1/630*a^3*(35*cot(1/2*f*x+1/2*e)^9-135*cot(1/2*f*x+1/2*e)^7+252*cot(1/2*f*x+1/2*e)^5-420*cot(1/2*f*x+1/2*e)^3+630*f*x+1260*cot(1/2*f*x+1/2*e))/c^5/f

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.29

$$\int\frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$$
$$=\frac{1051\,a^3\cos(fx+e)^5-1684\,a^3\cos(fx+e)^4+898\,a^3\cos(fx+e)^3+1468\,a^3\cos(fx+e)^2-1669\,a^3\cos(\ldots}{315\left(c^5f\cos(fx+e)^4-4\,c^5f\cos(fx+\ldots\right.}$$

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")

[Out] 1/315*(1051*a^3*cos(f*x + e)^5 - 1684*a^3*cos(f*x + e)^4 + 898*a^3*cos(f*x + e)^3 + 1468*a^3*cos(f*x + e)^2 - 1669*a^3*cos(f*x + e) + 496*a^3 + 315*(a

```
^3*f*x*cos(f*x + e)^4 - 4*a^3*f*x*cos(f*x + e)^3 + 6*a^3*f*x*cos(f*x + e)^2
 - 4*a^3*f*x*cos(f*x + e) + a^3*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)^4 -
 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c
^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{a^3\left(\int \frac{3\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx + \int \frac{3\sec^2(e+f}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)}\right.}{}$$

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
[Out] -a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*se
c(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*s
ec(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 -
10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(se
c(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2
 + 5*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**5 - 5*sec(e + f*x)**
4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x))/c**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(144) = 288$.

Time = 0.36 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.46

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3\left(\frac{10080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5} - \frac{\left(\frac{270\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{1008\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{2730\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{9765\sin(fx+e)^8}{(\cos(fx+e)+1)^8} - 35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}\right) - \frac{3a^3\left(\right.}{}}{}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
[Out] 1/5040*(a^3*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 3*a^3*(180*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f*x + e
) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 15*a^3*(18*sin(f
```

```
*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6
3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*
x + e)^9) - 7*a^3*(18*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{\frac{630\,(fx+e)a^3}{c^5}+\frac{1260\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-420\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+252\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-135\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^3}{c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}}{630\,f}$$

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/630*(630*(f*x + e)*a^3/c^5 + (1260*a^3*tan(1/2*f*x + 1/2*e)^8 - 420*a^3*t
an(1/2*f*x + 1/2*e)^6 + 252*a^3*tan(1/2*f*x + 1/2*e)^4 - 135*a^3*tan(1/2*f*
x + 1/2*e)^2 + 35*a^3)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f
```

Mupad [B] (verification not implemented)

Time = 16.94 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3\left(\frac{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{18}-\frac{3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{14}+\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{5}-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{3}+2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right.$$

$$c^5\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9$$

```
[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^5,x)
```

```
[Out] (a^3*(cos(e/2 + (f*x)/2)^9/18 + 2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^8 +
 sin(e/2 + (f*x)/2)^9*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2
)^6)/3 + (2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^4)/5 - (3*cos(e/2 + (f*
x)/2)^7*sin(e/2 + (f*x)/2)^2)/14))/(c^5*f*sin(e/2 + (f*x)/2)^9)
```

3.21 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{c^5x}{a^2} - \frac{47c^5\operatorname{arctanh}(\sin(e+fx))}{2a^2f} + \frac{13c^5\tan(e+fx)}{2a^2f} + \frac{112c^5\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{32c^5\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^5-c^5\sec(e+fx))\tan(e+fx)}{2a^2f}$$

```
[Out] c^5*x/a^2-47/2*c^5*arctanh(sin(f*x+e))/a^2/f+13/2*c^5*tan(f*x+e)/a^2/f+112/
3*c^5*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-32/3*c^5*tan(f*x+e)/f/(a+a*sec(f*x+e)
)^2+1/2*(c^5-c^5*sec(f*x+e))*tan(f*x+e)/a^2/f
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 26, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3989, 3971, 3554, 8, 2686, 2687, 30, 3852, 2701, 308, 213, 2700, 276, 294}

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = -\frac{47c^5\operatorname{arctanh}(\sin(e+fx))}{2a^2f} + \frac{7c^5\tan(e+fx)}{a^2f} - \frac{64c^5\cot^3(e+fx)}{3a^2f} - \frac{48c^5\cot(e+fx)}{a^2f} + \frac{131c^5\csc^3(e+fx)}{6a^2f} + \frac{33c^5\csc(e+fx)}{2a^2f} - \frac{c^5\csc^3(e+fx)\sec^2(e+fx)}{2a^2f} + \frac{c^5x}{a^2}$$

```
[In] Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^5*x)/a^2 - (47*c^5*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (48*c^5*Cot[e + f*
x])/(a^2*f) - (64*c^5*Cot[e + f*x]^3)/(3*a^2*f) + (33*c^5*Csc[e + f*x])/(2*
a^2*f) + (131*c^5*Csc[e + f*x]^3)/(6*a^2*f) - (c^5*Csc[e + f*x]^3*Sec[e + f
*x]^2)/(2*a^2*f) + (7*c^5*Tan[e + f*x])/(a^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \cot^4(e+fx)(c-c\sec(e+fx))^7\,dx}{a^2c^2}$$

$$= \frac{\int \left(c^7\cot^4(e+fx) - 7c^7\cot^3(e+fx)\csc(e+fx) + 21c^7\cot^2(e+fx)\csc^2(e+fx) - 35c^7\cot(e+\ldots\right.}{}$$

$$= \frac{c^5\int \cot^4(e+fx)\,dx}{a^2} - \frac{c^5\int \csc^4(e+fx)\sec^3(e+fx)\,dx}{a^2} - \frac{(7c^5)\int \cot^3(e+fx)\csc(e+fx)\,dx}{a^2} + \frac{(7c^5)\int \csc^4(e+fx)\sec^2(e+fx)\,dx}{a^2} + \frac{(21c^5)\int \cot^2(e+fx)\csc^2(e+fx)\,dx}{a^2} - \frac{(21c^5)\int \csc^4(e+fx)\sec(e+fx)\,dx}{a^2} - \frac{(35c^5)\int \cot(e+fx)\csc^3(e+fx)\,dx}{a^2} + \frac{(35c^5)\int \csc^4(e+fx)\,dx}{a^2}$$

$$= -\frac{c^5\cot^3(e+fx)}{3a^2f} - \frac{c^5\int \cot^2(e+fx)\,dx}{a^2} + \frac{c^5\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(e+fx)\right)}{a^2f} + \frac{(7c^5)\,\text{Subst}\left(\int (-1+x^2)\,dx, x, \csc(e+fx)\right)}{a^2f} + \frac{(7c^5)\,\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{(21c^5)\,\text{Subst}\left(\int x^2\,dx, x, -\cot(e+fx)\right)}{a^2f} + \frac{(21c^5)\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(e+fx)\right)}{a^2f} + \frac{(35c^5)\,\text{Subst}\left(\int x^2\,dx, x, \csc(e+fx)\right)}{a^2f} - \frac{(35c^5)\,\text{Subst}\left(\int (1+x^2)\,dx, x, \cot(e+fx)\right)}{a^2f}$$

$$= -\frac{34c^5\cot(e+fx)}{a^2f} - \frac{19c^5\cot^3(e+fx)}{a^2f} - \frac{7c^5\csc(e+fx)}{a^2f} + \frac{14c^5\csc^3(e+fx)}{a^2f} - \frac{c^5\csc^3(e+fx)\sec^2(e+fx)}{2a^2f} + \frac{c^5\int 1\,dx}{a^2} + \frac{(5c^5)\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(e+fx)\right)}{2a^2f} + \frac{(7c^5)\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx, x, \tan(e+fx)\right)}{a^2f} + \frac{(21c^5)\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(e+fx)\right)}{a^2f}$$

$$= \frac{c^5x}{a^2} - \frac{48c^5\cot(e+fx)}{a^2f} - \frac{64c^5\cot^3(e+fx)}{3a^2f} + \frac{14c^5\csc(e+fx)}{a^2f} + \frac{21c^5\csc^3(e+fx)}{a^2f} - \frac{c^5\csc^3(e+fx)\sec^2(e+fx)}{2a^2f} + \frac{7c^5\tan(e+fx)}{a^2f} + \frac{(5c^5)\operatorname{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(e+fx)\right)}{2a^2f} + \frac{(21c^5)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}dx, x, \csc(e+fx)\right)}{a^2f}$$

$$= \frac{c^5x}{a^2} - \frac{21c^5\operatorname{arctanh}(\sin(e+fx))}{a^2f} - \frac{48c^5\cot(e+fx)}{a^2f} - \frac{64c^5\cot^3(e+fx)}{3a^2f} + \frac{33c^5\csc(e+fx)}{2a^2f} + \frac{131c^5\csc^3(e+fx)}{6a^2f} - \frac{c^5\csc^3(e+fx)\sec^2(e+fx)}{2a^2f} + \frac{7c^5\tan(e+fx)}{a^2f} + \frac{(5c^5)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}dx, x, \csc(e+fx)\right)}{2a^2f}$$

$$= \frac{c^5x}{a^2} - \frac{47c^5\operatorname{arctanh}(\sin(e+fx))}{2a^2f} - \frac{48c^5\cot(e+fx)}{a^2f} - \frac{64c^5\cot^3(e+fx)}{3a^2f} + \frac{33c^5\csc(e+fx)}{2a^2f} + \frac{131c^5\csc^3(e+fx)}{6a^2f} - \frac{c^5\csc^3(e+fx)\sec^2(e+fx)}{2a^2f} + \frac{7c^5\tan(e+fx)}{a^2f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.18 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.58

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{c^{9/2}\tan(e+fx)\left(8\sqrt{a}\sqrt{c}+16\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\sec}\right.}{}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^(9/2)*Tan[e + f*x]*(8*Sqrt[a]*Sqrt[c] + 16*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hyper
geometric2F1[-7/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]]
 + 8*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-5/2, -3/2, -1/2, (1 + Sec[e
 + f*x])/2]*Sqrt[1 - Sec[e + f*x]] + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometr
ic2F1[-3/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] - 4*Sq
rt[a]*Sqrt[c]*Sec[e + f*x] - 4*Sqrt[a]*Sqrt[c]*Sec[e + f*x]^2 - 3*ArcTanh[S
qrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[-(a*c*Tan[e + f*x]^2)] -
 3*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f*x]*Sqrt
[-(a*c*Tan[e + f*x]^2)]))/(3*a^(5/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x
])^2)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{16c^5\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8}+\frac{1}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{15}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{47\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}{32}\right.}{f\,a^2}$
default	$\frac{16c^5\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8}+\frac{1}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{15}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{47\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}{32}\right.}{f\,a^2}$
parallelrisch	$\frac{125\left(\frac{94(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{125}+\frac{94(-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{125}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\cos(fx+e)+\frac{61\cos(2fx+}{75}\right.\right.}{4f\,a^2(1+\cos(2fx+2e))}$
risch	$\frac{c^5x}{a^2}+\frac{ic^5\left(99\,e^{6i(fx+e)}+435\,e^{5i(fx+e)}+484\,e^{4i(fx+e)}+930\,e^{3i(fx+e)}+575\,e^{2i(fx+e)}+507\,e^{i(fx+e)}+202\right)}{3f\,a^2\left(e^{i(fx+e)}+1\right)^3\left(1+e^{2i(fx+e)}\right)^2}-\frac{47c^5\ln(e^{i(}}{2a^2}$
norman	$\frac{\frac{c^5x}{a}+\frac{c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{a}+\frac{6c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{a}+\frac{45c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{491c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4a}$

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16/f*c^5/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)+1/8*arctan(tan(
1/2*f*x+1/2*e))+1/32/(tan(1/2*f*x+1/2*e)+1)^2-15/32/(tan(1/2*f*x+1/2*e)+1)-
47/32*ln(tan(1/2*f*x+1/2*e)+1)-1/32/(tan(1/2*f*x+1/2*e)-1)^2-15/32/(tan(1/2
*f*x+1/2*e)-1)+47/32*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.78

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{12\,c^5fx\cos\left(fx+e\right)^4+24\,c^5fx\cos\left(fx+e\right)^3+12\,c^5fx\cos\left(fx+e\right)^2-141\left(c^5\cos\left(fx+e\right)^4+2\,c^5\cos\left(fx\right.\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^5*f*x*cos(f*x + e)^4 + 24*c^5*f*x*cos(f*x + e)^3 + 12*c^5*f*x*co
s(f*x + e)^2 - 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x + e)^3 + c^5*cos(f*x
 + e)^2)*log(sin(f*x + e) + 1) + 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x +
e)^3 + c^5*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(202*c^5*cos(f*x + e)
^3 + 305*c^5*cos(f*x + e)^2 + 36*c^5*cos(f*x + e) - 3*c^5)*sin(f*x + e))/(a
^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^5\left(\int \frac{5\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int\left(-\frac{10\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx + \int \frac{10\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int\left(-\right.\right.}{a^2}$$

```
[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-5*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(
-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(127) = 254$.

Time = 0.35 (sec) , antiderivative size = 603, normalized size of antiderivative = 4.43

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{c^5\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}} + \frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2} + \frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right) + 5\,c}{}$$

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f
*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 5*c^5*((15*
sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2
- 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(co
s(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(co
s(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2)
- c^5*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 10*c^5*(3*sin
(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 5
*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)
^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{6\,(fx+e)c^5}{a^2} - \frac{141\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} + \frac{141\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} - \frac{6\left(15\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-13\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^2} + \frac{32\,(\ldots}{\ldots}}{6\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(f*x + e)*c^5/a^2 - 141*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 +
 141*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 - 6*(15*c^5*tan(1/2*f*x + 1
/2*e)^3 - 13*c^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^2)
 + 32*(a^4*c^5*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^5*tan(1/2*f*x + 1/2*e))/a^6
)/f
```

Mupad [B] (verification not implemented)

Time = 13.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{c^5\,x}{a^2} - \frac{15\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-13\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^2\right)} + \frac{32\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f} + \frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} - \frac{47\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

```
[In] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^2,x)
```

```
[Out] (c^5*x)/a^2 - (15*c^5*tan(e/2 + (f*x)/2)^3 - 13*c^5*tan(e/2 + (f*x)/2))/(f*
(a^2*tan(e/2 + (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) + (32*c^5*ta
n(e/2 + (f*x)/2))/(a^2*f) + (16*c^5*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) - (47*c
^5*atanh(tan(e/2 + (f*x)/2)))/(a^2*f)
```

3.22 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{c^4x}{a^2} - \frac{6c^4\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{16c^4\cot(e+fx)}{a^2f} - \frac{32c^4\cot^3(e+fx)}{3a^2f} + \frac{32c^4\csc^3(e+fx)}{3a^2f} + \frac{c^4\tan(e+fx)}{a^2f}$$

```
[Out] c^4*x/a^2-6*c^4*arctanh(sin(f*x+e))/a^2/f-16*c^4*cot(f*x+e)/a^2/f-32/3*c^4*
cot(f*x+e)^3/a^2/f+32/3*c^4*csc(f*x+e)^3/a^2/f+c^4*tan(f*x+e)/a^2/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3989, 3971, 3554, 8, 2686, 2687, 30, 3852, 2701, 308, 213, 2700, 276}

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{6c^4\text{arctanh}(\sin(e+fx))}{a^2f} + \frac{c^4\tan(e+fx)}{a^2f} - \frac{32c^4\cot^3(e+fx)}{3a^2f} - \frac{16c^4\cot(e+fx)}{a^2f} + \frac{32c^4\csc^3(e+fx)}{3a^2f} + \frac{c^4x}{a^2}$$

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^4*x)/a^2 - (6*c^4*ArcTanh[Sin[e + f*x]])/(a^2*f) - (16*c^4*Cot[e + f*x])
/(a^2*f) - (32*c^4*Cot[e + f*x]^3)/(3*a^2*f) + (32*c^4*Csc[e + f*x]^3)/(3*a
^2*f) + (c^4*Tan[e + f*x])/(a^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \cot^4(e+fx)(c-c\sec(e+fx))^6\,dx}{a^2c^2}$$

$$= \frac{\int \left(c^6\cot^4(e+fx) - 6c^6\cot^3(e+fx)\csc(e+fx) + 15c^6\cot^2(e+fx)\csc^2(e+fx) - 20c^6\cot(e+\right.}{}$$

$$= \frac{c^4\int \cot^4(e+fx)\,dx}{a^2} + \frac{c^4\int \csc^4(e+fx)\sec^2(e+fx)\,dx}{a^2} - \frac{(6c^4)\int \cot^3(e+fx)\csc(e+fx)\,dx}{a^2} - \frac{(6c^4)\int \csc^4(e+fx)\sec(e+fx)\,dx}{a^2} + \frac{(15c^4)\int \cot^2(e+fx)\csc^2(e+fx)\,dx}{a^2} + \frac{(15c^4)\int \csc^4(e+fx)\,dx}{a^2} - \frac{(20c^4)\int \cot(e+fx)\csc^3(e+fx)\,dx}{a^2}$$

$$= -\frac{c^4\cot^3(e+fx)}{3a^2f} - \frac{c^4\int\cot^2(e+fx)\,dx}{a^2} + \frac{c^4\text{Subst}\left(\int\frac{(1+x^2)^2}{x^4}\,dx,x,\tan(e+fx)\right)}{a^2f} + \frac{(6c^4)\,\text{Subst}\left(\int\frac{x^4}{-1+x^2}\,dx,x,\csc(e+fx)\right)}{a^2f} + \frac{(6c^4)\,\text{Subst}\left(\int(-1+x^2)\,dx,x,\csc(e+fx)\right)}{a^2f} + \frac{(15c^4)\,\text{Subst}\left(\int x^2\,dx,x,-\cot(e+fx)\right)}{a^2f} - \frac{(15c^4)\,\text{Subst}\left(\int(1+x^2)\,dx,x,\cot(e+fx)\right)}{a^2f} + \frac{(20c^4)\,\text{Subst}\left(\int x^2\,dx,x,\csc(e+fx)\right)}{a^2f}$$

$$= -\frac{14c^4\cot(e+fx)}{a^2f} - \frac{31c^4\cot^3(e+fx)}{3a^2f} - \frac{6c^4\csc(e+fx)}{a^2f} + \frac{26c^4\csc^3(e+fx)}{3a^2f} + \frac{c^4\int 1\,dx}{a^2} + \frac{c^4\text{Subst}\left(\int\left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx,x,\tan(e+fx)\right)}{a^2f} + \frac{(6c^4)\,\text{Subst}\left(\int\left(1+x^2+\frac{1}{-1+x^2}\right)dx,x,\csc(e+fx)\right)}{a^2f}$$

$$= \frac{c^4x}{a^2} - \frac{16c^4\cot(e+fx)}{a^2f} - \frac{32c^4\cot^3(e+fx)}{3a^2f} + \frac{32c^4\csc^3(e+fx)}{3a^2f} + \frac{c^4\tan(e+fx)}{a^2f} + \frac{(6c^4)\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\csc(e+fx)\right)}{a^2f}$$

$$= \frac{c^4x}{a^2} - \frac{6c^4\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{16c^4\cot(e+fx)}{a^2f} - \frac{32c^4\cot^3(e+fx)}{3a^2f} + \frac{32c^4\csc^3(e+fx)}{3a^2f} + \frac{c^4\tan(e+fx)}{a^2f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.44 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.92

$$\int\frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^{7/2}\tan(e+fx)\left(-8\sqrt{a}\sqrt{c}-8\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\text{se}}\right.}{}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(c^(7/2)*Tan[e + f*x]*(-8*Sqrt[a]*Sqrt[c] - 8*Sqrt[2]*Sqrt[a]*Sqrt[c]*
Hypergeometric2F1[-5/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e +
f*x]] - 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 +
Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] + 4*Sqrt[a]*Sqrt[c]*Sec[e + f*x] +
4*Sqrt[a]*Sqrt[c]*Sec[e + f*x]^2 + 3*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(S
qrt[a]*Sqrt[c])]*Sqrt[-(a*c*Tan[e + f*x]^2)] + 3*ArcTanh[Sqrt[-(a*c*Tan[e +
 f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(a^
(5/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{8c^4\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\right.}{f\,a^2}$
default	$\frac{8c^4\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\right.}{f\,a^2}$
parallelrisch	$\frac{6\left(\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+\frac{19\left(\cos(fx+e)+\frac{\cos(2fx+2e)}{4}+\frac{25}{76}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\right.}{18}\right.}{f\,a^2\cos(fx+e)}$
risch	$\frac{c^4x}{a^2}+\frac{2ic^4\left(51\,e^{3i(fx+e)}+25\,e^{2i(fx+e)}+57\,e^{i(fx+e)}+19\right)}{3f\,a^2\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}+1\right)^3}+\frac{6c^4\ln\left(e^{i(fx+e)}-i\right)}{a^2f}-\frac{6c^4\ln\left(e^{i(fx+e)}+i\right)}{a^2f}$
norman	$\frac{\frac{c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{a}-\frac{c^4x}{a}-\frac{10c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{76c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{18c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}+\frac{8c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3af}+\frac{3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{a}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3a}$

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/f*c^4/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+tan(1/2*f*x+1/2*e)-1/8/(tan(1/2*f*x+1
/2*e)+1)-3/4*ln(tan(1/2*f*x+1/2*e)+1)-1/8/(tan(1/2*f*x+1/2*e)-1)+3/4*ln(tan
(1/2*f*x+1/2*e)-1)+1/4*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(98) = 196$.

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.16

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{3\,c^4fx\cos\left(fx+e\right)^3+6\,c^4fx\cos\left(fx+e\right)^2+3\,c^4fx\cos\left(fx+e\right)-9\left(c^4\cos\left(fx+e\right)^3+2\,c^4\cos\left(fx+e\right)^2\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^4*f*x*cos(f*x + e)^3 + 6*c^4*f*x*cos(f*x + e)^2 + 3*c^4*f*x*cos(f*
x + e) - 9*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*l
og(sin(f*x + e) + 1) + 9*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*c
os(f*x + e))*log(-sin(f*x + e) + 1) + (19*c^4*cos(f*x + e)^2 + 38*c^4*cos(f
*x + e) + 3*c^4)*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)
^2 + a^2*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^2}\,dx = \frac{c^4\left(\int \left(-\frac{4\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx + \int \frac{6\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \left(-\frac{4\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx + \int \frac{\text{sec}}{}\right.}{a^2}$$

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(6*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-4*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(
1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(98) = 196$.

Time = 0.32 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.05

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^2}\,dx = \frac{c^4\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2} + \frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2} + \frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right) + 4\,c}{}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c^4*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 4*c^4*((9*sin(f*x + e
)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin
```

```
(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/a^2) - c^4*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) +
6*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/a^2 - 4*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{3(fx+e)c^4}{a^2} - \frac{18c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} + \frac{18c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} - \frac{6c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^2} + \frac{8\left(a^4c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3a\right.}{a^6}}{3f}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*c^4/a^2 - 18*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 +
18*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 - 6*c^4*tan(1/2*f*x + 1/2*e)/
((tan(1/2*f*x + 1/2*e)^2 - 1)*a^2) + 8*(a^4*c^4*tan(1/2*f*x + 1/2*e)^3 + 3*
a^4*c^4*tan(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{c^4x}{a^2} + \frac{8c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f} + \frac{8c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{3a^2f} - \frac{12c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f} - \frac{2c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-a^2\right)}$$

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^2,x)
```

```
[Out] (c^4*x)/a^2 + (8*c^4*tan(e/2 + (f*x)/2))/(a^2*f) + (8*c^4*tan(e/2 + (f*x)/2
)^3)/(3*a^2*f) - (12*c^4*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (2*c^4*tan(e/
2 + (f*x)/2))/(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2))
```

3.23 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{c^3x}{a^2}-\frac{c^3\operatorname{arctanh}(\sin(e+fx))}{a^2f} - \frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}+\frac{4c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

```
[Out] c^3*x/a^2-c^3*arctanh(sin(f*x+e))/a^2/f-8/3*c^3*tan(f*x+e)/a^2/f/(1+sec(f*x
+e))^2+4/3*c^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3988, 3862, 4004, 3879, 3881, 3882, 3884, 4083, 3855}

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^3\operatorname{arctanh}(\sin(e+fx))}{a^2f}+\frac{4c^3\tan(e+fx)}{3a^2f(\sec(e+fx)+1)} - \frac{8c^3\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{c^3x}{a^2}$$

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^3*x)/a^2 - (c^3*ArcTanh[Sin[e + f*x]])/(a^2*f) - (8*c^3*Tan[e + f*x])/(3
*a^2*f*(1 + Sec[e + f*x])^2) + (4*c^3*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f
*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

```
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{c^3}{(1+\sec(e+fx))^2} - \frac{3c^3\sec(e+fx)}{(1+\sec(e+fx))^2} + \frac{3c^3\sec^2(e+fx)}{(1+\sec(e+fx))^2} - \frac{c^3\sec^3(e+fx)}{(1+\sec(e+fx))^2}\right) dx}{a^2} \\
&= \frac{c^3\int \frac{1}{(1+\sec(e+fx))^2}\,dx}{a^2} - \frac{c^3\int \frac{\sec^3(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2} \\
&\quad - \frac{(3c^3)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2} + \frac{(3c^3)\int \frac{\sec^2(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2} \\
&= -\frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} - \frac{c^3\int \frac{-3+\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2} \\
&\quad - \frac{c^3\int \frac{\sec(e+fx)(-2+3\sec(e+fx))}{1+\sec(e+fx)}\,dx}{3a^2} - \frac{c^3\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{a^2} + \frac{(2c^3)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{a^2} \\
&= \frac{c^3x}{a^2} - \frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} + \frac{c^3\tan(e+fx)}{a^2f(1+\sec(e+fx))} \\
&\quad - \frac{c^3\int \sec(e+fx)\,dx}{a^2} - \frac{(4c^3)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2} + \frac{(5c^3)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2} \\
&= \frac{c^3x}{a^2} - \frac{c^3\operatorname{arctanh}(\sin(e+fx))}{a^2f} - \frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} + \frac{4c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.22

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{c^{5/2}\tan(e+fx)\left(4\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\sec(e+fx)}-\right.}{3a^{5/2}f(-1+\text{s}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^(5/2)*Tan[e + f*x]*(4*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-3/2, -3
/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] - 4*Sqrt[a]*Sqrt[c]*
(-2 + Sec[e + f*x] + Sec[e + f*x]^2) - 6*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)
]/(Sqrt[a]*Sqrt[c])]*Cos[(e + f*x)/2]^2*Sec[e + f*x]*Sqrt[-(a*c*Tan[e + f*x
]^2)]))/(3*a^(5/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{c^3\left(4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+3fx+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right)}{3a^2f}$
derivativedivides	$\frac{4c^3\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{f\,a^2}$
default	$\frac{4c^3\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{f\,a^2}$
risch	$\frac{c^3x}{a^2}-\frac{8ic^3\left(3\,e^{2i(fx+e)}+1\right)}{3f\,a^2\left(e^{i(fx+e)}+1\right)^3}+\frac{c^3\ln\left(e^{i(fx+e)}-i\right)}{a^2f}-\frac{c^3\ln\left(e^{i(fx+e)}+i\right)}{a^2f}$
norman	$\frac{\frac{c^3x}{a}+\frac{c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{a}+\frac{4c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{8c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3af}+\frac{4c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{3af}-\frac{2c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{a}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2a}+\frac{c^3\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{a^2f}$

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c^3*(4*tan(1/2*f*x+1/2*e)^3+3*f*x+3*ln(tan(1/2*f*x+1/2*e)-1)-3*ln(tan(1
/2*f*x+1/2*e)+1))/a^2/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(81) = 162.

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.04

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^2}\,dx$$
$$= \frac{6\,c^3fx\cos\left(fx+e\right)^2 + 12\,c^3fx\cos\left(fx+e\right) + 6\,c^3fx - 3\left(c^3\cos\left(fx+e\right)^2 + 2\,c^3\cos\left(fx+e\right) + c^3\right)\log\left(\sin\right.}{6\left(a^2f\cos\left(fx+\right.\right.}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*c^3*f*x*cos(f*x + e)^2 + 12*c^3*f*x*cos(f*x + e) + 6*c^3*f*x - 3*(c^
3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*log(sin(f*x + e) + 1) + 3*(c^3
*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*log(-sin(f*x + e) + 1) - 8*(c^3
*cos(f*x + e) - c^3)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x
+ e) + a^2*f)
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^2}\,dx =$$
$$-\frac{c^3\left(\int \frac{3\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int\left(-\frac{3\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx + \int \frac{\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int\left(-\right.\right.}{a^2}$$

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integr
al(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(81) = 162.

Time = 0.29 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.15

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^2}\,dx$$
$$= \frac{c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right) - c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{12\arctan\right.}{6f}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - c^3*((9*sin(f*x + e)/(cos(f*x + e) +
 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/a^2) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)/a^2 - 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1)
 - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{4\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}{a^2}+\frac{3\,(fx+e)c^3}{a^2}-\frac{3\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}+\frac{3\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}}{3\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(4*c^3*tan(1/2*f*x + 1/2*e)^3/a^2 + 3*(f*x + e)*c^3/a^2 - 3*c^3*log(abs
(tan(1/2*f*x + 1/2*e) + 1))/a^2 + 3*c^3*log(abs(tan(1/2*f*x + 1/2*e) - 1))/
a^2)/f
```

Mupad [B] (verification not implemented)

Time = 13.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.54

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{c^3\,x}{a^2}-\frac{c^3\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}\right)}{a^2\,f}$$

```
[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^2,x)
```

```
[Out] (c^3*x)/a^2 - (c^3*(2*atanh(tan(e/2 + (f*x)/2)) - (4*tan(e/2 + (f*x)/2)^3)/
3))/(a^2*f)
```

3.24 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx=\frac{c^2x}{a^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

```
[Out] c^2*x/a^2-4/3*c^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2-4/3*c^2*tan(f*x+e)/a^2/
f/(1+sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3988, 3862, 4004, 3879, 3881, 3882}

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx=-\frac{4c^2\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{4c^2\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{c^2x}{a^2}$$

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*x)/a^2 - (4*c^2*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2) - (4*c^2*
Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{c^2}{(1+\sec(e+fx))^2} - \frac{2c^2\sec(e+fx)}{(1+\sec(e+fx))^2} + \frac{c^2\sec^2(e+fx)}{(1+\sec(e+fx))^2}\right) dx}{a^2} \\
&= \frac{c^2 \int \frac{1}{(1+\sec(e+fx))^2} \, dx}{a^2} + \frac{c^2 \int \frac{\sec^2(e+fx)}{(1+\sec(e+fx))^2} \, dx}{a^2} - \frac{(2c^2) \int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2} \, dx}{a^2} \\
&= -\frac{4c^2\tan(e+fx)}{3a^2 f(1+\sec(e+fx))^2} - \frac{c^2 \int \frac{-3+\sec(e+fx)}{1+\sec(e+fx)} \, dx}{3a^2}
\end{aligned}$$

$$= \frac{c^2x}{a^2} - \frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} - \frac{(4c^2)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}$$
$$= \frac{c^2x}{a^2} - \frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} - \frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\left(\frac{2\arctan\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} - \frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f} + \frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}\right)}{a^2}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]
[Out] (c^2*((2*ArcTan[Tan[e/2 + (f*x)/2]])/f - (2*Tan[e/2 + (f*x)/2])/f + (2*Tan[
e/2 + (f*x)/2]^3)/(3*f)))/a^2
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{c^2\left(2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+3fx-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a^2f}$	41
derivativedivides	$\frac{2c^2\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$	47
default	$\frac{2c^2\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$	47
risch	$\frac{c^2x}{a^2}-\frac{8ic^2\left(3\,e^{2i(fx+e)}+3\,e^{i(fx+e)}+2\right)}{3f\,a^2\left(e^{i(fx+e)}+1\right)^3}$	59
norman	$\frac{\frac{c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{a}-\frac{c^2x}{a}+\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{8c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}+\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3af}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	113

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] 1/3*c^2*(2*tan(1/2*f*x+1/2*e)^3+3*f*x-6*tan(1/2*f*x+1/2*e))/a^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{3\,c^2fx\cos(fx+e)^2+6\,c^2fx\cos(fx+e)+3\,c^2fx-4\,(2\,c^2\cos(fx+e)+c^2)\sin(fx+e)}{3\,\left(a^2f\cos(fx+e)^2+2\,a^2f\cos(fx+e)+a^2f\right)}$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^2*f*x*cos(f*x + e)^2 + 6*c^2*f*x*cos(f*x + e) + 3*c^2*f*x - 4*(2*c
^2*cos(f*x + e) + c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*
x + e) + a^2*f)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\left(\int\left(-\frac{2\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\frac{1}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(63) = 126.

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.54

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{2\,c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+}{(\cos(fx+e}\right.}{a^2}}{6\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*si
n(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 +
2*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{3\,(fx+e)c^2}{a^2}+\frac{2\left(a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*c^2/a^2 + 2*(a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^2*ta
n(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,c^2\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{3\,f\,x}{2}\right)}{3\,a^2\,f}$$

```
[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^2,x)
```

```
[Out] (2*c^2*(tan(e/2 + (f*x)/2)^3 - 3*tan(e/2 + (f*x)/2) + (3*f*x)/2))/(3*a^2*f)
```

3.25 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx=\frac{cx}{a^2}-\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{5c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

```
[Out] c*x/a^2-2/3*c*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2-5/3*c*tan(f*x+e)/a^2/f/(1+s
ec(f*x+e))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3988, 3862, 4004, 3879, 3881}

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx=-\frac{5c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{cx}{a^2}$$

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c*x)/a^2 - (2*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2) - (5*c*Tan[e
+ f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{c}{(1+\sec(e+fx))^2}-\frac{c\sec(e+fx)}{(1+\sec(e+fx))^2}\right)dx}{a^2}\\
&= \frac{c\int \frac{1}{(1+\sec(e+fx))^2}\,dx}{a^2}-\frac{c\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2}\\
&= -\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{c\int \frac{-3+\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}-\frac{c\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}\\
&= \frac{cx}{a^2}-\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{(4c)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}\\
&= \frac{cx}{a^2}-\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{5c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(61) = 122.

Time = 0.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.28

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx = -\frac{\sqrt{c}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(-7+2\sec(e+fx)+5\sec^2(e+fx))+6\operatorname{arctanh}\left(\frac{\sqrt{-ac\tan^2(e+fx)}}{\sqrt{a}\sqrt{c}}\right)\cos^2\left(\frac{1}{2}(e+\right.\right.}{3a^{5/2}f(-1+\sec(e+fx))(1+\sec(e+fx))^2}$$

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(Sqrt[c]*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(-7 + 2*Sec[e + f*x] + 5*Sec[e
+ f*x]^2) + 6*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Cos[(e
 + f*x)/2]^2*Sec[e + f*x]*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(a^(5/2)*f*(-1 + Se
c[e + f*x])*(1 + Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+3fx-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a^2f}$	37
derivativedivides	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$	46
default	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$	46
norman	$\frac{\frac{cx}{a}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}}{a}$	50
risch	$\frac{cx}{a^2}-\frac{2ic\left(9\,e^{2i(fx+e)}+12\,e^{i(fx+e)}+7\right)}{3f\,a^2\left(e^{i(fx+e)}+1\right)^3}$	55

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c*(tan(1/2*f*x+1/2*e)^3+3*f*x-6*tan(1/2*f*x+1/2*e))/a^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \frac{3\,cfx\cos\left(fx+e\right)^2+6\,cfx\cos\left(fx+e\right)+3\,cfx-\left(7\,c\cos\left(fx+e\right)+5\,c\right)\sin\left(fx+e\right)}{3\,\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
[Out] 1/3*(3*c*f*x*cos(f*x + e)^2 + 6*c*f*x*cos(f*x + e) + 3*c*f*x - (7*c*cos(f*x
 + e) + 5*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a
^2*f)
```

Sympy [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\left(\int \frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \left(-\frac{1}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx\right)}{a^2}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
[Out] -c*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(57) = 114.

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6\,f}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
[Out] -1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + c*(3*sin(f*
x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^2}\, dx = \frac{\frac{3\,(fx+e)c}{a^2} + \frac{a^4 c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 6\,a^4 c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6}}{3\,f}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac")
[Out] 1/3*(3*(f*x + e)*c/a^2 + (a^4*c*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*c*tan(1/2*f*
x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 15.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^2}\, dx = \frac{c\,x}{a^2} - \frac{c\,\left(6\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right) - \tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^3\right)}{3\,a^2\,f}$$

```
[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^2,x)
[Out] (c*x)/a^2 - (c*(6*tan(e/2 + (f*x)/2) - tan(e/2 + (f*x)/2)^3))/(3*a^2*f)
```

3.26 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{x}{a^2c}+\frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} - \frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf}$$

```
[Out] x/a^2/c+1/3*cot(f*x+e)*(3-2*sec(f*x+e))/a^2/c/f-1/3*cot(f*x+e)^3*(1-sec(f*x
+e))/a^2/c/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3967, 8}

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} + \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf}+\frac{x}{a^2c}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] x/(a^2*c) + (Cot[e + f*x]*(3 - 2*Sec[e + f*x]))/(3*a^2*c*f) - (Cot[e + f*x]
^3*(1 - Sec[e + f*x]))/(3*a^2*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \cot^4(e+fx)(c-c\sec(e+fx))\,dx}{a^2c^2} \\
&= -\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} + \frac{\int \cot^2(e+fx)(-3c+2c\sec(e+fx))\,dx}{3a^2c^2} \\
&= \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} - \frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} + \frac{\int 3c\,dx}{3a^2c^2} \\
&= \frac{x}{a^2c} + \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} - \frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\cot^3(e+fx)\left(\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)-3\sec(e+fx)+2\sec^3(e+fx)\right)}{3a^2cf}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] -1/3*(Cot[e + f*x]^3*(Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2] - 3
*Sec[e + f*x] + 2*Sec[e + f*x]^3))/(a^2*c*f)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+12fx-12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{12a^{2}cf}$	50
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^{2}c}$	60
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^{2}c}$	60
risch	$\frac{x}{a^{2}c}-\frac{2i\left(3\,e^{3i(fx+e)}-5\,e^{i(fx+e)}-4\right)}{3f\,a^{2}c\left(e^{i(fx+e)}+1\right)^{3}\left(e^{i(fx+e)}-1\right)}$	72
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{ca}+\frac{1}{4acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{12acf}}{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	89

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(tan(1/2*f*x+1/2*e)^3+12*f*x-12*tan(1/2*f*x+1/2*e)+3*cot(1/2*f*x+1/2*e
))/a^2/c/f
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{4\cos(fx+e)^2+3\,(fx\cos(fx+e)+fx)\sin(fx+e)+\cos(fx+e)-2}{3\,(a^2cf\cos(fx+e)+a^2cf)\sin(fx+e)}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(f*x + e)^2 + 3*(f*x*cos(f*x + e) + f*x)*sin(f*x + e) + cos(f*x +
 e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{\sec^3(e+fx)+\sec^2(e+fx)-\sec(e+fx)-1}\,dx}{a^2c}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) - 1), x)/(a**
2*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2c}-\frac{24\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c}-\frac{3(\cos(fx+e)+1)}{a^2c\sin(fx+e)}}{12f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*((12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/(a^2*c) - 24*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c) - 3*(co
s(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\frac{12(fx+e)}{a^2c}+\frac{3}{a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-12a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^3}}{12f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(f*x + e)/(a^2*c) + 3/(a^2*c*tan(1/2*f*x + 1/2*e)) + (a^4*c^2*tan(
1/2*f*x + 1/2*e)^3 - 12*a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f
```

Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{x}{a^2\,c}+\frac{\frac{4\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3}-\frac{7\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{6}+\frac{1}{12}}{a^2\,c\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

```
[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))),x)
[Out] x/(a^2*c) + ((4*cos(e/2 + (f*x)/2)^4)/3 - (7*cos(e/2 + (f*x)/2)^2)/6 + 1/12
)/(a^2*c*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2))
```

3.27 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 46

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx=\frac{x}{a^2c^2}+\frac{\cot(e+fx)}{a^2c^2f}-\frac{\cot^3(e+fx)}{3a^2c^2f}$$

```
[Out] x/a^2/c^2+cot(f*x+e)/a^2/c^2/f-1/3*cot(f*x+e)^3/a^2/c^2/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3554, 8}

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx=-\frac{\cot^3(e+fx)}{3a^2c^2f}+\frac{\cot(e+fx)}{a^2c^2f}+\frac{x}{a^2c^2}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
[Out] x/(a^2*c^2) + Cot[e + f*x]/(a^2*c^2*f) - Cot[e + f*x]^3/(3*a^2*c^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \cot^4(e+fx)\,dx}{a^2c^2}\\ &= -\frac{\cot^3(e+fx)}{3a^2c^2f} - \frac{\int \cot^2(e+fx)\,dx}{a^2c^2}\\ &= \frac{\cot(e+fx)}{a^2c^2f} - \frac{\cot^3(e+fx)}{3a^2c^2f} + \frac{\int 1\,dx}{a^2c^2}\\ &= \frac{x}{a^2c^2} + \frac{\cot(e+fx)}{a^2c^2f} - \frac{\cot^3(e+fx)}{3a^2c^2f}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = -\frac{\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3a^2c^2f}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
[Out] -1/3*(Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(a^
2*c^2*f)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)-\frac{\pi}{2}+\operatorname{arccot}(\cot(fx+e))}{a^2c^2f}$	38
parallelrisch	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+24fx-15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{24f\,a^2c^2}$	63
risch	$\frac{x}{a^2c^2}+\frac{4i\left(3\,e^{4i(fx+e)}-3\,e^{2i(fx+e)}+2\right)}{3f\,a^2c^2\left(e^{i(fx+e)}+1\right)^3\left(e^{i(fx+e)}-1\right)^3}$	72
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{ca}-\frac{1}{24acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{8acf}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{24acf}}{ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	116

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] 1/a^2/c^2/f*(-1/3*cot(f*x+e)^3+cot(f*x+e)-1/2*Pi+arccot(cot(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{4\,\cos\left(fx+e\right)^3+3\left(fx\cos\left(fx+e\right)^2-fx\right)\sin\left(fx+e\right)-3\,\cos\left(fx+e\right)}{3\left(a^2c^2f\,\cos\left(fx+e\right)^2-a^2c^2f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
[Out] 1/3*(4*cos(f*x + e)^3 + 3*(f*x*cos(f*x + e)^2 - f*x)*sin(f*x + e) - 3*cos(f
*x + e))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{\sec^4(e+fx)-2\sec^2(e+fx)+1}\,dx}{a^2c^2}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
[Out] Integral(1/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\frac{3\,(fx+e)}{a^2c^2} + \frac{3\,\tan(fx+e)^2-1}{a^2c^2\tan(fx+e)^3}}{3\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)/(a^2*c^2) + (3*tan(f*x + e)^2 - 1)/(a^2*c^2*tan(f*x + e)^3
))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(44) = 88.

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\frac{24\,(fx+e)}{a^2c^2} + \frac{15\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1}{a^2c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3} + \frac{a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6c^6}}{24\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(f*x + e)/(a^2*c^2) + (15*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^2*c^2*tan
(1/2*f*x + 1/2*e)^3) + (a^4*c^4*tan(1/2*f*x + 1/2*e)^3 - 15*a^4*c^4*tan(1/2
*f*x + 1/2*e))/(a^6*c^6))/f
```

Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = -\frac{\cos\left(3\,e+3\,f\,x\right)+\frac{3\,\sin(3\,e+3\,f\,x)\,(e+f\,x)}{4}-\frac{9\,\sin(e+f\,x)\,(e+f\,x)}{4}}{3\,a^2\,c^2\,f\,\sin\left(e+f\,x\right)^3}$$

```
[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2),x)
```

```
[Out] -(cos(3*e + 3*f*x) + (3*sin(3*e + 3*f*x)*(e + f*x))/4 - (9*sin(e + f*x)*(e
+ f*x))/4)/(3*a^2*c^2*f*sin(e + f*x)^3)
```

3.28 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3} dx$

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Optimal result

Integrand size = 26, antiderivative size = 98

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{x}{a^2c^3}+\frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f}+\frac{\cot(e+fx)(15+8\sec(e+fx))}{15a^2c^3f}$$

```
[Out] x/a^2/c^3+1/5*cot(f*x+e)^5*(1+sec(f*x+e))/a^2/c^3/f-1/15*cot(f*x+e)^3*(5+4*
sec(f*x+e))/a^2/c^3/f+1/15*cot(f*x+e)*(15+8*sec(f*x+e))/a^2/c^3/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3967, 8}

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\cot^5(e+fx)(\sec(e+fx)+1)}{5a^2c^3f}-\frac{\cot^3(e+fx)(4\sec(e+fx)+5)}{15a^2c^3f} +\frac{\cot(e+fx)(8\sec(e+fx)+15)}{15a^2c^3f}+\frac{x}{a^2c^3}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] x/(a^2*c^3) + (Cot[e + f*x]^5*(1 + Sec[e + f*x]))/(5*a^2*c^3*f) - (Cot[e +
f*x]^3*(5 + 4*Sec[e + f*x]))/(15*a^2*c^3*f) + (Cot[e + f*x]*(15 + 8*Sec[e +
 f*x]))/(15*a^2*c^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))\,dx}{a^3c^3} \\
&= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\int \cot^4(e+fx)(-5a-4a\sec(e+fx))\,dx}{5a^3c^3} \\
&= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f} \\
&\quad - \frac{\int \cot^2(e+fx)(15a+8a\sec(e+fx))\,dx}{15a^3c^3} \\
&= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f} \\
&\quad + \frac{\cot(e+fx)(15+8\sec(e+fx))}{15a^2c^3f} - \frac{\int -15a\,dx}{15a^3c^3} \\
&= \frac{x}{a^2c^3} + \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} \\
&\quad - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f} + \frac{\cot(e+fx)(15+8\sec(e+fx))}{15a^2c^3f}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{\cot^5(e+fx)\left(3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)+15\sec(e+fx)-20\sec^3(e+fx)+8\sec^5(e+fx)\right)}{15a^2c^3f}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (Cot[e + f*x]^5*(3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2] + 15*S
ec[e + f*x] - 20*Sec[e + f*x]^3 + 8*Sec[e + f*x]^5))/(15*a^2*c^3*f)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result	size
parallelrisch	$\frac{3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5-30\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+240fx+240\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-90\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{240f\,a^2c^3}$	78
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{16}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16f\,a^2c^3}$	88
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{16}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16f\,a^2c^3}$	88
risch	$\frac{x}{a^2c^3}+\frac{2i\left(15\,e^{7i(fx+e)}+15\,e^{6i(fx+e)}-65\,e^{5i(fx+e)}+25\,e^{4i(fx+e)}+73\,e^{3i(fx+e)}-31\,e^{2i(fx+e)}-31\,e^{i(fx+e)}+23\right)}{15f\,a^2c^3\left(e^{i(fx+e)}-1\right)^5\left(e^{i(fx+e)}+1\right)^3}$	12
norman	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{acf}+\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{ca}+\frac{1}{80acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{8acf}-\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{48acf}}{a\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	13

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*(3*cot(1/2*f*x+1/2*e)^5-30*cot(1/2*f*x+1/2*e)^3+5*tan(1/2*f*x+1/2*e)^
3+240*f*x+240*cot(1/2*f*x+1/2*e)-90*tan(1/2*f*x+1/2*e))/f/a^2/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{23\cos(fx+e)^4 - 8\cos(fx+e)^3 - 27\cos(fx+e)^2 + 15\left(fx\cos(fx+e)^3 - fx\cos(fx+e)^2 - fx\cos(\right.}{15\left(a^2c^3f\cos(fx+e)^3 - a^2c^3f\cos(fx+e)^2 - a^2c^3f\cos(fx+e) + a^2c^3\right.}$$

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] 1/15*(23*cos(f*x + e)^4 - 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*cos(f*x + e)^3 - f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) + f*x)*sin(f*x + e) + 7*cos(f*x + e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{\sec^5(e+fx)-\sec^4(e+fx)-2\sec^3(e+fx)+2\sec^2(e+fx)+\sec(e+fx)-1}\,dx}{a^2c^3}$$

[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)

[Out] -Integral(1/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = -\frac{\frac{5\left(\frac{18\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^3} - \frac{480\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^3} + \frac{3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{80\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{a^2c^3\sin(fx+e)^5}}{240\,f}$$

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] -1/240*(5*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2*c^3) - 480*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c^3) + 3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 80*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x + e)^5))/f

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\frac{240\,(fx+e)}{a^2c^3} + \frac{3\left(80\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-10\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)}{a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5} + \frac{5\left(a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-18\,a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6c^9}}{240\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)/(a^2*c^3) + 3*(80*tan(1/2*f*x + 1/2*e)^4 - 10*tan(1/2*
f*x + 1/2*e)^2 + 1)/(a^2*c^3*tan(1/2*f*x + 1/2*e)^5) + 5*(a^4*c^6*tan(1/2*f
*x + 1/2*e)^3 - 18*a^4*c^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9))/f
```

Mupad [B] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.64

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-90\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+240\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-3}{240\,a^2\,c^3\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f}{2}\right.}$$

```
[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (3*cos(e/2 + (f*x)/2)^8 + 5*sin(e/2 + (f*x)/2)^8 - 90*cos(e/2 + (f*x)/2)^2*
sin(e/2 + (f*x)/2)^6 + 240*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^4 - 30*c
os(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^2 + 240*cos(e/2 + (f*x)/2)^3*sin(e/2
 + (f*x)/2)^5*(e + f*x))/(240*a^2*c^3*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x
)/2)^5)
```

3.29 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 166

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{x}{a^2c^4}+\frac{\cot(e+fx)}{a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{2\csc(e+fx)}{a^2c^4f}-\frac{2\csc^3(e+fx)}{a^2c^4f}+\frac{6\csc^5(e+fx)}{5a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}$$

```
[Out] x/a^2/c^4+cot(f*x+e)/a^2/c^4/f-1/3*cot(f*x+e)^3/a^2/c^4/f+1/5*cot(f*x+e)^5/
a^2/c^4/f-2/7*cot(f*x+e)^7/a^2/c^4/f+2*csc(f*x+e)/a^2/c^4/f-2*csc(f*x+e)^3/
a^2/c^4/f+6/5*csc(f*x+e)^5/a^2/c^4/f-2/7*csc(f*x+e)^7/a^2/c^4/f
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30}

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = -\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot(e+fx)}{a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{6\csc^5(e+fx)}{5a^2c^4f}-\frac{2\csc^3(e+fx)}{a^2c^4f}+\frac{2\csc(e+fx)}{a^2c^4f}+\frac{x}{a^2c^4}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] x/(a^2*c^4) + Cot[e + f*x]/(a^2*c^4*f) - Cot[e + f*x]^3/(3*a^2*c^4*f) + Cot
[e + f*x]^5/(5*a^2*c^4*f) - (2*Cot[e + f*x]^7)/(7*a^2*c^4*f) + (2*Csc[e + f
*x])/(a^2*c^4*f) - (2*Csc[e + f*x]^3)/(a^2*c^4*f) + (6*Csc[e + f*x]^5)/(5*a
^2*c^4*f) - (2*Csc[e + f*x]^7)/(7*a^2*c^4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^2\,dx}{a^4c^4}\\
&= \frac{\int \left(a^2\cot^8(e+fx)+2a^2\cot^7(e+fx)\csc(e+fx)+a^2\cot^6(e+fx)\csc^2(e+fx)\right)\,dx}{a^4c^4}\\
&= \frac{\int \cot^8(e+fx)\,dx}{a^2c^4}+\frac{\int \cot^6(e+fx)\csc^2(e+fx)\,dx}{a^2c^4}+\frac{2\int \cot^7(e+fx)\csc(e+fx)\,dx}{a^2c^4}\\
&= -\frac{\cot^7(e+fx)}{7a^2c^4f}-\frac{\int \cot^6(e+fx)\,dx}{a^2c^4}+\frac{\text{Subst}\left(\int x^6\,dx,x,-\cot(e+fx)\right)}{a^2c^4f}\\
&\quad-\frac{2\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\csc(e+fx)\right)}{a^2c^4f}\\
&= \frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{\int \cot^4(e+fx)\,dx}{a^2c^4}\\
&\quad-\frac{2\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\csc(e+fx)\right)}{a^2c^4f}\\
&= -\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{2\csc(e+fx)}{a^2c^4f}\\
&\quad-\frac{2\csc^3(e+fx)}{a^2c^4f}+\frac{6\csc^5(e+fx)}{5a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}-\frac{\int \cot^2(e+fx)\,dx}{a^2c^4}\\
&= \frac{\cot(e+fx)}{a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{2\csc(e+fx)}{a^2c^4f}\\
&\quad-\frac{2\csc^3(e+fx)}{a^2c^4f}+\frac{6\csc^5(e+fx)}{5a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{\int 1\,dx}{a^2c^4}\\
&= \frac{x}{a^2c^4}+\frac{\cot(e+fx)}{a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+fx)}{7a^2c^4f}\\
&\quad+\frac{2\csc(e+fx)}{a^2c^4f}-\frac{2\csc^3(e+fx)}{a^2c^4f}+\frac{6\csc^5(e+fx)}{5a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = -\frac{\cot^7(e+fx)\left(5+5\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(e+fx)\right)+70\sec(e+fx)-140\sec^3(e+\cdots\right.}{35a^2c^4f}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] -1/35*(Cot[e + f*x]^7*(5 + 5*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[e + f*x]
^2] + 70*Sec[e + f*x] - 140*Sec[e + f*x]^3 + 112*Sec[e + f*x]^5 - 32*Sec[e
+ f*x]^7))/(a^2*c^4*f)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.55

method	result
parallelrisch	$\frac{-15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7+147\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5+35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-770\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+3360fx-735\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4410\cot\left(\frac{fx}{2}+\cdots\right.}{3360f\,a^2c^4}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{22}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{42}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+64\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right.}{32f\,c^4a^2}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{22}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{42}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+64\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right.}{32f\,c^4a^2}$
risch	$\frac{x}{a^2c^4}+\frac{2i\left(210\,e^{9i(fx+e)}-315\,e^{8i(fx+e)}-420\,e^{7i(fx+e)}+1470\,e^{6i(fx+e)}-504\,e^{5i(fx+e)}-1204\,e^{4i(fx+e)}+1108\,e^{3i(fx+e)}\cdots\right.}{105f\,c^4a^2\left(e^{i(fx+e)}-1\right)^7\left(e^{i(fx+e)}+1\right)^3}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{ca}-\frac{1}{224acf}+\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{160acf}-\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{48acf}+\frac{21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{16acf}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{32acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{96acf}}{a\,c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3360*(-15*cot(1/2*f*x+1/2*e)^7+147*cot(1/2*f*x+1/2*e)^5+35*tan(1/2*f*x+1/
2*e)^3-770*cot(1/2*f*x+1/2*e)^3+3360*f*x-735*tan(1/2*f*x+1/2*e)+4410*cot(1/
2*f*x+1/2*e))/f/a^2/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{191\cos(fx+e)^5 - 172\cos(fx+e)^4 - 253\cos(fx+e)^3 + 258\cos(fx+e)^2 + 105\left(fx\cos(fx+e)^4 - 2\right.}{105\left(a^2c^4f\cos(fx+e)^4 - 2a^2c^4f\cos(fx+e)^3 + 2a^2c^4f\cos\right.}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(191*cos(f*x + e)^5 - 172*cos(f*x + e)^4 - 253*cos(f*x + e)^3 + 258*c
os(f*x + e)^2 + 105*(f*x*cos(f*x + e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(
f*x + e) - f*x)*sin(f*x + e) + 87*cos(f*x + e) - 96)/((a^2*c^4*f*cos(f*x +
e)^4 - 2*a^2*c^4*f*cos(f*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*f)*s
in(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{1}{\sec^6(e+fx)-2\sec^5(e+fx)-\sec^4(e+fx)+4\sec^3(e+fx)-\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{a^2c^4}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(1/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)**4 + 4*sec(e
 + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = -\frac{\frac{35\left(\frac{21\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^4} - \frac{6720\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^4} - \frac{\left(\frac{147\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{770\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4410\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+}{a^2c^4\sin(fx+e)^7}}{3360\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/3360*(35*(21*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a^2*c^4) - 6720*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c
^4) - (147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 770*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 4410*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e
) + 1)^7/(a^2*c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\frac{3360\,(fx+e)}{a^2c^4} + \frac{4410\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-770\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+147\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15}{a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7} + \frac{35\left(a^4c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-21\,a^4c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6c^{12}}}{3360\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3360*(3360*(f*x + e)/(a^2*c^4) + (4410*tan(1/2*f*x + 1/2*e)^6 - 770*tan(1
/2*f*x + 1/2*e)^4 + 147*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^2*c^4*tan(1/2*f*x +
 1/2*e)^7) + 35*(a^4*c^8*tan(1/2*f*x + 1/2*e)^3 - 21*a^4*c^8*tan(1/2*f*x +
1/2*e))/(a^6*c^12))/f
```

Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{35\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-15\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-735\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+4410\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right.}{3360\,a^2\,c^4\,f\cos}$$

```
[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4),x)
```

```
[Out] (35*sin(e/2 + (f*x)/2)^10 - 15*cos(e/2 + (f*x)/2)^10 - 735*cos(e/2 + (f*x)/
2)^2*sin(e/2 + (f*x)/2)^8 + 4410*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^6
- 770*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^4 + 147*cos(e/2 + (f*x)/2)^8*
sin(e/2 + (f*x)/2)^2 + 3360*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^7*(e +
f*x))/(3360*a^2*c^4*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^7)
```

3.30 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5} dx$

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Fricas [A] (verification not implemented) 266
Sympy [F] 267
Maxima [A] (verification not implemented) 267
Giac [A] (verification not implemented) 267
Mupad [B] (verification not implemented) 268

Optimal result

Integrand size = 26, antiderivative size = 210

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5} dx = \frac{x}{a^2c^5}+\frac{\cot(e+fx)}{a^2c^5f}-\frac{\cot^3(e+fx)}{3a^2c^5f}+\frac{\cot^5(e+fx)}{5a^2c^5f}-\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{3\csc(e+fx)}{a^2c^5f}-\frac{13\csc^3(e+fx)}{3a^2c^5f}+\frac{21\csc^5(e+fx)}{5a^2c^5f}-\frac{15\csc^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}$$

```
[Out] x/a^2/c^5+cot(f*x+e)/a^2/c^5/f-1/3*cot(f*x+e)^3/a^2/c^5/f+1/5*cot(f*x+e)^5/
a^2/c^5/f-1/7*cot(f*x+e)^7/a^2/c^5/f+4/9*cot(f*x+e)^9/a^2/c^5/f+3*csc(f*x+e
)/a^2/c^5/f-13/3*csc(f*x+e)^3/a^2/c^5/f+21/5*csc(f*x+e)^5/a^2/c^5/f-15/7*cs
c(f*x+e)^7/a^2/c^5/f+4/9*csc(f*x+e)^9/a^2/c^5/f
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5} dx = \frac{4\cot^9(e+fx)}{9a^2c^5f}-\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{\cot^5(e+fx)}{5a^2c^5f}-\frac{\cot^3(e+fx)}{3a^2c^5f}+\frac{\cot(e+fx)}{a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}-\frac{15\csc^7(e+fx)}{7a^2c^5f}+\frac{21\csc^5(e+fx)}{5a^2c^5f}-\frac{13\csc^3(e+fx)}{3a^2c^5f}+\frac{3\csc(e+fx)}{a^2c^5f}+\frac{x}{a^2c^5}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] x/(a^2*c^5) + Cot[e + f*x]/(a^2*c^5*f) - Cot[e + f*x]^3/(3*a^2*c^5*f) + Cot
[e + f*x]^5/(5*a^2*c^5*f) - Cot[e + f*x]^7/(7*a^2*c^5*f) + (4*Cot[e + f*x]^
9)/(9*a^2*c^5*f) + (3*Csc[e + f*x])/(a^2*c^5*f) - (13*Csc[e + f*x]^3)/(3*a^
2*c^5*f) + (21*Csc[e + f*x]^5)/(5*a^2*c^5*f) - (15*Csc[e + f*x]^7)/(7*a^2*c
^5*f) + (4*Csc[e + f*x]^9)/(9*a^2*c^5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\frac{\int \cot^{10}(e+fx)(a+a\sec(e+fx))^3\,dx}{a^5c^5}$$

$$= -\frac{\int \left(a^3\cot^{10}(e+fx)+3a^3\cot^9(e+fx)\csc(e+fx)+3a^3\cot^8(e+fx)\csc^2(e+fx)+a^3\cot^7(e+\right.}{a^5c^5}$$

$$= -\frac{\int \cot^{10}(e+fx)\,dx}{a^2c^5}-\frac{\int \cot^7(e+fx)\csc^3(e+fx)\,dx}{a^2c^5} - \frac{3\int \cot^9(e+fx)\csc(e+fx)\,dx}{a^2c^5}-\frac{3\int \cot^8(e+fx)\csc^2(e+fx)\,dx}{a^2c^5}$$

$$= \frac{\cot^9(e+fx)}{9a^2c^5f}+\frac{\int \cot^8(e+fx)\,dx}{a^2c^5}+\frac{\text{Subst}\left(\int x^2(-1+x^2)^3\,dx,x,\csc(e+fx)\right)}{a^2c^5f} - \frac{3\text{Subst}\left(\int x^8\,dx,x,-\cot(e+fx)\right)}{a^2c^5f}+\frac{3\text{Subst}\left(\int (-1+x^2)^4\,dx,x,\csc(e+fx)\right)}{a^2c^5f}$$

$$= -\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}-\frac{\int \cot^6(e+fx)\,dx}{a^2c^5} + \frac{\text{Subst}\left(\int (-x^2+3x^4-3x^6+x^8)\,dx,x,\csc(e+fx)\right)}{a^2c^5f} + \frac{3\text{Subst}\left(\int (1-4x^2+6x^4-4x^6+x^8)\,dx,x,\csc(e+fx)\right)}{a^2c^5f}$$

$$= \frac{\cot^5(e+fx)}{5a^2c^5f}-\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{3\csc(e+fx)}{a^2c^5f}-\frac{13\csc^3(e+fx)}{3a^2c^5f} + \frac{21\csc^5(e+fx)}{5a^2c^5f}-\frac{15\csc^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}+\frac{\int \cot^4(e+fx)\,dx}{a^2c^5}$$

$$= -\frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e+fx)}{9a^2c^5f} + \frac{3\csc(e+fx)}{a^2c^5f} - \frac{13\csc^3(e+fx)}{3a^2c^5f} + \frac{21\csc^5(e+fx)}{5a^2c^5f} - \frac{15\csc^7(e+fx)}{7a^2c^5f} + \frac{4\csc^9(e+fx)}{9a^2c^5f} - \frac{\int\cot^2(e+fx)\,dx}{a^2c^5}$$

$$= \frac{\cot(e+fx)}{a^2c^5f} - \frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e+fx)}{9a^2c^5f} + \frac{3\csc(e+fx)}{a^2c^5f} - \frac{13\csc^3(e+fx)}{3a^2c^5f} + \frac{21\csc^5(e+fx)}{5a^2c^5f} - \frac{15\csc^7(e+fx)}{7a^2c^5f} + \frac{4\csc^9(e+fx)}{9a^2c^5f} + \frac{\int 1\,dx}{a^2c^5}$$

$$= \frac{x}{a^2c^5} + \frac{\cot(e+fx)}{a^2c^5f} - \frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e+fx)}{9a^2c^5f} + \frac{3\csc(e+fx)}{a^2c^5f} - \frac{13\csc^3(e+fx)}{3a^2c^5f} + \frac{21\csc^5(e+fx)}{5a^2c^5f} - \frac{15\csc^7(e+fx)}{7a^2c^5f} + \frac{4\csc^9(e+fx)}{9a^2c^5f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.91 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{\cot^9(e+fx)\left(105+35\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(e+fx)\right)+945\sec(e+fx)-2415\sec^3(e\right.}{315a^2c^5f}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] (Cot[e + f*x]^9*(105 + 35*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2]
 + 945*Sec[e + f*x] - 2415*Sec[e + f*x]^3 + 2898*Sec[e + f*x]^5 - 1656*Sec[
e + f*x]^7 + 368*Sec[e + f*x]^9))/(315*a^2*c^5*f)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.50

method	result
parallelrisch	$\frac{35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-360\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+1827\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-6720\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+20160fx-2520\tan\left(\frac{fx}{2}\right.}{20160f\,a^{2}c^{5}}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+128\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{8}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{29}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{64}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}}{64f\,c^{5}a^{2}}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+128\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{8}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{29}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{64}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}}{64f\,c^{5}a^{2}}$
risch	$\frac{x}{a^{2}c^{5}}+\frac{2i\left(945\,e^{11i(fx+e)}-3150\,e^{10i(fx+e)}+2625\,e^{9i(fx+e)}+6300\,e^{8i(fx+e)}-13482\,e^{7i(fx+e)}+5292\,e^{6i(fx+e)}+10566\,e^{5}\right.}{315f\,c^{5}a^{2}\left(e^{i(fx+e)}-1\right)^{9}\left(e^{i(fx+e)}+\right.}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{ca}+\frac{1}{576acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{56acf}+\frac{29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{320acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{3acf}+\frac{99\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{64acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{192acf}}{a\,c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}$

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20160*(35*cot(1/2*f*x+1/2*e)^9-360*cot(1/2*f*x+1/2*e)^7+1827*cot(1/2*f*x+
1/2*e)^5+105*tan(1/2*f*x+1/2*e)^3-6720*cot(1/2*f*x+1/2*e)^3+20160*f*x-2520*
tan(1/2*f*x+1/2*e)+31185*cot(1/2*f*x+1/2*e))/f/a^2/c^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.10

$$\int\frac{1}{(a+a\sec(e+fx))^{2}(c-c\sec(e+fx))^{5}}\,dx$$
$$=\frac{668\cos(fx+e)^{6}-1059\cos(fx+e)^{5}-573\cos(fx+e)^{4}+1813\cos(fx+e)^{3}-393\cos(fx+e)^{2}+31}{315\left(a^{2}c^{5}f\cos(fx+e)^{5}-3a^{2}c^{5}f\cos(fx+e)^{4}+\right.}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(668*cos(f*x + e)^6 - 1059*cos(f*x + e)^5 - 573*cos(f*x + e)^4 + 1813
*cos(f*x + e)^3 - 393*cos(f*x + e)^2 + 315*(f*x*cos(f*x + e)^5 - 3*f*x*cos(
f*x + e)^4 + 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 - 3*f*x*cos(f*x +
e) + f*x)*sin(f*x + e) - 789*cos(f*x + e) + 368)/((a^2*c^5*f*cos(f*x + e)^5
 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^5*f*co
s(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$$
$$= -\frac{\int \frac{1}{\sec^7(e+fx)-3\sec^6(e+fx)+\sec^5(e+fx)+5\sec^4(e+fx)-5\sec^3(e+fx)-\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{a^2c^5}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
[Out] -Integral(1/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)**5 + 5*sec(
e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x)
/(a**2*c**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx =$$
$$-\frac{\frac{105\left(\frac{24\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^5}-\frac{40320\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^5}+\frac{\left(\frac{360\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1827\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6720\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{31185\sin(fx+}{(\cos(fx+e)+}}{a^2c^5\sin(fx+e)^9}}{20160\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
[Out] -1/20160*(105*(24*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2*c^5) - 40320*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^
2*c^5) + (360*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1827*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 6720*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 31185*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(a^2*c^5*sin(f*x +
 e)^9))/f
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{20160\,(fx+e)}{a^2c^5}+\frac{31185\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-6720\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+1827\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-360\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+35}{a^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}+\frac{105\left(a^4c^{10}\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.}{}}{20160\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/20160*(20160*(f*x + e)/(a^2*c^5) + (31185*tan(1/2*f*x + 1/2*e)^8 - 6720*t
an(1/2*f*x + 1/2*e)^6 + 1827*tan(1/2*f*x + 1/2*e)^4 - 360*tan(1/2*f*x + 1/2
*e)^2 + 35)/(a^2*c^5*tan(1/2*f*x + 1/2*e)^9) + 105*(a^4*c^10*tan(1/2*f*x +
1/2*e)^3 - 24*a^4*c^10*tan(1/2*f*x + 1/2*e))/(a^6*c^15))/f
```

Mupad [B] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{35\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+105\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-2520\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+31185\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\sin\left(\frac{e}{2}+\right.}{}$$

```
[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (35*cos(e/2 + (f*x)/2)^12 + 105*sin(e/2 + (f*x)/2)^12 - 2520*cos(e/2 + (f*x
)/2)^2*sin(e/2 + (f*x)/2)^10 + 31185*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2
)^8 - 6720*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^6 + 1827*cos(e/2 + (f*x)
/2)^8*sin(e/2 + (f*x)/2)^4 - 360*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)^2
 + 20160*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^9*(e + f*x))/(20160*a^2*c^
5*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^9)
```

3.31 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 162

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{c^5x}{a^3}+\frac{8c^5\text{arctanh}(\sin(e+fx))}{a^3f}+\frac{32c^5\cot(e+fx)}{a^3f}+\frac{128c^5\cot^3(e+fx)}{3a^3f}+\frac{128c^5\cot^5(e+fx)}{5a^3f}-\frac{16c^5\csc(e+fx)}{a^3f}+\frac{64c^5\csc^3(e+fx)}{3a^3f}-\frac{128c^5\csc^5(e+fx)}{5a^3f}-\frac{c^5\tan(e+fx)}{a^3f}$$

```
[Out] c^5*x/a^3+8*c^5*arctanh(sin(f*x+e))/a^3/f+32*c^5*cot(f*x+e)/a^3/f+128/3*c^5
*cot(f*x+e)^3/a^3/f+128/5*c^5*cot(f*x+e)^5/a^3/f-16*c^5*csc(f*x+e)/a^3/f+64
/3*c^5*csc(f*x+e)^3/a^3/f-128/5*c^5*csc(f*x+e)^5/a^3/f-c^5*tan(f*x+e)/a^3/f
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30, 14, 3852, 2701, 308, 213, 2700, 276}

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{8c^5\text{arctanh}(\sin(e+fx))}{a^3f}-\frac{c^5\tan(e+fx)}{a^3f}+\frac{128c^5\cot^5(e+fx)}{5a^3f}+\frac{128c^5\cot^3(e+fx)}{3a^3f}+\frac{32c^5\cot(e+fx)}{a^3f}-\frac{128c^5\csc^5(e+fx)}{5a^3f}+\frac{64c^5\csc^3(e+fx)}{3a^3f}-\frac{16c^5\csc(e+fx)}{a^3f}+\frac{c^5x}{a^3}$$

```
[In] Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^5*x)/a^3 + (8*c^5*ArcTanh[Sin[e + f*x]])/(a^3*f) + (32*c^5*Cot[e + f*x])
/(a^3*f) + (128*c^5*Cot[e + f*x]^3)/(3*a^3*f) + (128*c^5*Cot[e + f*x]^5)/(5
*a^3*f) - (16*c^5*Csc[e + f*x])/(a^3*f) + (64*c^5*Csc[e + f*x]^3)/(3*a^3*f)
 - (128*c^5*Csc[e + f*x]^5)/(5*a^3*f) - (c^5*Tan[e + f*x])/(a^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
```

```
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))^8\,dx}{a^3c^3}$$

$$= -\frac{\int \left(c^8\cot^6(e+fx) - 8c^8\cot^5(e+fx)\csc(e+fx) + 28c^8\cot^4(e+fx)\csc^2(e+fx) - 56c^8\cot^3(e\right.}{}$$

$$= -\frac{c^5\int \cot^6(e+fx)\,dx}{a^3} - \frac{c^5\int \csc^6(e+fx)\sec^2(e+fx)\,dx}{a^3} + \frac{(8c^5)\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3} + \frac{(8c^5)\int \csc^6(e+fx)\sec(e+fx)\,dx}{a^3} - \frac{(28c^5)\int \cot^4(e+fx)\csc^2(e+fx)\,dx}{a^3} - \frac{(28c^5)\int \csc^6(e+fx)\,dx}{a^3} + \frac{(56c^5)\int \cot^3(e+fx)\csc^3(e+fx)\,dx}{a^3} + \frac{(56c^5)\int \cot(e+fx)\csc^5(e+fx)\,dx}{a^3} - \frac{(70c^5)\int \cot^2(e+fx)\csc^4(e+fx)\,dx}{a^3}$$

$$= \frac{c^5\cot^5(e+fx)}{5a^3f} + \frac{c^5\int \cot^4(e+fx)\,dx}{a^3} - \frac{c^5\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6}\,dx, x, \tan(e+fx)\right)}{a^3f} - \frac{(8c^5)\,\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(e+fx)\right)}{a^3f} - \frac{(8c^5)\,\text{Subst}\left(\int (-1+x^2)^2\,dx, x, \csc(e+fx)\right)}{a^3f} - \frac{(28c^5)\,\text{Subst}\left(\int x^4\,dx, x, -\cot(e+fx)\right)}{a^3f} + \frac{(28c^5)\,\text{Subst}\left(\int (1+2x^2+x^4)\,dx, x, \cot(e+fx)\right)}{a^3f} - \frac{(56c^5)\,\text{Subst}\left(\int x^4\,dx, x, \csc(e+fx)\right)}{a^3f} - \frac{(56c^5)\,\text{Subst}\left(\int x^2(-1+x^2)\,dx, x, \csc(e+fx)\right)}{a^3f} - \frac{(70c^5)\,\text{Subst}\left(\int x^2(1+x^2)\,dx, x, -\cot(e+fx)\right)}{a^3f}$$

$$= \frac{28c^5\cot(e+fx)}{a^3f} + \frac{55c^5\cot^3(e+fx)}{3a^3f} + \frac{57c^5\cot^5(e+fx)}{5a^3f} - \frac{56c^5\csc^5(e+fx)}{5a^3f} - \frac{c^5\int\cot^2(e+fx)\,dx}{a^3} - \frac{c^5\text{Subst}\left(\int\left(1+\frac{1}{x^6}+\frac{3}{x^4}+\frac{3}{x^2}\right)\,dx,x,\tan(e+fx)\right)}{a^3f} - \frac{(8c^5)\,\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,\csc(e+fx)\right)}{a^3f} - \frac{(8c^5)\,\text{Subst}\left(\int\left(1+x^2+x^4+\frac{1}{-1+x^2}\right)\,dx,x,\csc(e+fx)\right)}{a^3f} - \frac{(56c^5)\,\text{Subst}\left(\int\left(-x^2+x^4\right)\,dx,x,\csc(e+fx)\right)}{a^3f} - \frac{(70c^5)\,\text{Subst}\left(\int\left(x^2+x^4\right)\,dx,x,-\cot(e+fx)\right)}{a^3f}$$

$$= \frac{32c^5\cot(e+fx)}{a^3f} + \frac{128c^5\cot^3(e+fx)}{3a^3f} + \frac{128c^5\cot^5(e+fx)}{5a^3f} - \frac{16c^5\csc(e+fx)}{a^3f} + \frac{64c^5\csc^3(e+fx)}{3a^3f} - \frac{128c^5\csc^5(e+fx)}{5a^3f} - \frac{c^5\tan(e+fx)}{a^3f} + \frac{c^5\int 1\,dx}{a^3} - \frac{(8c^5)\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\csc(e+fx)\right)}{a^3f}$$

$$= \frac{c^5x}{a^3} + \frac{8c^5\text{arctanh}(\sin(e+fx))}{a^3f} + \frac{32c^5\cot(e+fx)}{a^3f} + \frac{128c^5\cot^3(e+fx)}{3a^3f} + \frac{128c^5\cot^5(e+fx)}{5a^3f} - \frac{16c^5\csc(e+fx)}{a^3f} + \frac{64c^5\csc^3(e+fx)}{3a^3f} - \frac{128c^5\csc^5(e+fx)}{5a^3f} - \frac{c^5\tan(e+fx)}{a^3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.29 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.30

$$\int\frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^{9/2}\tan(e+fx)\left(-46\sqrt{a}\sqrt{c}-48\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1}\right.}{}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -1/15*(c^(9/2)*Tan[e + f*x]*(-46*Sqrt[a]*Sqrt[c] - 48*Sqrt[2]*Sqrt[a]*Sqrt[
c]*Hypergeometric2F1[-7/2, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e
 + f*x]] - 24*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-5/2, -5/2, -3/2, (
1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] - 2*Sqrt[a]*Sqrt[c]*Sec[e + f*x
```

```
] + 22*Sqrt[a]*Sqrt[c]*Sec[e + f*x]^2 + 26*Sqrt[a]*Sqrt[c]*Sec[e + f*x]^3 +
 15*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[-(a*c*Tan[e
 + f*x]^2)] + 30*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec
[e + f*x]*Sqrt[-(a*c*Tan[e + f*x]^2)] + 15*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^
2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f*x]^2*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(a^(7/2
)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{8c^5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}+\frac{}{8\tan(}\right.}{f\,a^3}$
default	$\frac{8c^5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}+\frac{}{8\tan(}\right.}{f\,a^3}$
parallelrisch	$-\frac{2113\left(\frac{1920\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)}{2113}-\frac{1920\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)}{2113}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\cos(fx+e)+\frac{954\cos(2fx+2e)}{2113}\right.\right.}{240\cos(fx+e)a^3f}$
risch	$\frac{c^5x}{a^3}-\frac{2ic^5\left(240\,\mathrm{e}^{6i(fx+e)}+735\,\mathrm{e}^{5i(fx+e)}+1835\,\mathrm{e}^{4i(fx+e)}+1750\,\mathrm{e}^{3i(fx+e)}+1894\,\mathrm{e}^{2i(fx+e)}+955\,\mathrm{e}^{i(fx+e)}+239\right)}{15f\,a^3\left(\mathrm{e}^{i(fx+e)}+1\right)^5\left(1+\mathrm{e}^{2i(fx+e)}\right)}+\frac{8c^5\,\mathrm{l}}{}$
norman	$\frac{\frac{c^5x}{a}+\frac{c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{a}+\frac{6c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{a}-\frac{18c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{202c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^{}}$

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/f*c^5/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5-1/3*tan(1/2*f*x+1/2*e)^3-2*tan(1/2*f
*x+1/2*e)+1/4*arctan(tan(1/2*f*x+1/2*e))+ln(tan(1/2*f*x+1/2*e)+1)+1/8/(tan(
1/2*f*x+1/2*e)+1)+1/8/(tan(1/2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.78

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{15\,c^5fx\cos{(fx+e)}^4+45\,c^5fx\cos{(fx+e)}^3+45\,c^5fx\cos{(fx+e)}^2+15\,c^5fx\cos{(fx+e)}+60\left(c^5\cos{(f}\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*c^5*f*x*cos(f*x + e)^4 + 45*c^5*f*x*cos(f*x + e)^3 + 45*c^5*f*x*co
s(f*x + e)^2 + 15*c^5*f*x*cos(f*x + e) + 60*(c^5*cos(f*x + e)^4 + 3*c^5*cos
(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f*x + e))*log(sin(f*x + e) + 1
) - 60*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 +
c^5*cos(f*x + e))*log(-sin(f*x + e) + 1) - (239*c^5*cos(f*x + e)^3 + 477*c^
5*cos(f*x + e)^2 + 349*c^5*cos(f*x + e) + 15*c^5)*sin(f*x + e))/(a^3*f*cos(
f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*
x + e))
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^5\left(\int \frac{5\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{10\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int \frac{}{\sec^3(e+fx)+}\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*sec(e + f*x)**3/(sec(e
+ f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-5*sec(e
 + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(154) = 308$.

Time = 0.31 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.47

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{3c^5\left(\frac{40\sin(fx+e)}{\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}+\frac{\frac{85\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin}{\cos}\right.}{}\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(3*c^5*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x
```

```
+ e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60
*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*
x + e) + 1) - 1)/a^3) + 5*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/
a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a^3) + c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) -
 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5)/a^3 - 120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 10*c^5*(15*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 5*c^5*(15*sin(f*x + e)/(cos(f*x
 + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5)/a^3 - 30*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)c^5}{a^3} + \frac{120\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3} - \frac{120\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3} + \frac{30\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3} - \frac{8\left(3\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 \cdots\right.}{\cdots}}{15\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*c^5/a^3 + 120*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3
 - 120*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 + 30*c^5*tan(1/2*f*x + 1/
2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^3) - 8*(3*a^12*c^5*tan(1/2*f*x + 1/2*e
)^5 + 5*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^5*tan(1/2*f*x + 1/2*e))
/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{c^5\,x}{a^3} - \frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f} - \frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f} - \frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f} + \frac{16\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f} + \frac{2\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)}$$

```
[In] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^3,x)
```

```
[Out] (c^5*x)/a^3 - (16*c^5*tan(e/2 + (f*x)/2))/(a^3*f) - (8*c^5*tan(e/2 + (f*x)/
2)^3)/(3*a^3*f) - (8*c^5*tan(e/2 + (f*x)/2)^5)/(5*a^3*f) + (16*c^5*atanh(ta
n(e/2 + (f*x)/2)))/(a^3*f) + (2*c^5*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2 + (
f*x)/2)^2 - a^3))
```

3.32 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{c^4x}{a^3}+\frac{c^4\text{arctanh}(\sin(e+fx))}{a^3f} - \frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3} - \frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{14c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2} - \frac{23c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))}$$

```
[Out] c^4*x/a^3+c^4*arctanh(sin(f*x+e))/a^3/f-3*c^4*tan(f*x+e)/a^3/f/(1+sec(f*x+e
))^3-1/5*c^4*sec(f*x+e)^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3+14/5*c^4*tan(f*
x+e)/a^3/f/(1+sec(f*x+e))^2-23/5*c^4*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882, 3884, 4085, 3901, 4093, 4083, 3855}

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{c^4\text{arctanh}(\sin(e+fx))}{a^3f} - \frac{c^4\tan(e+fx)\sec^2(e+fx)}{5a^3f(\sec(e+fx)+1)^3} - \frac{23c^4\tan(e+fx)}{5a^3f(\sec(e+fx)+1)} + \frac{14c^4\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^2} - \frac{3c^4\tan(e+fx)}{a^3f(\sec(e+fx)+1)^3} + \frac{c^4x}{a^3}$$

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^4*x)/a^3 + (c^4*ArcTanh[Sin[e + f*x]])/(a^3*f) - (3*c^4*Tan[e + f*x])/(a
^3*f*(1 + Sec[e + f*x])^3) - (c^4*Sec[e + f*x]^2*Tan[e + f*x])/(5*a^3*f*(1
+ Sec[e + f*x])^3) + (14*c^4*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^2) -
 (23*c^4*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
```

```
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{c^4}{(1+\sec(e+fx))^3}-\frac{4c^4\sec(e+fx)}{(1+\sec(e+fx))^3}+\frac{6c^4\sec^2(e+fx)}{(1+\sec(e+fx))^3}-\frac{4c^4\sec^3(e+fx)}{(1+\sec(e+fx))^3}+\frac{c^4\sec^4(e+fx)}{(1+\sec(e+fx))^3}\right)dx}{a^3}\\
&= \frac{c^4\int \frac{1}{(1+\sec(e+fx))^3}\,dx}{a^3}+\frac{c^4\int \frac{\sec^4(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}-\frac{(4c^4)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}\\
&\quad -\frac{(4c^4)\int \frac{\sec^3(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}+\frac{(6c^4)\int \frac{\sec^2(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}\\
&= -\frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3}-\frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}\\
&\quad -\frac{c^4\int \frac{(2-5\sec(e+fx))\sec^2(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3}-\frac{c^4\int \frac{-5+2\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3}\\
&\quad -\frac{(4c^4)\int \frac{\sec(e+fx)(-3+5\sec(e+fx))}{(1+\sec(e+fx))^2}\,dx}{5a^3}\\
&\quad -\frac{(8c^4)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3}+\frac{(18c^4)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3}\\
&= -\frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3}-\frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{14c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}\\
&\quad +\frac{c^4\int \frac{15-7\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3}+\frac{c^4\int \frac{\sec(e+fx)(-14+15\sec(e+fx))}{1+\sec(e+fx)}\,dx}{15a^3}\\
&\quad -\frac{(8c^4)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3}+\frac{(6c^4)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{5a^3}-\frac{(28c^4)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3}\\
&= \frac{c^4x}{a^3}-\frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3}-\frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}\\
&\quad +\frac{14c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}-\frac{6c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))}\\
&\quad +\frac{c^4\int \sec(e+fx)\,dx}{a^3}-\frac{(22c^4)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3}-\frac{(29c^4)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3}
\end{aligned}$$

$$= \frac{c^4x}{a^3} + \frac{c^4\operatorname{arctanh}(\sin(e+fx))}{a^3f} - \frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3} - \frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{14c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2} - \frac{23c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{2c^{7/2}\tan(e+fx)\left(12\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\sec(e+fx)}+\right.}{15a^7}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*c^(7/2)*Tan[e + f*x]*(12*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-5/2,
 -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] + Sqrt[a]*Sqrt[c]
*(23 + Sec[e + f*x] - 11*Sec[e + f*x]^2 - 13*Sec[e + f*x]^3) - 30*ArcTanh[S
qrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Cos[(e + f*x)/2]^4*Sec[e + f*
x]^2*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(15*a^(7/2)*f*(-1 + Sec[e + f*x])*(1 + S
ec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

method	result
parallelrisch	$\frac{c^4\left(-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+5fx+5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5a^3f}$
derivativedivides	$\frac{4c^4\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}\right)}{f\,a^3}$
default	$\frac{4c^4\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}\right)}{f\,a^3}$
risch	$\frac{c^4x}{a^3}-\frac{16ic^4\left(5\,e^{4i(fx+e)}+10\,e^{3i(fx+e)}+20\,e^{2i(fx+e)}+10\,e^{i(fx+e)}+3\right)}{5f\,a^3\left(e^{i(fx+e)}+1\right)^5}+\frac{c^4\ln\left(e^{i(fx+e)}+i\right)}{a^3f}-\frac{c^4\ln\left(e^{i(fx+e)}-i\right)}{a^3f}$
norman	$\frac{\frac{c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{a}-\frac{c^4x}{a}+\frac{4c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{12c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}+\frac{64c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5af}-\frac{32c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{5af}+\frac{12c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{5af}-}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3a^2}$

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*c^4*(-4*tan(1/2*f*x+1/2*e)^5+5*f*x+5*ln(tan(1/2*f*x+1/2*e)+1)-5*ln(tan(
1/2*f*x+1/2*e)-1)-20*tan(1/2*f*x+1/2*e))/a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.64

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{10\,c^4 fx\cos{(fx+e)}^3 + 30\,c^4 fx\cos{(fx+e)}^2 + 30\,c^4 fx\cos{(fx+e)} + 10\,c^4 fx + 5\,\left(c^4\cos{(fx+e)}^3 + 3\,c\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/10*(10*c^4*f*x*cos(f*x + e)^3 + 30*c^4*f*x*cos(f*x + e)^2 + 30*c^4*f*x*co
s(f*x + e) + 10*c^4*f*x + 5*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*
c^4*cos(f*x + e) + c^4)*log(sin(f*x + e) + 1) - 5*(c^4*cos(f*x + e)^3 + 3*c
^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*log(-sin(f*x + e) + 1) - 16*(
3*c^4*cos(f*x + e)^2 + 4*c^4*cos(f*x + e) + 3*c^4)*sin(f*x + e))/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{c^4\left(\int\left(-\frac{4\sec{(e+fx)}}{\sec^3{(e+fx)}+3\sec^2{(e+fx)}+3\sec{(e+fx)}+1}\right)dx + \int\frac{6\sec^2{(e+fx)}}{\sec^3{(e+fx)}+3\sec^2{(e+fx)}+3\sec{(e+fx)}+1}\,dx + \int\left(-\frac{}{\sec^3{(e+fx)}\cdots}\right.\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*x)**3/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*
x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
gral(1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(142) = 284$.

Time = 0.30 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.68

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^4\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+c^4\left(\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\cdots}{(\cos\cdots}\right.\cdots}{\cdots}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
 - 1)/a^3) + c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arc
tan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 4*c^4*(15*sin(f*x + e)/(cos(f*x
 + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5)/a^3 + 4*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^
3 - 18*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e
) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.69

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{5(fx+e)c^4}{a^3}+\frac{5c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^3}-\frac{5c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^3}-\frac{4\left(a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+5a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{15}}}{5f}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/5*(5*(f*x + e)*c^4/a^3 + 5*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 5
*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - 4*(a^12*c^4*tan(1/2*f*x + 1/2
*e)^5 + 5*a^12*c^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 14.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.34

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{c^4\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}+f\,x\right)}{a^3\,f}$$

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^3,x)
[Out] (c^4*(2*atanh(tan(e/2 + (f*x)/2)) - 4*tan(e/2 + (f*x)/2) - (4*tan(e/2 + (f*
x)/2)^5)/5 + f*x))/(a^3*f)
```

3.33 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{c^3x}{a^3} - \frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} - \frac{26c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))}$$

```
[Out] c^3*x/a^3-8/5*c^3*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3+4/15*c^3*tan(f*x+e)/a^3
/f/(1+sec(f*x+e))^2-26/15*c^3*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882, 3884, 4085}

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{26c^3\tan(e+fx)}{15a^3f(\sec(e+fx)+1)} + \frac{4c^3\tan(e+fx)}{15a^3f(\sec(e+fx)+1)^2} - \frac{8c^3\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3} + \frac{c^3x}{a^3}$$

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^3*x)/a^3 - (8*c^3*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) + (4*c^3*
Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^2) - (26*c^3*Tan[e + f*x])/(15*a
^3*f*(1 + Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
```

```
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = \frac{\int \left(\frac{c^3}{(1+\sec(e+fx))^3} - \frac{3c^3\sec(e+fx)}{(1+\sec(e+fx))^3} + \frac{3c^3\sec^2(e+fx)}{(1+\sec(e+fx))^3} - \frac{c^3\sec^3(e+fx)}{(1+\sec(e+fx))^3}\right)dx}{a^3}$$

$$= \frac{c^3\int \frac{1}{(1+\sec(e+fx))^3}\,dx}{a^3} - \frac{c^3\int \frac{\sec^3(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3} - \frac{(3c^3)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3} + \frac{(3c^3)\int \frac{\sec^2(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}$$

$$= -\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{c^3\int \frac{-5+2\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3} - \frac{c^3\int \frac{\sec(e+fx)(-3+5\sec(e+fx))}{(1+\sec(e+fx))^2}\,dx}{5a^3} - \frac{(6c^3)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3} + \frac{(9c^3)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3}$$

$$= -\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} + \frac{c^3\int \frac{15-7\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3} - \frac{(2c^3)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{5a^3} - \frac{(7c^3)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3} + \frac{(3c^3)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{5a^3}$$

$$= \frac{c^3x}{a^3} - \frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} - \frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))} - \frac{(22c^3)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3}$$

$$= \frac{c^3x}{a^3} - \frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} - \frac{26c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$
$$= -\frac{c^3\left(-\frac{2\arctan\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} + \frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f} - \frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f} + \frac{2\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}\right)}{a^3}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((c^3*((-2*ArcTan[Tan[e/2 + (f*x)/2]])/f + (2*Tan[e/2 + (f*x)/2])/f - (2*T
an[e/2 + (f*x)/2]^3)/(3*f) + (2*Tan[e/2 + (f*x)/2]^5)/(5*f)))/a^3)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

method	result
parallelrisch	$-\frac{c^3\left(6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-15fx+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15a^3f}$
derivativedivides	$\frac{2c^3\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^3}$
default	$\frac{2c^3\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^3}$
risch	$\frac{c^3x}{a^3}-\frac{4ic^3\left(45\,e^{4i(fx+e)}+90\,e^{3i(fx+e)}+140\,e^{2i(fx+e)}+70\,e^{i(fx+e)}+23\right)}{15f\,a^3\left(e^{i(fx+e)}+1\right)^5}$
norman	$\frac{\frac{c^3x}{a}+\frac{c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{a}-\frac{2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{14c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{56c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15af}+\frac{22c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{15af}-\frac{2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{5af}-}{a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}$

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*c^3*(6*tan(1/2*f*x+1/2*e)^5-10*tan(1/2*f*x+1/2*e)^3-15*f*x+30*tan(1/2
*f*x+1/2*e))/a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{15\,c^3fx\cos\left(fx+e\right)^3+45\,c^3fx\cos\left(fx+e\right)^2+45\,c^3fx\cos\left(fx+e\right)+15\,c^3fx-2\left(23\,c^3\cos\left(fx+e\right)^2+2\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right.}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*c^3*f*x*cos(f*x + e)^3 + 45*c^3*f*x*cos(f*x + e)^2 + 45*c^3*f*x*co
s(f*x + e) + 15*c^3*f*x - 2*(23*c^3*cos(f*x + e)^2 + 24*c^3*cos(f*x + e) +
13*c^3)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^
3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{}{\sec^3\left(e+fx\right)+3\,\mathrm{s}}\right.}{a^3}$$

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
 + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*
x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-1/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(90) = 180.

Time = 0.29 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.89

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{c^3\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a^3) + c^3*(15*sin(f*x + e)/(cos(f*x + e) +
1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5)/a^3 + 3*c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^
3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)c^3}{a^3}-\frac{2\left(3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-5\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{15\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*c^3/a^3 - 2*(3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 - 5*a^12*
c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{c^3\,x}{a^3}-\frac{\frac{46\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15}-\frac{22\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{15}+\frac{2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3}{5}}{a^3\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

```
[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^3,x)
```

```
[Out] (c^3*x)/a^3 - ((2*c^3*sin(e/2 + (f*x)/2))/5 - (22*c^3*cos(e/2 + (f*x)/2)^2*
sin(e/2 + (f*x)/2))/15 + (46*c^3*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/1
5)/(a^3*f*cos(e/2 + (f*x)/2)^5)
```

3.34 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{c^2x}{a^3} - \frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} - \frac{23c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))}$$

```
[Out] c^2*x/a^3-4/5*c^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3-8/15*c^2*tan(f*x+e)/a^3
/f/(1+sec(f*x+e))^2-23/15*c^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882}

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = -\frac{23c^2\tan(e+fx)}{15a^3f(\sec(e+fx)+1)} - \frac{8c^2\tan(e+fx)}{15a^3f(\sec(e+fx)+1)^2} - \frac{4c^2\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3} + \frac{c^2x}{a^3}$$

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^2*x)/a^3 - (4*c^2*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) - (8*c^2*
Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^2) - (23*c^2*Tan[e + f*x])/(15*a
^3*f*(1 + Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
```

```
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{c^2}{(1+\sec(e+fx))^3} - \frac{2c^2\sec(e+fx)}{(1+\sec(e+fx))^3} + \frac{c^2\sec^2(e+fx)}{(1+\sec(e+fx))^3}\right) dx}{a^3} \\
&= \frac{c^2 \int \frac{1}{(1+\sec(e+fx))^3}\, dx}{a^3} + \frac{c^2 \int \frac{\sec^2(e+fx)}{(1+\sec(e+fx))^3}\, dx}{a^3} - \frac{(2c^2) \int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\, dx}{a^3} \\
&= -\frac{4c^2\tan(e+fx)}{5a^3 f(1+\sec(e+fx))^3} - \frac{c^2 \int \frac{-5+2\sec(e+fx)}{(1+\sec(e+fx))^2}\, dx}{5a^3} \\
&\quad + \frac{(3c^2) \int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\, dx}{5a^3} - \frac{(4c^2) \int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\, dx}{5a^3} \\
&= -\frac{4c^2\tan(e+fx)}{5a^3 f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3 f(1+\sec(e+fx))^2} \\
&\quad + \frac{c^2 \int \frac{15-7\sec(e+fx)}{1+\sec(e+fx)}\, dx}{15a^3} + \frac{c^2 \int \frac{\sec(e+fx)}{1+\sec(e+fx)}\, dx}{5a^3} - \frac{(4c^2) \int \frac{\sec(e+fx)}{1+\sec(e+fx)}\, dx}{15a^3} \\
&= \frac{c^2 x}{a^3} - \frac{4c^2\tan(e+fx)}{5a^3 f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3 f(1+\sec(e+fx))^2} \\
&\quad - \frac{c^2\tan(e+fx)}{15a^3 f(1+\sec(e+fx))} - \frac{(22c^2) \int \frac{\sec(e+fx)}{1+\sec(e+fx)}\, dx}{15a^3} \\
&= \frac{c^2 x}{a^3} - \frac{4c^2\tan(e+fx)}{5a^3 f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3 f(1+\sec(e+fx))^2} - \frac{23c^2\tan(e+fx)}{15a^3 f(1+\sec(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.57

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\, dx =$$
$$-\frac{c^{3/2}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(-43-11\sec(e+fx)+31\sec^2(e+fx)+23\sec^3(e+fx))+60\text{arctanh}\left(\frac{\sqrt{-ac}}{}\right.\right.}{15a^{7/2}f(-1+\sec(e+fx))(1+\sec(e+fx)}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -1/15*(c^(3/2)*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(-43 - 11*Sec[e + f*x] + 31*Se
c[e + f*x]^2 + 23*Sec[e + f*x]^3) + 60*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/
(Sqrt[a]*Sqrt[c])]*Cos[(e + f*x)/2]^4*Sec[e + f*x]^2*Sqrt[-(a*c*Tan[e + f*x
]^2)]))/(a^(7/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$-\frac{c^{2}\left(3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-10 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}-15 f x+30 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{15 a^{3} f}$	54
derivativedivides	$\frac{c^{2}\left(-\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{2 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-2 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)+2 \arctan\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f\, a^{3}}$	61
default	$\frac{c^{2}\left(-\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{2 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-2 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)+2 \arctan\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f\, a^{3}}$	61
risch	$\frac{c^{2} x}{a^{3}}-\frac{2 i c^{2}\left(75 \,{\mathrm e}^{4 i (f x+e)}+180 \,{\mathrm e}^{3 i (f x+e)}+250 \,{\mathrm e}^{2 i (f x+e)}+140 \,{\mathrm e}^{i (f x+e)}+43\right)}{15 f\, a^{3} \left({\mathrm e}^{i (f x+e)}+1\right)^{5}}$	81
norman	$\frac{\frac{c^{2} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{a}-\frac{c^{2} x}{a}+\frac{2 c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{a f}-\frac{8 c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 a f}+\frac{13 c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{15 a f}-\frac{c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{5 a f}}{a^{2} \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)}$	135

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*c^2*(3*tan(1/2*f*x+1/2*e)^5-10*tan(1/2*f*x+1/2*e)^3-15*f*x+30*tan(1/2
*f*x+1/2*e))/a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{15\,c^{2} f x \cos\left(f x + e\right)^{3} + 45\,c^{2} f x \cos\left(f x + e\right)^{2} + 45\,c^{2} f x \cos\left(f x + e\right) + 15\,c^{2} f x - \left(43\,c^{2} \cos\left(f x + e\right)^{2} + 5}{15\,\left(a^{3} f \cos\left(f x + e\right)^{3} + 3\,a^{3} f \cos\left(f x + e\right)^{2} + 3\,a^{3} f \cos\left(f x + e\right) + a^{3} f}$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*c^2*f*x*cos(f*x + e)^3 + 45*c^2*f*x*cos(f*x + e)^2 + 45*c^2*f*x*co
s(f*x + e) + 15*c^2*f*x - (43*c^2*cos(f*x + e)^2 + 54*c^2*cos(f*x + e) + 23
*c^2)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*
f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^3}\,dx = \frac{c^2\left(\int \left(-\frac{2\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx + \int \frac{\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{}{\sec^3(e+fx)+3\sec^2}\right.}{a^3}$$

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(90) = 180$.

Time = 0.29 (sec), antiderivative size = 211, normalized size of antiderivative = 2.20

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^3}\,dx = -\frac{c^2\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1} - \frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right) + \frac{2c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1} - \frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right.}{a^3}}{60f}$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c^2*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a^3) + 2*c^2*(15*sin(f*x + e)/(cos(f*x + e)
+ 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)/a^3 - 3*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)c^2}{a^3} - \frac{3\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 10\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{15\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*c^2/a^3 - (3*a^12*c^2*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c
^2*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^2*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{c^2\,x}{a^3} - \frac{\frac{43\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15} - \frac{16\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{15} + \frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2}{5}}{a^3\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

```
[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^3,x)
```

```
[Out] (c^2*x)/a^3 - ((c^2*sin(e/2 + (f*x)/2))/5 - (16*c^2*cos(e/2 + (f*x)/2)^2*si
n(e/2 + (f*x)/2))/15 + (43*c^2*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/15)
/(a^3*f*cos(e/2 + (f*x)/2)^5)
```

3.35 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$

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Rubi [A] (verified) 298
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Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = \frac{cx}{a^3}-\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}-\frac{8c\tan(e+fx)}{5a^3f(1+\sec(e+fx))}$$

```
[Out] c*x/a^3-2/5*c*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3-3/5*c*tan(f*x+e)/a^3/f/(1+s
ec(f*x+e))^2-8/5*c*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3988, 3862, 4007, 4004, 3879, 3881}

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = -\frac{8c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)}-\frac{3c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^2}-\frac{2c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3}+\frac{cx}{a^3}$$

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c*x)/a^3 - (2*c*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) - (3*c*Tan[e
+ f*x])/(5*a^3*f*(1 + Sec[e + f*x])^2) - (8*c*Tan[e + f*x])/(5*a^3*f*(1 + S
ec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
```

```
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{\int \left(\frac{c}{(1+\sec(e+fx))^3} - \frac{c\sec(e+fx)}{(1+\sec(e+fx))^3}\right) dx}{a^3}$$

$$
\begin{aligned}
&= \frac{c\int \frac{1}{(1+\sec(e+fx))^3}\,dx}{a^3} - \frac{c\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3} \\
&= -\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{c\int \frac{-5+2\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3} - \frac{(2c)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3} \\
&= -\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2} \\
&\quad + \frac{c\int \frac{15-7\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3} - \frac{(2c)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3} \\
&= \frac{cx}{a^3} - \frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2} \\
&\quad - \frac{2c\tan(e+fx)}{15a^3f(1+\sec(e+fx))} - \frac{(22c)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3} \\
&= \frac{cx}{a^3} - \frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2} - \frac{8c\tan(e+fx)}{5a^3f(1+\sec(e+fx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$
\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = \frac{c\cot^5(e+fx)\left(16+3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)-60\sec(e+fx)+5\sec^2(e+fx)\right.}{15a^3f}
$$

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c*Cot[e + f*x]^5*(16 + 3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2]
 - 60*Sec[e + f*x] + 5*Sec[e + f*x]^2 + 60*Sec[e + f*x]^3 - 24*Sec[e + f*x]
^5))/(15*a^3*f)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

method	result	size
parallelrisch	$-\frac{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-10fx+20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10a^{3}f}$	50
derivativedivides	$\frac{c\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2fa^{3}}$	58
default	$\frac{c\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2fa^{3}}$	58
norman	$\frac{\frac{cx}{a}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2af}-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{10af}}{a^{2}}$	70
risch	$\frac{cx}{a^{3}}-\frac{2ic\left(20\,e^{4i(fx+e)}+55\,e^{3i(fx+e)}+75\,e^{2i(fx+e)}+45\,e^{i(fx+e)}+13\right)}{5fa^{3}\left(e^{i(fx+e)}+1\right)^{5}}$	77

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*c*(tan(1/2*f*x+1/2*e)^5-5*tan(1/2*f*x+1/2*e)^3-10*f*x+20*tan(1/2*f*x+
1/2*e))/a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{5\,cfx\cos\left(fx+e\right)^{3}+15\,cfx\cos\left(fx+e\right)^{2}+15\,cfx\cos\left(fx+e\right)+5\,cfx-\left(13\,c\cos\left(fx+e\right)^{2}+19\,c\cos\left(\right.\right.}{5\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}+3\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(5*c*f*x*cos(f*x + e)^3 + 15*c*f*x*cos(f*x + e)^2 + 15*c*f*x*cos(f*x +
e) + 5*c*f*x - (13*c*cos(f*x + e)^2 + 19*c*cos(f*x + e) + 8*c)*sin(f*x + e)
)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a
^3*f)
```

Sympy [F]

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^3} dx$$
$$= -\frac{c\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\, dx + \int \left(-\frac{1}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right) dx\right)}{a^3}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.81

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^3} dx =$$
$$-\frac{c\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1} - \frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right) + \frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1} - \frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(sin(
f*x + e)/(cos(f*x + e) + 1))/a^3) + c*(15*sin(f*x + e)/(cos(f*x + e) + 1) -
 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^3} dx = \frac{\frac{10(fx+e)c}{a^3} - \frac{a^{12}c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5 - 5a^{12}c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 20a^{12}c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}}}{10f}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/10*(10*(f*x + e)*c/a^3 - (a^12*c*tan(1/2*f*x + 1/2*e)^5 - 5*a^12*c*tan(1/
2*f*x + 1/2*e)^3 + 20*a^12*c*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 14.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{c\,x}{a^3}-\frac{\frac{13\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{5}-\frac{7\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{10}+\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{10}}{a^3\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

```
[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^3,x)
```

```
[Out] (c*x)/a^3 - ((c*sin(e/2 + (f*x)/2))/10 - (7*c*cos(e/2 + (f*x)/2)^2*sin(e/2
+ (f*x)/2))/10 + (13*c*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/5)/(a^3*f*c
os(e/2 + (f*x)/2)^5)
```

3.36 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$

Optimal result . . . 304
Rubi [A] (verified) . . . 304
Mathematica [C] (verified) . . . 306
Maple [A] (verified) . . . 307
Fricas [A] (verification not implemented) . . . 307
Sympy [F] . . . 308
Maxima [A] (verification not implemented) . . . 308
Giac [A] (verification not implemented) . . . 308
Mupad [B] (verification not implemented) . . . 309

Optimal result

Integrand size = 26, antiderivative size = 126

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{x}{a^3c}+\frac{\cot(e+fx)}{a^3cf}-\frac{\cot^3(e+fx)}{3a^3cf}+\frac{2\cot^5(e+fx)}{5a^3cf}-\frac{2\csc(e+fx)}{a^3cf}+\frac{4\csc^3(e+fx)}{3a^3cf}-\frac{2\csc^5(e+fx)}{5a^3cf}$$

```
[Out] x/a^3/c+cot(f*x+e)/a^3/c/f-1/3*cot(f*x+e)^3/a^3/c/f+2/5*cot(f*x+e)^5/a^3/c/
f-2*csc(f*x+e)/a^3/c/f+4/3*csc(f*x+e)^3/a^3/c/f-2/5*csc(f*x+e)^5/a^3/c/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30}

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{2\cot^5(e+fx)}{5a^3cf}-\frac{\cot^3(e+fx)}{3a^3cf}+\frac{\cot(e+fx)}{a^3cf}-\frac{2\csc^5(e+fx)}{5a^3cf}+\frac{4\csc^3(e+fx)}{3a^3cf}-\frac{2\csc(e+fx)}{a^3cf}+\frac{x}{a^3c}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] x/(a^3*c) + Cot[e + f*x]/(a^3*c*f) - Cot[e + f*x]^3/(3*a^3*c*f) + (2*Cot[e
+ f*x]^5)/(5*a^3*c*f) - (2*Csc[e + f*x])/(a^3*c*f) + (4*Csc[e + f*x]^3)/(3*
a^3*c*f) - (2*Csc[e + f*x]^5)/(5*a^3*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
```

```
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))^2\,dx}{a^3c^3}\\
&= -\frac{\int \left(c^2\cot^6(e+fx)-2c^2\cot^5(e+fx)\csc(e+fx)+c^2\cot^4(e+fx)\csc^2(e+fx)\right)\,dx}{a^3c^3}\\
&= -\frac{\int \cot^6(e+fx)\,dx}{a^3c}-\frac{\int \cot^4(e+fx)\csc^2(e+fx)\,dx}{a^3c}+\frac{2\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c}\\
&= \frac{\cot^5(e+fx)}{5a^3cf}+\frac{\int \cot^4(e+fx)\,dx}{a^3c}-\frac{\text{Subst}\left(\int x^4\,dx,x,-\cot(e+fx)\right)}{a^3cf}\\
&\quad -\frac{2\text{Subst}\left(\int (-1+x^2)^2\,dx,x,\csc(e+fx)\right)}{a^3cf}\\
&= -\frac{\cot^3(e+fx)}{3a^3cf}+\frac{2\cot^5(e+fx)}{5a^3cf}-\frac{\int \cot^2(e+fx)\,dx}{a^3c}\\
&\quad -\frac{2\text{Subst}\left(\int (1-2x^2+x^4)\,dx,x,\csc(e+fx)\right)}{a^3cf}\\
&= \frac{\cot(e+fx)}{a^3cf}-\frac{\cot^3(e+fx)}{3a^3cf}+\frac{2\cot^5(e+fx)}{5a^3cf}\\
&\quad -\frac{2\csc(e+fx)}{a^3cf}+\frac{4\csc^3(e+fx)}{3a^3cf}-\frac{2\csc^5(e+fx)}{5a^3cf}+\frac{\int 1\,dx}{a^3c}\\
&= \frac{x}{a^3c}+\frac{\cot(e+fx)}{a^3cf}-\frac{\cot^3(e+fx)}{3a^3cf}+\frac{2\cot^5(e+fx)}{5a^3cf}\\
&\quad -\frac{2\csc(e+fx)}{a^3cf}+\frac{4\csc^3(e+fx)}{3a^3cf}-\frac{2\csc^5(e+fx)}{5a^3cf}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$
\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx
= \frac{\cot^5(e+fx)\left(3+3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)-30\sec(e+fx)+40\sec^3(e+fx)-\right.}{15a^3cf}
$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] (Cot[e + f*x]^5*(3 + 3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2] -
30*Sec[e + f*x] + 40*Sec[e + f*x]^3 - 16*Sec[e + f*x]^5))/(15*a^3*c*f)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

method	result	size
parallelrisch	$\frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+25\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+120fx+15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-165\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{120f\,a^{3}c}$	65
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+16\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8f\,a^{3}c}$	73
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+16\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8f\,a^{3}c}$	73
risch	$\frac{x}{a^{3}c}-\frac{4i\left(15\,\mathrm{e}^{5i(fx+e)}+30\,\mathrm{e}^{4i(fx+e)}+10\,\mathrm{e}^{3i(fx+e)}-35\,\mathrm{e}^{2i(fx+e)}-37\,\mathrm{e}^{i(fx+e)}-13\right)}{15f\,a^{3}c\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}\left(\mathrm{e}^{i(fx+e)}-1\right)}$	105
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{ca}+\frac{1}{8acf}-\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{8acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{24acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{40acf}}{a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	111

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(-3*tan(1/2*f*x+1/2*e)^5+25*tan(1/2*f*x+1/2*e)^3+120*f*x+15*cot(1/2*f
*x+1/2*e)-165*tan(1/2*f*x+1/2*e))/f/a^3/c
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{26\cos(fx+e)^3+22\cos(fx+e)^2+15\left(fx\cos(fx+e)^2+2fx\cos(fx+e)+fx\right)\sin(fx+e)-17\,\text{co}}{15\left(a^3cf\cos(fx+e)^2+2a^3cf\cos(fx+e)+a^3cf\right)\sin(fx+e)}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/15*(26*cos(f*x + e)^3 + 22*cos(f*x + e)^2 + 15*(f*x*cos(f*x + e)^2 + 2*f*
x*cos(f*x + e) + f*x)*sin(f*x + e) - 17*cos(f*x + e) - 16)/((a^3*c*f*cos(f*
x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{\sec^4(e+fx)+2\sec^3(e+fx)-2\sec(e+fx)-1}\,dx}{a^3c}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
[Out] -Integral(1/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x) - 1), x)/
(a**3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\frac{\frac{165\sin(fx+e)}{\cos(fx+e)+1}-\frac{25\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c}-\frac{240\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c}-\frac{15(\cos(fx+e)+1)}{a^3c\sin(fx+e)}}{120f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")
[Out] -1/120*((165*sin(f*x + e)/(cos(f*x + e) + 1) - 25*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c) - 240*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/(a^3*c) - 15*(cos(f*x + e) + 1)/(a^3*c*sin(f
*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{\frac{120(fx+e)}{a^3c}+\frac{15}{a^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}-\frac{3a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-25a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+165a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^5}}{120f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] 1/120*(120*(f*x + e)/(a^3*c) + 15/(a^3*c*tan(1/2*f*x + 1/2*e)) - (3*a^12*c^
4*tan(1/2*f*x + 1/2*e)^5 - 25*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*a^12*c^
4*tan(1/2*f*x + 1/2*e))/(a^15*c^5))/f
```

Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{x}{a^3\,c}+\frac{\frac{26\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{15}-\frac{28\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15}+\frac{17\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{60}-\frac{1}{40}}{a^3\,c\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))),x)
```

```
[Out] x/(a^3*c) + ((17*cos(e/2 + (f*x)/2)^2)/60 - (28*cos(e/2 + (f*x)/2)^4)/15 +
(26*cos(e/2 + (f*x)/2)^6)/15 - 1/40)/(a^3*c*f*cos(e/2 + (f*x)/2)^5*sin(e/2
+ (f*x)/2))
```

3.37 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 100

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx &= \frac{x}{a^3c^2}+\frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f}\\ &\quad-\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f}+\frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f}\end{aligned}$$

```
[Out] x/a^3/c^2+1/15*cot(f*x+e)*(15-8*sec(f*x+e))/a^3/c^2/f-1/15*cot(f*x+e)^3*(5-
4*sec(f*x+e))/a^3/c^2/f+1/5*cot(f*x+e)^5*(1-sec(f*x+e))/a^3/c^2/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3967, 8}

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx &= \frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f}-\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f}\\ &\quad+\frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f}+\frac{x}{a^3c^2}\end{aligned}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] x/(a^3*c^2) + (Cot[e + f*x]*(15 - 8*Sec[e + f*x]))/(15*a^3*c^2*f) - (Cot[e
+ f*x]^3*(5 - 4*Sec[e + f*x]))/(15*a^3*c^2*f) + (Cot[e + f*x]^5*(1 - Sec[e
+ f*x]))/(5*a^3*c^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))\,dx}{a^3c^3}\\
&= \frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f} - \frac{\int \cot^4(e+fx)(-5c+4c\sec(e+fx))\,dx}{5a^3c^3}\\
&= -\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f} + \frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f}\\
&\quad - \frac{\int \cot^2(e+fx)(15c-8c\sec(e+fx))\,dx}{15a^3c^3}\\
&= \frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} - \frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f}\\
&\quad + \frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f} - \frac{\int -15c\,dx}{15a^3c^3}\\
&= \frac{x}{a^3c^2} + \frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f}\\
&\quad - \frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f} + \frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$$
$$= \frac{\cot^5(e+fx)\left(3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)-15\sec(e+fx)+20\sec^3(e+fx)-8\,\text{se}\right.}{15a^3c^2f}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Cot[e + f*x]^5*(3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2] - 15*S
ec[e + f*x] + 20*Sec[e + f*x]^3 - 8*Sec[e + f*x]^5))/(15*a^3*c^2*f)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$\frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-5\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+240fx+90\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{240f\,a^3c^2}$	78
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-16\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^2a^3}$	88
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-16\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^2a^3}$	88
risch	$\frac{x}{a^3c^2}-\frac{2i\left(15\,e^{7i(fx+e)}-15\,e^{6i(fx+e)}-65\,e^{5i(fx+e)}-25\,e^{4i(fx+e)}+73\,e^{3i(fx+e)}+31\,e^{2i(fx+e)}-31\,e^{i(fx+e)}-23\right)}{15f\,c^2a^3\left(e^{i(fx+e)}+1\right)^5\left(e^{i(fx+e)}-1\right)^3}$	127
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{ca}-\frac{1}{48acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{8acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{8acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{80acf}}{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	138

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*(-3*tan(1/2*f*x+1/2*e)^5-5*cot(1/2*f*x+1/2*e)^3+30*tan(1/2*f*x+1/2*e)
^3+240*f*x+90*cot(1/2*f*x+1/2*e)-240*tan(1/2*f*x+1/2*e))/f/a^3/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.54

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{23\cos(fx+e)^4+8\cos(fx+e)^3-27\cos(fx+e)^2+15\left(fx\cos(fx+e)^3+fx\cos(fx+e)^2-fx\cos\right.}{15\left(a^3c^2f\cos(fx+e)^3+a^3c^2f\cos(fx+e)^2-a^3c^2f\cos(fx+e)-a^3c\right.}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(f*x + e)^4 + 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*co
s(f*x + e)^3 + f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) - f*x)*sin(f*x + e) -
7*cos(f*x + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 -
 a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{\sec^5(e+fx)+\sec^4(e+fx)-2\sec^3(e+fx)-2\sec^2(e+fx)+\sec(e+fx)+1}\,dx}{a^3c^2}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**3 - 2*sec(e
 + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.46

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{\frac{3\left(\frac{80\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^2}-\frac{480\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^2}-\frac{5\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{a^3c^2\sin(fx+e)^3}}{240f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(3*(80*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) - 480*arctan(s
in(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^2) - 5*(18*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\frac{240\,(fx+e)}{a^3c^2}+\frac{5\left(18\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)}{a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}-\frac{3\left(a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+80\,a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}c^{10}}}{240\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)/(a^3*c^2) + 5*(18*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^2
*tan(1/2*f*x + 1/2*e)^3) - 3*(a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^8
*tan(1/2*f*x + 1/2*e)^3 + 80*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))/f
```

Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{5\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-30\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+240\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-\cdots}{240\,a^3\,c^2\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\cdots\right.}$$

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2),x)
```

```
[Out] -(5*cos(e/2 + (f*x)/2)^8 + 3*sin(e/2 + (f*x)/2)^8 - 30*cos(e/2 + (f*x)/2)^2
*sin(e/2 + (f*x)/2)^6 + 240*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^4 - 90*
cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^2 - 240*cos(e/2 + (f*x)/2)^5*sin(e/
2 + (f*x)/2)^3*(e + f*x))/(240*a^3*c^2*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*
x)/2)^3)
```

3.38 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$

Optimal result 315
Rubi [A] (verified) 315
Mathematica [C] (verified) 316
Maple [A] (verified) 317
Fricas [A] (verification not implemented) 317
Sympy [F] 318
Maxima [A] (verification not implemented) 318
Giac [B] (verification not implemented) 318
Mupad [B] (verification not implemented) 319

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{x}{a^3c^3}+\frac{\cot(e+fx)}{a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot^5(e+fx)}{5a^3c^3f}$$

```
[Out] x/a^3/c^3+cot(f*x+e)/a^3/c^3/f-1/3*cot(f*x+e)^3/a^3/c^3/f+1/5*cot(f*x+e)^5/
a^3/c^3/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3554, 8}

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\cot^5(e+fx)}{5a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot(e+fx)}{a^3c^3f}+\frac{x}{a^3c^3}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] x/(a^3*c^3) + Cot[e + f*x]/(a^3*c^3*f) - Cot[e + f*x]^3/(3*a^3*c^3*f) + Cot
[e + f*x]^5/(5*a^3*c^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \cot^6(e+fx)\,dx}{a^3c^3}\\
&= \frac{\cot^5(e+fx)}{5a^3c^3f}+\frac{\int \cot^4(e+fx)\,dx}{a^3c^3}\\
&= -\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot^5(e+fx)}{5a^3c^3f}-\frac{\int \cot^2(e+fx)\,dx}{a^3c^3}\\
&= \frac{\cot(e+fx)}{a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot^5(e+fx)}{5a^3c^3f}+\frac{\int 1\,dx}{a^3c^3}\\
&= \frac{x}{a^3c^3}+\frac{\cot(e+fx)}{a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot^5(e+fx)}{5a^3c^3f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$
\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx
= \frac{\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5a^3c^3f}
$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/(5*a^3*c
^3*f)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)+\frac{\pi}{2}-\operatorname{arccot}(\cot(fx+e))}{c^3a^3f}$	53
parallelrisch	$\frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5+35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+480fx-330\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+330\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{480f\,c^3a^3}$	91
risch	$\frac{x}{a^3c^3}+\frac{2i\left(45\,\mathrm{e}^{8i(fx+e)}-90\,\mathrm{e}^{6i(fx+e)}+140\,\mathrm{e}^{4i(fx+e)}-70\,\mathrm{e}^{2i(fx+e)}+23\right)}{15f\,c^3a^3\left(\mathrm{e}^{i(fx+e)}+1\right)^5\left(\mathrm{e}^{i(fx+e)}-1\right)^5}$	94
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{ca}+\frac{1}{160acf}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{96acf}+\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{16acf}-\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{16acf}+\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{96acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{160acf}}{a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	160

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3/a^3/f*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)+1/2*Pi-arccot(c
ot(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$$
$$=\frac{23\cos(fx+e)^5-35\cos(fx+e)^3+15\left(fx\cos(fx+e)^4-2\,fx\cos(fx+e)^2+fx\right)\sin(fx+e)+15\,c}{15\left(a^3c^3f\cos(fx+e)^4-2\,a^3c^3f\cos(fx+e)^2+a^3c^3f\right)\sin(fx+e)}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(f*x + e)^5 - 35*cos(f*x + e)^3 + 15*(f*x*cos(f*x + e)^4 - 2*f*
x*cos(f*x + e)^2 + f*x)*sin(f*x + e) + 15*cos(f*x + e))/((a^3*c^3*f*cos(f*x
 + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{\sec^6(e+fx)-3\sec^4(e+fx)+3\sec^2(e+fx)-1}\,dx}{a^3c^3}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
[Out] -Integral(1/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x)**2 - 1),
x)/(a**3*c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)}{a^3c^3} + \frac{15\,\tan(fx+e)^4-5\,\tan(fx+e)^2+3}{a^3c^3\tan(fx+e)^5}}{15\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
[Out] 1/15*(15*(f*x + e)/(a^3*c^3) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/(
a^3*c^3*tan(f*x + e)^5))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.93

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\frac{480\,(fx+e)}{a^3c^3} + \frac{330\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-35\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3}{a^3c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5} - \frac{3\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+330\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}c^{15}}}{480\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
[Out] 1/480*(480*(f*x + e)/(a^3*c^3) + (330*tan(1/2*f*x + 1/2*e)^4 - 35*tan(1/2*f
*x + 1/2*e)^2 + 3)/(a^3*c^3*tan(1/2*f*x + 1/2*e)^5) - (3*a^12*c^12*tan(1/2*
f*x + 1/2*e)^5 - 35*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 330*a^12*c^12*tan(1/
2*f*x + 1/2*e))/(a^15*c^15))/f
```

Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\frac{5\,\cos(e+f\,x)}{24}-\frac{5\,\cos(3\,e+3\,f\,x)}{48}+\frac{23\,\cos(5\,e+5\,f\,x)}{240}-\frac{5\,\sin(3\,e+3\,f\,x)\,(e+f\,x)}{16}+\frac{\sin(5\,e+5\,f\,x)\,(e+f\,x)}{16}+\frac{5\,\sin(e+f\,x)\,(e+f\,x)}{8}}{a^3\,c^3\,f\,{\sin\left(e+f\,x\right)}^5}$$

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3),x)
```

```
[Out] ((5*cos(e + f*x))/24 - (5*cos(3*e + 3*f*x))/48 + (23*cos(5*e + 5*f*x))/240
- (5*sin(3*e + 3*f*x)*(e + f*x))/16 + (sin(5*e + 5*f*x)*(e + f*x))/16 + (5*
sin(e + f*x)*(e + f*x))/8)/(a^3*c^3*f*sin(e + f*x)^5)
```

3.39 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$

Optimal result 320
Rubi [A] (verified) 320
Mathematica [C] (verified) 322
Maple [A] (verified) 323
Fricas [A] (verification not implemented) 323
Sympy [F] 324
Maxima [A] (verification not implemented) 324
Giac [A] (verification not implemented) 324
Mupad [B] (verification not implemented) 325

Optimal result

Integrand size = 26, antiderivative size = 129

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{x}{a^3c^4}-\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f}+\frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f}+\frac{\cot(e+fx)(35+16\sec(e+fx))}{35a^3c^4f}-\frac{\cot^3(e+fx)(35+24\sec(e+fx))}{105a^3c^4f}$$

```
[Out] x/a^3/c^4-1/7*cot(f*x+e)^7*(1+sec(f*x+e))/a^3/c^4/f+1/35*cot(f*x+e)^5*(7+6*
sec(f*x+e))/a^3/c^4/f+1/35*cot(f*x+e)*(35+16*sec(f*x+e))/a^3/c^4/f-1/105*co
t(f*x+e)^3*(35+24*sec(f*x+e))/a^3/c^4/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3967, 8}

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{\cot^7(e+fx)(\sec(e+fx)+1)}{7a^3c^4f}+\frac{\cot^5(e+fx)(6\sec(e+fx)+7)}{35a^3c^4f}-\frac{\cot^3(e+fx)(24\sec(e+fx)+35)}{105a^3c^4f}+\frac{\cot(e+fx)(16\sec(e+fx)+35)}{35a^3c^4f}+\frac{x}{a^3c^4}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] x/(a^3*c^4) - (Cot[e + f*x]^7*(1 + Sec[e + f*x]))/(7*a^3*c^4*f) + (Cot[e +
f*x]^5*(7 + 6*Sec[e + f*x]))/(35*a^3*c^4*f) + (Cot[e + f*x]*(35 + 16*Sec[e
+ f*x]))/(35*a^3*c^4*f) - (Cot[e + f*x]^3*(35 + 24*Sec[e + f*x]))/(105*a^3*
c^4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))\,dx}{a^4c^4} \\
&= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\int \cot^6(e+fx)(-7a-6a\sec(e+fx))\,dx}{7a^4c^4} \\
&= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f} \\
&\quad + \frac{\int \cot^4(e+fx)(35a+24a\sec(e+fx))\,dx}{35a^4c^4} \\
&= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f} \\
&\quad - \frac{\cot^3(e+fx)(35+24\sec(e+fx))}{105a^3c^4f} \\
&\quad + \frac{\int \cot^2(e+fx)(-105a-48a\sec(e+fx))\,dx}{105a^4c^4}
\end{aligned}$$

$$= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f} + \frac{\cot(e+fx)(35+16\sec(e+fx))}{35a^3c^4f} - \frac{\cot^3(e+fx)(35+24\sec(e+fx))}{105a^3c^4f} + \frac{\int 105a\,dx}{105a^4c^4}$$

$$= \frac{x}{a^3c^4} - \frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f} + \frac{\cot(e+fx)(35+16\sec(e+fx))}{35a^3c^4f} - \frac{\cot^3(e+fx)(35+24\sec(e+fx))}{105a^3c^4f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{\csc^7(e+fx)\left(-106+301\cos(2(e+fx))-70\cos(4(e+fx))+35\cos(6(e+fx))+160\cos^7(e+fx)\,\mathrm{H}\right.}{1120a^3c^4f}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] -1/1120*(Csc[e + f*x]^7*(-106 + 301*Cos[2*(e + f*x)] - 70*Cos[4*(e + f*x)]
+ 35*Cos[6*(e + f*x)] + 160*Cos[e + f*x]^7*Hypergeometric2F1[-7/2, 1, -5/2,
 -Tan[e + f*x]^2]))/(a^3*c^4*f)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{-15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}-21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+168\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+280\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-1015\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+6720fx-3045\tan\left(\frac{f}{2}\right.}{6720f\,a^{3}c^{4}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{8}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{29}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{64}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+128\,a}{64f\,c^{4}a^{3}}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{8}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{29}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{64}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+128\,a}{64f\,c^{4}a^{3}}$
risch	$\frac{x}{a^{3}c^{4}}+\frac{2i\left(105\,\mathrm{e}^{11i(fx+e)}+210\,\mathrm{e}^{10i(fx+e)}-735\,\mathrm{e}^{9i(fx+e)}+1638\,\mathrm{e}^{7i(fx+e)}-196\,\mathrm{e}^{6i(fx+e)}-1882\,\mathrm{e}^{5i(fx+e)}+880\,\mathrm{e}^{4i(fx+}\right.}{105f\,c^{4}a^{3}\left(\mathrm{e}^{i(fx+e)}-1\right)^{7}\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}}$
norman	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{acf}+\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{ca}-\frac{1}{448acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{40acf}-\frac{29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{192acf}-\frac{29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{64acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{24acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{320acf}}{a^{2}c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6720*(-15*cot(1/2*f*x+1/2*e)^7-21*tan(1/2*f*x+1/2*e)^5+168*cot(1/2*f*x+1/
2*e)^5+280*tan(1/2*f*x+1/2*e)^3-1015*cot(1/2*f*x+1/2*e)^3+6720*f*x-3045*tan
(1/2*f*x+1/2*e)+6720*cot(1/2*f*x+1/2*e))/f/a^3/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.80

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{176\,\cos\left(fx+e\right)^{6}-71\,\cos\left(fx+e\right)^{5}-335\,\cos\left(fx+e\right)^{4}+125\,\cos\left(fx+e\right)^{3}+225\,\cos\left(fx+e\right)^{2}+105}{105\,\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}-a^{3}c^{4}f\cos\left(fx+e\right)^{4}-2\,a\right.}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(176*cos(f*x + e)^6 - 71*cos(f*x + e)^5 - 335*cos(f*x + e)^4 + 125*co
s(f*x + e)^3 + 225*cos(f*x + e)^2 + 105*(f*x*cos(f*x + e)^5 - f*x*cos(f*x +
 e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 + f*x*cos(f*x + e) - f*
x)*sin(f*x + e) - 57*cos(f*x + e) - 48)/((a^3*c^4*f*cos(f*x + e)^5 - a^3*c^
4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*f*cos(f*x + e)^
2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{1}{\sec^7(e+fx)-\sec^6(e+fx)-3\sec^5(e+fx)+3\sec^4(e+fx)+3\sec^3(e+fx)-3\sec^2(e+fx)-\sec(e+fx)+1}\,dx}{a^3c^4}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(1/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)**5 + 3*sec(e
 + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f*x) + 1), x)/
(a**3*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{\frac{7\left(\frac{435\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^4}-\frac{13440\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^4}-\frac{\left(\frac{168\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1015\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6720\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\cdots}{a^3c^4\sin(fx+e)^7}}{6720f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/6720*(7*(435*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^4) - 13440*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^4) - (168*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 1015*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6720*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(a^3*c^4*sin(f*x + e)
^7))/f
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\frac{6720(fx+e)}{a^3c^4}+\frac{6720\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-1015\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+168\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-15}{a^3c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}-\frac{7\left(3a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-40a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}\cdots\right.\right.}{a^{15}c^{20}}}{6720f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/6720*(6720*(f*x + e)/(a^3*c^4) + (6720*tan(1/2*f*x + 1/2*e)^6 - 1015*tan(
1/2*f*x + 1/2*e)^4 + 168*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^3*c^4*tan(1/2*f*x
+ 1/2*e)^7) - 7*(3*a^12*c^16*tan(1/2*f*x + 1/2*e)^5 - 40*a^12*c^16*tan(1/2*
f*x + 1/2*e)^3 + 435*a^12*c^16*tan(1/2*f*x + 1/2*e))/(a^15*c^20))/f
```

Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.62

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{15\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+21\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-280\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+3045\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\sin\left(\frac{e}{2}+\right.}{}$$

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4),x)
```

```
[Out] -(15*cos(e/2 + (f*x)/2)^12 + 21*sin(e/2 + (f*x)/2)^12 - 280*cos(e/2 + (f*x)
/2)^2*sin(e/2 + (f*x)/2)^10 + 3045*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^
8 - 6720*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^6 + 1015*cos(e/2 + (f*x)/2
)^8*sin(e/2 + (f*x)/2)^4 - 168*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)^2 -
 6720*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^7*(e + f*x))/(6720*a^3*c^4*f*
cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^7)
```

3.40 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 210

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{x}{a^3c^5}+\frac{\cot(e+fx)}{a^3c^5f}-\frac{\cot^3(e+fx)}{3a^3c^5f}+\frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc(e+fx)}{a^3c^5f}-\frac{8\csc^3(e+fx)}{3a^3c^5f}+\frac{12\csc^5(e+fx)}{5a^3c^5f}-\frac{8\csc^7(e+fx)}{7a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}$$

```
[Out] x/a^3/c^5+cot(f*x+e)/a^3/c^5/f-1/3*cot(f*x+e)^3/a^3/c^5/f+1/5*cot(f*x+e)^5/
a^3/c^5/f-1/7*cot(f*x+e)^7/a^3/c^5/f+2/9*cot(f*x+e)^9/a^3/c^5/f+2*csc(f*x+e
)/a^3/c^5/f-8/3*csc(f*x+e)^3/a^3/c^5/f+12/5*csc(f*x+e)^5/a^3/c^5/f-8/7*csc(
f*x+e)^7/a^3/c^5/f+2/9*csc(f*x+e)^9/a^3/c^5/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30}

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{2\cot^9(e+fx)}{9a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^3(e+fx)}{3a^3c^5f}+\frac{\cot(e+fx)}{a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{8\csc^7(e+fx)}{7a^3c^5f}+\frac{12\csc^5(e+fx)}{5a^3c^5f}-\frac{8\csc^3(e+fx)}{3a^3c^5f}+\frac{2\csc(e+fx)}{a^3c^5f}+\frac{x}{a^3c^5}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] x/(a^3*c^5) + Cot[e + f*x]/(a^3*c^5*f) - Cot[e + f*x]^3/(3*a^3*c^5*f) + Cot
[e + f*x]^5/(5*a^3*c^5*f) - Cot[e + f*x]^7/(7*a^3*c^5*f) + (2*Cot[e + f*x]^
9)/(9*a^3*c^5*f) + (2*Csc[e + f*x])/(a^3*c^5*f) - (8*Csc[e + f*x]^3)/(3*a^3
*c^5*f) + (12*Csc[e + f*x]^5)/(5*a^3*c^5*f) - (8*Csc[e + f*x]^7)/(7*a^3*c^5
*f) + (2*Csc[e + f*x]^9)/(9*a^3*c^5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \cot^{10}(e+fx)(a+a\sec(e+fx))^2\,dx}{a^5c^5} \\
&= -\frac{\int \left(a^2\cot^{10}(e+fx)+2a^2\cot^9(e+fx)\csc(e+fx)+a^2\cot^8(e+fx)\csc^2(e+fx)\right)\,dx}{a^5c^5} \\
&= -\frac{\int \cot^{10}(e+fx)\,dx}{a^3c^5}-\frac{\int \cot^8(e+fx)\csc^2(e+fx)\,dx}{a^3c^5}-\frac{2\int \cot^9(e+fx)\csc(e+fx)\,dx}{a^3c^5} \\
&= \frac{\cot^9(e+fx)}{9a^3c^5f}+\frac{\int \cot^8(e+fx)\,dx}{a^3c^5}-\frac{\text{Subst}\left(\int x^8\,dx,x,-\cot(e+fx)\right)}{a^3c^5f} \\
&\quad+\frac{2\text{Subst}\left(\int \left(-1+x^2\right)^4\,dx,x,\csc(e+fx)\right)}{a^3c^5f} \\
&= -\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{2\cot^9(e+fx)}{9a^3c^5f}-\frac{\int \cot^6(e+fx)\,dx}{a^3c^5} \\
&\quad+\frac{2\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,x,\csc(e+fx)\right)}{a^3c^5f} \\
&= \frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc(e+fx)}{a^3c^5f}-\frac{8\csc^3(e+fx)}{3a^3c^5f} \\
&\quad+\frac{12\csc^5(e+fx)}{5a^3c^5f}-\frac{8\csc^7(e+fx)}{7a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}+\frac{\int \cot^4(e+fx)\,dx}{a^3c^5} \\
&= -\frac{\cot^3(e+fx)}{3a^3c^5f}+\frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc(e+fx)}{a^3c^5f} \\
&\quad-\frac{8\csc^3(e+fx)}{3a^3c^5f}+\frac{12\csc^5(e+fx)}{5a^3c^5f}-\frac{8\csc^7(e+fx)}{7a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{\int \cot^2(e+fx)\,dx}{a^3c^5} \\
&= \frac{\cot(e+fx)}{a^3c^5f}-\frac{\cot^3(e+fx)}{3a^3c^5f}+\frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{2\cot^9(e+fx)}{9a^3c^5f} \\
&\quad+\frac{2\csc(e+fx)}{a^3c^5f}-\frac{8\csc^3(e+fx)}{3a^3c^5f}+\frac{12\csc^5(e+fx)}{5a^3c^5f}-\frac{8\csc^7(e+fx)}{7a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f} \\
&\quad+\frac{\int 1\,dx}{a^3c^5}
\end{aligned}$$

$$= \frac{x}{a^3c^5} + \frac{\cot(e+fx)}{a^3c^5f} - \frac{\cot^3(e+fx)}{3a^3c^5f} + \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{2\cot^9(e+fx)}{9a^3c^5f} + \frac{2\csc(e+fx)}{a^3c^5f} - \frac{8\csc^3(e+fx)}{3a^3c^5f} + \frac{12\csc^5(e+fx)}{5a^3c^5f} - \frac{8\csc^7(e+fx)}{7a^3c^5f} + \frac{2\csc^9(e+fx)}{9a^3c^5f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\cot^9(e+fx)\left(35+35\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(e+fx)\right)+630\sec(e+fx)-1680\sec^3(e-\right.}{315a^3c^5f}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] (Cot[e + f*x]^9*(35 + 35*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2]
+ 630*Sec[e + f*x] - 1680*Sec[e + f*x]^3 + 2016*Sec[e + f*x]^5 - 1152*Sec[e
 + f*x]^7 + 256*Sec[e + f*x]^9))/(315*a^3*c^5*f)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.56

method	result
parallelrisch	$\frac{35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^9-405\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7-63\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+2331\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5+945\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-9765\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+40}{40320f\,a^3c^5}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+256\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{9}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{}{5\tan(}}{128f\,c^5a^3}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+256\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{9}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{}{5\tan(}}{128f\,c^5a^3}$
risch	$\frac{x}{a^3c^5}+\frac{4i\left(315\,e^{13i(fx+e)}-315\,e^{12i(fx+e)}-1470\,e^{11i(fx+e)}+3360\,e^{10i(fx+e)}+1113\,e^{9i(fx+e)}-6447\,e^{8i(fx+e)}+2028\,e^{7}\right.}{315f\,c^5a^3\left(e^{i(fx+e)}-\right.}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{ca}+\frac{1}{1152acf}-\frac{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{896acf}+\frac{37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{640acf}-\frac{31\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{128acf}+\frac{163\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{128acf}-\frac{37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{128acf}+\frac{3\tan}{}}{c^4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/40320*(35*cot(1/2*f*x+1/2*e)^9-405*cot(1/2*f*x+1/2*e)^7-63*tan(1/2*f*x+1/
2*e)^5+2331*cot(1/2*f*x+1/2*e)^5+945*tan(1/2*f*x+1/2*e)^3-9765*cot(1/2*f*x+
```

```
1/2*e)^3+40320*f*x-11655*tan(1/2*f*x+1/2*e)+51345*cot(1/2*f*x+1/2*e))/f/a^3
/c^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{598\cos(fx+e)^7-566\cos(fx+e)^6-1212\cos(fx+e)^5+1310\cos(fx+e)^4+860\cos(fx+e)^3-10}{315\left(a^3c^5f\cos(fx+e)^6-2a^3c^5f\cos(f\right.}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(598*cos(f*x + e)^7 - 566*cos(f*x + e)^6 - 1212*cos(f*x + e)^5 + 1310
*cos(f*x + e)^4 + 860*cos(f*x + e)^3 - 1014*cos(f*x + e)^2 + 315*(f*x*cos(f
*x + e)^6 - 2*f*x*cos(f*x + e)^5 - f*x*cos(f*x + e)^4 + 4*f*x*cos(f*x + e)^
3 - f*x*cos(f*x + e)^2 - 2*f*x*cos(f*x + e) + f*x)*sin(f*x + e) - 197*cos(f
*x + e) + 256)/((a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5 - a^
3*c^5*f*cos(f*x + e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f*x + e
)^2 - 2*a^3*c^5*f*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = -\frac{\int \frac{1}{\sec^8(e+fx)-2\sec^7(e+fx)-2\sec^6(e+fx)+6\sec^5(e+fx)-6\sec^3(e+fx)+2\sec^2(e+fx)+2\sec(e+fx)-1}\,dx}{a^3c^5}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -Integral(1/(sec(e + f*x)**8 - 2*sec(e + f*x)**7 - 2*sec(e + f*x)**6 + 6*se
c(e + f*x)**5 - 6*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + 2*sec(e + f*x) - 1)
, x)/(a**3*c**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = -\frac{\frac{63\left(\frac{185\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{15\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^5}-\frac{80640\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^5}+\frac{\left(\frac{405\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2331\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{9765\,\sin(fx+e}{(\cos(fx+e)+1}\right.}{a^3c^5\sin(fx}}{40320\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -1/40320*(63*(185*sin(f*x + e)/(cos(f*x + e) + 1) - 15*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - 80640*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^5) + (405*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 2331*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9765*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 51345*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35
)*(cos(f*x + e) + 1)^9/(a^3*c^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{\frac{40320\,(fx+e)}{a^3c^5}+\frac{51345\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-9765\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+2331\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-405\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35}{a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}-\frac{63\left(a^{12}c^{20}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}}{40320\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/40320*(40320*(f*x + e)/(a^3*c^5) + (51345*tan(1/2*f*x + 1/2*e)^8 - 9765*t
an(1/2*f*x + 1/2*e)^6 + 2331*tan(1/2*f*x + 1/2*e)^4 - 405*tan(1/2*f*x + 1/2
*e)^2 + 35)/(a^3*c^5*tan(1/2*f*x + 1/2*e)^9) - 63*(a^12*c^20*tan(1/2*f*x +
1/2*e)^5 - 15*a^12*c^20*tan(1/2*f*x + 1/2*e)^3 + 185*a^12*c^20*tan(1/2*f*x
+ 1/2*e))/(a^15*c^25))/f
```

Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{35\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}-63\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}+945\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-11655\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\sin\left(\frac{e}{2}+\frac{fx}{2}\right.}{}$$

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (35*cos(e/2 + (f*x)/2)^14 - 63*sin(e/2 + (f*x)/2)^14 + 945*cos(e/2 + (f*x)/
2)^2*sin(e/2 + (f*x)/2)^12 - 11655*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^
10 + 51345*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^8 - 9765*cos(e/2 + (f*x)
/2)^8*sin(e/2 + (f*x)/2)^6 + 2331*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)^
4 - 405*cos(e/2 + (f*x)/2)^12*sin(e/2 + (f*x)/2)^2 + 40320*cos(e/2 + (f*x)/
2)^5*sin(e/2 + (f*x)/2)^9*(e + f*x))/(40320*a^3*c^5*f*cos(e/2 + (f*x)/2)^5*
sin(e/2 + (f*x)/2)^9)
```

3.41 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 252

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx &= \frac{x}{a^3c^6}+\frac{\cot(e+fx)}{a^3c^6f}-\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}\\&+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{3\csc(e+fx)}{a^3c^6f}-\frac{16\csc^3(e+fx)}{3a^3c^6f}\\&+\frac{34\csc^5(e+fx)}{5a^3c^6f}-\frac{36\csc^7(e+fx)}{7a^3c^6f}+\frac{19\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}\end{aligned}$$

```
[Out] x/a^3/c^6+cot(f*x+e)/a^3/c^6/f-1/3*cot(f*x+e)^3/a^3/c^6/f+1/5*cot(f*x+e)^5/
a^3/c^6/f-1/7*cot(f*x+e)^7/a^3/c^6/f+1/9*cot(f*x+e)^9/a^3/c^6/f-4/11*cot(f*
x+e)^11/a^3/c^6/f+3*csc(f*x+e)/a^3/c^6/f-16/3*csc(f*x+e)^3/a^3/c^6/f+34/5*c
sc(f*x+e)^5/a^3/c^6/f-36/7*csc(f*x+e)^7/a^3/c^6/f+19/9*csc(f*x+e)^9/a^3/c^6
/f-4/11*csc(f*x+e)^11/a^3/c^6/f
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used

$= \{3989, 3971, 3554, 8, 2686, 200, 2687, 30, 276\}$

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$$
$$= -\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}$$
$$-\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot(e+fx)}{a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}+\frac{19\csc^9(e+fx)}{9a^3c^6f}$$
$$-\frac{36\csc^7(e+fx)}{7a^3c^6f}+\frac{34\csc^5(e+fx)}{5a^3c^6f}-\frac{16\csc^3(e+fx)}{3a^3c^6f}+\frac{3\csc(e+fx)}{a^3c^6f}+\frac{x}{a^3c^6}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] x/(a^3*c^6) + Cot[e + f*x]/(a^3*c^6*f) - Cot[e + f*x]^3/(3*a^3*c^6*f) + Cot
[e + f*x]^5/(5*a^3*c^6*f) - Cot[e + f*x]^7/(7*a^3*c^6*f) + Cot[e + f*x]^9/(
9*a^3*c^6*f) - (4*Cot[e + f*x]^11)/(11*a^3*c^6*f) + (3*Csc[e + f*x])/(a^3*c
^6*f) - (16*Csc[e + f*x]^3)/(3*a^3*c^6*f) + (34*Csc[e + f*x]^5)/(5*a^3*c^6*
f) - (36*Csc[e + f*x]^7)/(7*a^3*c^6*f) + (19*Csc[e + f*x]^9)/(9*a^3*c^6*f)
- (4*Csc[e + f*x]^11)/(11*a^3*c^6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cot^{12}(e+fx)(a+a\sec(e+fx))^3\,dx}{a^6c^6}\\
&= \frac{\int \left(a^3\cot^{12}(e+fx)+3a^3\cot^{11}(e+fx)\csc(e+fx)+3a^3\cot^{10}(e+fx)\csc^2(e+fx)+a^3\cot^9(e+\right.}{a^6c^6}\\
&= \frac{\int \cot^{12}(e+fx)\,dx}{a^3c^6}+\frac{\int \cot^9(e+fx)\csc^3(e+fx)\,dx}{a^3c^6}\\
&\quad+\frac{3\int \cot^{11}(e+fx)\csc(e+fx)\,dx}{a^3c^6}+\frac{3\int \cot^{10}(e+fx)\csc^2(e+fx)\,dx}{a^3c^6}\\
&= -\frac{\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\int \cot^{10}(e+fx)\,dx}{a^3c^6}-\frac{\text{Subst}\left(\int x^2(-1+x^2)^4\,dx,x,\csc(e+fx)\right)}{a^3c^6f}\\
&\quad+\frac{3\text{Subst}\left(\int x^{10}\,dx,x,-\cot(e+fx)\right)}{a^3c^6f}-\frac{3\text{Subst}\left(\int (-1+x^2)^5\,dx,x,\csc(e+fx)\right)}{a^3c^6f}
\end{aligned}
$$

$$= \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{4\cot^{11}(e+fx)}{11a^3c^6f} + \frac{\int \cot^8(e+fx)\,dx}{a^3c^6} - \frac{\text{Subst}\left(\int \left(x^2-4x^4+6x^6-4x^8+x^{10}\right)\,dx, x, \csc(e+fx)\right)}{a^3c^6f} - \frac{3\text{Subst}\left(\int \left(-1+5x^2-10x^4+10x^6-5x^8+x^{10}\right)\,dx, x, \csc(e+fx)\right)}{a^3c^6f}$$

$$= -\frac{\cot^7(e+fx)}{7a^3c^6f} + \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{4\cot^{11}(e+fx)}{11a^3c^6f} + \frac{3\csc(e+fx)}{a^3c^6f} - \frac{16\csc^3(e+fx)}{3a^3c^6f} + \frac{34\csc^5(e+fx)}{5a^3c^6f} - \frac{36\csc^7(e+fx)}{7a^3c^6f} + \frac{19\csc^9(e+fx)}{9a^3c^6f} - \frac{4\csc^{11}(e+fx)}{11a^3c^6f} - \frac{\int \cot^6(e+fx)\,dx}{a^3c^6}$$

$$= \frac{\cot^5(e+fx)}{5a^3c^6f} - \frac{\cot^7(e+fx)}{7a^3c^6f} + \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{4\cot^{11}(e+fx)}{11a^3c^6f} + \frac{3\csc(e+fx)}{a^3c^6f} - \frac{16\csc^3(e+fx)}{3a^3c^6f} + \frac{34\csc^5(e+fx)}{5a^3c^6f} - \frac{36\csc^7(e+fx)}{7a^3c^6f} + \frac{19\csc^9(e+fx)}{9a^3c^6f} - \frac{4\csc^{11}(e+fx)}{11a^3c^6f} + \frac{\int \cot^4(e+fx)\,dx}{a^3c^6}$$

$$= -\frac{\cot^3(e+fx)}{3a^3c^6f} + \frac{\cot^5(e+fx)}{5a^3c^6f} - \frac{\cot^7(e+fx)}{7a^3c^6f} + \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{4\cot^{11}(e+fx)}{11a^3c^6f} + \frac{3\csc(e+fx)}{a^3c^6f} - \frac{16\csc^3(e+fx)}{3a^3c^6f} + \frac{34\csc^5(e+fx)}{5a^3c^6f} - \frac{36\csc^7(e+fx)}{7a^3c^6f} + \frac{19\csc^9(e+fx)}{9a^3c^6f} - \frac{4\csc^{11}(e+fx)}{11a^3c^6f} - \frac{\int \cot^2(e+fx)\,dx}{a^3c^6}$$

$$= \frac{\cot(e+fx)}{a^3c^6f} - \frac{\cot^3(e+fx)}{3a^3c^6f} + \frac{\cot^5(e+fx)}{5a^3c^6f} - \frac{\cot^7(e+fx)}{7a^3c^6f} + \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{4\cot^{11}(e+fx)}{11a^3c^6f} + \frac{3\csc(e+fx)}{a^3c^6f} - \frac{16\csc^3(e+fx)}{3a^3c^6f} + \frac{34\csc^5(e+fx)}{5a^3c^6f} - \frac{36\csc^7(e+fx)}{7a^3c^6f} + \frac{19\csc^9(e+fx)}{9a^3c^6f} - \frac{4\csc^{11}(e+fx)}{11a^3c^6f} + \frac{\int 1\,dx}{a^3c^6}$$

$$= \frac{x}{a^3c^6} + \frac{\cot(e+fx)}{a^3c^6f} - \frac{\cot^3(e+fx)}{3a^3c^6f} + \frac{\cot^5(e+fx)}{5a^3c^6f} - \frac{\cot^7(e+fx)}{7a^3c^6f} + \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{4\cot^{11}(e+fx)}{11a^3c^6f} + \frac{3\csc(e+fx)}{a^3c^6f} - \frac{16\csc^3(e+fx)}{3a^3c^6f} + \frac{34\csc^5(e+fx)}{5a^3c^6f} - \frac{36\csc^7(e+fx)}{7a^3c^6f} + \frac{19\csc^9(e+fx)}{9a^3c^6f} - \frac{4\csc^{11}(e+fx)}{11a^3c^6f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 11.12 (sec) , antiderivative size = 787, normalized size of antiderivative = 3.12

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{\cot^9(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{11}{2},1,-\frac{9}{2},-\tan^2(e+fx)\right)}{11a^2c^5f(a+a\sec(e+fx))(c-c\sec(e+fx))} + \frac{16\tan(e+fx)}{55a^3f(c-c\sec(e+fx))^6} - \frac{2a^3\tan(e+fx)}{11f(a+a\sec(e+fx))^6(c-c\sec(e+fx))^6} - \frac{3a^3\sec(e+fx)\tan(e+fx)}{11f(a+a\sec(e+fx))^6(c-c\sec(e+fx))^6} - \frac{a^2\tan(e+fx)}{9f(a+a\sec(e+fx))^5(c-c\sec(e+fx))^6} - \frac{a\tan(e+fx)}{63f(a+a\sec(e+fx))^4(c-c\sec(e+fx))^6} - \frac{\tan(e+fx)}{35f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6} - \frac{8\tan(e+fx)}{105af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^6} - \frac{8\tan(e+fx)}{15a^2f(a+a\sec(e+fx))(c-c\sec(e+fx))^6} + \frac{16\tan(e+fx)}{99a^3cf(c-c\sec(e+fx))^5} - \frac{10a^2\sec(e+fx)\tan(e+fx)}{33cf(a+a\sec(e+fx))^5(c-c\sec(e+fx))^5} + \frac{64\tan(e+fx)}{693a^3c^2f(c-c\sec(e+fx))^4} - \frac{80a\sec(e+fx)\tan(e+fx)}{231c^2f(a+a\sec(e+fx))^4(c-c\sec(e+fx))^4} + \frac{64\tan(e+fx)}{1155a^3c^3f(c-c\sec(e+fx))^3} - \frac{32\sec(e+fx)\tan(e+fx)}{77c^3f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3} + \frac{128\tan(e+fx)}{3465a^3c^4f(c-c\sec(e+fx))^2} - \frac{128\sec(e+fx)\tan(e+fx)}{231ac^4f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2} + \frac{128\tan(e+fx)}{3465a^3c^5f(c-c\sec(e+fx))} - \frac{256\sec(e+fx)\tan(e+fx)}{231a^2c^5f(a+a\sec(e+fx))(c-c\sec(e+fx))}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] (Cot[e + f*x]^9*Hypergeometric2F1[-11/2, 1, -9/2, -Tan[e + f*x]^2])/(11*a^2
*c^5*f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])) + (16*Tan[e + f*x])/(55*a
^3*f*(c - c*Sec[e + f*x])^6) - (2*a^3*Tan[e + f*x])/(11*f*(a + a*Sec[e + f*
x])^6*(c - c*Sec[e + f*x])^6) - (3*a^3*Sec[e + f*x]*Tan[e + f*x])/(11*f*(a
+ a*Sec[e + f*x])^6*(c - c*Sec[e + f*x])^6) - (a^2*Tan[e + f*x])/(9*f*(a +
a*Sec[e + f*x])^5*(c - c*Sec[e + f*x])^6) - (a*Tan[e + f*x])/(63*f*(a + a*S
```

```
ec[e + f*x])^4*(c - c*Sec[e + f*x])^6) - Tan[e + f*x]/(35*f*(a + a*Sec[e +
f*x])^3*(c - c*Sec[e + f*x])^6) - (8*Tan[e + f*x])/(105*a*f*(a + a*Sec[e +
f*x])^2*(c - c*Sec[e + f*x])^6) - (8*Tan[e + f*x])/(15*a^2*f*(a + a*Sec[e +
 f*x])*(c - c*Sec[e + f*x])^6) + (16*Tan[e + f*x])/(99*a^3*c*f*(c - c*Sec[e
 + f*x])^5) - (10*a^2*Sec[e + f*x]*Tan[e + f*x])/(33*c*f*(a + a*Sec[e + f*x
])^5*(c - c*Sec[e + f*x])^5) + (64*Tan[e + f*x])/(693*a^3*c^2*f*(c - c*Sec[
e + f*x])^4) - (80*a*Sec[e + f*x]*Tan[e + f*x])/(231*c^2*f*(a + a*Sec[e + f
*x])^4*(c - c*Sec[e + f*x])^4) + (64*Tan[e + f*x])/(1155*a^3*c^3*f*(c - c*S
ec[e + f*x])^3) - (32*Sec[e + f*x]*Tan[e + f*x])/(77*c^3*f*(a + a*Sec[e + f
*x])^3*(c - c*Sec[e + f*x])^3) + (128*Tan[e + f*x])/(3465*a^3*c^4*f*(c - c*
Sec[e + f*x])^2) - (128*Sec[e + f*x]*Tan[e + f*x])/(231*a*c^4*f*(a + a*Sec[
e + f*x])^2*(c - c*Sec[e + f*x])^2) + (128*Tan[e + f*x])/(3465*a^3*c^5*f*(c
 - c*Sec[e + f*x])) - (256*Sec[e + f*x]*Tan[e + f*x])/(231*a^2*c^5*f*(a + a
*Sec[e + f*x])*(c - c*Sec[e + f*x]))
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.52

method	result
parallelrisch	$\frac{-315 \cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}+3850 \cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-22770 \cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}-693 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+90090 \cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+11550 \tan\left(\frac{fx}{2}+\right.}{887040 f\, a^3 c^6}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{10 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-46 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)+512 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{11 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{10}{9 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{}{7 \tan\left(\right.}}{256 f\, a^3 c^6}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{10 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-46 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)+512 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{11 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{10}{9 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{}{7 \tan\left(\right.}}{256 f\, a^3 c^6}$
risch	$\frac{x}{a^3 c^6}+\frac{2i\left(10395\, e^{15i(fx+e)}-31185\, e^{14i(fx+e)}+1155\, e^{13i(fx+e)}+148995\, e^{12i(fx+e)}-190113\, e^{11i(fx+e)}-117117\, e^{10i(fx+\right.}{}$
norman	$\frac{\frac{x \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{ca}-\frac{1}{2816acf}+\frac{5 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{1152acf}-\frac{23 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{896acf}+\frac{13 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{128acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{3acf}+\frac{191 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{128acf}-\frac{23 \tan\left(\right.}{12}}{c^5 a^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}$

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/887040*(-315*cot(1/2*f*x+1/2*e)^11+3850*cot(1/2*f*x+1/2*e)^9-22770*cot(1/
2*f*x+1/2*e)^7-693*tan(1/2*f*x+1/2*e)^5+90090*cot(1/2*f*x+1/2*e)^5+11550*ta
n(1/2*f*x+1/2*e)^3-295680*cot(1/2*f*x+1/2*e)^3+887040*f*x-159390*tan(1/2*f*
x+1/2*e)+1323630*cot(1/2*f*x+1/2*e))/f/a^3/c^6
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{7453\cos(fx+e)^8-11964\cos(fx+e)^7-11866\cos(fx+e)^6+30542\cos(fx+e)^5+90\cos(fx+e)}{3465\left(a^3c^6f\cos(fx+e)^7\right.}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/3465*(7453*cos(f*x + e)^8 - 11964*cos(f*x + e)^7 - 11866*cos(f*x + e)^6 +
 30542*cos(f*x + e)^5 + 90*cos(f*x + e)^4 - 26438*cos(f*x + e)^3 + 8539*cos
(f*x + e)^2 + 3465*(f*x*cos(f*x + e)^7 - 3*f*x*cos(f*x + e)^6 + f*x*cos(f*x
 + e)^5 + 5*f*x*cos(f*x + e)^4 - 5*f*x*cos(f*x + e)^3 - f*x*cos(f*x + e)^2
+ 3*f*x*cos(f*x + e) - f*x)*sin(f*x + e) + 7671*cos(f*x + e) - 3712)/((a^3*
c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(f*x + e)^6 + a^3*c^6*f*cos(f*x + e)^
5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3*c^6*f*cos(f*x + e)^3 - a^3*c^6*f*cos
(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e) - a^3*c^6*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\int \frac{1}{\sec^9(e+fx)-3\sec^8(e+fx)+8\sec^6(e+fx)-6\sec^5(e+fx)-6\sec^4(e+fx)+8\sec^3(e+fx)-3\sec(e+fx)+1}\,dx}{a^3c^6}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] Integral(1/(sec(e + f*x)**9 - 3*sec(e + f*x)**8 + 8*sec(e + f*x)**6 - 6*sec
(e + f*x)**5 - 6*sec(e + f*x)**4 + 8*sec(e + f*x)**3 - 3*sec(e + f*x) + 1),
 x)/(a**3*c**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = -\frac{\frac{231\left(\frac{690\sin(fx+e)}{\cos(fx+e)+1}-\frac{50\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^6}-\frac{1774080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^6}-\frac{5\left(\frac{770\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{4554\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{18018\sin}{(\cos(fx}\right.}{}}{887040\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -1/887040*(231*(690*sin(f*x + e)/(cos(f*x + e) + 1) - 50*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) - 1774
080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^6) - 5*(770*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 4554*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 18018*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 59136*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 + 264726*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1
)^11/(a^3*c^6*sin(f*x + e)^11))/f
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{\frac{887040\,(fx+e)}{a^3c^6} + \frac{5\left(264726\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10} - 59136\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8} + 18018\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6} - 4554\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4} + 770\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2} -}{a^3c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}}{887040\,f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="giac")
```

```
[Out] 1/887040*(887040*(f*x + e)/(a^3*c^6) + 5*(264726*tan(1/2*f*x + 1/2*e)^10 -
59136*tan(1/2*f*x + 1/2*e)^8 + 18018*tan(1/2*f*x + 1/2*e)^6 - 4554*tan(1/2*
f*x + 1/2*e)^4 + 770*tan(1/2*f*x + 1/2*e)^2 - 63)/(a^3*c^6*tan(1/2*f*x + 1/
2*e)^11) - 231*(3*a^12*c^24*tan(1/2*f*x + 1/2*e)^5 - 50*a^12*c^24*tan(1/2*f
*x + 1/2*e)^3 + 690*a^12*c^24*tan(1/2*f*x + 1/2*e))/(a^15*c^30))/f
```

Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx =$$
$$-\frac{315\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{16} + 693\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{16} - 11550\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{14} + 159390\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\sin}{}$$

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6),x)
```

```
[Out] -(315*cos(e/2 + (f*x)/2)^16 + 693*sin(e/2 + (f*x)/2)^16 - 11550*cos(e/2 + (
f*x)/2)^2*sin(e/2 + (f*x)/2)^14 + 159390*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*
x)/2)^12 - 1323630*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^10 + 295680*cos(
e/2 + (f*x)/2)^8*sin(e/2 + (f*x)/2)^8 - 90090*cos(e/2 + (f*x)/2)^10*sin(e/2
```

```
 + (f*x)/2)^6 + 22770*cos(e/2 + (f*x)/2)^12*sin(e/2 + (f*x)/2)^4 - 3850*cos
(e/2 + (f*x)/2)^14*sin(e/2 + (f*x)/2)^2 - 887040*cos(e/2 + (f*x)/2)^5*sin(e
/2 + (f*x)/2)^11*(e + f*x))/(887040*a^3*c^6*f*cos(e/2 + (f*x)/2)^5*sin(e/2
+ (f*x)/2)^11)
```

3.42 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \frac{2\sqrt{a}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^3c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{2a^4c^4\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

```
[Out] 2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c^4*t
an(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^4*tan(f*x+e)^3/f/(a+a*sec(f*x+
e))^(3/2)-2/5*a^3*c^4*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)+2/7*a^4*c^4*tan
(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

= {3989, 3972, 308, 209}

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \frac{2a^4c^4\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}} - \frac{2a^3c^4\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}} + \frac{2a^2c^4\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}} + \frac{2\sqrt{a}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f} - \frac{2ac^4\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*Sqrt[a]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^4*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^4*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^3*c^4*Tan[e + f*x]^5)/(5*f*(
a + a*Sec[e + f*x])^(5/2)) + (2*a^4*c^4*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e +
 f*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
```

```
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = \left(a^4c^4\right)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx$$
$$= -\frac{(2a^5c^4)\,\text{Subst}\left(\int \frac{x^8}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= -\frac{(2a^5c^4)\,\text{Subst}\left(\int \left(-\frac{1}{a^4}+\frac{x^2}{a^3}-\frac{x^4}{a^2}+\frac{x^6}{a}+\frac{1}{a^4(1+ax^2)}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= -\frac{2ac^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{2a^3c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{2a^4c^4\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}-\frac{(2ac^4)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^3c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}+\frac{2a^4c^4\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 5.73 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.70

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \frac{2ac^4\left(105\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(-176+122\sec(e+fx)-66\sec^2(e+fx)+15\right.}{105f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*a*c^4*(105*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c -
c*Sec[e + f*x]]*(-176 + 122*Sec[e + f*x] - 66*Sec[e + f*x]^2 + 15*Sec[e + f
*x]^3))*Tan[e + f*x])/(105*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e +
f*x]])
```

Maple [A] (warning: unable to verify)

Time = 7.13 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.33

method	result
default	$\frac{c^4\left(105\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{7}{2}}-758(1-\cos(fx+e))^7\csc(fx+e)^7+1078(1}{105f(-\cot(fx+e)+\csc(fx+e)-1)^3}$
parts	$\frac{2c^4\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{2c^4\left(16\cos(fx+e)^3+8\cos(fx+e)^2+6\cos(fx-}{35f(\cos}$

[In] int((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/105*c^4/f*(105*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(7/2)-758*(1-cos(f*x+e))^7*csc(f*x+e)^7+1078*(1-cos(f*x+e))^5*csc(f*x+e)^5-770*(1-cos(f*x+e))^3*csc(f*x+e)^3+210*csc(f*x+e)-210*cot(f*x+e))*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)-1)^3/(-cot(f*x+e)+csc(f*x+e)+1)^3

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.13

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$$

$$=\left[\frac{105\left(c^4\cos(fx+e)^4+c^4\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx-}{\cos(fx+e)+1}\right.}{105\left(f\cos(fx+\right.}\right.$$

$$-\frac{2\left(105\left(c^4\cos(fx+e)^4+c^4\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(176\,c^4\cos(fx+e)^3\right.\right.}{105\left(f\cos(fx+e)^4+f\cos(fx\right.}$$

[In] integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(176*c^4*cos(f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*cos(f*x + e) - 15*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*

```
x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(a)*ar
ctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x
+ e))) + (176*c^4*cos(f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*cos(f*x
+ e) - 15*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos
(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^4\,dx = c^4\left(\int \left(-4\sqrt{a\sec(e + fx) + a}\sec(e + fx)\right)dx + \int 6\sqrt{a\sec(e + fx) + a}\sec^2(e + fx)\,dx + \int \left(-4\sqrt{a\sec(e + fx) + a}\sec^3(e + fx)\right)dx + \int \sqrt{a\sec(e + fx) + a}\sec^4(e + fx)\,dx + \int \sqrt{a\sec(e + fx) + a}\,dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))**4*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**4*(Integral(-4*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(6*sq
rt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(-4*sqrt(a*sec(e + f*x
) + a)*sec(e + f*x)**3, x) + Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**4, x) + Integral(sqrt(a*sec(e + f*x) + a), x))
```

Maxima [F]

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^4\,dx = \int \sqrt{a\sec(fx + e) + a}(c\sec(fx + e) - c)^4\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/210*(105*((c^4*cos(2*f*x + 2*e)^2 + c^4*sin(2*f*x + 2*e)^2 + 2*c^4*cos(2
*f*x + 2*e) + c^4)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c^4*
cos(2*f*x + 2*e)^2 + c^4*sin(2*f*x + 2*e)^2 + 2*c^4*cos(2*f*x + 2*e) + c^4)
*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arct
```

```
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^4*f*cos(2*f*x + 2*
e)^2 + c^4*f*sin(2*f*x + 2*e)^2 + 2*c^4*f*cos(2*f*x + 2*e) + c^4*f)*integra
te((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x +
2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x +
 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*
f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos
(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*co
s(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*c
os(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*
cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((
cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) +
 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e)
- cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e)
 - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e
))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*
e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6
*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*
e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x +
 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*c
os(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e
))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*co
s(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2
 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f
*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
 cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*
f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 +
8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x +
8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*s
in(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f
*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6
*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*
x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
)*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2
*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)
^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*
f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*si
n(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f
*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12
*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x +
```

```
6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(
2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)
*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)),
 x) - 40*(c^4*f*cos(2*f*x + 2*e)^2 + c^4*f*sin(2*f*x + 2*e)^2 + 2*c^4*f*cos
(2*f*x + 2*e) + c^4*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4
*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*
e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6
*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*
e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2
*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x
+ 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x
+ 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*
x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2
*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(
2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos
(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*co
s(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(
2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos
(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) +
4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) +
 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(
4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2
+ 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x
+ 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))
*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4
*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) +
 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 1
0*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) +
sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*
x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*
sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)
^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(4*cos(
8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*
cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4
*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 +
12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x
+ 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + co
s(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x
 + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(
```

```
6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e
) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(
6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 28*(c^4*f*cos(2*f*x + 2*e)^2 + c^4*f*si
n(2*f*x + 2*e)^2 + 2*c^4*f*cos(2*f*x + 2*e) + c^4*f)*integrate((((cos(10*f*
x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*
f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f
*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(
2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*s
in(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*
sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)
*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e
)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e
)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x +
2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x +
10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x
+ 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x +
2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x +
 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f
*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6
*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*
f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x + 8*e)
 + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x +
 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e)
+ cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*
f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 1
6*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*
e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + si
n(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x
+ 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8
*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e)
 + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f
*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8
*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8
*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*co
s(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*
```

x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*
sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e
) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin
(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x +
 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin
(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)
*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) + 56*(c^4*f
*cos(2*f*x + 2*e)^2 + c^4*f*sin(2*f*x + 2*e)^2 + 2*c^4*f*cos(2*f*x + 2*e) +
 c^4*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*
e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4
*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x +
2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x +
 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x
+ 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*
x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f
*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*
f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin
(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*si
n(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*s
in(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*
sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (c
os(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) +
6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
 cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8
*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)))/(((2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) +
cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*
x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos
(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*
e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos
(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x +
6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f
*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e
))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(
2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e
)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) +
6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10
*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + c

```
os(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x
 + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*c
os(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^
2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2
*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6
*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*
x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)), x) + 14*(c^4*f*cos(2*f*x + 2*e)^2 + c^4*f*sin(2*f*x + 2*e)^
2 + 2*c^4*f*cos(2*f*x + 2*e) + c^4*f)*integrate((((cos(10*f*x + 10*e)*cos(2
*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + s
in(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) +
6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e)
 + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e
) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*
e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2
*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x +
10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x
+ 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x
 + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*
x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*
f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4
*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(
2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin
(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x
+ 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10
*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2
*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + co
s(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4
*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin
(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))
*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(
4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 +
 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x
 + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + s
in(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
```

```
^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos
(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x +
 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*
f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e)
+ 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*
f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e
) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x
+ 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*
sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2
 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x))*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(3/4)*sqrt(a) - 16*(7*(15*c^4*sin
(6*f*x + 6*e) + 50*c^4*sin(4*f*x + 4*e) + 37*c^4*sin(2*f*x + 2*e))*cos(7/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (105*c^4*cos(6*f*x + 6*e
) + 350*c^4*cos(4*f*x + 4*e) + 259*c^4*cos(2*f*x + 2*e) + 44*c^4)*sin(7/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a))/((f*cos(2*f*x + 2
*e)^2 + f*sin(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e) + f)*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(3/4))
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \int \sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^4\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^4 dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^4,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^4, x)
```

3.43 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 140

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = \frac{2\sqrt{a}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^3c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c^3*t
an(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^3*tan(f*x+e)^3/f/(a+a*sec(f*x+
e))^(3/2)-2/5*a^3*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 308, 209}

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = -\frac{2a^3c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}} + \frac{2a^2c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}} + \frac{2\sqrt{a}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f} - \frac{2ac^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*Sqrt[a]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^3*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^3*c^3*Tan[e + f*x]^5)/(5*f*(
a + a*Sec[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
```

```
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left((a^3c^3)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx\right)\\
&= \frac{(2a^4c^3)\operatorname{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{(2a^4c^3)\operatorname{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}-\frac{1}{a^3(1+ax^2)}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{2ac^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}\\
&\quad -\frac{2a^3c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}-\frac{(2ac^3)\operatorname{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{2\sqrt{a}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}\\
&\quad +\frac{2a^2c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{2a^3c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$
\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx
= \frac{2ac^3\left(15\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(-23+11\sec(e+fx)-3\sec^2(e+fx))\right)\tan(e}{15f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a*c^3*(15*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c - c
*Sec[e + f*x]]*(-23 + 11*Sec[e + f*x] - 3*Sec[e + f*x]^2))*Tan[e + f*x])/(1
5*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (warning: unable to verify)

Time = 6.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.51

method	result
default	$\frac{c^3\left(15\sqrt{2}\ \mathrm{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{5}{2}}-74(1-\cos(fx+e))^5\csc(fx+e)^5+80(1-\cos(\right.}{15f(-\cot(fx+e)+\csc(fx+e)-1)^2(-\cot(fx+e)+\csc(fx}$
parts	$\frac{2c^3\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \mathrm{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{6c^3\sqrt{a(\sec(fx+e)+1)}\,(\cot(fx+e)-\csc(fx+e))}{f}$

```
[In] int((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*c^3/f*(15*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1
/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)-74*(1
-cos(f*x+e))^5*csc(f*x+e)^5+80*(1-cos(f*x+e))^3*csc(f*x+e)^3-30*csc(f*x+e)+
30*cot(f*x+e))*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+
csc(f*x+e)-1)^2/(-cot(f*x+e)+csc(f*x+e)+1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.48

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$$

$$= \left[\frac{15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)}{\cos(fx+e)+1}\right.}{15\left(f\cos(fx+e)^3+f\cos(fx\right.}\right.$$

$$-\frac{2\left(15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(23\,c^3\cos(fx+e)^2-11\right.\right.}{15\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}$$

```
[In] integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(23*c^3*cos(f*x
 + e)^2 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(c^3*cos
(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/
```

```
cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (23*c^3*cos(f*x + e)^2
 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin
(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^3 \, dx$$
$$= -c^3 \left(\int 3\sqrt{a\sec(e + fx) + a}\sec(e + fx)\, dx + \int \left(-3\sqrt{a\sec(e + fx) + a}\sec^2(e + fx)\right) dx + \int \sqrt{a\sec(e + fx) + a}\sec^3(e + fx)\, dx + \int \left(-\sqrt{a\sec(e + fx) + a}\right) dx \right)$$

```
[In] integrate((c-c*sec(f*x+e))**3*(a+a*sec(f*x+e))**(1/2),x)
[Out] -c**3*(Integral(3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-3*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x)
+ a)*sec(e + f*x)**3, x) + Integral(-sqrt(a*sec(e + f*x) + a), x))
```

Maxima [F]

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^3 \, dx = \int -\sqrt{a\sec(fx + e) + a}(c\sec(fx + e) - c)^3 \, dx$$

```
[In] integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -1/30*(15*((c^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f
*x + 2*e) + c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c^3*co
s(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f*x + 2*e) + c^3)*a
rctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^3*f*cos(2*f*x + 2*e)
^2 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f)*integrate
((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e)
```

```
 + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8
*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x +
2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8
*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x +
4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x +
2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x +
 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x +
6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e)
 + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e
) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x +
6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2
*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*co
s(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 +
6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x +
6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2
*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*
e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 +
 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(3*cos(6*f*x + 6*e) + 3*cos
(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6
*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6
*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*
f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e
))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 +
6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 18*(c^3*f*cos(2*f*x + 2*e)^
2 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f)*integrate(
(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e)
+ 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*
e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2
*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*
e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4
*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2
*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x +
```

2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6
*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e)
+ 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
 + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6
*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*
e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*cos
(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6
*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6
*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*
f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e
))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 +
6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(3*cos(6*f*x + 6*e) + 3*cos(
4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*
(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*
e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f
*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e)
)*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x
 + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6
*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) + 10*(c^3*f*cos(2*f*x + 2*e)^2
 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f)*integrate((
((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e
)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*
e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*
e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e
)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*
e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*
e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2
*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*
e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) +
 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
+ cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*
e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e
)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*cos(
4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*

```
(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*
e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f
*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e)
)*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x
 + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6
*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(3*cos(6*f*x + 6*e) + 3*cos(4
*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*(
3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*e
)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*
x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))
*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x
+ 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) + 10*(c^3*f*cos(2*f*x + 2*e)^2
+ c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f)*integrate(((
(cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)
*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e
)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)
*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e
)*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e
)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*
e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e
)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) +
3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
 cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e
)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)
^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*cos(4
*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*(
3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*e
)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*
x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))
*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x
+ 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(3*cos(6*f*x + 6*e) + 3*cos(4*
f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*(3
*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)
```

```
^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
 + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*
sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x +
 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*s
in(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x))*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) - 4*(5*(9*c^3*sin(4*f*x +
 4*e) + 16*c^3*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)) - (45*c^3*cos(4*f*x + 4*e) + 80*c^3*cos(2*f*x + 2*e) + 23*c^
3)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a))/((f*c
os(2*f*x + 2*e)^2 + f*sin(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e) + f)*(cos(2
*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4))
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = \int -\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^3\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^3 dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3, x)
```

3.44 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx$

Optimal result 362
Rubi [A] (verified) 362
Mathematica [A] (verified) 364
Maple [A] (verified) 364
Fricas [A] (verification not implemented) 365
Sympy [F] 365
Maxima [F] 366
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Optimal result

Integrand size = 28, antiderivative size = 105

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \frac{2\sqrt{a}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c^2*t
an(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^2*tan(f*x+e)^3/f/(a+a*sec(f*x+
e))^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 308, 209}

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \frac{2a^2c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}} + \frac{2\sqrt{a}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f} - \frac{2ac^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*Sqrt[a]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^2*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx\\
&= -\frac{\left(2a^3c^2\right)\operatorname{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{\left(2a^3c^2\right)\operatorname{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{2ac^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}\\
&\quad-\frac{\left(2ac^2\right)\operatorname{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}
\end{aligned}
$$

$$= \frac{2\sqrt{a}c^2 \arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx$$
$$= \frac{2ac^2\left(3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+(-4+\sec(e+fx))\sqrt{c-c\sec(e+fx)}\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a*c^2*(3*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + (-4 + Sec[e
 + f*x])*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(3*f*Sqrt[a*(1 + Sec[e + f
*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.52

method	result
parts	$\frac{2c^2\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f} + \frac{2c^2\sqrt{a(\sec(fx+e)+1)}\,(2\sin(fx+e)+\tan(fx+e}{3f(\cos(fx+e)+1)}$
default	$-\frac{2c^2\sqrt{a(\sec(fx+e)+1)}\left(-3\ \text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \cos(fx+e)-3\ \text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{c}{co}}}\right.\right.}{3f(\cos(fx+e)+1)}$

```
[In] int((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^2/f*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh
(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+2/3*c^2/f*(a
*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(2*sin(f*x+e)+tan(f*x+e))+4*c^2/f*(a*
(sec(f*x+e)+1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.98

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \left[\frac{3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-\cdots}{\cos(fx+e)+1}\right)\cdots}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}\right.$$

$$-\frac{2\left(3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(4\,c^2\cos\left(fx+e\right)-c^2\right)\sqrt{\cdots}\right)}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}$$

```
[In] integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(4*c^2*cos(f*x + e)
 - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqr
t(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s
in(f*x + e))) + (4*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = c^2\left(\int\left(-2\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)\right)dx + \int\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)dx + \int\sqrt{a\sec\left(e+fx\right)+a}\,dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))**2*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(sqrt
(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x) + a
), x))
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \int \sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^2\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*c^2*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*
x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x
 + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x
 + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x
+ 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x
 + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x
+ 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x +
 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x +
2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x +
 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(2*co
s(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 +
4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*
e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f
*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
)^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f
*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x +
6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4)), x) + 4*c^2*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2
*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x
+ 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x +
 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x
+ 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x +
 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x
+ 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x +
 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e
) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x +
```

```
4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6
*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
+ cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x
+ 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x
+ 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos
(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*
e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*
f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 6*c^2*f*integrate((((cos(6*f*x + 6*
e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e
)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*
e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e
))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e
) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e)
 - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)
*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e)
+ 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*
x + 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*c
os(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x +
2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(
4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x +
2*e))*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(
4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e)
+ sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4
*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - c^2*arctan2((cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)) + 1) + c^2*arctan2((cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
```

```
os(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) - 1))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(3/4)*sqrt(a) + 8*(3*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))*sin(2*f*x + 2*e) - (3*c^2*cos(2*f*x + 2*e) + 2*c^2)*sin(3/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a))/((cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(3/4)*f)
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)-c\right)^2\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx=\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^2\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2, x)
```

3.45 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = \frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c*tan(f
*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3989, 3972, 327, 209}

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = \frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f} - \frac{2ac\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left((ac)\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx\right)\\
&= \frac{(2a^2c)\,\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{2ac\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(2ac)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$$
$$=\frac{2c\sqrt{a(1+\sec(e+fx))}\left(\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)-\sqrt{c-c\sec(e+fx)}\right)\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*c*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/S
qrt[c]] - Sqrt[c - c*Sec[e + f*x]])*Tan[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e +
 f*x]])
```

Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

method	result
default	$\frac{2c\sqrt{a(\sec(fx+e)+1)}\left(\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+\cot(fx+e)-\csc(fx+e)\right)}{f}$
parts	$\frac{2c\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{2c\sqrt{a(\sec(fx+e)+1)}\,(\cot(fx+e)-\csc(fx+e))}{f}$

```
[In] int((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c/f*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x
+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cot(f*x+e)-cs
c(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.55

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$$
$$= \left[\frac{(c\cos(fx+e)+c)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\ldots}{f\cos(fx+e)+f}\right.$$
$$\left.-\frac{2\left((c\cos(fx+e)+c)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{f\cos(fx+e)+f}\right]$$

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e
) - a)/(cos(f*x + e) + 1)) - 2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arctan(s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))
 + c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)
+ f)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = -c\left(\int \sqrt{a\sec(e+fx)+a}\sec(e+fx)\,dx + \int \left(-\sqrt{a\sec(e+fx)+a}\right)dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c*(Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-sqrt(a*s
ec(e + f*x) + a), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(58) = 116.

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.23

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$$
$$= \frac{\sqrt{a}c\arctan\left(\left(\cos\left(2\,fx+2\,e\right)^2+\sin\left(2\,fx+2\,e\right)^2+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,fx+2\,e\right)\right.\right.\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*c*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(
f*x + e))/f
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = \int -\sqrt{a\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)-c)\,dx$$

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = \int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)), x)
```

3.46 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c/f+2*cot(f*x+e
)*(a+a*sec(f*x+e))^(1/2)/c/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 331, 209}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}+\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^{3/2}\,dx}{ac} \\
&= \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} \\
&= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf} - \frac{(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} \\
&= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \frac{2\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\sec(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{cf}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]
[Out] (2*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]*Sqrt[a*(1
 + Sec[e + f*x])])/(c*f)
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+\cot(fx+e)\right)}{cf}$	87

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2/c/f*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x
+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.86

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \left[\frac{\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7\,a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+4\sqrt{}}{2\,cf\sin(fx+e)}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e
))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(
f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), (sqrt(a)*arctan(2*sqrt(a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos
(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c - c\sec(e + fx)}\,dx = -\frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx}{c}$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
[Out] -Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x)/c
```

Maxima [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c - c\sec(e + fx)}\,dx = \int -\frac{\sqrt{a\sec(fx+e) + a}}{c\sec(fx+e) - c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
[Out] -integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c), x)
```

Giac [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c - c\sec(e + fx)}\,dx = \int -\frac{\sqrt{a\sec(fx+e) + a}}{c\sec(fx+e) - c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c - c\sec(e + fx)}\,dx = \int \frac{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}{c - \frac{c}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x)), x)
```

3.47 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2} dx$

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Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2} dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^2f}$$

```
[Out] -2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a/c^2/f+2*arctan(a^(1/2)*tan(f*x+e
)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c^2/f+2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)
/c^2/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 331, 209}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2} dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f} - \frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^2f} + \frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^2f}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^2*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{5/2}\,dx}{a^2c^2}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^2f}$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^2f} + \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}$$

$$= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^2f} - \frac{(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}$$

$$= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^2f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = -\frac{2a\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))^2}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]
[Out] (-2*a*Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(3*f
*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.73

method	result
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)-3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(f}{\cos(fx}}}\right.\right.}{3c^2f(\cos(fx+e)-1)}$

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] 2/3/c^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*(3*arctanh(sin(f*x+e)/(co
s(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*cos(f*x+e)-3*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*
x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+4*cos(f*x+e)*cot(f*x+e)-
3*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = \left[\frac{3\sqrt{-a}(\cos(fx+e)-1)\log\left(-\frac{8a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right.}{6\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e)}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x
 + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(4*
cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(
(c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*sqrt(a)*(cos(f*x + e) -
1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(4*c
os(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((
c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{c^2}$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x
)/c**2
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(c\sec\left(fx+e\right)-c\right)^2}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(c\sec\left(fx+e\right)-c\right)^2}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^2, x)
```

3.48 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx$

Optimal result . . . 384
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Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}+\frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f}-\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^3f}+\frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f}$$

```
[Out] -2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a/c^3/f+2/5*cot(f*x+e)^5*(a+a*sec(
f*x+e))^(5/2)/a^2/c^3/f+2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))
*a^(1/2)/c^3/f+2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^3/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 331, 209}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^2c^3f}+\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^3f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^3f}+\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^3f}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^3*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a^2*c^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{7/2}\,dx}{a^3c^3}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^3f}$$

$$= \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f} - \frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^3f}$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^3f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f} + \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}$$

$$= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^3f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f} - \frac{(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}$$

$$= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^3f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(-\frac{5}{2}, 1, -\frac{3}{2}, 1-\sec(e+fx)\right)\tan(e+fx)}{5c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]
[Out] (2*a*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(5*c^
3*f*(-1 + Sec[e + f*x])^3*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(123) = 246.

Time = 2.65 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.90

method	result
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^2-30\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(}{\cos(f}}}\right.\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 2/15/c^3/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^2*(15*(-cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)
```

```
+1))^(1/2))*cos(f*x+e)^2-30*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/
(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+15*arc
tanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)+23*cos(f*x+e)^2*cot(f*x+e)-35*cos(f*x+e)*cot(f*x
+e)+15*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \left[\frac{15\left(\cos\left(fx+e\right)^2-2\,\cos\left(fx+e\right)+1\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\mathrm{si}}{\cos(fx+e)+1}\right.}{30\left(c^3 f\cos\left(fx+e\right)^2-2\,c^3\right.}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x
+ e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))
*sin(f*x + e) + 4*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x + e))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*
cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(cos(f*x + e)^2 - 2*cos(f*x +
 e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x
 + e) + 2*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x + e))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x
+ e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{\sqrt{a\sec\left(e+fx\right)+a}}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx}{c^3}$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3
*sec(e + f*x) - 1), x)/c**3
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \int -\frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(c\sec\left(fx+e\right)-c\right)^3}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
[Out] -integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \int -\frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(c\sec\left(fx+e\right)-c\right)^3}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^3, x)
```

3.49 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 174

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^4f} - \frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f}$$

```
[Out] -2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a/c^4/f+2/5*cot(f*x+e)^5*(a+a*sec(
f*x+e))^(5/2)/a^2/c^4/f-2/7*cot(f*x+e)^7*(a+a*sec(f*x+e))^(7/2)/a^3/c^4/f+2
*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c^4/f+2*cot(f*x+
e)*(a+a*sec(f*x+e))^(1/2)/c^4/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

= {3989, 3972, 331, 209}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = -\frac{2\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7a^3c^4f} + \frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^2c^4f} + \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^4f} - \frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^4f} + \frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^4f}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^4*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a^2*c^4*f) - (2*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/(7
*a^3*c^4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^{9/2}\,dx}{a^4c^4}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{x^8(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3c^4f}$$

$$= -\frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f} + \frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^4f}$$

$$= \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^4f} - \frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f} - \frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^4f}$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^4f} - \frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f} + \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f}$$

$$= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^4f} - \frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f} - \frac{(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f}$$

$$= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^4f} - \frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.34

$$
\begin{aligned}
&\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx \\
&= -\frac{2a\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\sec(e+fx)\right)\tan(e+fx)}{7f\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))^4}
\end{aligned}
$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (-2*a*Hypergeometric2F1[-7/2, 1, -5/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(7*f
*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(154) = 308.

Time = 2.80 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.00

method	result
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(105\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^3-315\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\frac{}{(\cos}\right.\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/c^4/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^3*(105*(-cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+
e)+1))^(1/2))*cos(f*x+e)^3-315*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(s
in(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)^2+3
15*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+176*cos(f*x+e)^3*cot(f*x+e)-105
*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)-406*cos(f*x+e)^2*cot(f*x+e)+350*cos(f*x+e)*c
ot(f*x+e)-105*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^4}\,dx$$
$$= \left[\frac{105\left(\cos\left(fx+e\right)^3 - 3\,\cos\left(fx+e\right)^2 + 3\,\cos\left(fx+e\right) - 1\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3 - 4\left(2\,\cos(fx+e)^2 - \cos(fx+\right.}{\mathrm{co}}\right.}{210\left(c^4 f\cos\left(fx+\right.\right.}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(cos(f*x + e)^3 - 3*cos(f*x + e)^2 + 3*cos(f*x + e) - 1)*sqrt(-
a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a
)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(176*cos(f*x + e)^4 - 406*cos(f*x +
e)^3 + 350*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f
*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(cos(f*x + e)^3 - 3*cos(f*x + e)
^2 + 3*cos(f*x + e) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x +
e) - a))*sin(f*x + e) + 2*(176*cos(f*x + e)^4 - 406*cos(f*x + e)^3 + 350*co
s(f*x + e)^2 - 105*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(
(c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4
*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^4}\,dx = \frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec^4(e+fx) - 4\sec^3(e+fx) + 6\sec^2(e+fx) - 4\sec(e+fx) + 1}\,dx}{c^4}$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*
sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)/c**4
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(c\sec\left(fx+e\right)-c\right)^4}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^4,x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^4, x)
```

3.50 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx$

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Mathematica [A] (verified) 397
Maple [A] (warning: unable to verify) 398
Fricas [A] (verification not implemented) 398
Sympy [F] 399
Maxima [F] 399
Giac [F] 406
Mupad [F(-1)] 407

Optimal result

Integrand size = 28, antiderivative size = 177

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = \frac{2a^{3/2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^4c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

```
[Out] 2*a^(3/2)*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c^3
*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^3*c^3*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)-2/5*a^4*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)-2/7*a^5*c^3*t
an(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3989, 3972, 470, 308, 209}

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = \frac{2a^{3/2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f} - \frac{2a^5c^3\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}} - \frac{2a^4c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}} + \frac{2a^3c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}} - \frac{2a^2c^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(3/2)*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^2*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*c^3*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^4*c^3*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) - (2*a^5*c^3*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx\right)$$

$$= \frac{(2a^5c^3)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} + \frac{(2a^5c^3)\,\text{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} + \frac{(2a^5c^3)\,\text{Subst}\left(\int \left(\frac{1}{a^3} - \frac{x^2}{a^2} + \frac{x^4}{a} - \frac{1}{a^3(1+ax^2)}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^2c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^4c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} - \frac{(2a^2c^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^{3/2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^4c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx =$$

$$-\frac{2a^2\left(-105c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right) + c^3\sqrt{c-c\sec(e+fx)}(146-32\sec(e+fx)-24\sec^2(e+fx)+1\ldots}{105f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (-2*a^2*(-105*c^(7/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + c^3*Sqrt[
c - c*Sec[e + f*x]]*(146 - 32*Sec[e + f*x] - 24*Sec[e + f*x]^2 + 15*Sec[e +
 f*x]^3))*Tan[e + f*x])/(105*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
+ f*x]])
```

Maple [A] (warning: unable to verify)

Time = 8.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.32

method	result
default	$\frac{a\,c^{3}\left(105\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^{2}\csc(fx+e)^{2}-1}}\right)\left((1-\cos(fx+e))^{2}\csc(fx+e)^{2}-1\right)^{\frac{7}{2}}-278(1-\cos(fx+e))^{7}\csc(fx+e)^{7}+1078(1}{105f(-\cot(fx+e)+\csc(fx+e)+1)^{3}(}$
parts	$\frac{2c^{3}a\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+}{\cos(fx+e}}}\right.\right.}{f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*a*c^3/f*(105*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1
)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(7/2)-2
78*(1-cos(f*x+e))^7*csc(f*x+e)^7+1078*(1-cos(f*x+e))^5*csc(f*x+e)^5-770*(1-
cos(f*x+e))^3*csc(f*x+e)^3+210*csc(f*x+e)-210*cot(f*x+e))*(-2*a/((1-cos(f*x
+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)+1)^3/(-cot(f*x+e)+csc
(f*x+e)-1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.18

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3}\,dx = \left[\frac{105\left(ac^{3}\cos(fx+e)^{4}+ac^{3}\cos(fx+e)^{3}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^{2}-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(}{\cos(fx+e)-}\right.}{}\right.$$

$$-\frac{2\left(105\left(ac^{3}\cos(fx+e)^{4}+ac^{3}\cos(fx+e)^{3}\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(146\,ac^{3}\cos(fx+e)^{3}-\right.\right.}{105\left(f\cos(fx+e)^{4}+f\cos(fx\right.}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e)^3)*sqrt(-a)*log((2*a
*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(146*a*c^
3*cos(f*x + e)^3 - 32*a*c^3*cos(f*x + e)^2 - 24*a*c^3*cos(f*x + e) + 15*a*c
^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
```

```
 + f*cos(f*x + e)^3), -2/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e
)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) + (146*a*c^3*cos(f*x + e)^3 - 32*a*c^3*cos(f*x + e)^2
 - 24*a*c^3*cos(f*x + e) + 15*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = -c^3\left(\int\left(-a\sqrt{a\sec(e+fx)+a}\right)dx+\int 2a\sqrt{a\sec(e+fx)+a}\sec(e+fx)\,dx + \int\left(-2a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\right)dx + \int a\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(2*a*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x), x) + Integral(-2*a*sqrt(a*sec(e + f*x) + a)*sec(
e + f*x)**3, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x))
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{3/2}(c - c\sec(e+fx))^3\,dx = \int -(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^3\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/210*(105*((a*c^3*cos(2*f*x + 2*e)^2 + a*c^3*sin(2*f*x + 2*e)^2 + 2*a*c^3
*cos(2*f*x + 2*e) + a*c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1)
 - (a*c^3*cos(2*f*x + 2*e)^2 + a*c^3*sin(2*f*x + 2*e)^2 + 2*a*c^3*cos(2*f*x
 + 2*e) + a*c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(a*c^
```

```
3*f*cos(2*f*x + 2*e)^2 + a*c^3*f*sin(2*f*x + 2*e)^2 + 2*a*c^3*f*cos(2*f*x +
 2*e) + a*c^3*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x
 + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
+ 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x
 + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x
 + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*
x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x
 + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*
x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x
 + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*
x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x
 + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*
x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x
 + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2
*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)
^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*c
os(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f
*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 +
 sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)
 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x
 + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x
+ 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos
(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))
*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)
 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2
*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin
(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x +
```

2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2), x) + 12*(a*c^3*f*cos(2*f*x + 2*e)^2 + a*c^3*f*sin(2*f*x + 2*e)^2 + 2*a*c^3*f*cos(2*f*x + 2*e) + a*c^3*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2

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*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4
 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8
*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2
*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*
e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin
(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*
x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2
*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6
*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*
e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f
*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*si
n(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + s
in(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x
+ 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos
(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x +
2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2
*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*
f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin
```

```
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
 + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (
cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x +
8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 +
 (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x
+ 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2), x) - 1
6*(a*c^3*f*cos(2*f*x + 2*e)^2 + a*c^3*f*sin(2*f*x + 2*e)^2 + 2*a*c^3*f*cos(
2*f*x + 2*e) + a*c^3*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*co
s(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos
(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*si
n(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*si
n(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*s
in(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*si
n(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*s
in(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*si
n(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*s
in(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (co
s(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*c
os(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*si
n(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/
((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
 + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2
 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(
cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
 + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(co
s(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos
```

```
(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^
3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
+ 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x
 + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*
f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(
2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9
*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f
*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)
^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6
*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f
*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3
*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f
*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(
cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f
*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x
 + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*s
in(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1))^2), x) + 6*(a*c^3*f*cos(2*f*x + 2*e)^2 + a*c
^3*f*sin(2*f*x + 2*e)^2 + 2*a*c^3*f*cos(2*f*x + 2*e) + a*c^3*f)*integrate((
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((
cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3
*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*
sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)
*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)
```

```
*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*
sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)
*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*co
s(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)
*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e
)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)
*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3
*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)
*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^
2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x +
2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 +
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*
x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*
e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + co
s(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)
*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))
*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*
x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(s
in(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)
^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(
2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
```

```
2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3
 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*
f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 +
 cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 +
cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*
e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x
+ 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2
*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*
e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*
f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*
e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(
4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2),
x))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(3/4
)*sqrt(a) - 8*(7*(15*a*c^3*sin(6*f*x + 6*e) + 25*a*c^3*sin(4*f*x + 4*e) + 2
9*a*c^3*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) - (105*a*c^3*cos(6*f*x + 6*e) + 175*a*c^3*cos(4*f*x + 4*e) + 203*a*
c^3*cos(2*f*x + 2*e) + 73*a*c^3)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)))*sqrt(a))/((f*cos(2*f*x + 2*e)^2 + f*sin(2*f*x + 2*e)^2 + 2*
f*cos(2*f*x + 2*e) + f)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(3/4))
```

Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx=\int -(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^3\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx=\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^3\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3, x)
```

3.51 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx$

Optimal result 408
Rubi [A] (verified) 408
Mathematica [A] (verified) 410
Maple [A] (verified) 410
Fricas [A] (verification not implemented) 411
Sympy [F] 411
Maxima [F] 412
Giac [F] 417
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Optimal result

Integrand size = 28, antiderivative size = 142

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx=\frac{2a^{3/2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^3c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}+\frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 2*a^(3/2)*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c^2
*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^3*c^2*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)+2/5*a^4*c^2*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3989, 3972, 470, 308, 209}

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx=\frac{2a^{3/2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2a^4c^2\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^3c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}-\frac{2a^2c^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(3/2)*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^2*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*c^2*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) + (2*a^4*c^2*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(a^2c^2\right)\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx\\ &= -\frac{(2a^4c^2)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\ &= \frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}-\frac{(2a^4c^2)\,\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\end{aligned}$$

$$=\frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}-\frac{(2a^4c^2)\operatorname{Subst}\left(\int\left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$=-\frac{2a^2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^3c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}+\frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}-\frac{(2a^2c^2)\operatorname{Subst}\left(\int\frac{1}{1+ax^2}dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$=\frac{2a^{3/2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^3c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}+\frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx=\frac{2a^2\left(15c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+c^2\sqrt{c-c\sec(e+fx)}(-17-\sec(e+fx)+3\sec^2$$
$$15f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^2*(15*c^(5/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + c^2*Sqrt[c -
 c*Sec[e + f*x]]*(-17 - Sec[e + f*x] + 3*Sec[e + f*x]^2))*Tan[e + f*x])/(15
*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.33

method	result
default	$\frac{2a\,c^2\sqrt{a(\sec(fx+e)+1)}\left(15\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+15\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{c}{co}}}\right.\right.}{15f(\cos(fx+e)+1)}$
parts	$\frac{2c^2a\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+}{\cos(fx+e}}}\right.\right.}{f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*a*c^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(15*arctanh(sin(f*x+e)
/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)+15*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(c
os(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-17*sin(f*x+e)-tan(f
*x+e)+3*sec(f*x+e)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.50

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \left[\frac{15\left(ac^2\cos(fx+e)^3+ac^2\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\right.}{\cos(fx+e)\right.}\right.}{15\,(f\cos}\right.$$

$$-\frac{2\left(15\left(ac^2\cos(fx+e)^3+ac^2\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(17\,ac^2\cos(fx+e)^2+\right.\right.}{15\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*c
os(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(17*a*c^2*c
os(f*x + e)^2 + a*c^2*cos(f*x + e) - 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(
a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (17*a*c^2*c
os(f*x + e)^2 + a*c^2*cos(f*x + e) - 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = c^2\left(\int a\sqrt{a\sec(e+fx)+a}\,dx + \int\left(-a\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx + \int\left(-a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\right)dx + \int a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(a*sqrt(a*sec(e + f*x) + a), x) + Integral(-a*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x), x) + Integral(-a*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x)**2, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x))
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^2\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*((a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*c
os(2*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) -
 (a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*cos(2*f*x +
 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(a*c^2*
f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*cos(2*f*x + 2
*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x +
 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x +
2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x +
6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2
*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x +
4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2
*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*
e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e
)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x +
 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*
e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
```

```
x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2
*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6
*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*
e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 +
 (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x
+ 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2
*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*
x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)
*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))
*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*
x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*
e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(
4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2),
x) + 6*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f
*cos(2*f*x + 2*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e
)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)
^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e
)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)
*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e
)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)
*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (
cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + c
os(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))
/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*
```

```
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*
cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
 + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) +
cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*
f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*
e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^
2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + co
s(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos
(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4
*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(
sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*
x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1))^2), x) - 6*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)
^2 + 2*a*c^2*f*cos(2*f*x + 2*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + s
in(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + si
n(2*f*x + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (c
os(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - co
s(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*c
os(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*co
s(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*
f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin
(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*
```

```
f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
 + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e
)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)
^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x
 + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (co
s(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*
e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2
 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos
(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x
 + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x
+ 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos
(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))
*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)
 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*
x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1))^2), x) + 2*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*si
n(2*f*x + 2*e)^2 + 2*a*c^2*f*cos(2*f*x + 2*e) + a*c^2*f)*integrate((cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f
*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*
x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f
*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*
x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*
x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x
 + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x +
```

```
2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x
 + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x
+ 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x
+ 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2
+ (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(c
os(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*co
s(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)
^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*
x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*
sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^
4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(
6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4
*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2
+ 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)
+ 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x
+ 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos
(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^
3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*si
n(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x +
2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2), x))*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) - 4*(5*(3*a*c^2*sin(4*f
*x + 4*e) + 4*a*c^2*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)) - (15*a*c^2*cos(4*f*x + 4*e) + 20*a*c^2*cos(2*f*x + 2*e
) + 17*a*c^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqr
t(a))/((f*cos(2*f*x + 2*e)^2 + f*sin(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e)
+ f)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
4))
```

Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^2\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2, x)
```

3.52 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx$

Optimal result . . . 418
Rubi [A] (verified) . . . 418
Mathematica [A] (verified) . . . 420
Maple [B] (verified) . . . 420
Fricas [A] (verification not implemented) . . . 421
Sympy [F] . . . 421
Maxima [B] (verification not implemented) . . . 422
Giac [F] . . . 423
Mupad [F(-1)] . . . 423

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = \frac{2a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*a^(3/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c*tan
(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*a^3*c*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^
(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3989, 3972, 470, 327, 209}

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = \frac{2a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f} - \frac{2a^3c\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}} - \frac{2a^2c\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a^2*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*a^3*c*Tan[e + f*x]^
3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\left((ac)\int \sqrt{a+a\sec(e+fx)}\tan^2(e+fx)\,dx\right)$$

$$= \frac{(2a^3c)\,\text{Subst}\left(\int \frac{x^2(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{(2a^3c)\,\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^2c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{(2a^2c)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = -\frac{2a^2c\left(-3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+(2+\sec(e+fx))\sqrt{c-c\sec(e+fx)}\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
[Out] (-2*a^2*c*(-3*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + (2 + Sec[
e + f*x])*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(3*f*Sqrt[a*(1 + Sec[e +
f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(89) = 178.

Time = 1.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.04

method	result
default	$\frac{2ca\sqrt{a(\sec(fx+e)+1)}\left(\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e}{\cos(fx+e)-}}}\right.\right.}{f(\cos(fx+e)+1)}$
parts	$\frac{2ca\sqrt{a(\sec(fx+e)+1)}\left(\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e}{\cos(fx+e)-}}}\right.\right.}{f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c/f*a*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f
*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+
arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e))/(cos(f*x+e)+1)-2/3*c/f*a*(a*(sec(
f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(5*sin(f*x+e)+tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.00

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = \left[\frac{3\left(ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\cos(fx+e)+1}\right.}{3\left(f\cos\left(fx+e\right)^2+f\right.}\right.$$

$$-\frac{2\left(3\left(ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(2\,ac\cos\left(fx+e\right)+ac\right)\sqrt{\frac{a}{}}\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(2*a*c*cos(f*x + e)
 + a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqr
t(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s
in(f*x + e))) + (2*a*c*cos(f*x + e) + a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = -c\left(\int\left(-a\sqrt{a\sec\left(e+fx\right)+a}\right)dx+\int a\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(a*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(89) = 178$.

Time = 0.44 (sec) , antiderivative size = 998, normalized size of antiderivative = 9.88

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 + 1) + a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - (a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))*sqrt(a))*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = \int -(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c - c\sec(e+fx))\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x)), x)
```

3.53 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx=\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{4a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+4*a*cot(f*x
+e)*(a+a*sec(f*x+e))^(1/2)/c/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 464, 209}

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx=\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}+\frac{4a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (4*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^{5/2}\,dx}{ac} \\
&= \frac{(2a)\text{Subst}\left(\int \frac{2+ax^2}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} \\
&= \frac{4a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} \\
&= \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{4a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \frac{2a\cot(e+fx)\sqrt{a(1+\sec(e+fx))}\left(2\sqrt{c}-\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)\sqrt{c-c\text{se}}\right.}{c^{3/2}f}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a*Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*(2*Sqrt[c] - ArcTanh[Sqrt[c -
c*Sec[e + f*x]]/Sqrt[c]]*Sqrt[c - c*Sec[e + f*x]]))/(c^(3/2)*f)
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+2\cot(fx+e)\right)}{cf}$	90

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/c/f*a*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f
*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*cot(f*x+e
))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.84

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \left[\frac{\sqrt{-a}a\log\left(-\frac{8a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\,\text{c}}{\cos(fx+e)+1}\right.}{2cf\sin(fx+e)}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*co
s(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), (a^(3/2)*arctan(2*sqr
t(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a
```

```
*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 4*a*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = -\frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx + \int \frac{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec(e+fx)-1}\,dx}{c}$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
[Out] -(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral(a*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x))/c
```

Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{c\sec(fx+e)-c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
[Out] -integrate((a*sec(f*x + e) + a)^(3/2)/(c*sec(f*x + e) - c), x)
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{c\sec(fx+e)-c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x)), x)
```

3.54 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx$

Optimal result 428
Rubi [A] (verified) 428
Mathematica [C] (verified) 430
Maple [A] (verified) 430
Fricas [A] (verification not implemented) 431
Sympy [F] 431
Maxima [F(-1)] 431
Giac [F] 432
Mupad [F(-1)] 432

Optimal result

Integrand size = 28, antiderivative size = 102

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{4\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f-4/3*cot(f
*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^2/f+2*a*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/
c^2/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3989, 3972, 464, 331, 209}

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f} - \frac{4\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^2f} + \frac{2a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^2f}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^2*f) - (4*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{7/2}\,dx}{a^2c^2}$$

$$= -\frac{2\text{Subst}\left(\int \frac{2+ax^2}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}$$

$$= -\frac{4\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f} + \frac{(2a)\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}$$

$$= \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{4\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}$$

$$= \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{4\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^2\left(-2+3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\sec(e+fx)\right)(-1+\sec(e+fx)\right)}{3c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^2*(-2 + 3*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]*(-1 + Sec[
e + f*x]))*Tan[e + f*x])/(3*c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[a*(1 + Sec[e +
 f*x])])
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.77

method	result
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)-3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx}{\cos(fx+}}}\right.\right.}{3c^2f(\cos(fx+e)-1)}$

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/c^2/f*a*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*(3*arctanh(sin(f*x+e)/(
cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*cos(f*x+e)-3*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(
f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cos(f*x+e)*cot(f*x+e
)-3*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.44

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \left[\frac{3\left(a\cos\left(fx+e\right)-a\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{a}}{\cos(fx+e)+1}\right.}{6\left(c^2f\right.}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a*cos(f*x + e) - a)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f
*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(
5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*(a*cos(f*x + e) -
a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e)
+ 2*(5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx + \int \frac{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx}{c^2}$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] (Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1)
, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x)**2 -
2*sec(e + f*x) + 1), x))/c**2
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^2}\,dx = \int \frac{(a\sec(fx + e) + a)^{\frac{3}{2}}}{(c\sec(fx + e) - c)^2}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^2}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^2, x)
```

3.55 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx$

Optimal result 433
Rubi [A] (verified) 433
Mathematica [C] (verified) 435
Maple [B] (verified) 436
Fricas [A] (verification not implemented) 436
Sympy [F] 437
Maxima [F(-1)] 437
Giac [F] 437
Mupad [F(-1)] 437

Optimal result

Integrand size = 28, antiderivative size = 137

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^3f} + \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3f}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^3/f-2/3*cot(f
*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^3/f+4/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2
)/a/c^3/f+2*a*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^3/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3989, 3972, 464, 331, 209}

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^3f} + \frac{4\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5ac^3f} - \frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^3f} + \frac{2a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^3f}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) - (2*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^3*f) + (4*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a*c^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{9/2}\,dx}{a^3c^3}$$

$$= \frac{2\text{Subst}\left(\int \frac{2+ax^2}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^3f}$$

$$= \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3f} - \frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^3f} + \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3f} + \frac{(2a)\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}$$

$$= \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^3f} + \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3f} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}$$

$$= \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^3f} + \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.96 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = -\frac{2a^2\left(-6+5\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx)\right)(-1+\sec(e+fx))\right)\tan(e+fx)}{15c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (-2*a^2*(-6 + 5*Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]]*(-1 + Se
c[e + f*x]))*Tan[e + f*x])/(15*c^3*f*(-1 + Sec[e + f*x])^3*Sqrt[a*(1 + Sec[
e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(121) = 242.

Time = 2.74 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.93

method	result
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^2-30\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos}{\cos(}}}\right.\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/c^3/f*a*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^2*(15*(-cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+
e)+1))^(1/2))*cos(f*x+e)^2-30*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e
)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+15*a
rctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)+26*cos(f*x+e)^2*cot(f*x+e)-35*cos(f*x+e)*cot(f
*x+e)+15*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.04

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \left[\frac{15\left(a\cos(fx+e)^2-2\,a\cos(fx+e)+a\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(fx-\right.}{}\right.}{}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(-a)*log(-(8*a*cos(
f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) +
 1))*sin(f*x + e) + 4*(26*a*cos(f*x + e)^3 - 35*a*cos(f*x + e)^2 + 15*a*cos
(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2
- 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(a*cos(f*x + e)^2 -
 2*a*cos(f*x + e) + a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e)
 - a))*sin(f*x + e) + 2*(26*a*cos(f*x + e)^3 - 35*a*cos(f*x + e)^2 + 15*a*c
os(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^
2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{c^3}$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)
[Out] -(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2
+ 3*sec(e + f*x) - 1), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(c\sec(fx+e)-c)^3}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^3, x)
```

3.56 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4} dx$

Optimal result 438
Rubi [A] (verified) 438
Mathematica [C] (verified) 441
Maple [B] (verified) 441
Fricas [A] (verification not implemented) 442
Sympy [F] 442
Maxima [F(-1)] 443
Giac [F] 443
Mupad [F(-1)] 443

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4} dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} - \frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^4/f-2/3*cot(f
*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^4/f+2/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2
)/a/c^4/f-4/7*cot(f*x+e)^7*(a+a*sec(f*x+e))^(7/2)/a^2/c^4/f+2*a*cot(f*x+e)*
(a+a*sec(f*x+e))^(1/2)/c^4/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3989, 3972, 464, 331, 209}

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4} dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^4f} - \frac{4\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7a^2c^4f} + \frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5ac^4f} - \frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^4f} + \frac{2a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^4f}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^4*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a*c^4*f) - (4*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/(7*a
^2*c^4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^{11/2}\,dx}{a^4c^4}$$

$$= -\frac{2\text{Subst}\left(\int \frac{2+ax^2}{x^8(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^4f}$$

$$= -\frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f} + \frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^4f}$$

$$= \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} - \frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f} - \frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f}$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} - \frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f} + \frac{(2a)\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f}$$

$$= \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} - \frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f}$$

$$= \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} - \frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.60 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{2a^2\left(-10+7\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\sec(e+fx)\right)(-1+\sec(e+\right.}{35c^4f(-1+\sec(e+fx))^4\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*a^2*(-10 + 7*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]]*(-1 + Se
c[e + f*x]))*Tan[e + f*x])/(35*c^4*f*(-1 + Sec[e + f*x])^4*Sqrt[a*(1 + Sec[
e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(152) = 304.

Time = 2.46 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.03

method	result
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(105\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^3-315\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\right.\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/c^4/f*a*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^3*(105*(-cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*
x+e)+1))^(1/2))*cos(f*x+e)^3-315*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh
(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)^2
+191*cos(f*x+e)^3*cot(f*x+e)+315*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*
x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-4
06*cos(f*x+e)^2*cot(f*x+e)-105*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+
e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+350*cos(f*x+e)
*cot(f*x+e)-105*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.88

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \left[\frac{105\left(a\cos\left(fx+e\right)^3-3\,a\cos\left(fx+e\right)^2+3\,a\cos\left(fx+e\right)-a\right)\sqrt{-a}\log\left(-\frac{8}{\cdots}\right.}{\cdots}\right.$$

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")

[Out] [1/210*(105*(a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(191*a*cos(f*x + e)^4 - 406*a*cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(191*a*cos(f*x + e)^4 - 406*a*cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx + \int \frac{a\sqrt{a\sec\left(e+fx\right)+\cdots}}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\,\mathrm{se}\cdots}}{c^4}$$

[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**4,x)

[Out] (Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(c\sec(fx+e)-c)^4}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^4,x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^4, x)
```

3.57 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx$

Optimal result 444
Rubi [A] (verified) 444
Mathematica [A] (verified) 446
Maple [A] (verified) 446
Fricas [A] (verification not implemented) 447
Sympy [F] 448
Maxima [F(-1)] 448
Giac [F] 449
Mupad [F(-1)] 449

Optimal result

Integrand size = 28, antiderivative size = 212

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = \frac{2a^{5/2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^3c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^4c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^5c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{6a^6c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} - \frac{2a^7c^3\tan^9(e+fx)}{9f(a+a\sec(e+fx))^{9/2}}$$

```
[Out] 2*a^(5/2)*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c^3
*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^4*c^3*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)-2/5*a^5*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)-6/7*a^6*c^3*t
an(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)-2/9*a^7*c^3*tan(f*x+e)^9/f/(a+a*sec(f*
x+e))^(9/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 472, 209}

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = \frac{2a^{5/2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f} - \frac{2a^7c^3\tan^9(e+fx)}{9f(a\sec(e+fx)+a)^{9/2}} - \frac{6a^6c^3\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}} - \frac{2a^5c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}} + \frac{2a^4c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}} - \frac{2a^3c^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(5/2)*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^3*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*c^3*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^5*c^3*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) - (6*a^6*c^3*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2)) - (2*a^7*c^3*Tan[e + f*x]^9)/(9*f*(a + a*Sec[e + f*x])^(9/2
))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx\right)$$

$$= \frac{\left(2a^6c^3\right)\text{Subst}\left(\int \frac{x^6\left(2+ax^2\right)^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{(2a^6c^3)\,\mathrm{Subst}\left(\int\left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+3x^6+ax^8-\frac{1}{a^3(1+ax^2)}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^3c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^4c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^5c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}-\frac{6a^6c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} - \frac{2a^7c^3\tan^9(e+fx)}{9f(a+a\sec(e+fx))^{9/2}}-\frac{(2a^3c^3)\,\mathrm{Subst}\left(\int\frac{1}{1+ax^2}dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^{5/2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^4c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^5c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}-\frac{6a^6c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}-\frac{2a^7c^3\tan^9(e+fx)}{9f(a+a\sec(e+fx))^{9/2}}$$

Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.63

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = \frac{2a^3c^3\left(315\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(-383-34\sec(e+fx)+1\right.}{315f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^3*c^3*(315*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c
- c*Sec[e + f*x]]*(-383 - 34*Sec[e + f*x] + 132*Sec[e + f*x]^2 + 5*Sec[e +
f*x]^3 - 35*Sec[e + f*x]^4))*Tan[e + f*x])/(315*f*Sqrt[a*(1 + Sec[e + f*x])
]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 89.61 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.05

method	result
default	$-\frac{2a^2c^3\sqrt{a(\sec(fx+e)+1)}\left(-315\,\mathrm{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)-315\,\mathrm{arctanh}\left(\frac{}{(\cos(fx+e)}\right.\right.}{}$
parts	$\frac{2c^3a^2\sqrt{a(\sec(fx+e)+1)}\left(3\,\mathrm{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+3\,\mathrm{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{c}{c}}}\right.\right.}{3f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*a^2*c^3/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(-315*arctanh(sin(
f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*cos(f*x+e)-315*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*
x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+383*sin(f*x+
e)+34*tan(f*x+e)-132*sec(f*x+e)*tan(f*x+e)-5*sec(f*x+e)^2*tan(f*x+e)+35*sec
(f*x+e)^3*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.08

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = \left[\frac{315\left(a^2c^3\cos(fx+e)^5+a^2c^3\cos(fx+e)^4\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{\cos(fx}\right.}{}\right.$$

$$-\frac{2\left(315\left(a^2c^3\cos(fx+e)^5+a^2c^3\cos(fx+e)^4\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(383\,a^2c^3\cos(fx+\right.\right.}{315\left(f\cos(fx+\right.}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(383*
a^2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos(f*x +
e)^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -2/315*(315*(
a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (383*a^
```

```
2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos(f*x + e)
^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4)]
```

Sympy [F]

$$\begin{aligned}\int(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx &= \\ -c^3\bigg(\int\left(-a^2\sqrt{a\sec(e+fx)+a}\right)\,dx &+\int a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)\,dx \\ &+\int 2a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx \\ &+\int\left(-2a^2\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\right)\,dx \\ &+\int\left(-a^2\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)\right)\,dx \\ &+\int a^2\sqrt{a\sec(e+fx)+a}\sec^5(e+fx)\,dx\bigg)\end{aligned}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**3,x)
[Out] -c**3*(Integral(-a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(a**2*sqrt(a*s
ec(e + f*x) + a)*sec(e + f*x), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a
)*sec(e + f*x)**2, x) + Integral(-2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x)**3, x) + Integral(-a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x) +
Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**5, x))
```

Maxima [F(-1)]

Timed out.

$$\int(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^3 \, dx = \int -(a\sec(fx + e) + a)^{\frac{5}{2}}(c\sec(fx + e) - c)^3 \, dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^3 \, dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{5/2} \left(c - \frac{c}{\cos(e + f\,x)}\right)^3 \, dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^3, x)
```

3.58 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx$

Optimal result 450
Rubi [A] (verified) 450
Mathematica [A] (verified) 452
Maple [A] (verified) 452
Fricas [A] (verification not implemented) 453
Sympy [F] 453
Maxima [F] 454
Giac [F] 460
Mupad [F(-1)] 460

Optimal result

Integrand size = 28, antiderivative size = 177

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \frac{2a^{5/2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^3c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^4c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{6a^5c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{2a^6c^2\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

```
[Out] 2*a^(5/2)*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c^2
*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^4*c^2*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)+6/5*a^5*c^2*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)+2/7*a^6*c^2*t
an(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 472, 209}

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \frac{2a^{5/2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f} + \frac{2a^6c^2\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}} + \frac{6a^5c^2\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}} + \frac{2a^4c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}} - \frac{2a^3c^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^3*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*c^2*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) + (6*a^5*c^2*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) + (2*a^6*c^2*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = (a^2c^2)\int \sqrt{a+a\sec(e+fx)}\tan^4(e+fx)\,dx$$

$$= -\frac{(2a^5c^2)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{(2a^5c^2)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+3x^4+ax^6+\frac{1}{a^2(1+ax^2)}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^3c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^4c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{6a^5c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{2a^6c^2\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} - \frac{(2a^3c^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^{5/2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^3c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^4c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{6a^5c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{2a^6c^2\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \frac{2a^3c^2\left(105\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right) + \sqrt{c-c\sec(e+fx)}(-92-46\sec(e+fx)+18\right.}{105f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+f}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*c^2*(105*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c
- c*Sec[e + f*x]]*(-92 - 46*Sec[e + f*x] + 18*Sec[e + f*x]^2 + 15*Sec[e + f
*x]^3))*Tan[e + f*x])/(105*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e +
f*x]])
```

Maple [A] (verified)

Time = 22.84 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.17

method	result
default	$-\frac{2a^2c^2\sqrt{a(\sec(fx+e)+1)}\left(-105\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)-105\,\text{arctanh}\left(\frac{\text{si}}{(\cos(fx+e)+}\right.\right.}{105f(\cos(fx+}$
parts	$\frac{2c^2a^2\sqrt{a(\sec(fx+e)+1)}\left(3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+3\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos}{\cos(}}}\right.\right.}{3f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*a^2*c^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(-105*arctanh(sin(
f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos
```

```
(f*x+e)+1))^(1/2)*cos(f*x+e)-105*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*
x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+92*sin(f*x+e
)+46*tan(f*x+e)-18*sec(f*x+e)*tan(f*x+e)-15*sec(f*x+e)^2*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.31

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \left[\frac{105\left(a^2c^2\cos(fx+e)^4+a^2c^2\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cdots}{\cos(fx\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{2\left(105\left(a^2c^2\cos(fx+e)^4+a^2c^2\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(92a^2c^2\cos(fx+e)\cdots\right.}{105\left(f\cos(fx+e)^4+f\cos(\cdots\right.}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(92*a
^2*c^2*cos(f*x + e)^3 + 46*a^2*c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x + e)
 - 15*a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos
(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*
c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (92*a^2*c^2*cos(f*x + e)^3 + 46*a^2*
c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x + e) - 15*a^2*c^2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)
]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = c^2\left(\int a^2\sqrt{a\sec(e+fx)+a}\,dx + \int\left(-2a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\right)dx + \int a^2\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(-2*a**2*sqrt(a*
sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(a**2*sqrt(a*sec(e + f*x) +
 a)*sec(e + f*x)**4, x))
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)^2\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/210*(105*((a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*a
^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) + 1) - (a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*a^2
*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
) - 1) - 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*f*sin(2*f*x + 2*e)^2 + 2
*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + s
in(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + si
n(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (c
os(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - co
s(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(
9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*c
os(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*co
s(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*
f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin
(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*
x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*
```

```
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*
e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e
)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (c
os(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6
*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^
2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*co
s(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*co
s(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*
x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x
 + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + co
s(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e)
)*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e
) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f
*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)), x) + 8*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*
f*sin(2*f*x + 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate(
(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
+ cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*
e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*
e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e
)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*c
os(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e
)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)
*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2
*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*
sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2
)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x +
2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
```

```
f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*
x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*
e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x +
4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2
*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2
+ cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x +
2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*
f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^
2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*si
n(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*
x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*
f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*
(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*
f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 12*(a^2*c^2*f*cos(
2*f*x + 2*e)^2 + a^2*c^2*f*sin(2*f*x + 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e
) + a^2*c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x
 + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x
+ 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x
+ 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x +
 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x
+ 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x +
 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x +
6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2
*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
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e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*
x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x +
 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*si
n(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*co
s(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x +
 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^
3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f
*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (co
s(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2
*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2
*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*
e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2
*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x +
 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*s
in(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2
)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4))
, x) + 8*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*f*sin(2*f*x + 2*e)^2 + 2*a
^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(2*
f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin
(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(
2*f*x + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos
(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(
6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos
(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(
4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
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+ 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*
x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
+ 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)
^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^
2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x
+ 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*
f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e
) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2
+ 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(
4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x
+ 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x +
 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(
2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*
cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)
+ (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x
 + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)), x) - 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*f*
sin(2*f*x + 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate(((
(cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)
*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)
*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*
sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos
```

(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(3/4)*sqrt(a) - 16*(7*(5*a^2*c^2*sin(4*f*x + 4*e) + 4*a^2*c^2*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (35*a^2*c^2*cos(4*f*x + 4*e) + 28*a^2*c^2*cos(2*f*x + 2*e) + 23*a^2*c^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*

```
x + 2*e) + 1)))*sqrt(a))/((f*cos(2*f*x + 2*e)^2 + f*sin(2*f*x + 2*e)^2 + 2*
f*cos(2*f*x + 2*e) + f)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(3/4))
```

Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)^2\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2, x)
```

3.59 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 132

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = \frac{2a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^3c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^4c\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{2a^5c\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 2*a^(5/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c*tan
(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2*a^4*c*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(3
/2)-2/5*a^5*c*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3989, 3972, 472, 209}

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = \frac{2a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f} - \frac{2a^5c\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}} - \frac{2a^4c\tan^3(e+fx)}{f(a\sec(e+fx)+a)^{3/2}} - \frac{2a^3c\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(5/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a^3*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*a^4*c*Tan[e + f*x]^
3)/(f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^5*c*Tan[e + f*x]^5)/(5*f*(a + a*Se
c[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left((ac)\int (a+a\sec(e+fx))^{3/2}\tan^2(e+fx)\,dx\right)\\
&= \frac{(2a^4c)\,\mathrm{Subst}\left(\int \frac{x^2(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{(2a^4c)\,\mathrm{Subst}\left(\int \left(\frac{1}{a}+3x^2+ax^4-\frac{1}{a(1+ax^2)}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{2a^3c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2a^4c\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}}\\
&\quad -\frac{2a^5c\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}-\frac{(2a^3c)\,\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}
\end{aligned}$$

$$= \frac{2a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^3c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^4c\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{2a^5c\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = -\frac{2a^3c\left(-5\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(1+3\sec(e+fx)+\sec^2(e+fx))\right)\tan(e+fx}{5f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] (-2*a^3*c*(-5*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c -
c*Sec[e + f*x]]*(1 + 3*Sec[e + f*x] + Sec[e + f*x]^2))*Tan[e + f*x])/(5*f*S
qrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (warning: unable to verify)

Time = 7.99 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.44

method	result
default	$\frac{a^2c\left(5\sqrt{2}\ \text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{5}{2}}+2(1-\cos(fx+e))^5\csc(fx+e)^5-10\csc(fx\cdots}{5f(-\cot(fx+e)+\csc(fx+e)+1)^2(-\cot(fx+e)+\csc(fx+e)-1)^2}$
parts	$\frac{2c\,a^2\sqrt{a(\sec(fx+e)+1)}\left(3\ \text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \cos(fx+e)+3\ \text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\cdots}}\cdots\right.\right.}{3f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^2*c/f*(5*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1
/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)+2*(1-
cos(f*x+e))^5*csc(f*x+e)^5-10*csc(f*x+e)+10*cot(f*x+e))*(-2*a/((1-cos(f*x+e
))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)+1)^2/(-cot(f*x+e)+csc(f
*x+e)-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.67

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = \left[\frac{5\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+\ldots}{\cos(fx+e)+1}\right.}{5\left(f\cos(fx\ldots\right.}\right.$$

$$-\frac{2\left(5\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(a^2c\cos(fx+e)^2+3\,a^2c\ldots\right.}{5\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/5*(5*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(a^2*c*cos(f*
x + e)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/5*(5*(a^2*c*c
os(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c*cos(f*x + e
)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = -c\left(\int\left(-a^2\sqrt{a\sec(e+fx)+a}\right)dx+\int\left(-a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx + \int a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx + \int a^2\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(-a**2*sqrt(a*sec
(e + f*x) + a)*sec(e + f*x), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*se
```

```
c(e + f*x)**2, x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3,
 x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. 2(118) = 236.

Time = 0.42 (sec) , antiderivative size = 1396, normalized size of antiderivative = 10.58

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 2*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e
) - 3*a^2*sin(2*f*x + 2*e) - 4*(3*a^2*cos(2*f*x + 2*e) + 4*a^2)*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) + (12*a^2*sin(2*f*x + 2*e)*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2*cos(2*f*x + 2*e) - a^2 + 4*(3*a^2*cos
(2*f*x + 2*e) + 4*a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a) + 3*((a
^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*f*x + 2*e) + a
^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))) + 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(
2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e
)^2 + 2*a^2*cos(2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
```

```
*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)) + 1) + (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*
f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*sqrt(a)
)*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx=\int -(a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x)), x)
```

3.60 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx$

Optimal result 467
Rubi [A] (verified) 467
Mathematica [C] (verified) 469
Maple [A] (verified) 469
Fricas [A] (verification not implemented) 469
Sympy [F] 470
Maxima [F] 470
Giac [F] 470
Mupad [F(-1)] 471

Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{8a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf} - \frac{2a^3\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+8*a^2*cot(f
*x+e)*(a+a*sec(f*x+e))^(1/2)/c/f-2*a^3*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2
)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 472, 209}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf} - \frac{2a^3\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}} + \frac{8a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (8*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f) - (2*a^3*Tan[e + f*x])
/(c*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^{7/2}\,dx}{ac}\\
&= \frac{(2a^2)\,\text{Subst}\left(\int \frac{(2+ax^2)^2}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\
&= \frac{(2a^2)\,\text{Subst}\left(\int \left(a+\frac{4}{x^2}-\frac{a}{1+ax^2}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\
&= \frac{8a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}-\frac{2a^3\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}}\\
&\quad -\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}
\end{aligned}
$$

$$=\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{8a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}-\frac{2a^3\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx=\frac{2a^2\csc(e+fx)\left(-1+4\cos(e+fx)+\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}\right.\right.}{cf}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]
[Out] (2*a^2*Csc[e + f*x]*(-1 + 4*Cos[e + f*x] + Cos[e + f*x]*Hypergeometric2F1[-
1/2, 1, 1/2, 1 - Sec[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x])])/(c*f)
```

Maple [A] (verified)

Time = 5.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+5\cot(fx+e)-\csc(fx+e)\right)}{cf}$	100

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2/c/f*a^2*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos
(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cot(f*x
+e)-csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.83

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx=\left[\frac{\sqrt{-a}a^2\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7}{\cos(fx+e)+1}\right.}{2\,cf\sin\left(fx\right.}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a^2*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x
 + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*
cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(5*a^2*cos(f*x + e)
- a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*sin(f*x + e)), (a^(5/2
)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(5*a^
2*cos(f*x + e) - a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*sin(f*x
 + e))]
```

Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{c - c\sec(e + fx)}\,dx = -\frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx + \int \frac{2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)}{\sec(e+fx)-1}\,dx}{c}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -(Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral(
2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integ
ral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2/(sec(e + f*x) - 1), x))/c
```

Maxima [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{c - c\sec(e + fx)}\,dx = \int -\frac{(a\sec(fx + e) + a)^{\frac{5}{2}}}{c\sec(fx + e) - c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^(5/2)/(c*sec(f*x + e) - c), x)
```

Giac [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{c - c\sec(e + fx)}\,dx = \int -\frac{(a\sec(fx + e) + a)^{\frac{5}{2}}}{c\sec(fx + e) - c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x)), x)
```

3.61 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx$

Optimal result 472
Rubi [A] (verified) 472
Mathematica [C] (verified) 474
Maple [B] (verified) 474
Fricas [B] (verification not implemented) 474
Sympy [F] 475
Maxima [F(-1)] 475
Giac [F] 475
Mupad [F(-1)] 476

Optimal result

Integrand size = 28, antiderivative size = 74

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} - \frac{8a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f-8/3*a*cot
(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^2/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 472, 209}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f} - \frac{8a\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^2f}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 - (8*a*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{9/2}\,dx}{a^2c^2} \\
&= -\frac{(2a)\text{Subst}\left(\int \frac{(2+ax^2)^2}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} \\
&= -\frac{(2a)\text{Subst}\left(\int \left(\frac{4}{x^4}+\frac{a^2}{1+ax^2}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} \\
&= -\frac{8a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} \\
&= \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} - \frac{8a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = -\frac{2a^3\left(\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx)\right)+3\sec(e+fx)\right)\tan(e+fx)}{3c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a^3*(Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]] + 3*Sec[e + f*x
])*Tan[e + f*x])/(3*c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(64) = 128.

Time = 13.92 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.36

method	result
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(3\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)-3\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(f}{\cos(fx}}}\right.\right.}{3c^2f(\cos(fx+e)-1)}$

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/c^2/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*(3*arctanh(sin(f*x+e)
/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)-3*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(co
s(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+4*cos(f*x+e)*cot(f*x
+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(64) = 128.

Time = 0.32 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.58

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \left[\frac{16\,a^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)^2+3\left(a^2\cos(fx+e)-a^2\right)\sqrt{-a}\log\left(-\frac{8\,a\cos}{}\right.}{6\left(c^2f\cos(fx+\right.}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(16*a^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)^2 + 3*(a^
2*cos(f*x + e) - a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)
^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
 + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e))/((c^2*f*cos
(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(8*a^2*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)^2 + 3*(a^2*cos(f*x + e) - a^2)*sqrt(a)*arctan(2*sqrt
(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*
cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e))/((c^2*f*cos(f*x + e) -
c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^2}\,dx = \frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{a^2\sqrt{a\sec(e+fx)}}{\sec^2(e+fx)-2}}{c^2}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] (Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) +
 1), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*
x)**2 - 2*sec(e + f*x) + 1), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*se
c(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^2}\,dx = \int \frac{(a\sec(fx + e) + a)^{\frac{5}{2}}}{(c\sec(fx + e) - c)^2}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^2}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^2, x)
```

3.62 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx$

Optimal result 477
Rubi [A] (verified) 477
Mathematica [C] (verified) 479
Maple [B] (verified) 479
Fricas [B] (verification not implemented) 480
Sympy [F] 480
Maxima [F(-1)] 481
Giac [F] 481
Mupad [F(-1)] 481

Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx=\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}+\frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f}+\frac{8\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^3f}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^3/f+8/5*cot(f
*x+e)^5*(a+a*sec(f*x+e))^(5/2)/c^3/f+2*a^2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2
)/c^3/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 472, 209}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx=\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^3f}+\frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^3f}+\frac{8\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5c^3f}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) + (8*Cot[e + f*x]^
5*(a + a*Sec[e + f*x])^(5/2))/(5*c^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{11/2}\,dx}{a^3c^3}\\
&= \frac{2\text{Subst}\left(\int \frac{(2+ax^2)^2}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{4}{x^6}+\frac{a^2}{x^2}-\frac{a^3}{1+ax^2}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}\\
&= \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f}+\frac{8\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^3f}\\
&\quad -\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}
\end{aligned}
$$

$$= \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} + \frac{8\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \frac{2a^3\left(4+3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\sec(e+fx)\right)+5\sec(e+fx)\right)}{15c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^3*(4 + 3*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]] + 5*Sec[e
+ f*x])*Tan[e + f*x])/(15*c^3*f*(-1 + Sec[e + f*x])^3*Sqrt[a*(1 + Sec[e + f
*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(92) = 184.

Time = 70.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.57

method	result
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(5\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^2-10\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{c}{co}}}\right.\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/c^3/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^2*(5*(-cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+
e)+1))^(1/2))*cos(f*x+e)^2-10*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e
)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+9*co
s(f*x+e)^2*cot(f*x+e)+5*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos
(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-10*cos(f*x+e)*cot(f*x
+e)+5*cot(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(92) = 184.

Time = 0.31 (sec) , antiderivative size = 441, normalized size of antiderivative = 4.24

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \left[\frac{5\left(a^2\cos(fx+e)^2-2\,a^2\cos(fx+e)+a^2\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(f\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/10*(5*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*a
*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x +
 e) + 1))*sin(f*x + e) + 4*(9*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x + e)^2 +
5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*
x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/5*(5*(a^2*cos(f*x
 + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a
*cos(f*x + e) - a))*sin(f*x + e) + 2*(9*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x
 + e)^2 + 5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^
3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{a^2\sqrt{a\sec(e+fx)+a}\sec^2\cdots}{\sec^3(e+fx)-3\sec^2(e+fx)+3\,\mathrm{s}\cdots}\cdots}{c^3}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -(Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)*
*2 + 3*sec(e + f*x) - 1), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a)*sec
(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) +
Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2/(sec(e + f*x)**3 - 3
*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(c\sec(fx+e)-c)^3}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^3, x)
```

3.63 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx$

Optimal result 482
Rubi [A] (verified) 482
Mathematica [C] (verified) 484
Maple [B] (verified) 484
Fricas [B] (verification not implemented) 485
Sympy [F(-1)] 485
Maxima [F(-1)] 486
Giac [F] 486
Mupad [F(-1)] 486

Optimal result

Integrand size = 28, antiderivative size = 140

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} - \frac{8\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7ac^4f}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^4/f-2/3*a*cot
(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^4/f-8/7*cot(f*x+e)^7*(a+a*sec(f*x+e))^(7
/2)/a/c^4/f+2*a^2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^4/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 472, 209}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^4f} + \frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^4f} - \frac{8\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7ac^4f} - \frac{2a\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^4f}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*a*Cot[e + f*x
]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^4*f) - (8*Cot[e + f*x]^7*(a + a*Sec[e
+ f*x])^(7/2))/(7*a*c^4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^{13/2}\,dx}{a^4c^4}\\
&= -\frac{2\text{Subst}\left(\int \frac{(2+ax^2)^2}{x^8(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^4f}\\
&= -\frac{2\text{Subst}\left(\int \left(\frac{4}{x^8}+\frac{a^2}{x^4}-\frac{a^3}{x^2}+\frac{a^4}{1+ax^2}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^4f}
\end{aligned}$$

$$= \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} - \frac{8\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7ac^4f} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f}$$

$$= \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} - \frac{8\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7ac^4f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = -\frac{2a^3\left(8+5\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\sec(e+fx)\right)+7\sec(e+fx)\right)\tan(e+fx)}{35c^4f(-1+\sec(e+fx))^4\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
[Out] (-2*a^3*(8 + 5*Hypergeometric2F1[-7/2, 1, -5/2, 1 - Sec[e + f*x]] + 7*Sec[e
 + f*x])*Tan[e + f*x])/(35*c^4*f*(-1 + Sec[e + f*x])^4*Sqrt[a*(1 + Sec[e +
f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(124) = 248.

Time = 1.64 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.51

$$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(21\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^3 - 63\sqrt{-\frac{\cos(}{\cos(f}}\right.}{}$$

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x)
[Out] 2/21/c^4/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^3*(21*(-cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*
x+e)+1))^(1/2))*cos(f*x+e)^3-63*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(
sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)^2+
40*cos(f*x+e)^3*cot(f*x+e)+63*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e
)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-77*c
```

```
os(f*x+e)^2*cot(f*x+e)-21*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(c
os(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+70*cos(f*x+e)*cot(f
*x+e)-21*cot(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(124) = 248.

Time = 0.34 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.76

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \left[\frac{21\left(a^2\cos(fx+e)^3-3\,a^2\cos(fx+e)^2+3\,a^2\cos(fx+e)-a^2\right)\sqrt{-a}\log}{}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
[Out] [1/42*(21*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) -
 a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e
))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(
f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(40*a^2*cos(f*x + e)^4 -
 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a^2*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*
x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/21*(21*(a^2*cos(f
*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) - a^2)*sqrt(a)*arctan
(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(40*a^2*cos(f
*x + e)^4 - 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a^2*cos(f*x
+ e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c
^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**4,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(c\sec(fx+e)-c)^4}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^4,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^4, x)
```

3.64 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 172

$$
\begin{aligned}
\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx &= \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^5f} \\
&+ \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^5f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^5f} \\
&+ \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^5f} + \frac{8\cot^9(e+fx)(a+a\sec(e+fx))^{9/2}}{9a^2c^5f}
\end{aligned}
$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^5/f-2/3*a*cot
(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^5/f+2/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5
/2)/c^5/f+8/9*cot(f*x+e)^9*(a+a*sec(f*x+e))^(9/2)/a^2/c^5/f+2*a^2*cot(f*x+e
)*(a+a*sec(f*x+e))^(1/2)/c^5/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 472, 209}

$$
\begin{aligned}
\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx &= \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^5f} \\
&+ \frac{8\cot^9(e+fx)(a\sec(e+fx)+a)^{9/2}}{9a^2c^5f} + \frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^5f} \\
&+ \frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5c^5f} - \frac{2a\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^5f}
\end{aligned}
$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^5*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^5*f) - (2*a*Cot[e + f*x
]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^5*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e
+ f*x])^(5/2))/(5*c^5*f) + (8*Cot[e + f*x]^9*(a + a*Sec[e + f*x])^(9/2))/(9
*a^2*c^5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \cot^{10}(e+fx)(a+a\sec(e+fx))^{15/2}\,dx}{a^5c^5} \\
&= \frac{2\text{Subst}\left(\int \frac{(2+ax^2)^2}{x^{10}(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^5f} \\
&= \frac{2\text{Subst}\left(\int \left(\frac{4}{x^{10}}+\frac{a^2}{x^6}-\frac{a^3}{x^4}+\frac{a^4}{x^2}-\frac{a^5}{1+ax^2}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^5f}
\end{aligned}
$$

$$= \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^5f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^5f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^5f} + \frac{8\cot^9(e+fx)(a+a\sec(e+fx))^{9/2}}{9a^2c^5f} - \frac{(2a^3)\,\mathrm{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^5f}$$

$$= \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^5f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^5f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^5f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^5f} + \frac{8\cot^9(e+fx)(a+a\sec(e+fx))^{9/2}}{9a^2c^5f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int\frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \frac{2a^3\left(12+7\,\mathrm{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},1-\sec(e+fx)\right)+9\sec(e+fx\right.}{63c^5f(-1+\sec(e+fx))^5\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (2*a^3*(12 + 7*Hypergeometric2F1[-9/2, 1, -7/2, 1 - Sec[e + f*x]] + 9*Sec[e
 + f*x])*Tan[e + f*x])/(63*c^5*f*(-1 + Sec[e + f*x])^5*Sqrt[a*(1 + Sec[e +
f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(152) = 304.

Time = 1.35 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.53

$$\frac{2a^2\sqrt{a\left(\sec\left(fx+e\right)+1\right)}\left(45\,\mathrm{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos\left(fx+e\right)^4-180\sqrt{-\frac{}{c}}\right.}{}$$

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 2/45/c^5/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)^4*(45*arctanh(sin(f*
x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*cos(f*x+e)^4-180*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh
(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)^3
```

```
+270*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(
-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)^2+89*cos(f*x+e)^4*cot(f*x+e)-
180*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(
-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-243*cos(f*x+e)^3*cot(f*x+e)+45
*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)+324*cos(f*x+e)^2*cot(f*x+e)-195*cos(f*x+e)*c
ot(f*x+e)+45*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.49

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \left[\frac{45\left(a^2\cos(fx+e)^4-4a^2\cos(fx+e)^3+6a^2\cos(fx+e)^2-4a^2\cos(fx+\right.}{}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/90*(45*(a^2*cos(f*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^2
 - 4*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f
*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(
89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324*a^2*cos(f*x + e)^3 - 1
95*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e)))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*
x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e)), 1/45*(45*(a^2*cos(f
*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^2 - 4*a^2*cos(f*x + e
) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x
 + e) + 2*(89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324*a^2*cos(f*x
 + e)^3 - 195*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e)))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c
^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**5,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(c\sec(fx+e)-c)^5}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^5}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^5,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^5, x)
```

3.65 $\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result . . . 492
Rubi [A] (verified) . . . 492
Mathematica [A] (verified) . . . 495
Maple [A] (warning: unable to verify) . . . 495
Fricas [A] (verification not implemented) . . . 496
Sympy [F] . . . 497
Maxima [F] . . . 497
Giac [F(-2)] . . . 497
Mupad [F(-1)] . . . 498

Optimal result

Integrand size = 28, antiderivative size = 185

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{16\sqrt{2}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} + \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-16*c^4*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/
2)+14*c^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2*a*c^4*tan(f*x+e)^3/f/(a+a*s
ec(f*x+e))^(3/2)+2/5*a^2*c^4*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3989, 3972, 490, 596, 536, 209}

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2a^2c^4\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}} + \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f} - \frac{16\sqrt{2}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f} - \frac{2ac^4\tan^3(e+fx)}{f(a\sec(e+fx)+a)^{3/2}} + \frac{14c^4\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (16*Sqrt[2]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(Sqrt[a]*f) + (14*c^4*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
 - (2*a*c^4*Tan[e + f*x]^3)/(f*(a + a*Sec[e + f*x])^(3/2)) + (2*a^2*c^4*Tan
[e + f*x]^5)/(5*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= (a^4c^4)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx\\
&= -\frac{(2a^4c^4)\,\text{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}+\frac{(2a^2c^4)\,\text{Subst}\left(\int \frac{x^4(10+15ax^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{5f}\\
&= -\frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}}+\frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}\\
&\quad-\frac{(2c^4)\,\text{Subst}\left(\int \frac{x^2(90a+105a^2x^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{15f}\\
&= \frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}}+\frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}\\
&\quad+\frac{(2c^4)\,\text{Subst}\left(\int \frac{210a^2+225a^3x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{15a^2f}\\
&= \frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}}\\
&\quad+\frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}-\frac{(2c^4)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&\quad+\frac{(32c^4)\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{16\sqrt{2}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}\\
&\quad+\frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}}+\frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.43 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{(c - c\sec(e + fx))^4}{\sqrt{a + a\sec(e + fx)}}\,dx$$
$$= \frac{c^4 \cot\left(\frac{1}{2}(e + fx)\right)\left(100 - 155\cos(e + fx) + 96\cos(2(e + fx)) - 41\cos(3(e + fx)) + 20\arctan\left(\sqrt{-1 + \cdots}\right.\right.}{1\cdots}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^4*Cot[(e + f*x)/2]*(100 - 155*Cos[e + f*x] + 96*Cos[2*(e + f*x)] - 41*Co
s[3*(e + f*x)] + 20*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3*Sqrt[-1
+ Sec[e + f*x]] - 160*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e
 + f*x]^3*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^3)/(10*f*Sqrt[a*(1 + Sec[e
+ f*x])])
```

Maple [A] (warning: unable to verify)

Time = 6.49 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.52

method	result
default	$\frac{c^4\left(5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{5}{2}}-80\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{(1-\cos(f\cdots}\right.\right.}{5f\cdots}$
parts	$-\frac{c^4\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\cdots\right.}{fa}$

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*c^4/f/a*(5*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1
/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)-80*ln
(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x
+e))^2*csc(f*x+e)^2-1)^(5/2)+98*(1-cos(f*x+e))^5*csc(f*x+e)^5-160*(1-cos(f*
x+e))^3*csc(f*x+e)^3+70*csc(f*x+e)-70*cot(f*x+e))*(-2*a/((1-cos(f*x+e))^2*c
sc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)+1)^2/(-cot(f*x+e)+csc(f*x+e)-
1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.98

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[\frac{40\sqrt{2}\left(ac^4\cos(fx+e)^3+ac^4\cos(fx+e)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{}\right.$$

$$-\frac{2\left(5\left(c^4\cos(fx+e)^3+c^4\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(41c^4\cos(fx+e)^2-7c\right.\right.}{5\left(af\cos(fx+\right.}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/5*(40*sqrt(2)*(a*c^4*cos(f*x + e)^3 + a*c^4*cos(f*x + e)^2)*sqrt(-1/a)*l
og((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x +
e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 +
2*cos(f*x + e) + 1)) - 5*(c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*sqrt(-a)
*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*
(41*c^4*cos(f*x + e)^2 - 7*c^4*cos(f*x + e) + c^4)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^2), -2
/5*(5*(c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (41*c^4*
cos(f*x + e)^2 - 7*c^4*cos(f*x + e) + c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e) - 40*sqrt(2)*(a*c^4*cos(f*x + e)^3 + a*c^4*cos(f*x + e
)^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^4}{\sqrt{a + a\sec(e + fx)}}\,dx = c^4\left(\int \left(-\frac{4\sec(e + fx)}{\sqrt{a\sec(e + fx) + a}}\right)dx + \int \frac{6\sec^2(e + fx)}{\sqrt{a\sec(e + fx) + a}}\,dx + \int \left(-\frac{4\sec^3(e + fx)}{\sqrt{a\sec(e + fx) + a}}\right)dx + \int \frac{\sec^4(e + fx)}{\sqrt{a\sec(e + fx) + a}}\,dx + \int \frac{1}{\sqrt{a\sec(e + fx) + a}}\,dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(6*se
c(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(-4*sec(e + f*x)**3/sq
rt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**4/sqrt(a*sec(e + f*x) +
 a), x) + Integral(1/sqrt(a*sec(e + f*x) + a), x))
```

Maxima [F]

$$\int \frac{(c - c\sec(e + fx))^4}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{(c\sec(fx + e) - c)^4}{\sqrt{a\sec(fx + e) + a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^4/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^4}{\sqrt{a + a\sec(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^4}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^4}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(1/2), x)
```

3.66 $\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result 499
Rubi [A] (verified) 499
Mathematica [A] (verified) 501
Maple [A] (warning: unable to verify) 502
Fricas [A] (verification not implemented) 502
Sympy [F] 504
Maxima [F] 504
Giac [F(-2)] 504
Mupad [F(-1)] 505

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{8\sqrt{2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-8*c^3*arc
tan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2
)+6*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*a*c^3*tan(f*x+e)^3/f/(a+a*s
ec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3989, 3972, 490, 596, 536, 209}

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f} - \frac{8\sqrt{2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f} - \frac{2ac^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}} + \frac{6c^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (8*Sqrt[2]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
```

```
f*x]])])/(Sqrt[a]*f) + (6*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) -
(2*a*c^3*Tan[e + f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
```

```
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\left((a^3c^3)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx\right)$$
$$= \frac{(2a^3c^3)\,\text{Subst}\left(\int \frac{x^6}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= -\frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{(2ac^3)\,\text{Subst}\left(\int \frac{x^2(6+9ax^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{3f}$$
$$= \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{(2c^3)\,\text{Subst}\left(\int \frac{18a+21a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{3af}$$
$$= \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{(2c^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{(16c^3)\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{8\sqrt{2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.09

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$= \frac{4c^3\cos\left(\frac{e}{2}\right)\cos(e)\cot\left(\frac{1}{2}(e+fx)\right)\left(-6+11\cos(e+fx)-5\cos(2(e+fx))+3\arctan\left(\sqrt{-1+\sec(e+f\right.\right.}{3f\left(\cos\left(\frac{e}{2}\right)+\text{co}\right.}$$

[In] Integrate[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]

[Out] (4*c^3*Cos[e/2]*Cos[e]*Cot[(e + f*x)/2]*(-6 + 11*Cos[e + f*x] - 5*Cos[2*(e + f*x)] + 3*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]] - 12*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(3*f*(Cos[e/2] + Cos[(3*e)/2])*Sqrt[a*(1 + Sec[e + f*x])])

Maple [A] (warning: unable to verify)

Time = 5.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.71

method	result
default	$\frac{c^3\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}-24\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{(1-\cos(fx\cdots\right.\right.}{3fa(-\cot(fx+e)+\mathrm{cs}\cdots}$
parts	$-\frac{c^3\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-\cdots\right.}{fa}$

[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*c^3/f/a*(3*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)-24*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)+22*(1-cos(f*x+e))^3*csc(f*x+e)^3-18*csc(f*x+e)+18*cot(f*x+e))*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)+1)/(-cot(f*x+e)+csc(f*x+e)-1)

Fricas [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.41

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[\frac{12\sqrt{2}\left(ac^3\cos(fx+e)^2+ac^3\cos(fx+e)\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{}\right.$$

$$-\frac{2\left(3\left(c^3\cos(fx+e)^2+c^3\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-(10c^3\cos(fx+e)-c^3)\right.}{3\left(af\cos(fx+e)^2+af\right.}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(12*sqrt(2)*(a*c^3*cos(f*x + e)^2 + a*c^3*cos(f*x + e))*sqrt(-1/a)*log
((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)
*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) - 3*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)*log
((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(10*
c^3*cos(f*x + e) - c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e
))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -2/3*(3*(c^3*cos(f*x + e)^2 + c
^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)/(sqrt(a)*sin(f*x + e))) - (10*c^3*cos(f*x + e) - c^3)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 12*sqrt(2)*(a*c^3*cos(f*x + e)^
2 + a*c^3*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^2 + a*f
*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = -c^3\left(\int \frac{3\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int \left(-\frac{3\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)dx + \int \frac{\sec^3(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int \left(-\frac{1}{\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-3*s
ec(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**3/sqrt
(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(a*sec(e + f*x) + a), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int -\frac{(c\sec(fx+e)-c)^3}{\sqrt{a\sec(fx+e)+a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)^3/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)
```

3.67 $\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result . . . 506
Rubi [A] (verified) . . . 506
Mathematica [A] (verified) . . . 508
Maple [B] (verified) . . . 508
Fricas [A] (verification not implemented) . . . 509
Sympy [F] . . . 510
Maxima [F] . . . 510
Giac [F(-2)] . . . 510
Mupad [F(-1)] . . . 511

Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{4\sqrt{2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}+\frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-4*c^2*arc
tan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2
)+2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3989, 3972, 490, 536, 209}

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{4\sqrt{2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}+\frac{2c^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (4*Sqrt[2]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f) + (2*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = \left(a^2c^2\right)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$$

$$= -\frac{(2a^2c^2)\,\mathrm{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{(2c^2)\,\mathrm{Subst}\left(\int \frac{2+3ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(2c^2)\,\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{(8c^2)\,\mathrm{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{4\sqrt{2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^2\left(\operatorname{arctanh}\left(\sqrt{1-\sec(e+fx)}\right) - 2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right) + \sqrt{1-\sec(e+fx)}\right)\tan(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^2*(ArcTanh[Sqrt[1 - Sec[e + f*x]]] - 2*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[e
+ f*x]]/Sqrt[2]] + Sqrt[1 - Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt[1 - Sec[e
+ f*x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(102) = 204.

Time = 5.18 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.73

method	result
default	$-\frac{2c^2\sqrt{a(\sec(fx+e)+1)}\left(2\sqrt{2}\cos(fx+e)\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\sqrt{-}\right.}{}$
parts	$-\frac{c^2\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\right.}{fa}$

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c^2/f/a*(a*(sec(f*x+e)+1))^(1/2)*(2*2^(1/2)*cos(f*x+e)*ln(csc(f*x+e)-cot
(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))*(-cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)+2*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*l
n(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-
1)^(1/2))-arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1
/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-arctanh(sin(f*x+e)/(cos(
f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)-sin(f*x+e))/(cos(f*x+e)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.68

$$\int \frac{(c - c\sec(e + fx))^2}{\sqrt{a + a\sec(e + fx)}}\,dx$$

$$= \left[\frac{2\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e) + 2\sqrt{2}(ac^2\cos(fx+e) + ac^2)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(}{\cos(fx+e)^2+2}\right.}{}\right.$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 2*sqrt(2)*(a
*c^2*cos(f*x + e) + a*c^2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 +
 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - (c^2*cos(f*x
+ e) + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos
```

```
(f*x + e) + 1)))/(a*f*cos(f*x + e) + a*f), 2*(c^2*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2
*sqrt(2)*(a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*
x + e) + a*f)]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = c^2\left(\int \left(-\frac{2\sec{(e+fx)}}{\sqrt{a\sec{(e+fx)}+a}}\right)dx + \int \frac{\sec^2{(e+fx)}}{\sqrt{a\sec{(e+fx)}+a}}\,dx + \int \frac{1}{\sqrt{a\sec{(e+fx)}+a}}\,dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
[Out] c**2*(Integral(-2*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(
e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(1/sqrt(a*sec(e + f*x) +
 a), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c\sec{(fx+e)}-c)^2}{\sqrt{a\sec{(fx+e)}+a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((c*sec(f*x + e) - c)^2/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^2}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^2}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2), x)
```

3.68 $\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result 512
Rubi [A] (verified) 512
Mathematica [A] (verified) 514
Maple [A] (verified) 514
Fricas [A] (verification not implemented) 515
Sympy [F] 515
Maxima [C] (verification not implemented) 516
Giac [F(-2)] 516
Mupad [F(-1)] 517

Optimal result

Integrand size = 26, antiderivative size = 87

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-2*c*arctan(
1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3989, 3972, 492, 209}

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}$$

```
[In] Int[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f) -
 (2*Sqrt[2]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x
]])]])/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left((ac)\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx\right)\\
&= \frac{(2ac)\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{(2c)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{(4c)\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^{3/2}\left(\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\right)\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^(3/2)*(ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] - Sqrt[2]*ArcTanh[Sqr
t[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])])*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec[
e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

method	result
default	$-\frac{2c\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-\right.}{fa}$
parts	$-\frac{c\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-2\right.}{fa}$

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c/f/a*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2^(1/
2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e
)^2-1)^(1/2))-arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.43

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\,dx$$
$$= \left[\frac{\sqrt{2}ac\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right) - \sqrt{-a}c\log\left(\frac{2a\cos(fx+e)}{\cdots}\right.}{af}\right.$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a*c*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x +
e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(-a)*c*log((2*a*cos(f*
x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*
sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f), 2*(sqrt(2)*s
qrt(a)*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e))) - sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\,dx = -c\left(\int \frac{\sec(e + fx)}{\sqrt{a\sec(e + fx) + a}}\,dx + \int \left(-\frac{1}{\sqrt{a\sec(e + fx) + a}}\right)dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c*(Integral(sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(a
*sec(e + f*x) + a), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 699, normalized size of antiderivative = 8.03

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\left(\sqrt{2}\sqrt{a}\arctan\left(\frac{\left(\left|2\,e^{(i\,fx+i\,e)}+2\right|^4+16\,\cos(fx+e)^4+16\,\sin(fx+e)^4+8\left(\cos(fx+e)^2-\sin(fx+e)^2-2\,\cos(fx+e)+1\right)\left|2\,e^{(i\,fx+i\,e)}+2\right|^2 \cdots}{\cdots}\right.\right.}{\cdots}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*f*x + I*e) + 2)^4 + 16*cos(f*x + e)^
4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^2 - 2*cos(f*x + e)
 + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 + 32*(cos(f*x + e)^2
 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)^2 - 64*cos(f*x + e)
 + 16)^(1/4)*sin(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(f*x + e)/abs(2*e^(I*f
*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*cos(f*x + e)^2 - 4*sin(
f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*e) + 2)^2)) + 2*sin(f*x
 + e))/abs(2*e^(I*f*x + I*e) + 2), ((abs(2*e^(I*f*x + I*e) + 2)^4 + 16*cos(
f*x + e)^4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^2 - 2*cos
(f*x + e) + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 + 32*(cos(f
*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)^2 - 64*cos
(f*x + e) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(f*x + e)/abs
(2*e^(I*f*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*cos(f*x + e)^2
 - 4*sin(f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*e) + 2)^2)) +
2*cos(f*x + e) - 2)/abs(2*e^(I*f*x + I*e) + 2)) - sqrt(a)*arctan2((cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(f*x + e), (cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(f*x + e)))*c/(a*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{c-\frac{c}{\cos(e+f\,x)}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2), x)
```

3.69 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal result 518
Rubi [A] (verified) 518
Mathematica [C] (verified) 520
Maple [A] (verified) 521
Fricas [A] (verification not implemented) 521
Sympy [F] 522
Maxima [F] 522
Giac [F(-2)] 522
Mupad [F(-1)] 523

Optimal result

Integrand size = 28, antiderivative size = 121

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f/a^(1/2)-1/2*arctan(
1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2)+
cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3989, 3972, 491, 536, 209}

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{acf}$$

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c*f) -
 ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(Sqrt[2]
*Sqrt[a]*c*f) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(a*c*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\frac{\int \cot^2(e+fx)\sqrt{a+a\sec(e+fx)}\,dx}{ac}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{acf}$$
$$= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf} + \frac{\text{Subst}\left(\int \frac{-3a-a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{acf}$$
$$= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf} + \frac{\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}$$
$$= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\cot\left(\frac{1}{2}(e+fx)\right)\left(\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right) - 2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1\right.\right.}{cf\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
[Out] -((Cot[(e + f*x)/2]*(Hypergeometric2F1[-1/2, 1, 1/2, (1 - Sec[e + f*x])/2]
- 2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]))/(c*f*Sqrt[a*(1 + Se
c[e + f*x])]))
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-\right.}{2cfa}$

```
[In] int(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c/f/a*(a*(sec(f*x+e)+1))^(1/2)*(2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x
+e)^2-1)^(1/2))-4*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e
)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-2*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.60

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)-2\sqrt{-a}\log\cdots}{\cdots}\right.$$

```
[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x
+ e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) - 2*sqrt(-a)*
log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(
cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*
sin(f*x + e) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))
*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(a*
c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)

[Out] -Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*sec(e + f*x) + a)), x)/c

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int -\frac{1}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)}\,dx$$

[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx=\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))), x)
```

3.70 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx$

Optimal result 524
Rubi [A] (verified) 524
Mathematica [C] (verified) 527
Maple [A] (verified) 527
Fricas [A] (verification not implemented) 527
Sympy [F] 528
Maxima [F] 528
Giac [F(-2)] 529
Mupad [F(-1)] 529

Optimal result

Integrand size = 28, antiderivative size = 161

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c^2f} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}\sqrt{a}c^2f} + \frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f}$$

```
[Out] -1/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^2/c^2/f+2*arctan(a^(1/2)*tan(f*x
+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f/a^(1/2)-1/4*arctan(1/2*a^(1/2)*tan(f*x+e)
*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c^2/f*2^(1/2)/a^(1/2)+3/2*cot(f*x+e)*(a+a*
sec(f*x+e))^(1/2)/a/c^2/f
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3989, 3972, 491, 597, 536, 209}

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = -\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3a^2c^2f} + \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}c^2f} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}\sqrt{a}c^2f} + \frac{3\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{2ac^2f}$$

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c^2*f)
 - ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(2*Sqr
t[2]*Sqrt[a]*c^2*f) + (3*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(2*a*c^2*f)
 - (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(3*a^2*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{3/2}\,dx}{a^2c^2} \\
&= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^2f} \\
&= -\frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f} - \frac{\text{Subst}\left(\int \frac{-9a-3a^2x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{3a^2c^2f} \\
&= \frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-21a^2-9a^3x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{6a^2c^2f} \\
&= \frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2c^2f} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} \\
&= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c^2f} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}\sqrt{a}c^2f} \\
&\quad + \frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx$$
$$= \frac{\left(\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)-2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx\right.\right.}{3c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] ((Hypergeometric2F1[-3/2, 1, -1/2, (1 - Sec[e + f*x])/2] - 2*Hypergeometric
2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]])*Tan[e + f*x])/(3*c^2*f*(-1 + Sec[e +
f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(3\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\right.}{1}$

```
[In] int(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/c^2/f/a*(a*(sec(f*x+e)+1))^(1/2)*(3*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+cs
c(f*x+e)^2-1)^(1/2))-24*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos
(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+22*cot(f*x+e)^3+4*csc
(f*x+e)*cot(f*x+e)^2-18*csc(f*x+e)^2*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.23

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx$$
$$= \left[-\frac{3\sqrt{2}\sqrt{-a}(\cos(fx+e)-1)\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin\right.$$

```
[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/24*(3*sqrt(2)*sqrt(-a)*(cos(f*x + e) - 1)*log(-(2*sqrt(2)*sqrt(-a)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x
 + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(
f*x + e) + 12*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 + 4*(2*c
os(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) -
 4*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)), 1/12*(3*sqrt(2)*sqrt
(a)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 12*sqrt(a)*(cos(f*x
+ e) - 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) +
 2*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{\sqrt{a\sec{(e+fx)}+a}\sec^2{(e+fx)}-2\sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}+\sqrt{a\sec{(e+fx)}+a}}\,dx}{c^2}$$

```
[In] integrate(1/(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - 2*sqrt(a*sec(e + f*x
) + a)*sec(e + f*x) + sqrt(a*sec(e + f*x) + a)), x)/c**2
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a\sec{(fx+e)}+a}(c\sec{(fx+e)}-c)^2}\,dx$$

```
[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2), x)
```

3.71 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx$

Optimal result 530
Rubi [A] (verified) 531
Mathematica [C] (verified) 533
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Sympy [F] 535
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Giac [F(-2)] 536
Mupad [F(-1)] 536

Optimal result

Integrand size = 28, antiderivative size = 196

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c^3f} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}\sqrt{a}c^3f} + \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4ac^3f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f} + \frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f}$$

```
[Out] -1/2*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^2/c^3/f+1/5*cot(f*x+e)^5*(a+a*se
c(f*x+e))^(5/2)/a^3/c^3/f+2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2
))/c^3/f/a^(1/2)-1/8*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))
^(1/2))/c^3/f*2^(1/2)/a^(1/2)+7/4*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c^3/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3989, 3972, 491, 597, 536, 209}

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx\\
&=\frac{\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^3c^3f}-\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{2a^2c^3f}\\
&+\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}c^3f}-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{4\sqrt{2}\sqrt{a}c^3f}\\
&+\frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4ac^3f}
\end{aligned}$$

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c^3*f)
 - ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(4*Sqr
t[2]*Sqrt[a]*c^3*f) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*a*c^3*f)
 - (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(2*a^2*c^3*f) + (Cot[e + f*x
]^5*(a + a*Sec[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{5/2}\,dx}{a^3c^3} \\
&= \frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3c^3f} \\
&= \frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f} + \frac{\text{Subst}\left(\int \frac{-15a-5a^2x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{5a^3c^3f} \\
&= -\frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f} + \frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-105a^2-45a^3x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{30a^3c^3f}
\end{aligned}$$

$$= \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4ac^3f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f} + \frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f} + \frac{\text{Subst}\left(\int \frac{-225a^3-105a^4x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{60a^3c^3f}$$

$$= \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4ac^3f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f} + \frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f} + \frac{\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{4c^3f} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}$$

$$= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c^3f} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}\sqrt{a}c^3f} + \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4ac^3f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f} + \frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = -\frac{\left(\text{Hypergeometric2F1}\left(-\frac{5}{2}, 1, -\frac{3}{2}, \frac{1}{2}(1-\sec(e+fx))\right) - 2\,\text{Hypergeometric2F1}\left(-\frac{5}{2}, 1, -\frac{3}{2}, 1-\sec(e+\right.\right.}{5c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
[Out] -1/5*((Hypergeometric2F1[-5/2, 1, -3/2, (1 - Sec[e + f*x])/2] - 2*Hypergeom
etric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]])*Tan[e + f*x])/(c^3*f*(-1 + Sec[e
 + f*x])^3*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(167) = 334.

Time = 2.85 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.91

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(5\sqrt{2}\cos(fx+e)\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\sqrt{-\frac{\cos}{\cos(}}\right.}{}$

[In] int(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/40/c^3/f/a*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*(5*2^(1/2)*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-80*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-5*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))+80*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+98*cos(f*x+e)*cot(f*x+e)^3-62*cot(f*x+e)^3-90*csc(f*x+e)*cot(f*x+e)^2+70*csc(f*x+e)^2*cot(f*x+e))

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.10

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx$$

$$= \left[-\frac{5\sqrt{2}\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{}\right.$$

[In] integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/80*(5*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 40*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(49*cos(f*x + e)^3 - 80*cos(f*x +

```
e)^2 + 35*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a*c^3*f*
cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e)), 1/40*(5*s
qrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f
*x + e) + 40*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos
(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(49*cos(f*x + e)^3 - 80
*cos(f*x + e)^2 + 35*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))
/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e))
]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)-3\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+3\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c^3}$$

```
[In] integrate(1/(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - 3*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2 + 3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*
sec(e + f*x) + a)), x)/c**3
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = \int -\frac{1}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^3}\,dx$$

```
[In] integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] -integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^3}\,dx = \int \frac{1}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}\,\left(c - \frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3), x)
```

3.72 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 203

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} + \frac{12\sqrt{2}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f+12*c^4*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2)
/f-14*c^4*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+8/3*c^4*tan(f*x+e)^3/f/(a+a
*sec(f*x+e))^(3/2)-a*c^4*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)*tan(f*x+e)^4/f/(a+
a*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

$= \{3989, 3972, 481, 596, 536, 209\}$

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f} + \frac{12\sqrt{2}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f} + \frac{8c^4\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}} - \frac{14c^4\tan(e+fx)}{af\sqrt{a\sec(e+fx)+a}} - \frac{ac^4\sin(e+fx)\tan^4(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{f(a\sec(e+fx)+a)^{5/2}}$$

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 + (12*Sqrt[2]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(a^(3/2)*f) - (14*c^4*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]
]) + (8*c^4*Tan[e + f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (a*c^4*Sec[(
e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^4)/(f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
```

```
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = (a^4c^4)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{11/2}}\,dx$$

$$= -\frac{(2a^3c^4)\,\text{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{f(a+a\sec(e+fx))^{5/2}} - \frac{(ac^4)\,\text{Subst}\left(\int \frac{x^4(10+8ax^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{f(a+a\sec(e+fx))^{5/2}} + \frac{c^4\text{Subst}\left(\int \frac{x^2(48a+42a^2x^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{3af}$$

$$= -\frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{f(a+a\sec(e+fx))^{5/2}} - \frac{c^4\text{Subst}\left(\int \frac{84a^2+78a^3x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{3a^3f}$$

$$= -\frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{f(a+a\sec(e+fx))^{5/2}} - \frac{(2c^4)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af} - \frac{(24c^4)\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$

$$= \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} + \frac{12\sqrt{2}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+f}{f(a+a\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.97

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c^4\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\left(-22+20\cos(e+fx)-26\cos(2(e+fx))\right.}{}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^4*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(-22 + 20*Cos[e + f*x] - 26*Cos[2*(e
 + f*x)] + 28*Cos[3*(e + f*x)] + 6*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(Cos[(e
+ f*x)/2] + Cos[(3*(e + f*x))/2])^2*Sqrt[-1 + Sec[e + f*x]] + 36*Sqrt[2]*Ar
cTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*(Cos[(e + f*x)/2] + Cos[(3*(e + f*x))
/2])^2*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(12*a*f*Sqrt[a*(1 + Sec[e +
 f*x])])
```

Maple [A] (warning: unable to verify)

Time = 7.02 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.39

method	result
default	$\frac{c^4\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}+12(1-\cos(fx+e))^5\csc(fx+e)^5+36\ln\left(\csc\right.\right.}{3a^2}$
parts	Expression too large to display

[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/3/a^2*c^4/f*(3*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^
(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)+12*
(1-cos(f*x+e))^5*csc(f*x+e)^5+36*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2
*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)-58*(1-cos(f
*x+e))^3*csc(f*x+e)^3+42*csc(f*x+e)-42*cot(f*x+e))*(-2*a/((1-cos(f*x+e))^2*
csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x+e)-1)/(-cot(f*x+e)+csc(f*x+e)+1
)

Fricas [A] (verification not implemented)

none

Time = 1.06 (sec) , antiderivative size = 634, normalized size of antiderivative = 3.12

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[\frac{18\sqrt{2}\left(ac^4\cos(fx+e)^3+2\,ac^4\cos(fx+e)^2+ac^4\cos(fx+e)\right)\sqrt{-\frac{1}{a}}\log}{\cdots}\right.$$

$$-\frac{2\left(3\left(c^4\cos(fx+e)^3+2\,c^4\cos(fx+e)^2+c^4\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(28\,c^4\cos\cdots\right.\right.}{3\left(a^2f\cos(f\cdots\right.}$$

[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")

```
[Out] [1/3*(18*sqrt(2)*(a*c^4*cos(f*x + e)^3 + 2*a*c^4*cos(f*x + e)^2 + a*c^4*cos
(f*x + e))*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e)
 + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 3*(c^4*cos(f*x + e)^3 + 2*c^
4*cos(f*x + e)^2 + c^4*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*s
qrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) +
 a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(28*c^4*cos(f*x + e)^2 + 15*c^
4*cos(f*x + e) - c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))
/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2*f*cos(f*x + e)), -2/3
*(3*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*sqrt(a)*
arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*
x + e))) + (28*c^4*cos(f*x + e)^2 + 15*c^4*cos(f*x + e) - c^4)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 18*sqrt(2)*(a*c^4*cos(f*x + e)^3
 + 2*a*c^4*cos(f*x + e)^2 + a*c^4*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(
a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = c^4\left(\int\left(-\frac{4\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx\right.$$
$$+\int\frac{6\sec^2(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx$$
$$+\int\left(-\frac{4\sec^3(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx$$
$$+\int\frac{\sec^4(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx$$
$$\left.+\int\frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
*sqrt(a*sec(e + f*x) + a)), x) + Integral(6*sec(e + f*x)**2/(a*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-4*s
ec(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e +
f*x) + a)), x) + Integral(sec(e + f*x)**4/(a*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(c\sec(fx+e)-c)^4}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sec(f*x + e) - c)^4/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(3/2), x)
```

3.73 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 169

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} + \frac{2\sqrt{2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f+2*c^3*arc
tan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/
f-4*c^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+c^3*sec(1/2*f*x+1/2*e)^2*sin(
f*x+e)*tan(f*x+e)^2/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3989, 3972, 481, 596, 536, 209}

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f} + \frac{2\sqrt{2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f} - \frac{4c^3\tan(e+fx)}{af\sqrt{a\sec(e+fx)+a}} + \frac{c^3\sin(e+fx)\tan^2(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{f(a\sec(e+fx)+a)^{3/2}}$$

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 + (2*Sqrt[2]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(a^(3/2)*f) - (4*c^3*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]])
 + (c^3*Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^2)/(f*(a + a*Sec[e + f
*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx\right)\\
&= \frac{\left(2a^2c^3\right)\text{Subst}\left(\int \frac{x^6}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}}\\
&\quad+\frac{c^3\text{Subst}\left(\int \frac{x^2(6+4ax^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}}+\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}}\\
&\quad-\frac{c^3\text{Subst}\left(\int \frac{8a+6a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f}\\
&= -\frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}}+\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}}\\
&\quad-\frac{\left(2c^3\right)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}\\
&\quad-\frac{\left(4c^3\right)\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}\\
&= \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}+\frac{2\sqrt{2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}\\
&\quad-\frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}}+\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78

$$\int \frac{(c - c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^3\left(-3+\arctan\left(\sqrt{-1+\sec(e+fx)}\right)\cot^2\left(\frac{1}{2}(e+fx)\right)\sqrt{-1+\sec(e+fx)}\right.}{}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^3*(-3 + ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cot[(e + f*x)/2]^2*Sqrt[-1 + S
ec[e + f*x]] + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cot[(e + f*x
)/2]^2*Sqrt[-1 + Sec[e + f*x]] - Sec[e + f*x])*Tan[(e + f*x)/2])/(a*f*Sqrt[
a*(1 + Sec[e + f*x])])
```

Maple [A] (warning: unable to verify)

Time = 5.67 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.31

method	result
default	$\frac{c^3\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}+2(1-\cos(fx+\right.}{}$
parts	$-\frac{c^3\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)-\sqrt{(1-c\right.}{4fa^2}$

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*c^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(2^(1/2)*arctanh
(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*
((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)+2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*l
n(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2)-4*csc(f*x+e)+4*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.25

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[\frac{\sqrt{2}\left(ac^3\cos(fx+e)^2+2\,ac^3\cos(fx+e)+ac^3\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+}{\cos(fx+e)}}}{}\right.}{}\right.$$

$$-\frac{2\left(\left(c^3\cos(fx+e)^2+2\,c^3\cos(fx+e)+c^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(3\,c^3\cos(fx+e)+c^3\right)\sqrt{}\right.}{a^2f\cos(fx+e)^2+2\,a^2f\cos(}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*sqrt(-1/a)*
log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x
+ e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2
+ 2*cos(f*x + e) + 1)) - (c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sq
rt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)
) - 2*(3*c^3*cos(f*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -2*((c^3
*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (3*c^3*cos(f
*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + sqrt(
2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*arctan(sqrt(2)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/s
qrt(a))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx =$$
$$-c^3\left(\int \frac{3\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{3\sec^2(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx + \int \frac{\sec^3(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)
[Out] -c**3*(Integral(3*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
*sqrt(a*sec(e + f*x) + a)), x) + Integral(-3*sec(e + f*x)**2/(a*sqrt(a*sec(
e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec
(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*
x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqr
t(a*sec(e + f*x) + a)), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \int -\frac{(c\sec(fx+e)-c)^3}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] -integrate((c*sec(f*x + e) - c)^3/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2), x)
```

3.74 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result 551
Rubi [A] (verified) 551
Mathematica [A] (verified) 553
Maple [A] (verified) 554
Fricas [B] (verification not implemented) 554
Sympy [F] 555
Maxima [F] 555
Giac [F(-2)] 556
Mupad [F(-1)] 556

Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{\sqrt{2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-c^2*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/f-
2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3989, 3972, 481, 12, 400, 209}

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f} - \frac{\sqrt{2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f} - \frac{c^2\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{af\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 - (Sqrt[2]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f
```

```
*x]])])/(a^(3/2)*f) - (c^2*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(a*f*Sqrt[a + a
*Sec[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = (a^2c^2)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx$$
$$= -\frac{(2ac^2)\,\text{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= -\frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{c^2\text{Subst}\left(\int \frac{2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$
$$= -\frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$
$$= -\frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af} + \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$
$$= \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{\sqrt{2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{af\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{c^2\left(2\sqrt{1-\sec(e+fx)} - 2\text{arctanh}\left(\sqrt{1-\sec(e+fx)}\right)(1+\sec(e+fx)) + \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)(1}{f\sqrt{1-\sec(e+fx)}(a(1+\sec(e+fx)))^{3/2}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
[Out] -((c^2*(2*Sqrt[1 - Sec[e + f*x]] - 2*ArcTanh[Sqrt[1 - Sec[e + f*x]]]*(1 + S
ec[e + f*x]) + Sqrt[2]*ArcTanh[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]]*(1 + Sec[e +
 f*x]))*Tan[e + f*x])/(f*Sqrt[1 - Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(3/2
)))
```

Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.68

method	result
default	$\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(fx+}\right.}{a^2f}$
parts	$\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(fx}\right.}{4fa^2}$

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*c^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*(2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2)*(-cot(f*x+e)+csc(f*x+e))-ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(102) = 204.

Time = 0.49 (sec) , antiderivative size = 542, normalized size of antiderivative = 4.55

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\sqrt{2}\left(ac^2\cos(fx+e)^2+2ac^2\right.}{}\right.$$

$$-\frac{2c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+2\left(c^2\cos(fx+e)^2+2c^2\cos(fx+e)+c^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(}{\cos}}}{\sqrt{}}\right.}{a^2f\cos(fx+e)^2+2a^2f\cos(fx+}$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
 e) - sqrt(2)*(a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqrt(-1
/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f
*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)
^2 + 2*cos(f*x + e) + 1)) + 2*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^
```

```
2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e)
 + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*c^2*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 2*(c^2*cos(
f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(2)*(a*c^2*co
s(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a^2
*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]

$$\int \frac{(c - c\sec(e+fx))^2}{(a + a\sec(e+fx))^{3/2}}\,dx = c^2\left(\int \left(-\frac{2\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a\sqrt{a\sec(e+fx)+a}}\right)dx + \int \frac{\sec^2(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a\sqrt{a\sec(e+fx)+a}}\,dx + \int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a\sqrt{a\sec(e+fx)+a}}\,dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**2/(a*sqrt(a*sec(e +
 f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F]

$$\int \frac{(c - c\sec(e+fx))^2}{(a + a\sec(e+fx))^{3/2}}\,dx = \int \frac{(c\sec(fx+e) - c)^2}{(a\sec(fx+e) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^2}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2),x)

[Out] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2), x)

3.75 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}-\frac{3c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{3/2}f}-\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-3/2*c*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)-
c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3989, 3972, 482, 536, 209}

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{3c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}a^{3/2}f}-\frac{c\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{2af\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f) -
 (3*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(S
```

```
qrt[2]*a^(3/2)*f) - (c*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(2*a*f*Sqrt[a + a*S
ec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\left((ac)\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx\right)$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{2af\sqrt{a+a\sec(e+fx)}} - \frac{c\text{Subst}\left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$

$$= -\frac{c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{2af\sqrt{a+a\sec(e+fx)}} - \frac{(2c)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af} + \frac{(3c)\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$

$$= \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{3c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{3/2}f} - \frac{c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{2af\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.28

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\left(4c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)(1+\sec(e+fx)) - 3\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e}}{\sqrt{2}\sqrt{}}\right.\right.}{2f(a(1+\sec(e+fx)))^{3/2}\sqrt{c}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
[Out] ((4*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]]*(1 + Sec[e + f*x]) -
3*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(1 +
Sec[e + f*x]) - 2*c*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(2*f*(a*(1 + Se
c[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(96) = 192$.

Time = 2.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.78

method	result
default	$-\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)-\sqrt{(1-\text{co}}\right.}{2a^2f}$
parts	$\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(4\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(f}\right.}{4fa^2}$

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c/a^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)^(1/2)*(-2*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(
f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))+3*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(96) = 192$.

Time $= 0.39$ (sec) , antiderivative size $= 505$, normalized size of antiderivative $= 4.47$

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\,c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3\sqrt{2}\left(c\cos(fx+e)^2+2\,c\cos\right.}{}\right.$$

$$-\frac{2\,c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3\sqrt{2}\left(c\cos(fx+e)^2+2\,c\cos(fx+e)+c\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\,\mathrm{co}}{\mathrm{c}}}}{}\right.}{2\left(a^2f\cos(fx+e)^2+2\,a^2f\,\mathrm{co}\right.}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + 3*sqrt(2)*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(
f*x + e) + 1)) + 4*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log((
2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a^2*f*co
s(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/2*(2*c*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*sqrt(2)*(c*cos(f*x + e)^
2 + 2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 4*(c*cos(f*x + e)^2 +
2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos
(f*x + e) + a^2*f)]
```

Sympy [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = -c\left(\int \frac{\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int \left(-\frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)
[Out] -c*(Integral(sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt
(a*sec(e + f*x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e +
 f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int -\frac{c\sec(fx+e)-c}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] -integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{c - \frac{c}{\cos(e+f\,x)}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)
```

3.76 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}cf} - \frac{7\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{3/2}cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf} + \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{4a^2cf}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c/f-7/8*arctan(
1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c/f*2^(1/2)+
1/4*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^2/c/f+1/4*cos(f*x+e)*cot(f*x+e)*sec
(1/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(1/2)/a^2/c/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3989, 3972, 483, 597, 536, 209}

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}cf} - \frac{7\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{4\sqrt{2}a^{3/2}cf} + \frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4a^2cf} + \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a\sec(e+fx)+a}}{4a^2cf}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c*f) -
 (7*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(4*S
qrt[2]*a^(3/2)*c*f) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*a^2*c*f) +
 (Cos[e + f*x]*Cot[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[a + a*Sec[e + f*x]])/(4
*a^2*c*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{ac} \\
&= \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2cf} \\
&= \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{4a^2cf} \\
&\quad+ \frac{\text{Subst}\left(\int \frac{a-3a^2x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2a^3cf} \\
&= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf} \\
&\quad+ \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{4a^2cf} \\
&\quad- \frac{\text{Subst}\left(\int \frac{9a^2+a^3x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{4a^3cf} \\
&= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf} \\
&\quad+ \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{4a^2cf} \\
&\quad+ \frac{7\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{4acf} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{acf} \\
&= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}cf} - \frac{7\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{3/2}cf} \\
&\quad+ \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf} \\
&\quad+ \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{4a^2cf}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx =$$
$$-\frac{\left(-2+7\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))-8\,\text{Hypergeometric2F1}\left(-\frac{1}{2}\right.\right.}{4cf(-1+\sec(e+fx))(a(1+\sec(e+fx)))^{3/2}}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] -1/4*((-2 + 7*Hypergeometric2F1[-1/2, 1, 1/2, (1 - Sec[e + f*x])/2]*(1 + Se
c[e + f*x]) - 8*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]*(1 + Sec[
e + f*x]))*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*x])*(a*(1 + Sec[e + f*x]))^(3
/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(151) = 302.

Time = 2.18 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.92

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(7\sqrt{2}\cos(fx+e)\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\sqrt{-\frac{\cos}{\cos(}}\right.}{}$

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/c/f/a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(7*2^(1/2)*cos(f*x+e)*
ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2
-1)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+7*2^(1/2)*(-cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x
+e)+csc(f*x+e)^2-1)^(1/2))-16*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e
)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-16*a
rctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)-6*cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.90

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \left[-\frac{7\sqrt{2}\sqrt{-a}(\cos(fx+e)+1)\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{}\right.}{}\right.$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/16*(7*sqrt(2)*sqrt(-a)*(cos(f*x + e) + 1)*log(-(2*sqrt(2)*sqrt(-a)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x
 + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(
f*x + e) + 8*sqrt(-a)*(cos(f*x + e) + 1)*log(-(8*a*cos(f*x + e)^3 + 4*(2*co
s(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) -
4*(3*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
)/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e)), 1/8*(7*sqrt(2)*sqrt(a)*(
cos(f*x + e) + 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 8*sqrt(a)*(cos(f*x + e) +
 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(3*
cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a
^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)-a\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e +
f*x) + a)), x)/c
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx=\int -\frac{1}{(a\sec{(fx+e)}+a)^{\frac{3}{2}}(c\sec{(fx+e)}-c)}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))), x)
```

3.77 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx$

Optimal result 569
Rubi [A] (verified) 569
Mathematica [C] (verified) 572
Maple [A] (verified) 573
Fricas [A] (verification not implemented) 573
Sympy [F] 574
Maxima [F] 574
Giac [F(-2)] 574
Mupad [F(-1)] 574

Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^2f} - \frac{9\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{3/2}c^2f} + \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{8a^2c^2f} + \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f} - \frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{4a^3c^2f}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^2/f+1/12*cot(
f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^3/c^2/f-1/4*cos(f*x+e)*cot(f*x+e)^3*sec(1
/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(3/2)/a^3/c^2/f-9/16*arctan(1/2*a^(1/2)*ta
n(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^2/f*2^(1/2)+7/8*cot(f*x+
e)*(a+a*sec(f*x+e))^(1/2)/a^2/c^2/f
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used

$= \{3989, 3972, 483, 597, 536, 209\}$

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}c^2f} - \frac{9\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{8\sqrt{2}a^{3/2}c^2f} + \frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{12a^3c^2f} - \frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a\sec(e+fx)+a)^{3/2}}{4a^3c^2f} + \frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{8a^2c^2f}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c^2*f)
 - (9*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(8
*Sqrt[2]*a^(3/2)*c^2*f) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(8*a^2*
c^2*f) + (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(12*a^3*c^2*f) - (Cos[
e + f*x]*Cot[e + f*x]^3*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^(3/2))/(4*a
^3*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \cot^4(e+fx)\sqrt{a+a\sec(e+fx)}\,dx}{a^2c^2}\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3c^2f}\\
&= -\frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{4a^3c^2f}\\
&\quad -\frac{\text{Subst}\left(\int \frac{-a-5a^2x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2a^4c^2f}\\
&= \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f}\\
&\quad -\frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{4a^3c^2f}\\
&\quad +\frac{\text{Subst}\left(\int \frac{21a^2-3a^3x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{12a^4c^2f}
\end{aligned}$$

$$= \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{8a^2c^2f} + \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f} - \frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{4a^3c^2f} - \frac{\text{Subst}\left(\int \frac{69a^3+21a^4x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{24a^4c^2f}$$

$$= \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{8a^2c^2f} + \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f} - \frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{4a^3c^2f} + \frac{9\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{8ac^2f} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^2f}$$

$$= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^2f} - \frac{9\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{3/2}c^2f} + \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{8a^2c^2f} + \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f} - \frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{4a^3c^2f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{\left(-6+9\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-\sec(e+fx\right.\right.}{1}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] ((-6 + 9*Hypergeometric2F1[-3/2, 1, -1/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e
+ f*x]) - 8*Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]]*(1 + Sec[e +
 f*x]))*Tan[e + f*x])/(12*c^2*f*(-1 + Sec[e + f*x])^2*(a*(1 + Sec[e + f*x])
)^(3/2))
```

Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.94

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(27\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right.\right.}{4}$

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/c^2/f/a^2*(a*(sec(f*x+e)+1))^(1/2)*(27*2^(1/2)*(-cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)
+csc(f*x+e)^2-1)^(1/2))-96*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(
cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+62*cot(f*x+e)^3-4*
csc(f*x+e)*cot(f*x+e)^2-42*csc(f*x+e)^2*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \left[-\frac{27\sqrt{2}\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx}{\cos(fx}}}{}\right.}{}\right.$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [-1/96*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(
f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*s
in(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4
*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x +
 e) - 4*(31*cos(f*x + e)^3 - 2*cos(f*x + e)^2 - 21*cos(f*x + e))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(
f*x + e)), 1/48*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(sqrt(2)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*s
in(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 +
 a*cos(f*x + e) - a))*sin(f*x + e) + 2*(31*cos(f*x + e)^3 - 2*cos(f*x + e)^
2 - 21*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^2*f*c
os(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{a\sqrt{a\sec{(e+fx)}+a}\sec^3{(e+fx)}-a\sqrt{a\sec{(e+fx)}+a}\sec^2{(e+fx)}-a\sqrt{a\sec{(e}}\cdots}{c^2}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - a*sqrt(a*sec(e + f
*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt
(a*sec(e + f*x) + a)), x)/c**2
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{(a\sec{(fx+e)}+a)^{\frac{3}{2}}(c\sec{(fx+e)}-c)^2}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2), x)
```

3.78 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx$

Optimal result . . . 575
Rubi [A] (verified) . . . 576
Mathematica [C] (verified) . . . 579
Maple [A] (verified) . . . 579
Fricas [A] (verification not implemented) . . . 580
Sympy [F] . . . 580
Maxima [F] . . . 581
Giac [F(-2)] . . . 581
Mupad [F(-1)] . . . 581

Optimal result

Integrand size = 28, antiderivative size = 249

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx &= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^3f} \\ &- \frac{11\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}a^{3/2}c^3f} + \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^2c^3f} \\ &- \frac{5\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{24a^3c^3f} - \frac{3\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{20a^4c^3f} \\ &+ \frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{5/2}}{4a^4c^3f}\end{aligned}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^3/f-5/24*cot(
f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^3/c^3/f-3/20*cot(f*x+e)^5*(a+a*sec(f*x+e)
)^(5/2)/a^4/c^3/f+1/4*cos(f*x+e)*cot(f*x+e)^5*sec(1/2*f*x+1/2*e)^2*(a+a*sec
(f*x+e))^(5/2)/a^4/c^3/f-11/32*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*s
ec(f*x+e))^(1/2))/a^(3/2)/c^3/f*2^(1/2)+21/16*cot(f*x+e)*(a+a*sec(f*x+e))^(
1/2)/a^2/c^3/f
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3989, 3972, 483, 597, 536, 209}

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}c^3f}-\frac{11\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{16\sqrt{2}a^{3/2}c^3f}-\frac{3\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{20a^4c^3f}+\frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a\sec(e+fx)+a)^{5/2}}{4a^4c^3f}-\frac{5\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{24a^3c^3f}+\frac{21\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{16a^2c^3f}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c^3*f)
 - (11*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(
16*Sqrt[2]*a^(3/2)*c^3*f) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(16*
a^2*c^3*f) - (5*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(24*a^3*c^3*f) -
 (3*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/(20*a^4*c^3*f) + (Cos[e + f*
x]*Cot[e + f*x]^5*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^(5/2))/(4*a^4*c^3
*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{3/2}\,dx}{a^3c^3}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^4c^3f}\\
&= \frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{5/2}}{4a^4c^3f}\\
&\quad+\frac{\text{Subst}\left(\int \frac{-3a-7a^2x^2}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2a^5c^3f}
\end{aligned}$$

$$=-\frac{3\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{20a^4c^3f}+\frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{5/2}}{4a^4c^3f}-\frac{\text{Subst}\left(\int\frac{25a^2-15a^3x^2}{x^4(1+ax^2)(2+ax^2)}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{20a^5c^3f}$$

$$=-\frac{5\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{24a^3c^3f}-\frac{3\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{20a^4c^3f}+\frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{5/2}}{4a^4c^3f}+\frac{\text{Subst}\left(\int\frac{315a^3+75a^4x^2}{x^2(1+ax^2)(2+ax^2)}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{120a^5c^3f}$$

$$=\frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^2c^3f}-\frac{5\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{24a^3c^3f}-\frac{3\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{20a^4c^3f}+\frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{5/2}}{4a^4c^3f}-\frac{\text{Subst}\left(\int\frac{795a^4+315a^5x^2}{(1+ax^2)(2+ax^2)}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{240a^5c^3f}$$

$$=\frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^2c^3f}-\frac{5\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{24a^3c^3f}-\frac{3\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{20a^4c^3f}+\frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{5/2}}{4a^4c^3f}+\frac{11\text{Subst}\left(\int\frac{1}{2+ax^2}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{16ac^3f}-\frac{2\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^3f}$$

$$= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^3f} - \frac{11\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}a^{3/2}c^3f} + \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^2c^3f} - \frac{5\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{24a^3c^3f} - \frac{3\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{20a^4c^3f} + \frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{5/2}}{4a^4c^3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = -\frac{\left(-10+11\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))-8\,\text{Hypergeometric2F}}{20c^3f(-1+\sec(e+fx))^3(a(1+\sec(e+fx)))}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] -1/20*((-10 + 11*Hypergeometric2F1[-5/2, 1, -3/2, (1 - Sec[e + f*x])/2]*(1
+ Sec[e + f*x]) - 8*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]]*(1 +
 Sec[e + f*x]))*Tan[e + f*x])/(c^3*f*(-1 + Sec[e + f*x])^3*(a*(1 + Sec[e +
f*x]))^(3/2))
```

Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\left(165\sin(fx+e)^5\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-960\right.}{}$

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/480/c^3/f/a^2*(165*sin(f*x+e)^5*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e
)^2-1)^(1/2))-960*sin(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(s
in(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))-898*cos(f*x+e)
^5-196*cos(f*x+e)^4+1432*cos(f*x+e)^3+100*cos(f*x+e)^2-630*cos(f*x+e))*(a*(
sec(f*x+e)+1))^(1/2)*csc(f*x+e)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.87

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \left[-\frac{165\sqrt{2}\left(\cos\left(fx+e\right)^3-\cos\left(fx+e\right)^2-\cos\left(fx+e\right)+\right.}{}\right.$$

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] [-1/960*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 480*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(449*cos(f*x + e)^4 - 351*cos(f*x + e)^3 - 365*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)), 1/480*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 480*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(449*cos(f*x + e)^4 - 351*cos(f*x + e)^3 - 365*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))]

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)-2a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)+2a\sqrt{a\sec(e+fx)+a}\sec(e+fx)-a\sqrt{a\sec(e+fx)+a}}\,dx}{c^3}$$

[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)

[Out] -Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4 - 2*a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 + 2*a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - a*sqrt(a*sec(e + f*x) + a)), x)/c**3

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \int -\frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^3}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] -integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3), x)
```

3.79 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result . . . 582
Rubi [A] (verified) . . . 583
Mathematica [A] (verified) . . . 586
Maple [A] (warning: unable to verify) . . . 587
Fricas [A] (verification not implemented) . . . 587
Sympy [F] . . . 588
Maxima [F(-1)] . . . 589
Giac [F(-2)] . . . 589
Mupad [F(-1)] . . . 589

Optimal result

Integrand size = 28, antiderivative size = 260

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^5\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23\sqrt{2}c^5\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} + \frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}} - \frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} + \frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e+fx)}{4f(a+a\sec(e+fx))^{7/2}}$$

```
[Out] 2*c^5*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-23*c^5*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(5/2)
/f+21*c^5*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)-19/6*c^5*tan(f*x+e)^3/a/f
/(a+a*sec(f*x+e))^(3/2)+3/4*c^5*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)*tan(f*x+e)^
4/f/(a+a*sec(f*x+e))^(5/2)+1/4*a*c^5*sec(1/2*f*x+1/2*e)^4*sin(f*x+e)^2*tan(
f*x+e)^5/f/(a+a*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3989, 3972, 481, 592, 596, 536, 209}

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{2c^5\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{23\sqrt{2}c^5\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}+\frac{21c^5\tan(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}}-\frac{19c^5\tan^3(e+fx)}{6af(a\sec(e+fx)+a)^{3/2}}+\frac{ac^5\sin^2(e+fx)\tan^5(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{4f(a\sec(e+fx)+a)^{7/2}}+\frac{3c^5\sin(e+fx)\tan^4(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{4f(a\sec(e+fx)+a)^{5/2}}$$

```
[In] Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^5*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (23*Sqrt[2]*c^5*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(a^(5/2)*f) + (21*c^5*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*
x]]) - (19*c^5*Tan[e + f*x]^3)/(6*a*f*(a + a*Sec[e + f*x])^(3/2)) + (3*c^5*
Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^4)/(4*f*(a + a*Sec[e + f*x])^(
5/2)) + (a*c^5*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*x]^5)/(4*f*(a +
a*Sec[e + f*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\left(\left(a^5c^5\right)\int \frac{\tan^{10}(e+fx)}{(a+a\sec(e+fx))^{15/2}}\,dx\right)$$

$$=\frac{(2a^3c^5)\,\text{Subst}\left(\int\frac{x^{10}}{(1+ax^2)(2+ax^2)^3}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$=\frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e+fx)}{4f(a+a\sec(e+fx))^{7/2}}+\frac{(ac^5)\,\text{Subst}\left(\int\frac{x^6(14+10ax^2)}{(1+ax^2)(2+ax^2)^2}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2f}$$

$$=\frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}+\frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e+fx)}{4f(a+a\sec(e+fx))^{7/2}}-\frac{c^5\text{Subst}\left(\int\frac{x^4(-30a-38a^2x^2)}{(1+ax^2)(2+ax^2)}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{4af}$$

$$=-\frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}}+\frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}+\frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e+fx)}{4f(a+a\sec(e+fx))^{7/2}}+\frac{c^5\text{Subst}\left(\int\frac{x^2(-228a^2-252a^3x^2)}{(1+ax^2)(2+ax^2)}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{12a^3f}$$

$$=\frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}}-\frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}}+\frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}+\frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e+fx)}{4f(a+a\sec(e+fx))^{7/2}}-\frac{c^5\text{Subst}\left(\int\frac{-504a^3-528a^4x^2}{(1+ax^2)(2+ax^2)}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{12a^5f}$$

$$= \frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}} - \frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} + \frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e+fx)}{4f(a+a\sec(e+fx))^{7/2}} - \frac{(2c^5)\operatorname{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f} + \frac{(46c^5)\operatorname{Subst}\left(\int\frac{1}{2+ax^2}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f}$$

$$= \frac{2c^5\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23\sqrt{2}c^5\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} + \frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}} - \frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} + \frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e+fx)}{4f(a+a\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 8.45 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.69

$$\int\frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c^5\cot\left(\frac{1}{2}(e+fx)\right)\left((81-30\cos(e+fx)+52\cos(2(e+fx))-66\cos(3(e+f\right.}{}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c^5*Cot[(e + f*x)/2]*((81 - 30*Cos[e + f*x] + 52*Cos[2*(e + f*x)] - 66*Cos
[3*(e + f*x)] - 37*Cos[4*(e + f*x)])*Sec[(e + f*x)/2]^4 + 96*ArcTan[Sqrt[-1
 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]] - 1104*Sqrt[2]*Arc
Tan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]]
)*Sec[e + f*x]^2)/(48*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] (warning: unable to verify)

Time = 7.71 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.17

method	result
default	$\frac{c^5\left(-6(1-\cos(fx+e))^7\csc(fx+e)^7+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}-9(1-\cos(\right.}{}$
parts	Expression too large to display

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c^5/a^3/f*(-6*(1-cos(f*x+e))^7*csc(f*x+e)^7+3*2^(1/2)*arctanh(2^(1/2)/(
(1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(3/2)-9*(1-cos(f*x+e))^5*csc(f*x+e)^5-69*ln(csc(f*x
+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))^2*c
sc(f*x+e)^2-1)^(3/2)+82*(1-cos(f*x+e))^3*csc(f*x+e)^3-63*csc(f*x+e)+63*cot(
f*x+e))*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(-cot(f*x+e)+csc(f*x
+e)-1)/(-cot(f*x+e)+csc(f*x+e)+1)
```

Fricas [A] (verification not implemented)

none

Time = 1.69 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.85

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[\frac{69\sqrt{2}\left(ac^5\cos(fx+e)^4+3\,ac^5\cos(fx+e)^3+3\,ac^5\cos(fx+e)^2+ac^5\operatorname{co}\right.}{}\right.$$

$$-\frac{6\left(c^5\cos(fx+e)^4+3\,c^5\cos(fx+e)^3+3\,c^5\cos(fx+e)^2+c^5\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(f}{\sqrt{a}\sin(fx+e)}\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(69*sqrt(2)*(a*c^5*cos(f*x + e)^4 + 3*a*c^5*cos(f*x + e)^3 + 3*a*c^5*c
os(f*x + e)^2 + a*c^5*cos(f*x + e))*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x
 + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 6*(c
```

```
^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f
*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*
x + e) + 1)) + 4*(37*c^5*cos(f*x + e)^3 + 70*c^5*cos(f*x + e)^2 + 20*c^5*co
s(f*x + e) - c^5)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^
3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*
f*cos(f*x + e)), -1/3*(6*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5
*cos(f*x + e)^2 + c^5*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(37*c^5*cos(f*x +
e)^3 + 70*c^5*cos(f*x + e)^2 + 20*c^5*cos(f*x + e) - c^5)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e) - 69*sqrt(2)*(a*c^5*cos(f*x + e)^4 + 3*
a*c^5*cos(f*x + e)^3 + 3*a*c^5*cos(f*x + e)^2 + a*c^5*cos(f*x + e))*arctan(
sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f
*x + e)))/sqrt(a))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 + a^3*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx =$$

$$-c^5\left(\int \frac{5\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\right.$$

$$+\int\left(-\frac{10\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\right.$$

$$+\int\frac{10\sec^3(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

$$+\int\left(-\frac{5\sec^4(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\right.$$

$$+\int\frac{\sec^5(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

$$+\int\left(-\frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\right.$$

```
[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] -c**5*(Integral(5*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
) + a)), x) + Integral(-10*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*s
ec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a
*sec(e + f*x) + a)), x) + Integral(10*sec(e + f*x)**3/(a**2*sqrt(a*sec(e +
```

```
f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(-5*sec(e + f*x)**4/(a**2*sqrt
(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(
e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**5/(a
**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) +
 a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a**2*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*s
ec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^5}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^(5/2), x)
```

3.80 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result 590
Rubi [A] (verified) 590
Mathematica [A] (verified) 593
Maple [A] (warning: unable to verify) 594
Fricas [A] (verification not implemented) 594
Sympy [F] 595
Maxima [F(-1)] 596
Giac [F(-2)] 596
Mupad [F(-1)] 596

Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{5/2}f} + \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^3(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-11/2*c^4*
arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^(
1/2)+7/2*c^4*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)-1/4*c^4*sec(1/2*f*x+1/
2*e)^2*sin(f*x+e)*tan(f*x+e)^2/a/f/(a+a*sec(f*x+e))^(3/2)-1/4*c^4*sec(1/2*f
*x+1/2*e)^4*sin(f*x+e)^2*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

= {3989, 3972, 481, 592, 596, 536, 209}

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f} - \frac{11c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}a^{5/2}f} + \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a\sec(e+fx)+a}} - \frac{c^4\sin^2(e+fx)\tan^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{4f(a\sec(e+fx)+a)^{5/2}} - \frac{c^4\sin(e+fx)\tan^2(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{4af(a\sec(e+fx)+a)^{3/2}}$$

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (11*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(Sqrt[2]*a^(5/2)*f) + (7*c^4*Tan[e + f*x])/(2*a^2*f*Sqrt[a + a*Sec[e + f
*x]]) - (c^4*Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^2)/(4*a*f*(a + a*
Sec[e + f*x])^(3/2)) - (c^4*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*x]^
3)/(4*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^4c^4\right)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{13/2}}\,dx\\
&= -\frac{\left(2a^2c^4\right)\text{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^3(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}\\
&\quad -\frac{c^4\text{Subst}\left(\int \frac{x^4(10+6ax^2)}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2f}
\end{aligned}$$

$$= -\frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^3(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} + \frac{c^4\text{Subst}\left(\int \frac{x^2(-6a-14a^2x^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{4a^2f}$$

$$= \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^3(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{c^4\text{Subst}\left(\int \frac{-28a^2-36a^3x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{4a^4f}$$

$$= \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^3(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(2c^4)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f} + \frac{(11c^4)\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f}$$

$$= \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{5/2}f} + \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^3(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 5.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c^4\cot\left(\frac{1}{2}(e+fx)\right)\left((-4+19\cos(e+fx)-12\cos(2(e+fx))-3\cos(3(e+\right.}{}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c^4*Cot[(e + f*x)/2]*((-4 + 19*Cos[e + f*x] - 12*Cos[2*(e + f*x)] - 3*Cos[
3*(e + f*x)])*Sec[(e + f*x)/2]^4 + 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e
 + f*x]*Sqrt[-1 + Sec[e + f*x]] - 88*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]
/Sqrt[2]]*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x])/(16*a^2*f*Sqr
t[a*(1 + Sec[e + f*x])])
```

Maple [A] (warning: unable to verify)

Time = 6.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.07

method	result
default	$\frac{c^4\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left(-2(1-\cos(fx+e))^5\csc(fx+e)^5+2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{(1-\cos(fx+e)}\right.}{}$
parts	Expression too large to display

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c^4/a^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-2*(1-cos(f*x
+e))^5*csc(f*x+e)^5+2*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2
)-(1-cos(f*x+e))^3*csc(f*x+e)^3-11*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)+7*csc(f*x
+e)-7*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 1.39 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.86

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[-\frac{11\sqrt{2}\left(c^4\cos(fx+e)^3+3c^4\cos(fx+e)^2+3c^4\cos(fx+e)+c^4\right)\sqrt{-a}\,\mathrm{lo}}{}\right.$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*
x + e) + c^4)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x +
 e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 4*(c^4*cos(f*x + e)^3 + 3
*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(3*c^4*cos(f*x + e)
^2 + 9*c^4*cos(f*x + e) + 2*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
```

```
n(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*
x + e) + a^3*f), 1/2*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2
 + 3*c^4*cos(f*x + e) + c^4)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 4*(c^4*cos(f*x + e)
^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(a)*arctan(sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*
(3*c^4*cos(f*x + e)^2 + 9*c^4*cos(f*x + e) + 2*c^4)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)
^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^{5/2}}\,dx = c^4 \left(\int \left(-\frac{4\sec(e + fx)}{a^2\sqrt{a\sec(e + fx) + a}\sec^2(e + fx) + 2a^2\sqrt{a\sec(e + fx) + a}\sec \cdots} \right.\right.$$

$$+ \int \frac{6\sec^2(e + fx)}{a^2\sqrt{a\sec(e + fx) + a}\sec^2(e + fx) + 2a^2\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a^2\sqrt{a\sec(e + fx) + a}}\,dx$$

$$+ \int \left(-\frac{4\sec^3(e + fx)}{a^2\sqrt{a\sec(e + fx) + a}\sec^2(e + fx) + 2a^2\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a^2\sqrt{a\sec(e + fx) + \cdots}} \right.$$

$$+ \int \frac{\sec^4(e + fx)}{a^2\sqrt{a\sec(e + fx) + a}\sec^2(e + fx) + 2a^2\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a^2\sqrt{a\sec(e + fx) + a}}\,dx$$

$$+ \int \frac{1}{a^2\sqrt{a\sec(e + fx) + a}\sec^2(e + fx) + 2a^2\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a^2\sqrt{a\sec(e + fx) + a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
) + a)), x) + Integral(6*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*sec
(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*s
ec(e + f*x) + a)), x) + Integral(-4*sec(e + f*x)**3/(a**2*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*
*2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**4/(a**2*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a**2*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^{5/2}}\, dx = \text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^{5/2}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^{5/2}}\, dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^4}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\, dx$$

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(5/2), x)
```

3.81 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result 597
Rubi [A] (verified) 597
Mathematica [A] (verified) 600
Maple [A] (warning: unable to verify) 600
Fricas [A] (verification not implemented) 601
Sympy [F] 602
Maxima [F(-1)] 602
Giac [F(-2)] 603
Mupad [F(-1)] 603

Optimal result

Integrand size = 28, antiderivative size = 191

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}-\frac{7c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{5/2}f}-\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}}+\frac{c^3\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-7/4*c^3*a
rctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^(1
/2)-1/4*c^3*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)+1/
4*c^3*sec(1/2*f*x+1/2*e)^4*sin(f*x+e)^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/
2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

$= \{3989, 3972, 481, 592, 536, 209\}$

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f} - \frac{7c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}a^{5/2}f} - \frac{c^3\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{4a^2f\sqrt{a\sec(e+fx)+a}} + \frac{c^3\sin^2(e+fx)\tan(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{4af(a\sec(e+fx)+a)^{3/2}}$$

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (7*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/(2*Sqrt[2]*a^(5/2)*f) - (c^3*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(4*a^2*f*Sq
rt[a + a*Sec[e + f*x]]) + (c^3*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*
x])/(4*a*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
```

```
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{11/2}}\,dx\right)\\
&= \frac{(2ac^3)\,\text{Subst}\left(\int \frac{x^6}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{c^3\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}}\\
&\quad+\frac{c^3\text{Subst}\left(\int \frac{x^2(6+2ax^2)}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2af}\\
&= -\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}}+\frac{c^3\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}}\\
&\quad-\frac{c^3\text{Subst}\left(\int \frac{2a-6a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{4a^3f}
\end{aligned}$$

$$= -\frac{c^3 \sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2 f\sqrt{a+a\sec(e+fx)}} + \frac{c^3 \sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{(2c^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2 f} + \frac{(7c^3)\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2a^2 f}$$

$$= \frac{2c^3 \arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2} f} - \frac{7c^3 \arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{5/2} f} - \frac{c^3 \sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2 f\sqrt{a+a\sec(e+fx)}} + \frac{c^3 \sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.71

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^3 \cot\left(\frac{1}{2}(e+fx)\right)\left((-5+8\cos(e+fx)-3\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right) - 32\arctan\left(\sqrt{-1+\sec(e+\right.\right.}{16a^2 f\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/16*(c^3*Cot[(e + f*x)/2]*((-5 + 8*Cos[e + f*x] - 3*Cos[2*(e + f*x)])*Sec
[(e + f*x)/2]^4 - 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*x]
] + 28*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*
x]]))/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] (warning: unable to verify)

Time = 3.89 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.31

method	result
default	$-\frac{c^3\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2(1-\cos(fx+e))^3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\csc(fx\right.}{}$
parts	Expression too large to display

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^3*c^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2)*(2*(1-cos(f*x+e))^3*((1-cos(f*x+e))^2*csc(f*x+e)
```

```
^2-1)^(1/2)*csc(f*x+e)^3-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+
e)+csc(f*x+e))-4*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^
(1/2)*(-cot(f*x+e)+csc(f*x+e)))+7*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 1.12 (sec) , antiderivative size = 645, normalized size of antiderivative = 3.38

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[-\frac{7\sqrt{2}\left(c^3\cos{(fx+e)}^3+3\,c^3\cos{(fx+e)}^2+3\,c^3\cos{(fx+e)}+c^3\right)\sqrt{-a}\,\mathrm{lo}}{}\right.$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(7*sqrt(2)*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x
 + e) + c^3)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x +
e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^3 + 3*
c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x +
e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(3*c^3*cos(f*x + e)^
2 - c^3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))
/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^
3*f), 1/4*(7*sqrt(2)*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos
(f*x + e) + c^3)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 8*(c^3*cos(f*x + e)^3 + 3*c^3*c
os(f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(3*c^3*cos(f
*x + e)^2 - c^3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f
*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x +
 e) + a^3*f)]
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^{5/2}}\,dx =$$

$$-c^3\left(\int \frac{3\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}\right.$$

$$+\int\left(-\frac{3\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\right.$$

$$+\int \frac{\sec^3(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

$$+\int\left(-\frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\right.$$

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
) + a)), x) + Integral(-3*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*se
c(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*
sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**3/(a**2*sqrt(a*sec(e + f*x)
 + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2
*sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*x) + a)
*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt
(a*sec(e + f*x) + a)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e+fx))^3}{(a + a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e+fx))^3}{(a + a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^3}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2),x)

[Out] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)

3.82 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result 604
Rubi [A] (verified) 604
Mathematica [A] (verified) 607
Maple [A] (warning: unable to verify) 607
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Sympy [F] 609
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Mupad [F(-1)] 610

Optimal result

Integrand size = 28, antiderivative size = 189

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{5/2}f} - \frac{3c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-11/8*c^2*
arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^(
1/2)-3/8*c^2*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)-1
/4*c^2*cos(f*x+e)*sec(1/2*f*x+1/2*e)^4*sin(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1
/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used

= {3989, 3972, 481, 541, 536, 209}

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f} - \frac{11c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{4\sqrt{2}a^{5/2}f} - \frac{3c^2\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{8a^2f\sqrt{a\sec(e+fx)+a}} - \frac{c^2\sin(e+fx)\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{4a^2f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (11*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(4*Sqrt[2]*a^(5/2)*f) - (3*c^2*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(8*a^2*f
*Sqrt[a + a*Sec[e + f*x]]) - (c^2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f
*x])/(4*a^2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = (a^2c^2)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx$$

$$= -\frac{(2c^2)\,\text{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{c^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^2\text{Subst}\left(\int \frac{2-2ax^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2a^2f}$$

$$= -\frac{3c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^2\text{Subst}\left(\int \frac{10a-6a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{8a^3f}$$

$$= -\frac{3c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}} - \frac{(2c^2)\,\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f} + \frac{(11c^2)\,\text{Subst}\left(\int\frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{4a^2f}$$

$$= \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{5/2}f} - \frac{3c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.77

$$\int\frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^2\left(22\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx) - 8\text{arctanh}\left(\sqrt{1-\sec(e+fx)}\right)(1+\sec(e\right.}{4f\sqrt{1-\sec(e+fx)}(a(1+\sec(e+fx)))^{5/2}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/4*(c^2*(22*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]]*Cos[(e + f*x)
/2]^4*Sec[e + f*x]^2 - 8*ArcTanh[Sqrt[1 - Sec[e + f*x]]]*(1 + Sec[e + f*x])
^2 + Sqrt[1 - Sec[e + f*x]]*(7 + 3*Sec[e + f*x]))*Tan[e + f*x])/(f*Sqrt[1 -
 Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2))
```

Maple [A] (warning: unable to verify)

Time = 3.55 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.30

method	result
default	$-\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}(-\cot(fx+e)+\csc(f\right.}{}$
parts	$-\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2(1-\cos(fx+e))^3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\csc(f\right.}{}$

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

[Out] -1/8/a^3*c^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2)*(2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-cot
(f*x+e)+csc(f*x+e))-3*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+
csc(f*x+e))-8*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*(-cot(f*x+e)+csc(f*x+e)))+11*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*
csc(f*x+e)^2-1)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.84 (sec) , antiderivative size = 645, normalized size of antiderivative = 3.41

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[-\frac{11\sqrt{2}\left(c^2\cos(fx+e)^3+3\,c^2\cos(fx+e)^2+3\,c^2\cos(fx+e)+c^2\right)\sqrt{-a}\,\mathrm{lo}}{}\right.$$

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [-1/16*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f
*x + e) + c^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x
+ e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 16*(c^2*cos(f*x + e)^3 +
 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f*x
 + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(7*c^2*cos(f*x +
e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e)
 + a^3*f), 1/8*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c
^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 16*(c^2*cos(f*x + e)^3 +
3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(7*c^
2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*
cos(f*x + e) + a^3*f)]

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = c^2\left(\int\left(-\frac{2\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec\right.\right.$$
$$+\int \frac{\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$
$$+\int \frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
) + a)), x) + Integral(sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec
(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(c\sec(fx+e)-c)^2}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2/(a*sec(f*x + e) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^2}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)
```

3.83 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{5/2}f} - \frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}} - \frac{7c\tan(e+fx)}{8af(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-23/16*c*arc
tan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2
)-1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-7/8*c*tan(f*x+e)/a/f/(a+a*sec(f
*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3989, 3972, 482, 541, 536, 209}

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f} - \frac{23c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{8\sqrt{2}a^{5/2}f} - \frac{7c\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{16a^2f\sqrt{a\sec(e+fx)+a}} - \frac{c\sin(e+fx)\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{8a^2f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f) -
 (23*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(
8*Sqrt[2]*a^(5/2)*f) - (7*c*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(16*a^2*f*Sqrt
[a + a*Sec[e + f*x]]) - (c*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f*x])/(8
*a^2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\left((ac)\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx\right)$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$

$$= -\frac{c\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\text{Subst}\left(\int \frac{1-3ax^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2a^2f}$$

$$= -\frac{7c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{16a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\text{Subst}\left(\int \frac{9a-7a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{8a^3f}$$

$$= -\frac{7c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{16a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{(2c)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f} + \frac{(23c)\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{8a^2f}$$

$$= \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{5/2}f} - \frac{7c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{16a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{\left(-64c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)+46\sqrt{2}c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\cos^4\left(\frac{1}{2}\right.\right.}{8f(a(1+\sec(e+fx)))^{5/2}\sqrt{c-c\sec}}$$

[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]

[Out] -1/8*((-64*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]]*Cos[(e + f*x)/2]^4*Sec[e + f*x]^2 + 46*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Cos[(e + f*x)/2]^4*Sec[e + f*x]^2 + c*(11 + 7*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(f*(a*(1 + Sec[e + f*x]))^(5/2)*Sqrt[c - c*Sec[e + f*x]])

Maple [A] (warning: unable to verify)

Time = 2.66 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.64

method	result
default	$\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-2\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}(-\cot(fx+e)+\csc(fx+\right.}{}$
parts	$-\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2(1-\cos(fx+e))^3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\csc(fx+\right.}{}$

[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/16/a^3*c/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-cot(f*x+e)+csc(f*x+e))+16*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+7*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))-23*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(123) = 246.

Time = 0.56 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.09

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[-\frac{23\sqrt{2}\left(c\cos\left(fx+e\right)^{3}+3\,c\cos\left(fx+e\right)^{2}+3\,c\cos\left(fx+e\right)+c\right)\sqrt{-a}\log\right.}{}\right.$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(23*sqrt(2)*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e
) + c)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a
)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 32*(c*cos(f*x + e)^3 + 3*c*cos(f
*x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt
(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*
cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(11*c*cos(f*x + e)^2 + 7*c*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/16*(23
*sqrt(2)*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) + c)*sqr
t(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - 32*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos
(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(11*c*cos(f*x + e)^2 + 7*c*cos(f*x + e)
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)
^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx =$$

$$-c\left(\int \frac{\sec\left(e+fx\right)}{a^{2}\sqrt{a\sec\left(e+fx\right)+a}\sec^{2}\left(e+fx\right)+2a^{2}\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a^{2}\sqrt{a\sec\left(e+fx\right)+a}}\right.$$

$$+\int\left(-\frac{1}{a^{2}\sqrt{a\sec\left(e+fx\right)+a}\sec^{2}\left(e+fx\right)+2a^{2}\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a^{2}\sqrt{a\sec\left(e+fx\right)+}\right.}\right.$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] -c*(Integral(sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 +
2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a
```

```
)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**
2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x
))
```

Maxima [F]

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\,dx = \int -\frac{c\sec\left(fx + e\right) - c}{\left(a\sec\left(fx + e\right) + a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] -integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{c - \frac{c}{\cos(e+f\,x)}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2), x)
```

3.84 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx$

Optimal result 617
Rubi [A] (verified) 617
Mathematica [C] (verified) 621
Maple [B] (warning: unable to verify) 621
Fricas [A] (verification not implemented) 622
Sympy [F] 622
Maxima [F] 623
Giac [F(-2)] 623
Mupad [F(-1)] 623

Optimal result

Integrand size = 28, antiderivative size = 230

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}cf} - \frac{71\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}a^{5/2}cf} - \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{32a^3cf} + \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{32a^3cf} + \frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{16a^3cf}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c/f-71/64*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c/f*2^(1/2
)-7/32*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^3/c/f+13/32*cos(f*x+e)*cot(f*x+e
)*sec(1/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(1/2)/a^3/c/f+1/16*cos(f*x+e)^2*cot
(f*x+e)*sec(1/2*f*x+1/2*e)^4*(a+a*sec(f*x+e))^(1/2)/a^3/c/f
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{3989, 3972, 483, 593, 597, 536, 209\}$

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}cf} - \frac{71\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{32\sqrt{2}a^{5/2}cf} - \frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{32a^3cf} + \frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a\sec(e+fx)+a}}{16a^3cf} + \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a\sec(e+fx)+a}}{32a^3cf}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*c*f) -
 (71*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(32
*Sqrt[2]*a^(5/2)*c*f) - (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(32*a^3*c
*f) + (13*Cos[e + f*x]*Cot[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[a + a*Sec[e + f
*x]])/(32*a^3*c*f) + (Cos[e + f*x]^2*Cot[e + f*x]*Sec[(e + f*x)/2]^4*Sqrt[a
 + a*Sec[e + f*x]])/(16*a^3*c*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{\cot^2(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3cf}$$

$$
= \frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{16a^3cf}
+ \frac{\text{Subst}\left(\int \frac{3a-5a^2x^2}{x^2(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{4a^4cf}
$$

$$
= \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{32a^3cf}
+ \frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{16a^3cf}
+ \frac{\text{Subst}\left(\int \frac{-7a^2-39a^3x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{16a^5cf}
$$

$$
= -\frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{32a^3cf}
+ \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{32a^3cf}
+ \frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{16a^3cf}
- \frac{\text{Subst}\left(\int \frac{57a^3-7a^4x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{32a^5cf}
$$

$$
= -\frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{32a^3cf}
+ \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{32a^3cf}
+ \frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{16a^3cf}
- \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2cf}
+ \frac{71\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{32a^2cf}
$$

$$
= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}cf}
- \frac{71\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}a^{5/2}cf}
- \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{32a^3cf}
+ \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{32a^3cf}
+ \frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+fx)}}{16a^3cf}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = -\frac{\left(71\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^2-2\left(17+13\sec(e+fx)+32\,\text{H}\right.\right.}{32cf(-1+\sec(e+fx))(a(1+\sec(e}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] -1/32*((71*Hypergeometric2F1[-1/2, 1, 1/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e
 + f*x])^2 - 2*(17 + 13*Sec[e + f*x] + 32*Hypergeometric2F1[-1/2, 1, 1/2, 1
 - Sec[e + f*x]]*(1 + Sec[e + f*x])^2))*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*
x])*(a*(1 + Sec[e + f*x]))^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. 2(199) = 398.

Time = 2.48 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.17

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\left(24\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{9}{2}}\sin(fx+e)-24(1-\cos\right.}{}$

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192/c/f/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2)/(1-cos(f*x+e))*(24*((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)^(9/2)*sin(f*x+e)-24*(1-cos(f*x+e))^2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(7/2)*csc(f*x+e)+28*(1-cos(f*x+e))^2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/
2)*csc(f*x+e)-4*(1-cos(f*x+e))^6*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*cs
c(f*x+e)^5-35*(1-cos(f*x+e))^2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*csc(
f*x+e)+25*(1-cos(f*x+e))^4*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*csc(f*x+
e)^3-192*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-
cot(f*x+e)+csc(f*x+e)))*(1-cos(f*x+e))-42*(1-cos(f*x+e))^2*((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*csc(f*x+e)+213*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x
+e))^2*csc(f*x+e)^2-1)^(1/2))*(1-cos(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \left[-\frac{71\sqrt{2}\left(\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)+1\right)\sqrt{-a}\log\left(-\right.}{}\right.$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/128*(71*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(2*
sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f
*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*co
s(f*x + e) + 1))*sin(f*x + e) + 64*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sq
rt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(
-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e)
 + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(27*cos(f*x + e)^3 + 12*cos(f*x
+ e)^2 - 7*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*c*f
*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)), 1/64*(71
*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin
(f*x + e) + 64*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(
a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*c
os(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(27*cos(f*x + e)^3 +
12*cos(f*x + e)^2 - 7*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
)/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)
)]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{a^{2}\sqrt{a\sec\left(e+fx\right)+a}\sec^{3}\left(e+fx\right)+a^{2}\sqrt{a\sec\left(e+fx\right)+a}\sec^{2}\left(e+fx\right)-a^{2}\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)-a^{2}\sqrt{a\sec\left(e+fx\right)+a}}\,dx}{c}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 + a**2*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x)**2 - a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
) - a**2*sqrt(a*sec(e + f*x) + a)), x)/c
```

Maxima [F]

$$\int \frac{1}{(a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))}\,dx = \int -\frac{1}{(a\sec(fx + e) + a)^{\frac{5}{2}}(c\sec(fx + e) - c)}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
[Out] -integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))}\,dx = \int \frac{1}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}\,\left(c - \frac{c}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))),x)
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))), x)
```

3.85 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx$

Optimal result 624
Rubi [A] (verified) 625
Mathematica [C] (verified) 628
Maple [A] (verified) 629
Fricas [A] (verification not implemented) 629
Sympy [F] 630
Maxima [F] 630
Giac [F(-2)] 630
Mupad [F(-1)] 631

Optimal result

Integrand size = 28, antiderivative size = 269

$$\begin{aligned}
\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx &= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}c^2f} \\
&- \frac{107\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{64\sqrt{2}a^{5/2}c^2f} + \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{64a^3c^2f} \\
&+ \frac{43\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{96a^4c^2f} \\
&- \frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{32a^4c^2f} \\
&- \frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{16a^4c^2f}
\end{aligned}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c^2/f+43/96*cot
(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-15/32*cos(f*x+e)*cot(f*x+e)^3*se
c(1/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-1/16*cos(f*x+e)^2*cot(f
*x+e)^3*sec(1/2*f*x+1/2*e)^4*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-107/128*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c^2/f*2^(1
/2)+21/64*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^3/c^2/f
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3989, 3972, 483, 593, 597, 536, 209}

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}c^2f} - \frac{107\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{64\sqrt{2}a^{5/2}c^2f} + \frac{43\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{96a^4c^2f} - \frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a\sec(e+fx)+a)^{3/2}}{16a^4c^2f} - \frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a\sec(e+fx)+a)^{3/2}}{32a^4c^2f} + \frac{21\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{64a^3c^2f}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*c^2*f)
 - (107*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/
(64*Sqrt[2]*a^(5/2)*c^2*f) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(64
*a^3*c^2*f) + (43*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(96*a^4*c^2*f)
 - (15*Cos[e + f*x]*Cot[e + f*x]^3*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^
(3/2))/(32*a^4*c^2*f) - (Cos[e + f*x]^2*Cot[e + f*x]^3*Sec[(e + f*x)/2]^4*(
a + a*Sec[e + f*x])^(3/2))/(16*a^4*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{a^2c^2}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^4c^2f}$$

$$= -\frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{16a^4c^2f} - \frac{\text{Subst}\left(\int \frac{a-7a^2x^2}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{4a^5c^2f}$$

$$= -\frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{32a^4c^2f} - \frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{16a^4c^2f} - \frac{\text{Subst}\left(\int \frac{-43a^2-75a^3x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{16a^6c^2f}$$

$$= \frac{43\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{96a^4c^2f} - \frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{32a^4c^2f} - \frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{16a^4c^2f} + \frac{\text{Subst}\left(\int \frac{63a^3-129a^4x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{96a^6c^2f}$$

$$= \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{64a^3c^2f} + \frac{43\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{96a^4c^2f} - \frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{32a^4c^2f} - \frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{16a^4c^2f} - \frac{\text{Subst}\left(\int \frac{447a^4+63a^5x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{192a^6c^2f}$$

$$
\begin{aligned}
&= \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{64a^3c^2f} + \frac{43\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{96a^4c^2f} \\
&\quad - \frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{32a^4c^2f} \\
&\quad - \frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{16a^4c^2f} \\
&\quad + \frac{107\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{64a^2c^2f} \\
&\quad - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^2f}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}c^2f} - \frac{107\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{64\sqrt{2}a^{5/2}c^2f} \\
&\quad + \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{64a^3c^2f} + \frac{43\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{96a^4c^2f} \\
&\quad - \frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{32a^4c^2f} \\
&\quad - \frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+fx))^{3/2}}{16a^4c^2f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.42

$$
\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{\cot^3(e+fx)\left(107\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-\text{s}\right.\right.}{}
$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Cot[e + f*x]^3*(107*Hypergeometric2F1[-3/2, 1, -1/2, (1 - Sec[e + f*x])/2]
*(1 + Sec[e + f*x])^2 - 2*(57 + 45*Sec[e + f*x] + 32*Hypergeometric2F1[-3/2
, 1, -1/2, 1 - Sec[e + f*x]]*(1 + Sec[e + f*x])^2)))/(96*a^2*c^2*f*Sqrt[a*(
1 + Sec[e + f*x])])
```

Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(321\sqrt{2}\cos(fx+e)\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)\sqrt{-}\right.}{}$

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/384/c^2/f/a^3*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(321*2^(1/2)*cos(f
*x+e)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*
x+e)^2-1)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-768*arctanh(sin(f*x+e)/
(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*cos(f*x+e)+321*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(csc
(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1
/2))-768*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/
2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+410*cos(f*x+e)*cot(f*x+e)^3+142*cot(
f*x+e)^3-298*csc(f*x+e)*cot(f*x+e)^2-126*csc(f*x+e)^2*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \left[-\frac{321\sqrt{2}\left(\cos(fx+e)^3+\cos(fx+e)^2-\cos(fx+e)\right.}{}\right.$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [-1/768*(321*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*s
qrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f
*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 384*(cos(f*x + e)^3 + cos(f
*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(
f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*
(205*cos(f*x + e)^4 + 71*cos(f*x + e)^3 - 149*cos(f*x + e)^2 - 63*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 +
a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)
), 1/384*(321*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*
```

```
sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 384*(cos(f*x + e)^3 + cos(f*x + e)^
2 - cos(f*x + e) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e)
- a))*sin(f*x + e) + 2*(205*cos(f*x + e)^4 + 71*cos(f*x + e)^3 - 149*cos(f*
x + e)^2 - 63*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*
c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) -
a^3*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)-2a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+a^2\sqrt{a}}}{c^2}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4 - 2*a**2*sqrt(a*s
ec(e + f*x) + a)*sec(e + f*x)**2 + a**2*sqrt(a*sec(e + f*x) + a)), x)/c**2
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)^2}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2),x)
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2), x)
```

3.86 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result 632
Rubi [A] (verified) 632
Mathematica [A] (verified) 634
Maple [A] (verified) 635
Fricas [A] (verification not implemented) 635
Sympy [F(-1)] 636
Maxima [B] (verification not implemented) 636
Giac [F] 637
Mupad [F(-1)] 637

Optimal result

Integrand size = 30, antiderivative size = 185

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx = \frac{ac^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -1/2*a*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-1/3*a
*c*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+a*c^4*ln(cos(
f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a*c^3*(c
-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used

= {3991, 3990, 3556}

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx = \frac{ac^4\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{ac^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}} - \frac{ac^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}} - \frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a*c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a
 + a*Sec[e + f*x]]) - (a*c^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*
Sqrt[a + a*Sec[e + f*x]]) - (a*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(
3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rubi steps

$$\text{integral} = -\frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + c\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx$$

$$= -\frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + c^2\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx$$

$$= -\frac{ac^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + c^3\int\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$

$$= -\frac{ac^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} - \frac{(ac^4\tan(e+fx))\int\tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{ac^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.45

$$\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx = \frac{ac^4(6\log(\cos(e+fx))+18\sec(e+fx)-9\sec^2(e+fx)+2\sec^3(e+fx))\tan(e+fx)}{6f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2),x]
[Out] (a*c^4*(6*Log[Cos[e + f*x]] + 18*Sec[e + f*x] - 9*Sec[e + f*x]^2 + 2*Sec[e
+ f*x]^3)*Tan[e + f*x])/(6*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e +
f*x]])
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^3\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^3\left(6\cos(fx+e)^3\ln(-\cot(fx+e)+\csc(fx+e)+1)-6\cos(fx+e)^3\ln\left(\frac{2}{\cos(fx+}\right.\right.}{6f(\cos(fx+e)-1}$
risch	$\frac{c^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(-18ie^{5i(fx+e)}-3\,e^{6i(fx+e)}fx-18ie^{i(fx+e)}-6\,e^{6i(fx+e)}e-9\,e^{4i(fx+e)}fx-9ie^{4i(fx+e)}\ln(1\right.}{}$

```
[In] int((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*c^3*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)
^3*(6*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1)-6*cos(f*x+e)^3*ln(2/(cos(f*
x+e)+1))+6*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)-1)+29*cos(f*x+e)^3+18*cos
(f*x+e)^2-9*cos(f*x+e)+2)/(cos(f*x+e)-1)^3*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.48

$$\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx = \left[-\frac{\left(11\,c^3\cos(fx+e)^2-7\,c^3\cos(fx+e)+2\,c^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)-3}{6\,(}\right.$$

$$-\frac{\left(11\,c^3\cos(fx+e)^2-7\,c^3\cos(fx+e)+2\,c^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)-6\left(c^3\cos(fx+e\right.}{6\left(f\cos(fx+e)^3+f\cos(fx+\right.}$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/6*((11*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
e) - 3*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(a*c*co
s(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e)
 + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -1/6*((11*c
^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*x + e) + a)/c
```

```
os(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - 6*(c^3*
cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e)^3 + f*cos(
f*x + e)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))**(7/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(165) = 330$.

Time = 0.46 (sec) , antiderivative size = 1289, normalized size of antiderivative = 6.97

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{7/2}\,dx = \text{Too large to display}$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*c^3*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*c^3*cos(4*f*x + 4*e
)^2 + 27*(f*x + e)*c^3*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*c^3*sin(6*f*x + 6*e
)^2 + 27*(f*x + e)*c^3*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*c^3*sin(2*f*x + 2*
e)^2 + 18*(f*x + e)*c^3*cos(2*f*x + 2*e) + 3*(f*x + e)*c^3 + 18*c^3*sin(2*f
*x + 2*e) - 3*(c^3*cos(6*f*x + 6*e)^2 + 9*c^3*cos(4*f*x + 4*e)^2 + 9*c^3*co
s(2*f*x + 2*e)^2 + c^3*sin(6*f*x + 6*e)^2 + 9*c^3*sin(4*f*x + 4*e)^2 + 18*c
^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*c^3*sin(2*f*x + 2*e)^2 + 6*c^3*cos
(2*f*x + 2*e) + c^3 + 2*(3*c^3*cos(4*f*x + 4*e) + 3*c^3*cos(2*f*x + 2*e) +
c^3)*cos(6*f*x + 6*e) + 6*(3*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x + 4*e) +
 6*(c^3*sin(4*f*x + 4*e) + c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 6*(3*(f*x + e)*c^3*cos(4*f*x + 4*
e) + 3*(f*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 + 3*c^3*sin(4*f*x + 4
*e) + 3*c^3*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*c^3*cos(2*
f*x + 2*e) + (f*x + e)*c^3)*cos(4*f*x + 4*e) + 18*(c^3*sin(6*f*x + 6*e) + 3
*c^3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 44*(c^3*sin(6*f*x + 6*e) + 3*c^3*sin(4*f*x + 4*
e) + 3*c^3*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
```

```
2*e))) + 18*(c^3*sin(6*f*x + 6*e) + 3*c^3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*
x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 18*((f*x +
 e)*c^3*sin(4*f*x + 4*e) + (f*x + e)*c^3*sin(2*f*x + 2*e) - c^3*cos(4*f*x +
 4*e) - c^3*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 18*(3*(f*x + e)*c^3*sin(2*
f*x + 2*e) + c^3)*sin(4*f*x + 4*e) - 18*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4
*f*x + 4*e) + 3*c^3*cos(2*f*x + 2*e) + c^3)*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 44*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4*f*x + 4*e) +
 3*c^3*cos(2*f*x + 2*e) + c^3)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 18*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4*f*x + 4*e) + 3*c^3*cos(2*
f*x + 2*e) + c^3)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqr
t(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*cos(2*f*x + 2*e) + 1)*cos(6*f*x +
6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 9
*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^2 + 6*(sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18
*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2
*e) + 1)*f)
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx=\int \sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^{\frac{7}{2}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(7/2),x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(7/2), x)
```

3.87 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result 638
Rubi [A] (verified) 638
Mathematica [A] (verified) 640
Maple [A] (verified) 640
Fricas [A] (verification not implemented) 641
Sympy [F(-1)] 641
Maxima [B] (verification not implemented) 642
Giac [F] 642
Mupad [F(-1)] 643

Optimal result

Integrand size = 30, antiderivative size = 139

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\frac{ac^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{ac^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -1/2*a*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+a*c^3*l
n(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a*
c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3991, 3990, 3556}

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\frac{ac^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a
 + a*Sec[e + f*x]]) - (a*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sq
rt[a + a*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} \\
&\quad + c\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx \\
&= -\frac{ac^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} \\
&\quad + c^2\int\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{ac^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(ac^3\tan(e+fx))\int\tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= \frac{ac^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{ac^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.52

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{ac^3(-2\log(\cos(e+fx))-4\sec(e+fx)+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(a*c^3*(-2*Log[Cos[e + f*x]] - 4*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e
+ f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.13

method	result
default	$-\frac{c^2(\sec(fx+e)-1)^2\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\,\left(2\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)-1)-2\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx\cdots}\right.\right.}{2f(\cos(fx+e)-1)^2}$
risch	$-\frac{c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(ie^{4i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+e^{4i(fx+e)}fx+2ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+2\,e^{4i(fx+e)}e+2\,e^{2\cdots}}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\left(1+e^{2i(fx+e)}\right)\cdots}$

```
[In] int((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*c^2*(sec(f*x+e)-1)^2*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1
/2)*(2*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)-1)-2*cos(f*x+e)^2*ln(2/(cos(f
*x+e)+1))+2*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+5*cos(f*x+e)^2+4*cos(
f*x+e)-1)/(cos(f*x+e)-1)^2*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.06

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{5/2}\,dx = \left[-\frac{(3\,c^2\cos(fx+e) - c^2)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e) - (c^2\cos(fx+e)^2 + c^2\cos \cdots}{2\,(f\cos(fx+ \cdots}\right.$$

$$-\frac{(3\,c^2\cos(fx+e) - c^2)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e) - 2\,(c^2\cos(fx+e)^2 + c^2\cos(fx+e))\sqrt{a \cdots}}{2\,(f\cos(fx+e)^2 + f\cos(fx+e))}$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e)^2 +
 c^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3
 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*co
s(f*x + e)^2 + f*cos(f*x + e)), -1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f
*x + e) - 2*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(a*c)*arctan(sqrt(a
*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x
+ e)^2 + f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))**(5/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(125) = 250.

Time = 0.39 (sec) , antiderivative size = 710, normalized size of antiderivative = 5.11

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{5/2}\, dx =$$
$$-\frac{((fx+e)c^2\cos(4fx+4e)^2 + 4(fx+e)c^2\cos(2fx+2e)^2 + (fx+e)c^2\sin(4fx+4e)^2 + 4(fx+e)c^2\sin}{}$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*cos(2*f*x + 2*e)^2 + (
f*x + e)*c^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*c^2*cos(2*f*x + 2*e) + (f*x + e)*c^2 + 2*c^2*sin(2*f*x + 2*e) - (c^
2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 +
4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*
cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*c^2*cos(2
*f*x + 2*e) + (f*x + e)*c^2 + c^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(c
^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)*c^2*
sin(2*f*x + 2*e) - c^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) - 4*(c^2*cos(4*f*
x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2
*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f
*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*
sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{5/2}\, dx = \int \sqrt{a\sec(fx+e) + a}(-c\sec(fx+e) + c)^{\frac{5}{2}}\, dx$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2),x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2), x)
```

3.88 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result . . . 644
Rubi [A] (verified) . . . 644
Mathematica [A] (verified) . . . 645
Maple [A] (verified) . . . 646
Fricas [A] (verification not implemented) . . . 646
Sympy [F] . . . 647
Maxima [B] (verification not implemented) . . . 647
Giac [F] . . . 648
Mupad [F(-1)] . . . 648

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{ac^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{ac\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] a*c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-a*c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3991, 3990, 3556}

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a +
 a*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&=-\frac{ac\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+c\int\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx\\
&=-\frac{ac\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{\left(ac^2\tan(e+fx)\right)\int\tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&=\frac{ac^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{ac\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{ac^2(\log(\cos(e+fx))+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*c^2*(Log[Cos[e + f*x]] + Sec[e + f*x])*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec[
e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.42

method	result
default	$\frac{c\sqrt{a(\sec(fx+e)+1)}\,(\sec(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}\,\left(\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)-1)+\cos(fx+e)\ln(-\cot(fx+e)+\csc\right.}{f(\cos(fx+e)-1)}$
risch	$\frac{c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)} - \frac{2c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(fx+e)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f} - \frac{2ic\sqrt{\frac{a(}{1}}}{}$

```
[In] int((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c*(a*(sec(f*x+e)+1))^(1/2)*(sec(f*x+e)-1)*(-c*(sec(f*x+e)-1))^(1/2)*(co
s(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)-1)+cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+
1)-cos(f*x+e)*ln(2/(cos(f*x+e)+1))+cos(f*x+e)+1)/(cos(f*x+e)-1)*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.76

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx = \left[-\frac{2\,c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)-\sqrt{-ac}(c\cos(fx+e)+c)\log\left(\frac{ac\cos(fx+e)^4-\left(\cos\right.}{}\right.}{2\,(f\cos(fx+e)+f)}\right.$$

$$-\frac{c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)-\sqrt{ac}(c\cos(fx+e)+c)\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(}{ac\cos(fx+e)^2+ac}\right.}{f\cos(fx+e)+f}$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e))*sin(f*x + e) - sqrt(-a*c)*(c*cos(f*x + e) + c)*log(1/2*(a*c
*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*
```

```
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
 e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), -(c*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - s
qrt(a*c)*(c*cos(f*x + e) + c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e) + f)]
```

Sympy [F]

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{3/2}\,dx = \int \sqrt{a\left(\sec\left(e + fx\right) + 1\right)}\left(-c(\sec\left(e + fx\right) - 1)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(85) = 170.

Time = 0.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.61

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{3/2}\,dx = -\frac{((fx + e)c\cos\left(2\,fx + 2\,e\right)^2 + (fx + e)c\sin\left(2\,fx + 2\,e\right)^2 + 2\,(fx + e)c\cos\left(2\,fx + 2\,e\right) + 2\,c\cos\left(\frac{1}{2}\,\arctan\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c*cos(2*f*x + 2*e)^2 + (f*x + e)*c*sin(2*f*x + 2*e)^2 + 2*(f*x
+ e)*c*cos(2*f*x + 2*e) + 2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))*sin(2*f*x + 2*e) + (f*x + e)*c - (c*cos(2*f*x + 2*e)^2 + c*sin(2*f*
x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1) - 2*(c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}(-c\sec\left(fx+e\right)+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2), x)
```

3.89 $\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result 649
Rubi [A] (verified) 649
Mathematica [A] (verified) 650
Maple [A] (verified) 650
Fricas [B] (verification not implemented) 651
Sympy [F] 651
Maxima [A] (verification not implemented) 651
Giac [F] 652
Mupad [F(-1)] 652

Optimal result

Integrand size = 30, antiderivative size = 48

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx=\frac{ac\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a*c*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3990, 3556}

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx=\frac{ac\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
```

```
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(ac\tan(e+fx))\int\tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= \frac{ac\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \frac{c\log(\cos(e+fx))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
[Out] (c*Log[Cos[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[c
 - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.81

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,\left(\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\ln(-\cot(fx+e)+\csc(fx+e)-1)\right)\cot}{f}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})(fx+e)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}-\frac{i\sqrt{\frac{a\left(e^{i(fx\right.}}{1+e^{2i}}}}{}$

```
[In] int((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/f*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)*(ln(-cot(f*x+e)+csc
(f*x+e)+1)-ln(2/(cos(f*x+e)+1))+ln(-cot(f*x+e)+csc(f*x+e)-1))*cot(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(44) = 88.

Time = 0.33 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.17

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\left[\frac{\sqrt{-ac}\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)}{2f}, \frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e
))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2)/f, sqrt(a*c)*arctan(
sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))/f]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$=-\frac{\left(fx+e-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)\sqrt{a}\sqrt{c}}{f}$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(f*x + e - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sqrt(a)*sqrt(c
)/f
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int \sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx=\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2), x)
```

3.90 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result 653
Rubi [A] (verified) 653
Mathematica [A] (verified) 654
Maple [A] (verified) 654
Fricas [F] 655
Sympy [F] 655
Maxima [A] (verification not implemented) 655
Giac [F] 656
Mupad [F(-1)] 656

Optimal result

Integrand size = 30, antiderivative size = 51

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a*ln(1-cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3996, 31}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\tan(e+fx)\log(1-\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
```

```
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{-c+cx}\,dx, x, \cos(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &= \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$$
$$= \frac{(\log(\cos(e+fx))+\log(1-\sec(e+fx)))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]
[Out] ((Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x])]*Tan
[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

method	result
default	$\frac{\sqrt{a(\sec(fx+e)+1)}\left(\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\ln(-\cot(fx+e)+\csc(fx+e))\right)(\cot(fx+e)-\csc(fx+e))}{f\sqrt{-c(\sec(fx+e)-1)}}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)x}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}-\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(sec(f*x+e)+1))^(1/2)*(ln(2/(cos(f*x+e)+1))-2*ln(-cot(f*x+e)+csc(f*x
+e)))/(-c*(sec(f*x+e)-1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\sqrt{a\sec(fx+e)+a}}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e
) - c), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\sqrt{a(\sec(e+fx)+1)}}{\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(-c*(sec(e + f*x) - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\frac{2\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}}-\frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}}}{f}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] (2*sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1))/sqrt(c) - sqrt(-a)*log(sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c))/f
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(1/2), x)
```

3.91 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result 657
Rubi [A] (verified) 657
Mathematica [A] (verified) 658
Maple [A] (verified) 659
Fricas [F] 659
Sympy [F] 659
Maxima [B] (verification not implemented) 660
Giac [F] 660
Mupad [F(-1)] 661

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a*ln(1-cos(f*
x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3992, 3996, 31}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)))
 + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}\\
&= -\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\
&\quad+\frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{a\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))+\frac{1}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2),x]
[Out] (a*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + (-1 + Sec[e + f*x])^(-1))*T
an[e + f*x])/(c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\left(4\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))-2\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-4\ln(-\cot(fx+e)+\csc(fx+e))+2\ln\left(\frac{2}{\cos(fx+e)+1}\right)-}{2f\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx+e)-1)(\cos(fx+e)+1)}$
risch	$-\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(2ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+e^{2i(fx+e)}fx-4ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2e^{2i(fx+e)}e-2e^{i(fx+e)}fx-2ie^{i(fx+e}\right.}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(4*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e))-2*cos(f*x+e)*ln(2/(cos(f*x+
e)+1))-4*ln(-cot(f*x+e)+csc(f*x+e))+2*ln(2/(cos(f*x+e)+1))-cos(f*x+e)-1)*(a
*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f*x+e)-1)/(cos(f*x+
e)+1)*tan(f*x+e)
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x +
e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(88) = 176.

Time = 0.37 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.16

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\left((fx+e)\cos\left(2\,fx+2\,e\right)^2+4\,(fx+e)\cos\left(fx+e\right)^2+(fx+e)\sin\left(2\,fx+2\,e\right)^2+4\,(fx+e)\sin\left(fx+e\right)^2\right.}{}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin
(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(2*cos(f*x + e) -
 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*x + e)^2 - sin(2*f*x +
2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e)^2 + 4*cos(f*x + e
) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 2*(f*x + e)*cos(f
*x + e) + e + sin(f*x + e))*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x + e) - 2
*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e + 2*sin(f*x
 + e))*sqrt(a)*sqrt(c)/((c^2*cos(2*f*x + 2*e)^2 + 4*c^2*cos(f*x + e)^2 + c^
2*sin(2*f*x + 2*e)^2 - 4*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2*sin(f*x
+ e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(f*x + e) - c^2)*cos(2*f*x
+ 2*e))*f)
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(3/2),x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(3/2), x)
```

3.92 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result 662
Rubi [A] (verified) 662
Mathematica [A] (verified) 664
Maple [A] (verified) 664
Fricas [F] 665
Sympy [F] 665
Maxima [B] (verification not implemented) 665
Giac [F] 666
Mupad [F(-1)] 666

Optimal result

Integrand size = 30, antiderivative size = 142

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx=-\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}-\frac{a\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a*tan(f*x
+e)/c/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a*ln(1-cos(f*x+e))*ta
n(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3992, 3996, 31}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx=\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
)) - (a*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3
/2)) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}+\frac{\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c}\\
&= -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{a\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c^2}\\
&= -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{a\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\
&\quad+\frac{(a\tan(e+fx))\text{Subst}\left(\int\frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\left(-2\log(\cos(e+fx))-2\log(1-\sec(e+fx))+\frac{3-2\sec(e+fx)}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{2c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(a*(-2*Log[Cos[e + f*x]] - 2*Log[1 - Sec[e + f*x]] + (3 - 2*Sec[e + f*
x])/(-1 + Sec[e + f*x])^2)*Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*
Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.67

method	result
default	$-\frac{\sqrt{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(8\ln\left((1-\cos(fx+e))^2\csc(fx+e)^2+1\right)(1-\cos(fx+e))^4\csc(fx+e)^4-16\ln(-\mathrm{co}\right.}{16f\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^2\left(\frac{c(1-\cos(fx+}{(1-\cos(fx+e}\right.}$
risch	$-\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(-4ie^{i(fx+e)}+e^{4i(fx+e)}fx-8ie^{3i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2e^{4i(fx+e)}e-4e^{3i(fx+e)}fx+12ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}\right.\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/f*2^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(8*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1
)*(1-cos(f*x+e))^4*csc(f*x+e)^4-16*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-6*(1-cos(f*x+e))^2*csc(f*x+e)^2+1)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x +
 e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(128) = 256$.

Time = 0.48 (sec) , antiderivative size = 1173, normalized size of antiderivative = 8.26

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x
+ e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f*x
 + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2*e)
^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(4*cos(3*f*x + 3*e) - 6*cos(2
*f*x + 2*e) + 4*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8
*(6*cos(2*f*x + 2*e) - 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x
+ 3*e)^2 + 12*(4*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2
 - 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f
*x + e))*sin(4*f*x + 4*e) - sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2
*sin(f*x + e))*sin(3*f*x + 3*e) - 16*sin(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*
e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) - 16*sin(f*x + e)^2 + 8*cos(f*x + e
```

```
) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 4*(f*x + e)*cos(3
*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x + e) + e +
 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*cos(4*f*x + 4*e)
 - 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x + e) + e)*co
s(3*f*x + 3*e) + 12*(f*x - 4*(f*x + e)*cos(f*x + e) + e)*cos(2*f*x + 2*e) -
 8*(f*x + e)*cos(f*x + e) - 2*(4*(f*x + e)*sin(3*f*x + 3*e) - 6*(f*x + e)*s
in(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x + 3*e) - 3*cos(2*f
*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) - 4*(12*(f*x + e)*sin(2*f*x +
2*e) - 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e) - 6*(8*(f*x + e)*sin(
f*x + e) + 1)*sin(2*f*x + 2*e) + e + 4*sin(f*x + e))*sqrt(a)*sqrt(c)/((c^3*
cos(4*f*x + 4*e)^2 + 16*c^3*cos(3*f*x + 3*e)^2 + 36*c^3*cos(2*f*x + 2*e)^2
+ 16*c^3*cos(f*x + e)^2 + c^3*sin(4*f*x + 4*e)^2 + 16*c^3*sin(3*f*x + 3*e)^
2 + 36*c^3*sin(2*f*x + 2*e)^2 - 48*c^3*sin(2*f*x + 2*e)*sin(f*x + e) + 16*c
^3*sin(f*x + e)^2 - 8*c^3*cos(f*x + e) + c^3 - 2*(4*c^3*cos(3*f*x + 3*e) -
6*c^3*cos(2*f*x + 2*e) + 4*c^3*cos(f*x + e) - c^3)*cos(4*f*x + 4*e) - 8*(6*
c^3*cos(2*f*x + 2*e) - 4*c^3*cos(f*x + e) + c^3)*cos(3*f*x + 3*e) - 12*(4*c
^3*cos(f*x + e) - c^3)*cos(2*f*x + 2*e) - 4*(2*c^3*sin(3*f*x + 3*e) - 3*c^3
*sin(2*f*x + 2*e) + 2*c^3*sin(f*x + e))*sin(4*f*x + 4*e) - 16*(3*c^3*sin(2*
f*x + 2*e) - 2*c^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(5/2), x)
```

3.93 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal result 667
Rubi [A] (verified) 667
Mathematica [A] (verified) 669
Maple [A] (verified) 670
Fricas [F] 670
Sympy [F(-1)] 671
Maxima [B] (verification not implemented) 671
Giac [F] 673
Mupad [F(-1)] 673

Optimal result

Integrand size = 30, antiderivative size = 188

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = & -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} \\ & -\frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} \\ & -\frac{a\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} \\ & +\frac{a\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

```
[Out] -1/3*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a*tan
(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a*tan(f*x+e)/c^2/
f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a*ln(1-cos(f*x+e))*tan(f*x+
e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used

$= \{3992, 3996, 31\}$

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{a\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{a\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}} - \frac{a\tan(e+fx)}{2cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}} - \frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -1/3*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2
)) - (a*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^
(5/2)) - (a*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(3/2)) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\text{integral} = -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} + \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx}{c}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}dx}{c^2}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}dx}{c^3}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{(a\tan(e+fx))\text{Subst}\left(\int\frac{1}{-c+cx}dx,x,\cos(e+fx)\right)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}dx = -\frac{a\left(-6\log(\cos(e+fx))-6\log(1-\sec(e+fx))+\frac{-11+15\sec(e+fx)-6\sec^2(e+fx)}{(-1+\sec(e+fx))^3}\right)\tan(e+fx)}{6c^3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
```

[Out] -1/6*(a*(-6*Log[Cos[e + f*x]] - 6*Log[1 - Sec[e + f*x]] + (-11 + 15*Sec[e + f*x] - 6*Sec[e + f*x]^2)/(-1 + Sec[e + f*x])^3)*Tan[e + f*x])/(c^3*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.38

method	result
default	$\frac{\sqrt{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(48\ln(-\cot(fx+e)+\csc(fx+e))(1-\cos(fx+e))^6\csc(fx+e)^6-24\ln\left((1-\cos(fx+\right.\right.}{48f\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^3}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)x}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)(fx+e)}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}+\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(9\,e^{5i(fx+e)}-27\,e^{4i(fx+e)}+4\right.}{3c^3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^5}$

[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/48/f*2^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^3/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)^2)^(7/2)*(1-cos(f*x+e))*(48*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*x+e))^6*csc(f*x+e)^6-24*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*x+e))^6*csc(f*x+e)^6+21*(1-cos(f*x+e))^4*csc(f*x+e)^4-6*(1-cos(f*x+e))^2*csc(f*x+e)^2+1)*csc(f*x+e)

Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{\sqrt{a\sec(fx+e)+a}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x + e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c^4), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(7/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2444 vs. $2(168) = 336$.

Time = 1.91 (sec) , antiderivative size = 2444, normalized size of antiderivative = 13.00

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
[Out] -1/3*(3*(f*x + e)*cos(6*f*x + 6*e)^2 + 108*(f*x + e)*cos(5*f*x + 5*e)^2 + 6
75*(f*x + e)*cos(4*f*x + 4*e)^2 + 1200*(f*x + e)*cos(3*f*x + 3*e)^2 + 675*(
f*x + e)*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*cos(f*x + e)^2 + 3*(f*x + e)*si
n(6*f*x + 6*e)^2 + 108*(f*x + e)*sin(5*f*x + 5*e)^2 + 675*(f*x + e)*sin(4*f
*x + 4*e)^2 + 1200*(f*x + e)*sin(3*f*x + 3*e)^2 + 675*(f*x + e)*sin(2*f*x +
 2*e)^2 + 108*(f*x + e)*sin(f*x + e)^2 + 3*f*x + 6*(2*(6*cos(5*f*x + 5*e) -
 15*cos(4*f*x + 4*e) + 20*cos(3*f*x + 3*e) - 15*cos(2*f*x + 2*e) + 6*cos(f*
x + e) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 + 12*(15*cos(4*f*x + 4*e)
 - 20*cos(3*f*x + 3*e) + 15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)*cos(5*f*
x + 5*e) - 36*cos(5*f*x + 5*e)^2 + 30*(20*cos(3*f*x + 3*e) - 15*cos(2*f*x +
 2*e) + 6*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - 225*cos(4*f*x + 4*e)^2 + 40*
(15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 400*cos(3*f*x
 + 3*e)^2 + 30*(6*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 225*cos(2*f*x + 2*e)
^2 - 36*cos(f*x + e)^2 + 2*(6*sin(5*f*x + 5*e) - 15*sin(4*f*x + 4*e) + 20*s
in(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x + e))*sin(6*f*x + 6*e) -
sin(6*f*x + 6*e)^2 + 12*(15*sin(4*f*x + 4*e) - 20*sin(3*f*x + 3*e) + 15*sin
(2*f*x + 2*e) - 6*sin(f*x + e))*sin(5*f*x + 5*e) - 36*sin(5*f*x + 5*e)^2 +
30*(20*sin(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x + e))*sin(4*f*x +
 4*e) - 225*sin(4*f*x + 4*e)^2 + 120*(5*sin(2*f*x + 2*e) - 2*sin(f*x + e))*
sin(3*f*x + 3*e) - 400*sin(3*f*x + 3*e)^2 - 225*sin(2*f*x + 2*e)^2 + 180*si
n(2*f*x + 2*e)*sin(f*x + e) - 36*sin(f*x + e)^2 + 12*cos(f*x + e) - 1)*arct
an2(sin(f*x + e), cos(f*x + e) - 1) + 2*(3*f*x - 18*(f*x + e)*cos(5*f*x + 5
*e) + 45*(f*x + e)*cos(4*f*x + 4*e) - 60*(f*x + e)*cos(3*f*x + 3*e) + 45*(f
*x + e)*cos(2*f*x + 2*e) - 18*(f*x + e)*cos(f*x + e) + 3*e + 9*sin(5*f*x +
```

```
5*e) - 27*sin(4*f*x + 4*e) + 40*sin(3*f*x + 3*e) - 27*sin(2*f*x + 2*e) + 9*
sin(f*x + e))*cos(6*f*x + 6*e) - 6*(6*f*x + 90*(f*x + e)*cos(4*f*x + 4*e) -
 120*(f*x + e)*cos(3*f*x + 3*e) + 90*(f*x + e)*cos(2*f*x + 2*e) - 36*(f*x +
 e)*cos(f*x + e) + 6*e - 9*sin(4*f*x + 4*e) + 20*sin(3*f*x + 3*e) - 9*sin(2
*f*x + 2*e))*cos(5*f*x + 5*e) + 6*(15*f*x - 300*(f*x + e)*cos(3*f*x + 3*e)
+ 225*(f*x + e)*cos(2*f*x + 2*e) - 90*(f*x + e)*cos(f*x + e) + 15*e + 20*si
n(3*f*x + 3*e) - 9*sin(f*x + e))*cos(4*f*x + 4*e) - 120*(f*x + 15*(f*x + e)
*cos(2*f*x + 2*e) - 6*(f*x + e)*cos(f*x + e) + e + sin(2*f*x + 2*e) - sin(f
*x + e))*cos(3*f*x + 3*e) + 18*(5*f*x - 30*(f*x + e)*cos(f*x + e) + 5*e - 3
*sin(f*x + e))*cos(2*f*x + 2*e) - 36*(f*x + e)*cos(f*x + e) - 2*(18*(f*x +
e)*sin(5*f*x + 5*e) - 45*(f*x + e)*sin(4*f*x + 4*e) + 60*(f*x + e)*sin(3*f*
x + 3*e) - 45*(f*x + e)*sin(2*f*x + 2*e) + 18*(f*x + e)*sin(f*x + e) + 9*co
s(5*f*x + 5*e) - 27*cos(4*f*x + 4*e) + 40*cos(3*f*x + 3*e) - 27*cos(2*f*x +
 2*e) + 9*cos(f*x + e))*sin(6*f*x + 6*e) - 6*(90*(f*x + e)*sin(4*f*x + 4*e)
 - 120*(f*x + e)*sin(3*f*x + 3*e) + 90*(f*x + e)*sin(2*f*x + 2*e) - 36*(f*x
 + e)*sin(f*x + e) + 9*cos(4*f*x + 4*e) - 20*cos(3*f*x + 3*e) + 9*cos(2*f*x
 + 2*e) - 3)*sin(5*f*x + 5*e) - 6*(300*(f*x + e)*sin(3*f*x + 3*e) - 225*(f*
x + e)*sin(2*f*x + 2*e) + 90*(f*x + e)*sin(f*x + e) + 20*cos(3*f*x + 3*e) -
 9*cos(f*x + e) + 9)*sin(4*f*x + 4*e) - 40*(45*(f*x + e)*sin(2*f*x + 2*e) -
 18*(f*x + e)*sin(f*x + e) - 3*cos(2*f*x + 2*e) + 3*cos(f*x + e) - 2)*sin(3
*f*x + 3*e) - 54*(10*(f*x + e)*sin(f*x + e) - cos(f*x + e) + 1)*sin(2*f*x +
 2*e) + 3*e + 18*sin(f*x + e))*sqrt(a)*sqrt(c)/((c^4*cos(6*f*x + 6*e)^2 + 3
6*c^4*cos(5*f*x + 5*e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 400*c^4*cos(3*f*x +
 3*e)^2 + 225*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos(f*x + e)^2 + c^4*sin(6*f*
x + 6*e)^2 + 36*c^4*sin(5*f*x + 5*e)^2 + 225*c^4*sin(4*f*x + 4*e)^2 + 400*c
^4*sin(3*f*x + 3*e)^2 + 225*c^4*sin(2*f*x + 2*e)^2 - 180*c^4*sin(2*f*x + 2*
e)*sin(f*x + e) + 36*c^4*sin(f*x + e)^2 - 12*c^4*cos(f*x + e) + c^4 - 2*(6*
c^4*cos(5*f*x + 5*e) - 15*c^4*cos(4*f*x + 4*e) + 20*c^4*cos(3*f*x + 3*e) -
15*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(6*f*x + 6*e) - 12*(
15*c^4*cos(4*f*x + 4*e) - 20*c^4*cos(3*f*x + 3*e) + 15*c^4*cos(2*f*x + 2*e)
 - 6*c^4*cos(f*x + e) + c^4)*cos(5*f*x + 5*e) - 30*(20*c^4*cos(3*f*x + 3*e)
 - 15*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(4*f*x + 4*e) - 4
0*(15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(3*f*x + 3*e) - 3
0*(6*c^4*cos(f*x + e) - c^4)*cos(2*f*x + 2*e) - 2*(6*c^4*sin(5*f*x + 5*e) -
 15*c^4*sin(4*f*x + 4*e) + 20*c^4*sin(3*f*x + 3*e) - 15*c^4*sin(2*f*x + 2*e
) + 6*c^4*sin(f*x + e))*sin(6*f*x + 6*e) - 12*(15*c^4*sin(4*f*x + 4*e) - 20
*c^4*sin(3*f*x + 3*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(f*x + e))*sin(5
*f*x + 5*e) - 30*(20*c^4*sin(3*f*x + 3*e) - 15*c^4*sin(2*f*x + 2*e) + 6*c^4
*sin(f*x + e))*sin(4*f*x + 4*e) - 120*(5*c^4*sin(2*f*x + 2*e) - 2*c^4*sin(f
*x + e))*sin(3*f*x + 3*e))*f)
```

Giac [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^{7/2}}\,dx = \int \frac{\sqrt{a\sec\left(fx + e\right) + a}}{\left(-c\sec\left(fx + e\right) + c\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^{7/2}}\,dx = \int \frac{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(7/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(7/2), x)
```

3.94 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

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Rubi [A] (verified) 674
Mathematica [A] (verified) 676
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Optimal result

Integrand size = 30, antiderivative size = 190

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a^2c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -1/2*a^2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/3*a
^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+a^2*c^3*ln(co
s(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a^2*c^
2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec), antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used

$= \{3994, 3991, 3990, 3556\}$

$$\int (a + a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a^2c^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}} - \frac{a^2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}} + \frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^2*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) - (a^2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[a + a*Sec[e + f*x]]) - (a^2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
2*f*Sqrt[a + a*Sec[e + f*x]]) + (a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3994

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a, Int[Sqrt[a + b
*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n
```

```
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} \\
&\quad + a\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx \\
&= -\frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} \\
&\quad + (ac)\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx \\
&= -\frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \left(ac^2\right)\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^2c^3\tan(e+fx)\right)\int \tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= \frac{a^2c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$
\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a^2c^3(2+6\log(\cos(e+fx))+6\sec(e+fx)+3\sec^2(e+fx)-2\sec^3(e+fx))\tan(e+fx)}{6f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
[Out] (a^2*c^3*(2 + 6*Log[Cos[e + f*x]] + 6*Sec[e + f*x] + 3*Sec[e + f*x]^2 - 2*S
ec[e + f*x]^3)*Tan[e + f*x])/(6*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec
[e + f*x]])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.87

method	result
default	$-\frac{a\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^2c^2\sqrt{a(\sec(fx+e)+1)}\left(6\cos(fx+e)^3\ln(-\cot(fx+e)+\csc(fx+e)+1)+6\cos(fx+e)^3\ln(-\cot\right.}{6f(\cos(fx+e)-}$
risch	$-\frac{a\,c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(6ie^{5i(fx+e)}+3\,e^{6i(fx+e)}fx+6ie^{i(fx+e)}+6\,e^{6i(fx+e)}e+9\,e^{4i(fx+e)}fx+6ie^{4i(fx+e)}+18\,e\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*a*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^2*c^2*(a*(sec(f*x+e)+1))^
(1/2)*(6*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1)+6*cos(f*x+e)^3*ln(-cot(f
*x+e)+csc(f*x+e)-1)-6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+cos(f*x+e)^3+6*cos(
f*x+e)^2+3*cos(f*x+e)-2)/(cos(f*x+e)-1)^2*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.46

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \left[-\frac{\left(7\,ac^2\cos\left(fx+e\right)^2+ac^2\cos\left(fx+e\right)-2\,ac^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(f\right.}{}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [-1/6*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
 + e) - 3*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*
(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f
*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -1/6
*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e)
 - 6*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*
c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x +
 e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1356 vs. $2(170) = 340$.

Time = 0.48 (sec) , antiderivative size = 1356, normalized size of antiderivative = 7.14

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
[Out] -1/3*(3*(f*x + e)*a*c^2*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*a*c^2*cos(4*f*x +
 4*e)^2 + 27*(f*x + e)*a*c^2*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*a*c^2*sin(6*f
*x + 6*e)^2 + 27*(f*x + e)*a*c^2*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*a*c^2*si
n(2*f*x + 2*e)^2 + 18*(f*x + e)*a*c^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a*c^2
- 6*a*c^2*sin(2*f*x + 2*e) - 3*(a*c^2*cos(6*f*x + 6*e)^2 + 9*a*c^2*cos(4*f*
x + 4*e)^2 + 9*a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(6*f*x + 6*e)^2 + 9*a*c^
2*sin(4*f*x + 4*e)^2 + 18*a*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a*c^2
*sin(2*f*x + 2*e)^2 + 6*a*c^2*cos(2*f*x + 2*e) + a*c^2 + 2*(3*a*c^2*cos(4*f
*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*cos(6*f*x + 6*e) + 6*(3*a*c^2
*cos(2*f*x + 2*e) + a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(4*f*x + 4*e) + a
*c^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1) + 6*(3*(f*x + e)*a*c^2*cos(4*f*x + 4*e) + 3*(f*x + e)*a*c^2*c
os(2*f*x + 2*e) + (f*x + e)*a*c^2 - a*c^2*sin(4*f*x + 4*e) - a*c^2*sin(2*f*
x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*a*c^2*cos(2*f*x + 2*e) + (f*x
+ e)*a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*
x + 4*e) + 3*a*c^2*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 4*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 3*a
*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 6*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 3*a*c^2*sin(2*f*x
 + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(3*(f*x +
 e)*a*c^2*sin(4*f*x + 4*e) + 3*(f*x + e)*a*c^2*sin(2*f*x + 2*e) + a*c^2*cos
(4*f*x + 4*e) + a*c^2*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(9*(f*x + e)*a
*c^2*sin(2*f*x + 2*e) - a*c^2)*sin(4*f*x + 4*e) - 6*(a*c^2*cos(6*f*x + 6*e)
```

```
 + 3*a*c^2*cos(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a*c^2*cos(6*f*x + 6*e) + 3*a
*c^2*cos(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(a*c^2*cos(6*f*x + 6*e) + 3*a*c^2*c
os(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^
2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x +
6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*si
n(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2), x)
```

3.95 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result 680
Rubi [A] (verified) 680
Mathematica [A] (verified) 681
Maple [A] (verified) 682
Fricas [A] (verification not implemented) 682
Sympy [F] 683
Maxima [B] (verification not implemented) 683
Giac [F] 684
Mupad [F(-1)] 684

Optimal result

Integrand size = 30, antiderivative size = 103

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{a^2c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a^2*c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+1/2*a^2*c^2*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3990, 3554, 3556}

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{a^2c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^2*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (a^2*c^2*Tan[e + f*x]^3)/(2*f*Sqrt[a + a*Sec[e + f*x]
]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(a^2c^2\tan(e+fx)\right)\int\tan^3(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= \frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\left(a^2c^2\tan(e+fx)\right)\int\tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= \frac{a^2c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

$$\int(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a^2c^2(2\log(\cos(e+fx))+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^2*c^2*(2*Log[Cos[e + f*x]] + Sec[e + f*x]^2)*Tan[e + f*x])/(2*f*Sqrt[a*(
1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.39

method	result
default	$\frac{a\left(2\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)-1)+2\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)+1)-2\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\sin(\ldots\right.}{2f(\cos(fx+e)-1)}$
risch	$-\frac{ac\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(ie^{4i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+e^{4i(fx+e)}fx+2e^{4i(fx+e)}e+2ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+2e^{2}\ldots\right.}{\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*a*(2*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)-1)+2*cos(f*x+e)^2*ln(-cot
(f*x+e)+csc(f*x+e)+1)-2*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+sin(f*x+e)^2)*(-c
*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)*c*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e
)-1)*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.36

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \left[\frac{\sqrt{-ac}ac\cos(fx+e)\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)\ldots}{\cos(fx+e)}}\ldots}{2\cos(fx+e)^2}\right.}{2f\cos(fx+e)}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(sqrt(-a*c)*a*c*cos(f*x + e)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x +
e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
t((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2) -
a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*sin(f*x + e))/(f*cos(f*x + e)), 1/2*(2*sqrt(a*c)*a*c*arctan(sqrt(a
*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))*cos(f*x + e)
 - a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \int (a(\sec(e+fx)+1))^{\frac{3}{2}}\,(-c(\sec(e+fx)-1))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(3/2),x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(93) = 186.

Time = 0.40 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.63

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{((fx+e)ac\cos(4fx+4e)^2+4(fx+e)ac\cos(2fx+2e)^2+(fx+e)ac\sin(4fx+4e)^2+4(fx+e)a}{}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
[Out] -((f*x + e)*a*c*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a*c*cos(2*f*x + 2*e)^2 + (
f*x + e)*a*c*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a*c*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*a*c*cos(2*f*x + 2*e) + (f*x + e)*a*c - 2*a*c*sin(2*f*x + 2*e) - (a*
c*cos(4*f*x + 4*e)^2 + 4*a*c*cos(2*f*x + 2*e)^2 + a*c*sin(4*f*x + 4*e)^2 +
4*a*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a*c*sin(2*f*x + 2*e)^2 + 4*a*c*
cos(2*f*x + 2*e) + a*c + 2*(2*a*c*cos(2*f*x + 2*e) + a*c)*cos(4*f*x + 4*e))
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a*c*cos(2
*f*x + 2*e) + (f*x + e)*a*c - a*c*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(2
*(f*x + e)*a*c*sin(2*f*x + 2*e) + a*c*cos(2*f*x + 2*e))*sin(4*f*x + 4*e))*s
qrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x +
4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2),x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2), x)
```

3.96 $\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result 685
Rubi [A] (verified) 685
Mathematica [A] (verified) 686
Maple [A] (verified) 687
Fricas [A] (verification not implemented) 687
Sympy [F] 688
Maxima [B] (verification not implemented) 688
Giac [F] 688
Mupad [F(-1)] 689

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = \frac{a^2c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a^2*c*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-a*c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3991, 3990, 3556}

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = \frac{a^2c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^2*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rubi steps

integral

$$= -\frac{ac\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + a\int\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$

$$= -\frac{ac\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{(a^2c\tan(e+fx))\int\tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{a^2c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{ac\sqrt{a(1+\sec(e+fx))}(-\log(\cos(e+fx))+\sec(e+fx))\tan(e+fx)}{f(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((a*c*Sqrt[a*(1 + Sec[e + f*x])]*(-Log[Cos[e + f*x]] + Sec[e + f*x])*Tan[e
 + f*x])/(f*(1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.27

method	result
default	$-\frac{a\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\,\left(\ln(-\cot(fx+e)+\csc(fx+e)-1)\cot(fx+e)+\ln(-\cot(fx+e)+\csc(fx+e)+1)\cot(fx+e\right.}{f}$
risch	$\frac{a\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)}-\frac{2a\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(fx+e)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}+\frac{2ia\sqrt{}}{}$

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*a*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*(ln(-cot(f*x+e)+c
sc(f*x+e)-1)*cot(f*x+e)+ln(-cot(f*x+e)+csc(f*x+e)+1)*cot(f*x+e)-cot(f*x+e)*
ln(2/(cos(f*x+e)+1))-cot(f*x+e)-csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.73

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\left[\frac{2\,a\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)+\sqrt{-ac}(a\cos\left(fx+e\right)+a)}{2\left(f\cos\right.}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(2*a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e))*sin(f*x + e) + sqrt(-a*c)*(a*cos(f*x + e) + a)*log(1/2*(a*c*
cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), (a*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + sqr
t(a*c)*(a*cos(f*x + e) + a)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e) + f)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\int (a(\sec{(e+fx)}+1))^{\frac{3}{2}}\sqrt{-c\left(\sec{(e+fx)}-1\right)}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(85) = 170$.

Time = 0.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.61

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{((fx+e)a\cos{(2\,fx+2\,e)}^2+(fx+e)a\sin{(2\,fx+2\,e)}^2+2\,(fx+e)a\cos{(2\,fx+2\,e)}-2\,a\cos{\left(\frac{1}{2}\arctan\right.}}{}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] -((f*x + e)*a*cos(2*f*x + 2*e)^2 + (f*x + e)*a*sin(2*f*x + 2*e)^2 + 2*(f*x
+ e)*a*cos(2*f*x + 2*e) - 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))*sin(2*f*x + 2*e) + (f*x + e)*a - (a*cos(2*f*x + 2*e)^2 + a*sin(2*f*
x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1) + 2*(a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\int (a\sec{(fx+e)}+a)^{\frac{3}{2}}\sqrt{-c\sec{(fx+e)}+c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2),x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2), x)
```

3.97 $\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result 690
Rubi [A] (verified) 690
Mathematica [A] (verified) 691
Maple [A] (verified) 692
Fricas [F] 692
Sympy [F] 692
Maxima [A] (verification not implemented) 693
Giac [F] 693
Mupad [F(-1)] 693

Optimal result

Integrand size = 30, antiderivative size = 104

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+2*a^2*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3997, 78}

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{a^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{a+ax}{x(c-cx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(-\frac{2a}{c(-1+x)}+\frac{a}{cx}\right)\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= \frac{a^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a(\log(\cos(e+fx))+2\log(1-\sec(e+fx)))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+\right.}{f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*(Log[Cos[e + f*x]] + 2*Log[1 - Sec[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x])]
*Tan[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

method	result
default	$\frac{a\sqrt{a(\sec(fx+e)+1)}\left(\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\ln(-\cot(fx+e)+\csc(fx+e)+1)-4\ln(-\cot(fx+e)+\csc(fx+e))+\ln(-\cot(fx+e)+\csc(fx+e\right.}{f\sqrt{-c(\sec(fx+e)-1)}}$
risch	$\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)x}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}-\frac{4ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e}\right.}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/f*a*(a*(sec(f*x+e)+1))^(1/2)*(ln(2/(cos(f*x+e)+1))+ln(-cot(f*x+e)+csc(f*x
+e)+1)-4*ln(-cot(f*x+e)+csc(f*x+e))+ln(-cot(f*x+e)+csc(f*x+e)-1))/(-c*(sec(
f*x+e)-1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
[Out] integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x +
 e) - c), x)
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/sqrt(-c*(sec(e + f*x) - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{((fx+e)a+a\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)-4\,a\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right))}{\sqrt{c}f}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a + a*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*a*arc
tan2(sin(f*x + e), cos(f*x + e) - 1))*sqrt(a)/(sqrt(c)*f)
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec\left(fx+e\right)+a)^{\frac{3}{2}}}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(1/2), x)
```

3.98 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result . . . 694
Rubi [A] (verified) . . . 694
Mathematica [A] (verified) . . . 695
Maple [A] (verified) . . . 696
Fricas [F] . . . 696
Sympy [F] . . . 696
Maxima [A] (verification not implemented) . . . 697
Giac [F] . . . 697
Mupad [F(-1)] . . . 697

Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^2*ln(1-
cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3993, 3996, 31}

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3993

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}\\
&= -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\
&\quad+\frac{(a^2\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int\frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{a^2\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))+\frac{2}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2),x]
[Out] (a^2*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + 2/(-1 + Sec[e + f*x]))*Ta
n[e + f*x])/(c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.45

method	result
default	$\frac{a\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))-\ln\left(\frac{2}{\cos(fx+e)+1}\right)+2\ln(-\cot(fx+e)+\csc(fx+e))+\cos(fx}{f\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx+e)-1)(\cos(fx+e)+1)}$
risch	$-\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\left(2ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+e^{2i(fx+e)}fx-4ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2\,e^{2i(fx+e)}e-2\,e^{i(fx+e)}fx-4ie^{i(fx+e)}\right.}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,f}$

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+
e))-ln(2/(cos(f*x+e)+1))+2*ln(-cot(f*x+e)+csc(f*x+e))+cos(f*x+e)+1)*(a*(sec
(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f*x+e)-1)/(cos(f*x+e)+1)
*tan(f*x+e)
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x
 + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\frac{2\sqrt{-a}a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{-a}a\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}} + \frac{\sqrt{-a}a(\cos(fx+e)+1)^2}{c^{\frac{3}{2}}\sin(fx+e)^2}}{f}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] (2*sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(3/2) - sqrt(-a)*a*log
(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + sqrt(-a)*a*(cos(f*x + e
) + 1)^2/(c^(3/2)*sin(f*x + e)^2))/f
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(3/2), x)
```

3.99 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result 698
Rubi [A] (verified) 698
Mathematica [A] (verified) 700
Maple [A] (verified) 700
Fricas [F] 701
Sympy [F] 701
Maxima [B] (verification not implemented) 701
Giac [F] 703
Mupad [F(-1)] 703

Optimal result

Integrand size = 30, antiderivative size = 146

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^2*tan(f*x
+e)/c/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^2*ln(1-cos(f*x+e))*
tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3993, 3992, 3996, 31}

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{a^2\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}} - \frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)
)) - (a^2*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^
(3/2)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3993

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{a\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c} \\
&= -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} \\
&\quad -\frac{a^2\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} \\
&\quad - \frac{a^2\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} \\
&\quad + \frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} \\
&\quad - \frac{a^2\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} \\
&\quad + \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.57

$$
\int\frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^2\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))+\frac{-2+\sec(e+fx)}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + (-2 + Sec[e + f*x])/(-1 +
 Sec[e + f*x])^2)*Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.63

method	result
default	Numerator: $\sqrt{2}\,a\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\Big(8\ln(-\cot(fx+e)+\csc(fx+e))(1-\cos(fx+e))^4\csc(fx+e)^4-4\ln\Big((1-\cos(fx+e$ … Denominator: $8f\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^2\Big(\frac{c(1-\cos(fx+e))}{(1-\cos(fx+e))^2}$ …
risch	Numerator: $a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(6ie^{i(fx+e)}-e^{4i(fx+e)}fx-8ie^{2i(fx+e)}-2\,e^{4i(fx+e)}e+4\,e^{3i(fx+e)}fx-2ie^{4i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+8\,e^{3i(fx+e)}e-6$ …

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*2^(1/2)*a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(8*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(
f*x+e))^4*csc(f*x+e)^4-4*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*x+e))
^4*csc(f*x+e)^4+4*(1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x
 + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(-c(\sec(e+fx)-1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(-c*(sec(e + f*x) - 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. $2(134) = 268$.

Time = 0.53 (sec) , antiderivative size = 1786, normalized size of antiderivative = 12.23

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 16*(
f*x + e)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x
 + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x + e)*
a*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*a*sin(2*f*x + 2*e)^2 + 16*(f*x + e)*a*s
```

```
in(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*a*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*a*cos(2*f
*x + 2*e) + (f*x + e)*a - 2*(a*cos(4*f*x + 4*e)^2 + 36*a*cos(2*f*x + 2*e)^2
 + 16*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*sin(4*f*x + 4*e)^2 +
12*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a*sin(2*f*x + 2*e)^2 + 16*a*sin
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*a*cos(2*f*x + 2*e) + a)*cos(4
*f*x + 4*e) + 12*a*cos(2*f*x + 2*e) - 8*(a*cos(4*f*x + 4*e) + 6*a*cos(2*f*x
 + 2*e) - 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*cos
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a*cos(4*f*x + 4*e) +
 6*a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 8*(a*sin(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e) - 4*a*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 8*(a*sin(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e))*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*arctan2(sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 1) + 2*(6*(f*x + e)*a*cos(2*f*x + 2*e) + (f*x + e)*a - 4*a*sin
(2*f*x + 2*e))*cos(4*f*x + 4*e) - 2*(4*(f*x + e)*a*cos(4*f*x + 4*e) + 24*(f
*x + e)*a*cos(2*f*x + 2*e) - 16*(f*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*(f*x + e)*a + 3*a*sin(4*f*x + 4*e) + 2*a*sin(2*f*
x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(4*(f*x
+ e)*a*cos(4*f*x + 4*e) + 24*(f*x + e)*a*cos(2*f*x + 2*e) + 4*(f*x + e)*a +
 3*a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(3*(f*x + e)*a*sin(2*f*x + 2*e) + 2*a*cos(2*f*x
 + 2*e))*sin(4*f*x + 4*e) - 8*a*sin(2*f*x + 2*e) - 2*(4*(f*x + e)*a*sin(4*f
*x + 4*e) + 24*(f*x + e)*a*sin(2*f*x + 2*e) - 16*(f*x + e)*a*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*a*cos(4*f*x + 4*e) - 2*a*cos(2*f
*x + 2*e) - 3*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(
4*(f*x + e)*a*sin(4*f*x + 4*e) + 24*(f*x + e)*a*sin(2*f*x + 2*e) - 3*a*cos(
4*f*x + 4*e) - 2*a*cos(2*f*x + 2*e) - 3*a)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 36*c^3*cos
(2*f*x + 2*e)^2 + 16*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 16*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^3*
sin(4*f*x + 4*e)^2 + 12*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c^3*sin(
2*f*x + 2*e)^2 + 16*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 16*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*c^
3*cos(2*f*x + 2*e) + c^3 + 2*(6*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x + 4*e
) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e)
+ c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^3*sin(4*
f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e) - 4*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*
```

```
f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a\sec\left(fx+e\right)+a)^{\frac{3}{2}}}{(-c\sec\left(fx+e\right)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(5/2),x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(5/2), x)
```

3.100 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}} dx$

Optimal result . . . 704
Rubi [A] (verified) . . . 704
Mathematica [A] (verified) . . . 707
Maple [A] (warning: unable to verify) . . . 707
Fricas [F] . . . 708
Sympy [F(-1)] . . . 708
Maxima [B] (verification not implemented) . . . 708
Giac [F] . . . 711
Mupad [F(-1)] . . . 711

Optimal result

Integrand size = 30, antiderivative size = 196

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx &= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} \\ &- \frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} \\ &- \frac{a^2\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} \\ &+ \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

```
[Out] -2/3*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a^2
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^2*tan(f*x+e
)/c^2/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^2*ln(1-cos(f*x+e))*
tan(f*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used

$= \{3993, 3992, 3996, 31\}$

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{a^2\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}} - \frac{a^2\tan(e+fx)}{2cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}} - \frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-2*a^2*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7
/2)) - (a^2*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(5/2)) - (a^2*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x])^(3/2)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3993

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}+\frac{a\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx}{c}\\
&= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\
&\quad-\frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}+\frac{a\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c^2}\\
&= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\
&\quad-\frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{a^2\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c^3}\\
&= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\
&\quad-\frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{a^2\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\
&\quad+\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\
&\quad-\frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{a^2\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\
&\quad+\frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.51 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a^2\left(-6\log(\cos(e+fx))-6\log(1-\sec(e+fx))+\frac{-13+15\sec(e+fx)-6\sec^2(e+fx)}{(-1+\sec(e+fx))^3}\right)\tan(e+fx)}{6c^3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -1/6*(a^2*(-6*Log[Cos[e + f*x]] - 6*Log[1 - Sec[e + f*x]] + (-13 + 15*Sec[e
 + f*x] - 6*Sec[e + f*x]^2)/(-1 + Sec[e + f*x])^3)*Tan[e + f*x])/(c^3*f*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (warning: unable to verify)

Time = 2.11 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.33

method	result
default	$\frac{\sqrt{2}\,a\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(48\ln(-\cot(fx+e)+\csc(fx+e))(1-\cos(fx+e))^6\csc(fx+e)^6-24\ln\left((1-\cos(f\right.\right.}{48f\left((1-\cos(fx+e))^2\csc(fx+e)^2-\right.}$
risch	$\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)x}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}+\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(12e^{5i(fx+e)}-33e^{4i(f\right.}{3c^3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e\right.}}$

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f*2^(1/2)*a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*x
+e))^2*csc(f*x+e)^2-1)^3/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)*csc(f*x+e)^2)^(7/2)*(1-cos(f*x+e))*(48*ln(-cot(f*x+e)+csc(f*x+e))*(1-co
s(f*x+e))^6*csc(f*x+e)^6-24*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*x+
e))^6*csc(f*x+e)^6+24*(1-cos(f*x+e))^4*csc(f*x+e)^4-9*(1-cos(f*x+e))^2*csc(
f*x+e)^2+2)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^{7/2}}\, dx = \int \frac{(a\sec(fx + e) + a)^{\frac{3}{2}}}{(-c\sec(fx + e) + c)^{\frac{7}{2}}}\, dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x
+ e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) +
 c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^{7/2}}\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3480 vs. $2(176) = 352$.

Time = 1.85 (sec) , antiderivative size = 3480, normalized size of antiderivative = 17.76

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^{7/2}}\, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a*cos(4*f*x + 4*e)^2
 + 675*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a*cos(5/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a*sin(6*f*x + 6*e)^2 + 675*(
f*x + e)*a*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a*sin(2*f*x + 2*e)^2 + 108*(f
*x + e)*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*
x + e)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x
+ e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*(f*x + e
)*a*cos(2*f*x + 2*e) + 3*(f*x + e)*a - 6*(a*cos(6*f*x + 6*e)^2 + 225*a*cos(
```

```
4*f*x + 4*e)^2 + 225*a*cos(2*f*x + 2*e)^2 + 36*a*cos(5/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 400*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 36*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + a*sin(6*f*x + 6*e)^2 + 225*a*sin(4*f*x + 4*e)^2 + 450*a*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + 225*a*sin(2*f*x + 2*e)^2 + 36*a*sin(5/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a*sin(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 36*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + 2*(15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*cos(6*f*x
 + 6*e) + 30*(15*a*cos(2*f*x + 2*e) + a)*cos(4*f*x + 4*e) + 30*a*cos(2*f*x
+ 2*e) - 12*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x +
2*e) - 20*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*a*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*cos(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x +
 4*e) + 15*a*cos(2*f*x + 2*e) - 6*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + a)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
2*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(a*sin(4*f*x + 4*
e) + a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 12*(a*sin(6*f*x + 6*e) + 15*a*s
in(4*f*x + 4*e) + 15*a*sin(2*f*x + 2*e) - 20*a*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 6*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a*sin(
6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) + 15*a*sin(2*f*x + 2*e) - 6*a*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 12*(a*sin(6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) +
15*a*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + a)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 6*(15*(f*x + e)*a*cos(
4*f*x + 4*e) + 15*(f*x + e)*a*cos(2*f*x + 2*e) + (f*x + e)*a - 11*a*sin(4*f
*x + 4*e) - 11*a*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 90*(15*(f*x + e)*a*co
s(2*f*x + 2*e) + (f*x + e)*a)*cos(4*f*x + 4*e) - 12*(3*(f*x + e)*a*cos(6*f*
x + 6*e) + 45*(f*x + e)*a*cos(4*f*x + 4*e) + 45*(f*x + e)*a*cos(2*f*x + 2*e
) - 60*(f*x + e)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
8*(f*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(f*x
 + e)*a + 2*a*sin(6*f*x + 6*e) - 3*a*sin(4*f*x + 4*e) - 3*a*sin(2*f*x + 2*e
) + 10*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(6*(f*x + e)*a*cos(6*f*x + 6*
e) + 90*(f*x + e)*a*cos(4*f*x + 4*e) + 90*(f*x + e)*a*cos(2*f*x + 2*e) - 36
*(f*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(f*x
+ e)*a + 5*a*sin(6*f*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e)
 - 6*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(3*(f*x + e)*a*cos(6*f*x + 6*e)
 + 45*(f*x + e)*a*cos(4*f*x + 4*e) + 45*(f*x + e)*a*cos(2*f*x + 2*e) + 3*(f
*x + e)*a + 2*a*sin(6*f*x + 6*e) - 3*a*sin(4*f*x + 4*e) - 3*a*sin(2*f*x + 2
*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(15*(f*x + e)
*a*sin(4*f*x + 4*e) + 15*(f*x + e)*a*sin(2*f*x + 2*e) + 11*a*cos(4*f*x + 4*
```

```
e) + 11*a*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(225*(f*x + e)*a*sin(2*f*x
 + 2*e) - 11*a)*sin(4*f*x + 4*e) - 66*a*sin(2*f*x + 2*e) - 12*(3*(f*x + e)*
a*sin(6*f*x + 6*e) + 45*(f*x + e)*a*sin(4*f*x + 4*e) + 45*(f*x + e)*a*sin(2
*f*x + 2*e) - 60*(f*x + e)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 18*(f*x + e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 2*a*cos(6*f*x + 6*e) + 3*a*cos(4*f*x + 4*e) + 3*a*cos(2*f*x + 2*e) - 10
*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*a)*sin(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(6*(f*x + e)*a*sin(6*f*x + 6*
e) + 90*(f*x + e)*a*sin(4*f*x + 4*e) + 90*(f*x + e)*a*sin(2*f*x + 2*e) - 36
*(f*x + e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*a*cos
(6*f*x + 6*e) - 9*a*cos(4*f*x + 4*e) - 9*a*cos(2*f*x + 2*e) + 6*a*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*a)*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 12*(3*(f*x + e)*a*sin(6*f*x + 6*e) + 45*(f*x
 + e)*a*sin(4*f*x + 4*e) + 45*(f*x + e)*a*sin(2*f*x + 2*e) - 2*a*cos(6*f*x
+ 6*e) + 3*a*cos(4*f*x + 4*e) + 3*a*cos(2*f*x + 2*e) - 2*a)*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^4*cos(6*f*x + 6*
e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 225*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*c^4*cos(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + c^4*sin(6*f*x + 6*e)^2 + 225*c^4*sin(4*f*x
 + 4*e)^2 + 450*c^4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*c^4*sin(2*f*x +
 2*e)^2 + 36*c^4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4
00*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 30*c^4*cos(2*f*x + 2*e
) + c^4 + 2*(15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) + c^4)*cos(6
*f*x + 6*e) + 30*(15*c^4*cos(2*f*x + 2*e) + c^4)*cos(4*f*x + 4*e) - 12*(c^4
*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) - 20*
c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*c^4*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(5/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x +
 4*e) + 15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + c^4)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x
+ 2*e) + c^4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(c^
4*sin(4*f*x + 4*e) + c^4*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 12*(c^4*sin(6
*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*e) - 20*c^4*si
n(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*c^4*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - 40*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c
^4*sin(2*f*x + 2*e) - 6*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(c^4*sin(6
*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*e))*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac"
)
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(7/2),x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(7/2), x)
```

3.101 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result 712
Rubi [A] (verified) 712
Mathematica [A] (verified) 714
Maple [A] (verified) 714
Fricas [A] (verification not implemented) 714
Sympy [F(-1)] 715
Maxima [B] (verification not implemented) 715
Giac [F] 716
Mupad [F(-1)] 717

Optimal result

Integrand size = 30, antiderivative size = 153

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a^3c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a^3*c^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+1/2*a^3*c^3*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-1/4*a^3*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3990, 3554, 3556}

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{a^3c^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^3*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (a^3*c^3*Tan[e + f*x]^3)/(2*f*Sqrt[a + a*Sec[e + f*x]
]*Sqrt[c - c*Sec[e + f*x]]) - (a^3*c^3*Tan[e + f*x]^5)/(4*f*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(a^3c^3\tan(e+fx)\right)\int\tan^5(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\left(a^3c^3\tan(e+fx)\right)\int\tan^3(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad-\frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad-\frac{\left(a^3c^3\tan(e+fx)\right)\int\tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{a^3c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad-\frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{a^3c^3(-4\log(\cos(e+fx))-4\sec^2(e+fx)+\sec^4(e+fx))\tan(e+fx)}{4f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/4*(a^3*c^3*(-4*Log[Cos[e + f*x]] - 4*Sec[e + f*x]^2 + Sec[e + f*x]^4)*Ta
n[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 30.57 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.10

method	result
default	$-\frac{a^2(\sec(fx+e)-1)^2\left(4\cos(fx+e)^4\ln(-\cot(fx+e)+\csc(fx+e)-1)-4\cos(fx+e)^4\ln\left(\frac{2}{\cos(fx+e)+1}\right)+4\cos(fx+e)^4\ln(-\cot(fx+e)+\right.}{4f(\cos(fx+e)-1)^{}}$
risch	$-\frac{a^2c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(4ie^{4i(fx+e)}+e^{8i(fx+e)}fx+4ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+2e^{8i(fx+e)}e+4e^{6i(fx+e)}fx+4\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*a^2*(sec(f*x+e)-1)^2*(4*cos(f*x+e)^4*ln(-cot(f*x+e)+csc(f*x+e)-1)-4*
cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))+4*cos(f*x+e)^4*ln(-cot(f*x+e)+csc(f*x+e)+
1)-3*cos(f*x+e)^4+4*cos(f*x+e)^2-1)*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e
)-1))^(1/2)*c^2/(cos(f*x+e)-1)^2*sec(f*x+e)*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.65

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \left[\frac{2\sqrt{-ac}a^2c^2\cos(fx+e)^3\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos}{\cos}}}{2\cos(fx+e)^2}\right.}{4f\,c}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/4*(2*sqrt(-a*c)*a^2*c^2*cos(f*x + e)^3*log(1/2*(a*c*cos(f*x + e)^4 - (co
s(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x +
e)^2) - (3*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
+ e)^3), 1/4*(4*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*
x + e)/(a*c*cos(f*x + e)^2 + a*c))*cos(f*x + e)^3 - (3*a^2*c^2*cos(f*x + e)
^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1619 vs. $2(137) = 274$.

Time = 0.46 (sec) , antiderivative size = 1619, normalized size of antiderivative = 10.58

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a^2*c^2*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*a^2*c^2*cos(6*f*x + 6
*e)^2 + 36*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*a^2*c^2*cos(
2*f*x + 2*e)^2 + (f*x + e)*a^2*c^2*sin(8*f*x + 8*e)^2 + 16*(f*x + e)*a^2*c^
2*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e)^2 + 16*(f*x +
e)*a^2*c^2*sin(2*f*x + 2*e)^2 + 8*(f*x + e)*a^2*c^2*cos(2*f*x + 2*e) + (f*x
 + e)*a^2*c^2 - 4*a^2*c^2*sin(2*f*x + 2*e) - (a^2*c^2*cos(8*f*x + 8*e)^2 +
16*a^2*c^2*cos(6*f*x + 6*e)^2 + 36*a^2*c^2*cos(4*f*x + 4*e)^2 + 16*a^2*c^2*
cos(2*f*x + 2*e)^2 + a^2*c^2*sin(8*f*x + 8*e)^2 + 16*a^2*c^2*sin(6*f*x + 6*
e)^2 + 36*a^2*c^2*sin(4*f*x + 4*e)^2 + 48*a^2*c^2*sin(4*f*x + 4*e)*sin(2*f*
```

```
x + 2*e) + 16*a^2*c^2*sin(2*f*x + 2*e)^2 + 8*a^2*c^2*cos(2*f*x + 2*e) + a^2
*c^2 + 2*(4*a^2*c^2*cos(6*f*x + 6*e) + 6*a^2*c^2*cos(4*f*x + 4*e) + 4*a^2*c
^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(8*f*x + 8*e) + 8*(6*a^2*c^2*cos(4*f*x +
4*e) + 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(6*f*x + 6*e) + 12*(4*a^2*c
^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(4*f*x + 4*e) + 4*(2*a^2*c^2*sin(6*f*x +
6*e) + 3*a^2*c^2*sin(4*f*x + 4*e) + 2*a^2*c^2*sin(2*f*x + 2*e))*sin(8*f*x +
 8*e) + 16*(3*a^2*c^2*sin(4*f*x + 4*e) + 2*a^2*c^2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(4*(f*x + e
)*a^2*c^2*cos(6*f*x + 6*e) + 6*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e) + 4*(f*x
+ e)*a^2*c^2*cos(2*f*x + 2*e) + (f*x + e)*a^2*c^2 - 2*a^2*c^2*sin(6*f*x + 6
*e) - 2*a^2*c^2*sin(4*f*x + 4*e) - 2*a^2*c^2*sin(2*f*x + 2*e))*cos(8*f*x +
8*e) + 8*(6*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e) + 4*(f*x + e)*a^2*c^2*cos(2*
f*x + 2*e) + (f*x + e)*a^2*c^2 + a^2*c^2*sin(4*f*x + 4*e))*cos(6*f*x + 6*e)
 + 4*(12*(f*x + e)*a^2*c^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2*c^2 - 2*a^2*c
^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(2*(f*x + e)*a^2*c^2*sin(6*f*x +
6*e) + 3*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e) + 2*(f*x + e)*a^2*c^2*sin(2*f*x
 + 2*e) + a^2*c^2*cos(6*f*x + 6*e) + a^2*c^2*cos(4*f*x + 4*e) + a^2*c^2*cos
(2*f*x + 2*e))*sin(8*f*x + 8*e) + 4*(12*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e)
+ 8*(f*x + e)*a^2*c^2*sin(2*f*x + 2*e) - 2*a^2*c^2*cos(4*f*x + 4*e) - a^2*c
^2)*sin(6*f*x + 6*e) + 4*(12*(f*x + e)*a^2*c^2*sin(2*f*x + 2*e) + 2*a^2*c^2
*cos(2*f*x + 2*e) - a^2*c^2)*sin(4*f*x + 4*e))*sqrt(a)*sqrt(c)/((2*(4*cos(6
*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)
 + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos
(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f
*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x
+ 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*
f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e
) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{5/2}\,dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{5/2}\left(c - \frac{c}{\cos(e + f\,x)}\right)^{5/2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2),x)
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2), x)
```

3.102 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result . . . 718
Rubi [A] (verified) . . . 718
Mathematica [A] (verified) . . . 720
Maple [A] (verified) . . . 721
Fricas [A] (verification not implemented) . . . 721
Sympy [F(-1)] . . . 722
Maxima [B] (verification not implemented) . . . 722
Giac [F] . . . 723
Mupad [F(-1)] . . . 723

Optimal result

Integrand size = 30, antiderivative size = 190

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{a^3c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + \frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*a*c^2*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+1/3*c
^2*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+a^3*c^2*ln(co
s(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a^2*c^
2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used

$= \{3994, 3991, 3990, 3556\}$

$$\int (a + a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{a^3c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}} + \frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^3*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) - (a^2*c^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[c - c*Sec[e + f*x]]) - (a*c^2*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
2*f*Sqrt[c - c*Sec[e + f*x]]) + (c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3994

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a, Int[Sqrt[a + b
*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n
```

```
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} \\
&\quad + c\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{ac^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + \frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} \\
&\quad + (ac)\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{a^2c^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} \\
&\quad + (a^2c)\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{a^2c^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{(a^3c^2\tan(e+fx))\int \tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= \frac{a^3c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{ac^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + \frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$
\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{a^3c^2(2+6\log(\cos(e+fx))-6\sec(e+fx)+3\sec^2(e+fx)+2\sec^3(e+fx))\tan(e+fx)}{6f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
[Out] (a^3*c^2*(2 + 6*Log[Cos[e + f*x]] - 6*Sec[e + f*x] + 3*Sec[e + f*x]^2 + 2*S
ec[e + f*x]^3)*Tan[e + f*x])/(6*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec
[e + f*x]])
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^2\left(6\cos(fx+e)^3\ln\left(\frac{2}{\cos(fx+e)+1}\right)-6\cos(fx+e)^3\ln(-\cot(fx+e)+\csc(fx+e)-1)-6\cos(fx+e)^3\ln(-\cot(fx+e)+\csc(fx+e)+1)}{6f(\cos(fx}$
risch	$-\frac{a^2c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(-6ie^{5i(fx+e)}+3\,e^{6i(fx+e)}fx+6\,e^{6i(fx+e)}e-6ie^{i(fx+e)}+9\,e^{4i(fx+e)}fx+18\,e^{4i(fx+e)}e+\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*a^2*(6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e)^3*ln(-cot(f*x+
e)+csc(f*x+e)-1)-6*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1)+7*cos(f*x+e)^3
+6*cos(f*x+e)^2-3*cos(f*x+e)-2)*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)*c*
(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*sec(f*x+e)*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.46

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \left[\frac{\left(a^2c\cos(fx+e)^2-5\,a^2c\cos(fx+e)-2\,a^2c\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx}{}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/6*((a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
+ e) + 3*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(
a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*
x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), 1/6*(
(a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) +
 6*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*c)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e
)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1356 vs. 2(170) = 340.

Time = 0.43 (sec) , antiderivative size = 1356, normalized size of antiderivative = 7.14

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a^2*c*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*a^2*c*cos(4*f*x +
 4*e)^2 + 27*(f*x + e)*a^2*c*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*a^2*c*sin(6*f
*x + 6*e)^2 + 27*(f*x + e)*a^2*c*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*a^2*c*si
n(2*f*x + 2*e)^2 + 18*(f*x + e)*a^2*c*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2*c
- 6*a^2*c*sin(2*f*x + 2*e) - 3*(a^2*c*cos(6*f*x + 6*e)^2 + 9*a^2*c*cos(4*f*
x + 4*e)^2 + 9*a^2*c*cos(2*f*x + 2*e)^2 + a^2*c*sin(6*f*x + 6*e)^2 + 9*a^2*
c*sin(4*f*x + 4*e)^2 + 18*a^2*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a^2*c
*sin(2*f*x + 2*e)^2 + 6*a^2*c*cos(2*f*x + 2*e) + a^2*c + 2*(3*a^2*c*cos(4*f
*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*cos(6*f*x + 6*e) + 6*(3*a^2*c
*cos(2*f*x + 2*e) + a^2*c)*cos(4*f*x + 4*e) + 6*(a^2*c*sin(4*f*x + 4*e) + a
^2*c*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1) + 6*(3*(f*x + e)*a^2*c*cos(4*f*x + 4*e) + 3*(f*x + e)*a^2*c*c
os(2*f*x + 2*e) + (f*x + e)*a^2*c - a^2*c*sin(4*f*x + 4*e) - a^2*c*sin(2*f*
x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*a^2*c*cos(2*f*x + 2*e) + (f*x
+ e)*a^2*c)*cos(4*f*x + 4*e) - 6*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*
x + 4*e) + 3*a^2*c*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 4*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 3*a
^2*c*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 6*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 3*a^2*c*sin(2*f*x
 + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(3*(f*x +
 e)*a^2*c*sin(4*f*x + 4*e) + 3*(f*x + e)*a^2*c*sin(2*f*x + 2*e) + a^2*c*cos
(4*f*x + 4*e) + a^2*c*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(9*(f*x + e)*a
^2*c*sin(2*f*x + 2*e) - a^2*c)*sin(4*f*x + 4*e) + 6*(a^2*c*cos(6*f*x + 6*e)
```

```
 + 3*a^2*c*cos(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*c*cos(6*f*x + 6*e) + 3*a
^2*c*cos(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(a^2*c*cos(6*f*x + 6*e) + 3*a^2*c*c
os(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^
2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x +
6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*si
n(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2), x)
```

3.103 $\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result . . . 724
Rubi [A] (verified) . . . 724
Mathematica [A] (verified) . . . 726
Maple [A] (verified) . . . 726
Fricas [A] (verification not implemented) . . . 726
Sympy [F(-1)] . . . 727
Maxima [B] (verification not implemented) . . . 727
Giac [F] . . . 728
Mupad [F(-1)] . . . 728

Optimal result

Integrand size = 30, antiderivative size = 139

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\frac{a^3c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*a*c*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+a^3*c*l
n(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a^
2*c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3991, 3990, 3556}

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\frac{a^3c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^3*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a^2*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]]) - (a*c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sq
rt[c - c*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} \\
&\quad + a\int(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{a^2c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} \\
&\quad + a^2\int\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{a^2c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{(a^3c\tan(e+fx))\int\tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= \frac{a^3c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{a^2c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.52

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{a^3c(-2\log(\cos(e+fx))+4\sec(e+fx)+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -1/2*(a^3*c*(-2*Log[Cos[e + f*x]] + 4*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e
+ f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.98

method	result
default	$-\frac{a^2\sqrt{-c(\sec(fx+e)-1)}\sqrt{a(\sec(fx+e)+1)}\left(2\ln(-\cot(fx+e)+\csc(fx+e)-1)\cot(fx+e)+2\ln(-\cot(fx+e)+\csc(fx+e)+1)\cot(fx\right.}{2f}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(-ie^{4i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)-e^{4i(fx+e)}fx-2ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)-2e^{4i(fx+e)}e-2e^{2i}\right.}{\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*a^2*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*(2*ln(-cot(f*
x+e)+csc(f*x+e)-1)*cot(f*x+e)+2*ln(-cot(f*x+e)+csc(f*x+e)+1)*cot(f*x+e)-2*c
ot(f*x+e)*ln(2/(cos(f*x+e)+1))-3*cot(f*x+e)-4*csc(f*x+e)-sec(f*x+e)*csc(f*x
+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 420, normalized size of antiderivative = 3.02

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \left[\frac{(5a^2\cos(fx+e)+a^2)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+(a^2\cos\right.}{}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + (a^2*cos(f*x + e)^2 +
a^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3
+ cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos
(f*x + e)^2 + f*cos(f*x + e)), 1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
 + e) + 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt(a*c)*arctan(sqrt(a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x +
e)^2 + f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(125) = 250.

Time = 0.40 (sec) , antiderivative size = 710, normalized size of antiderivative = 5.11

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{\left((fx+e)a^2\cos(4\,fx+4\,e)^2+4\,(fx+e)a^2\cos(2\,fx+2\,e)^2+(fx+e)a^2\sin(4\,fx+4\,e)^2+4\,(fx+e)a^2\right.}{}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + (
f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 + 2*a^2*sin(2*f*x + 2*e) - (a^
2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 +
4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*
cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))
```

```
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a^2*cos(2
*f*x + 2*e) + (f*x + e)*a^2 + a^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(a
^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)*a^2*
sin(2*f*x + 2*e) - a^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + 4*(a^2*cos(4*f*
x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2
*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f
*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*
sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}\sqrt{-c\sec(fx+e)+c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2),x)
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2), x)
```

3.104 $\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result 729
Rubi [A] (verified) 729
Mathematica [A] (verified) 730
Maple [A] (verified) 731
Fricas [F] 731
Sympy [F(-1)] 731
Maxima [F(-2)] 732
Giac [F] 732
Mupad [F(-1)] 732

Optimal result

Integrand size = 30, antiderivative size = 152

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{a^3\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+4*a^3*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)+a^3*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e
))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^3\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{a^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
```

```
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (a^3*Sec[e + f*x]*Tan[e + f*x])/(
f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^2}{x(c-cx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(-\frac{a^2}{c}-\frac{4a^2}{c(-1+x)}+\frac{a^2}{cx}\right)\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{a^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{a^3\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.45

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^3(\log(\cos(e+fx))+4\log(1-\sec(e+fx))+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]
[Out] (a^3*(Log[Cos[e + f*x]] + 4*Log[1 - Sec[e + f*x]] + Sec[e + f*x])*Tan[e + f
*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.02

method	result
default	$-\frac{a^{2}\sqrt{a(\sec(fx+e)+1)}\left(\ln\left(\frac{2}{\cos(fx+e)+1}\right)\sin(fx+e)-8\ln(-\cot(fx+e)+\csc(fx+e))\sin(fx+e)+3\ln(-\cot(fx+e)+\csc(fx+e)-1)\right.}{f(\cos(fx+e)+1)\sqrt{-c(\sec(fx+e)-1)}}$
risch	$-\frac{a^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\left(-2ie^{i(fx+e)}+3ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+e^{3i(fx+e)}fx+8ie^{3i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2ie^{2i(fx+e)}-3ie^{i(fx}\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)/(-c*(sec(f*x+e)-1))^(1/2)*
(ln(2/(cos(f*x+e)+1))*sin(f*x+e)-8*ln(-cot(f*x+e)+csc(f*x+e))*sin(f*x+e)+3*
ln(-cot(f*x+e)+csc(f*x+e)-1)*sin(f*x+e)+3*ln(-cot(f*x+e)+csc(f*x+e)+1)*sin(
f*x+e)-sin(f*x+e)-tan(f*x+e))
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(1/2), x)
```

3.105 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}} dx$

Optimal result 733
Rubi [A] (verified) 733
Mathematica [A] (verified) 734
Maple [B] (verified) 735
Fricas [B] (verification not implemented) 735
Sympy [F(-1)] 736
Maxima [F(-2)] 736
Giac [F] 736
Mupad [F(-1)] 736

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}} dx = -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -4*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(co
s(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3995, 3990, 3556}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}} dx = \frac{a^3\tan(e+fx)\log(\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-4*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) + (a^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} \\
&\quad+\frac{a^2\int\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx}{c^2} \\
&= -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}-\frac{(a^3\tan(e+fx))\int\tan(e+fx)\,dx}{c\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a^3\log(\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int\frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{a^3\left(\log(\cos(e+fx))+\frac{4}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^3*(Log[Cos[e + f*x]] + 4/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(c*f*Sqrt[a*
(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(88) = 176$.

Time = 2.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.99

method	result
default	$\frac{a^2\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)-1)-\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln\left(\frac{\cdots}{\cos(f\cdots}\right.\right.}{f\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(f\cdots}$
risch	$-\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+e^{2i(fx+e)}fx-2ie^{i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+2\,e^{2i(fx+e)}e-2\,e^{i(fx+e)}fx-8ie^{i(f\cdots}\right.}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^2*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))-cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+
e)-1)-cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-ln(2/(cos(f*x+e)+1))+ln(-cot(
f*x+e)+csc(f*x+e)-1)+ln(-cot(f*x+e)+csc(f*x+e)+1)+2*cos(f*x+e)+2)*(a*(sec(f
*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f*x+e)-1)/(cos(f*x+e)+1)*t
an(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(88) = 176$.

Time = 0.33 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.60

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^{3/2}}\,dx = \left[\frac{(a^2c\cos(fx + e) - a^2c)\sqrt{-\frac{a}{c}}\log\left(\frac{a\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-\frac{a}{c}}\sqrt{\frac{a\cos\cdots}{\cos\cdots}}\cdots}{2\cos(fx+e)^2\cdots}\right.\cdots}{2\cdots}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/2*((a^2*c*cos(f*x + e) - a^2*c)*sqrt(-a/c)*log(1/2*(a*cos(f*x + e)^4 - (
cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a)/cos(f*x +
e)^2)*sin(f*x + e) + 4*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*
f*cos(f*x + e) - c^2*f)*sin(f*x + e)), ((a^2*c*cos(f*x + e) - a^2*c)*sqrt(a
/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*cos(f*x + e)^2 + a))
*sin(f*x + e) + 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos
(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(3/2), x)
```

3.106 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result 737
Rubi [A] (verified) 737
Mathematica [A] (verified) 738
Maple [B] (verified) 739
Fricas [F] 739
Sympy [F(-1)] 739
Maxima [A] (verification not implemented) 740
Giac [F] 740
Mupad [F(-1)] 740

Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-
cos(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3995, 3996, 31}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c^2}\\
&= -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{(a^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}+\frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int\frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\frac{a^3\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))-\frac{2}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] - 2/(-1 + Sec[e + f*x])^2)*
Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(92) = 184$.

Time = 2.01 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.40

method	result
default	$\frac{\sqrt{2}\, a^{2} \sqrt{-\frac{2a}{(1-\cos(fx+e))^{2} \csc(fx+e)^{2}-1}}\, (1-\cos(fx+e)) \left(4 \ln(-\cot(fx+e)+\csc(fx+e)) (1-\cos(fx+e))^{4} \csc(fx+e)^{4}-2 \ln\left((1-\cos(fx\right.\right.}{4f\left((1-\cos(fx+e))^{2} \csc(fx+e)^{2}-1\right)^{2} \left(\frac{c(1-\cos(fx+}{(1-\cos(fx+e)}\right.}$
risch	$\frac{a^{2} \sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\, (8i e^{i(fx+e)}-e^{4i(fx+e)} fx-8i e^{2i(fx+e)}-2\, e^{4i(fx+e)} e+4\, e^{3i(fx+e)} fx+8i e^{3i(fx+e)} \ln\left(e^{i(fx+e)}-1\right)+8\, e^{3i(fx+e)} e}{}$

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*2^(1/2)*a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(4*ln(-cot(f*x+e)+csc(f*x+e))*(1-co
s(f*x+e))^4*csc(f*x+e)^4-2*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*x+e
))^4*csc(f*x+e)^4+2*(1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^
2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.39

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\frac{4\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{5}{2}}} - \frac{2\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{5}{2}}} - \frac{\left(\sqrt{-a}a^2\sqrt{c} - \frac{2\sqrt{-a}a^2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right.}{c^3\sin(fx+e)^4}}{2f}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/2*(4*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) - 2*sqrt(-
a)*a^2*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) - (sqrt(-a)*a^2
*sqrt(c) - 2*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos
(f*x + e) + 1)^4/(c^3*sin(f*x + e)^4))/f
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(5/2), x)
```

3.107 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 148

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a^3\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -4/3*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-a^3*tan
(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-cos(f*
x+e))*tan(f*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3995, 3992, 3996, 31}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{a^3\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}} - \frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-4*a^3*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7
/2)) - (a^3*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec
[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}+\frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c^2}\\
&= -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c^3}
\end{aligned}$$

$$= -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a^3\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{(a^3\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a^3\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

$$\int\frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a^3\left(-3\log(\cos(e+fx))-3\log(1-\sec(e+fx))+\frac{-4-3(-1+\sec(e+fx))^2}{(-1+\sec(e+fx))^3}\right)\tan(e+fx)}{3c^3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2),x]
[Out] -1/3*(a^3*(-3*Log[Cos[e + f*x]] - 3*Log[1 - Sec[e + f*x]] + (-4 - 3*(-1 + S
ec[e + f*x])^2)/(-1 + Sec[e + f*x])^3)*Tan[e + f*x])/(c^3*f*Sqrt[a*(1 + Sec
[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (warning: unable to verify)

Time = 2.20 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.77

method	result
default	$\frac{\sqrt{2}\,a^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\big(12\ln(-\cot(fx+e)+\csc(fx+e))(1-\cos(fx+e))^6\csc(fx+e)^6-6\ln\big((1-\cos(f}{12f\big((1-\cos(fx+e))^2\csc(fx+e)^2-1}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)x}{c^3(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)(fx+e)}{c^3(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}+\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(15\,e^{5i(fx+e)}-36\,e^{4}}{3c^3(e^{i(fx+e)}+1)(e^{i(fx}}$

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/f*2^(1/2)*a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^3/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)*csc(f*x+e)^2)^(7/2)*(1-cos(f*x+e))*(12*ln(-cot(f*x+e)+csc(f*x+e))*(1-
cos(f*x+e))^6*csc(f*x+e)^6-6*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*x
+e))^6*csc(f*x+e)^6+6*(1-cos(f*x+e))^4*csc(f*x+e)^4-3*(1-cos(f*x+e))^2*csc(
f*x+e)^2+1)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x + e)^4 - 4*c^4*sec(f*x + e)^3
 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3738 vs. $2(134) = 268$.

Time = 1.85 (sec) , antiderivative size = 3738, normalized size of antiderivative = 25.26

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a^2*cos(4*f*x + 4*
e)^2 + 675*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a^2*cos(5/2*arc
```

```
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a^2*cos(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a^2*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a^2*sin(6*f*x +
6*e)^2 + 675*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a^2*sin(2*f*x
 + 2*e)^2 + 108*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + 1200*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 108*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 90*(f*x + e)*a^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2 - 72*a^2*s
in(2*f*x + 2*e) - 6*(a^2*cos(6*f*x + 6*e)^2 + 225*a^2*cos(4*f*x + 4*e)^2 +
225*a^2*cos(2*f*x + 2*e)^2 + 36*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 + 400*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 36*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2
*sin(6*f*x + 6*e)^2 + 225*a^2*sin(4*f*x + 4*e)^2 + 450*a^2*sin(4*f*x + 4*e)
*sin(2*f*x + 2*e) + 225*a^2*sin(2*f*x + 2*e)^2 + 36*a^2*sin(5/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a^2*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + 36*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + 30*a^2*cos(2*f*x + 2*e) + a^2 + 2*(15*a^2*cos(4*f*x + 4*e)
 + 15*a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) + 30*(15*a^2*cos(2*f*x +
 2*e) + a^2)*cos(4*f*x + 4*e) - 12*(a^2*cos(6*f*x + 6*e) + 15*a^2*cos(4*f*x
 + 4*e) + 15*a^2*cos(2*f*x + 2*e) - 20*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 6*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + a^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a^
2*cos(6*f*x + 6*e) + 15*a^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) - 6*
a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2)*cos(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(a^2*cos(6*f*x + 6*e) + 15*a
^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 30*(a^2*sin(4*f*x + 4*e) + a^2*sin(2*f*x +
 2*e))*sin(6*f*x + 6*e) - 12*(a^2*sin(6*f*x + 6*e) + 15*a^2*sin(4*f*x + 4*e
) + 15*a^2*sin(2*f*x + 2*e) - 20*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 6*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a^2*sin(6*f*x
+ 6*e) + 15*a^2*sin(4*f*x + 4*e) + 15*a^2*sin(2*f*x + 2*e) - 6*a^2*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 12*(a^2*sin(6*f*x + 6*e) + 15*a^2*sin(4*f*x + 4*e
) + 15*a^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 6*(15*(f*x + e)*a^2*
cos(4*f*x + 4*e) + 15*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 - 12*a
^2*sin(4*f*x + 4*e) - 12*a^2*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 90*(15*(f
*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2)*cos(4*f*x + 4*e) - 6*(6*(f*x
+ e)*a^2*cos(6*f*x + 6*e) + 90*(f*x + e)*a^2*cos(4*f*x + 4*e) + 90*(f*x + e
)*a^2*cos(2*f*x + 2*e) - 120*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 36*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 6*(f*x + e)*a^2 + 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*
f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e) + 16*a^2*sin(3/2*arctan2(sin(2*f*x + 2*
```

```
e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 4*(30*(f*x + e)*a^2*cos(6*f*x + 6*e) + 450*(f*x + e)*a^2*cos(4*f*x + 4*
e) + 450*(f*x + e)*a^2*cos(2*f*x + 2*e) - 180*(f*x + e)*a^2*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(f*x + e)*a^2 + 29*a^2*sin(6*f*x
 + 6*e) + 75*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*e) - 24*a^2*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - 6*(6*(f*x + e)*a^2*cos(6*f*x + 6*e) + 90*(f*x +
 e)*a^2*cos(4*f*x + 4*e) + 90*(f*x + e)*a^2*cos(2*f*x + 2*e) + 6*(f*x + e)*
a^2 + 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2
*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 18*(5*(f*x + e)
*a^2*sin(4*f*x + 4*e) + 5*(f*x + e)*a^2*sin(2*f*x + 2*e) + 4*a^2*cos(4*f*x
+ 4*e) + 4*a^2*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 18*(75*(f*x + e)*a^2*si
n(2*f*x + 2*e) - 4*a^2)*sin(4*f*x + 4*e) - 6*(6*(f*x + e)*a^2*sin(6*f*x + 6
*e) + 90*(f*x + e)*a^2*sin(4*f*x + 4*e) + 90*(f*x + e)*a^2*sin(2*f*x + 2*e)
 - 120*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 36*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*
a^2*cos(6*f*x + 6*e) - 3*a^2*cos(4*f*x + 4*e) - 3*a^2*cos(2*f*x + 2*e) - 16
*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*a^2)*sin(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(30*(f*x + e)*a^2*sin(6*f*
x + 6*e) + 450*(f*x + e)*a^2*sin(4*f*x + 4*e) + 450*(f*x + e)*a^2*sin(2*f*x
 + 2*e) - 180*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 29*a^2*cos(6*f*x + 6*e) - 75*a^2*cos(4*f*x + 4*e) - 75*a^2*cos(2*f*
x + 2*e) + 24*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 29
*a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(6*(f*x + e)
*a^2*sin(6*f*x + 6*e) + 90*(f*x + e)*a^2*sin(4*f*x + 4*e) + 90*(f*x + e)*a^
2*sin(2*f*x + 2*e) - 5*a^2*cos(6*f*x + 6*e) - 3*a^2*cos(4*f*x + 4*e) - 3*a^
2*cos(2*f*x + 2*e) - 5*a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))))*sqrt(a)*sqrt(c)/((c^4*cos(6*f*x + 6*e)^2 + 225*c^4*cos(4*f*x + 4*e)^
2 + 225*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos(5/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + 400*c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + 36*c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 c^4*sin(6*f*x + 6*e)^2 + 225*c^4*sin(4*f*x + 4*e)^2 + 450*c^4*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + 225*c^4*sin(2*f*x + 2*e)^2 + 36*c^4*sin(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*c^4*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 30*c^4*cos(2*f*x + 2*e) + c^4 + 2*(15*c^4*cos(4*f*x +
4*e) + 15*c^4*cos(2*f*x + 2*e) + c^4)*cos(6*f*x + 6*e) + 30*(15*c^4*cos(2*f
*x + 2*e) + c^4)*cos(4*f*x + 4*e) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4
*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) - 20*c^4*cos(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 6*c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + c^4)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40
*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e)
- 6*c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(c^4*cos(6*f*x + 6*e) +
15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) + c^4)*cos(1/2*arctan2(si
```

```
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(c^4*sin(4*f*x + 4*e) + c^4*sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) - 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x +
 4*e) + 15*c^4*sin(2*f*x + 2*e) - 20*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 6*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*sin(6*
f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x +
 4*e) + 15*c^4*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*f)
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac"
)
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(7/2),x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(7/2), x)
```

3.108 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}} dx$

Optimal result 748
Rubi [A] (verified) 748
Mathematica [A] (verified) 751
Maple [A] (warning: unable to verify) 751
Fricas [F] 752
Sympy [F(-1)] 752
Maxima [B] (verification not implemented) 752
Giac [F] 756
Mupad [F(-1)] 756

Optimal result

Integrand size = 30, antiderivative size = 194

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = & -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}} \\ & -\frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} \\ & -\frac{a^3\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} \\ & +\frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

```
[Out] -a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a^3*tan
(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^3*tan(f*x+e)/
c^3/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-cos(f*x+e))*ta
n(f*x+e)/c^4/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used

$= \{3995, 3992, 3996, 31\}$

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^4f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{a^3\tan(e+fx)}{c^3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}} - \frac{a^3\tan(e+fx)}{2c^2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}} - \frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -((a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(9/2)
)) - (a^3*Tan[e + f*x])/(2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(5/2)) - (a^3*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^4*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}}+\frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx}{c^2}\\
&= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}+\frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c^3}\\
&= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c^4}\\
&= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\
&\quad+\frac{(a^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\
&\quad+\frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.55

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a^3\left(-2\log(\cos(e+fx))-2\log(1-\sec(e+fx))+\frac{2+(-1+\sec(e+fx))^2-2(-1+\sec(e+fx))^3}{(-1+\sec(e+fx))^4}\right)\tan(e+fx)}{2c^4f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -1/2*(a^3*(-2*Log[Cos[e + f*x]] - 2*Log[1 - Sec[e + f*x]] + (2 + (-1 + Sec[
e + f*x])^2 - 2*(-1 + Sec[e + f*x])^3)/(-1 + Sec[e + f*x])^4)*Tan[e + f*x])
/(c^4*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (warning: unable to verify)

Time = 2.45 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\sqrt{2}\,a^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(16\ln\left((1-\cos(fx+e))^2\csc(fx+e)^2+1\right)(1-\cos(fx+e))^8\csc(fx+e)^8-32\ln\right.}{32f\left((1-\cos(fx+\right.}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)x}{c^4\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)(fx+e)}{c^4\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}+\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(6\,e^{7i(fx+e)}-23\,e^{6i}}{c^4\left(e^{i(fx+\right.}}$

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/f*2^(1/2)*a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^4/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e
)^2-1)*csc(f*x+e)^2)^(9/2)*(1-cos(f*x+e))*(16*ln((1-cos(f*x+e))^2*csc(f*x+e
)^2+1)*(1-cos(f*x+e))^8*csc(f*x+e)^8-32*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f
*x+e))^8*csc(f*x+e)^8-16*(1-cos(f*x+e))^6*csc(f*x+e)^6+8*(1-cos(f*x+e))^4*c
sc(f*x+e)^4-4*(1-cos(f*x+e))^2*csc(f*x+e)^2+1)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^5*sec(f*x + e)^5 - 5*c^5*sec(f*x + e)^
4 + 10*c^5*sec(f*x + e)^3 - 10*c^5*sec(f*x + e)^2 + 5*c^5*sec(f*x + e) - c^
5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6134 vs. $2(176)=352$.

Time $=11.57$ (sec) , antiderivative size $=6134$, normalized size of antiderivative $=31.62$

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a^2*cos(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 +
 4900*(f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*cos(2*f*x + 2*e)
^2 + 64*(f*x + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 3136*(f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + 3136*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 64*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + (f*x + e)*a^2*sin(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*sin(6*f*x + 6*e)^
2 + 4900*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*sin(2*f*x + 2
*e)^2 + 64*(f*x + e)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
```

```
))^2 + 3136*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 3136*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 64*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 56*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 - 46*a^2*sin(2*f*
x + 2*e) - 2*(a^2*cos(8*f*x + 8*e)^2 + 784*a^2*cos(6*f*x + 6*e)^2 + 4900*a^
2*cos(4*f*x + 4*e)^2 + 784*a^2*cos(2*f*x + 2*e)^2 + 64*a^2*cos(7/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(5/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 + 64*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + a^2*sin(8*f*x + 8*e)^2 + 784*a^2*sin(6*f*x + 6*e)^2 + 4900*a^2*
sin(4*f*x + 4*e)^2 + 3920*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*a^2*s
in(2*f*x + 2*e)^2 + 64*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 3136*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
3136*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*a^2*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*a^2*cos(2*f*x + 2
*e) + a^2 + 2*(28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x + 4*e) + 28*a^2*c
os(2*f*x + 2*e) + a^2)*cos(8*f*x + 8*e) + 56*(70*a^2*cos(4*f*x + 4*e) + 28*
a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) + 140*(28*a^2*cos(2*f*x + 2*e)
 + a^2)*cos(4*f*x + 4*e) - 16*(a^2*cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*
e) + 70*a^2*cos(4*f*x + 4*e) + 28*a^2*cos(2*f*x + 2*e) - 56*a^2*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*a^2*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 8*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + a^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 112*(a^2*cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x +
4*e) + 28*a^2*cos(2*f*x + 2*e) - 56*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 8*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + a^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(a^2*
cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x + 4*e) + 28*a
^2*cos(2*f*x + 2*e) - 8*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(a^2
*cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x + 4*e) + 28*
a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 28*(2*a^2*sin(6*f*x + 6*e) + 5*a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f
*x + 2*e))*sin(8*f*x + 8*e) + 784*(5*a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x
 + 2*e))*sin(6*f*x + 6*e) - 16*(a^2*sin(8*f*x + 8*e) + 28*a^2*sin(6*f*x + 6
*e) + 70*a^2*sin(4*f*x + 4*e) + 28*a^2*sin(2*f*x + 2*e) - 56*a^2*sin(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*a^2*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) - 8*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112
*(a^2*sin(8*f*x + 8*e) + 28*a^2*sin(6*f*x + 6*e) + 70*a^2*sin(4*f*x + 4*e)
+ 28*a^2*sin(2*f*x + 2*e) - 56*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 8*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(a^2*sin(8*f*x +
 8*e) + 28*a^2*sin(6*f*x + 6*e) + 70*a^2*sin(4*f*x + 4*e) + 28*a^2*sin(2*f*
x + 2*e) - 8*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(
```

3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(a^2*sin(8*f*x + 8*e)
 + 28*a^2*sin(6*f*x + 6*e) + 70*a^2*sin(4*f*x + 4*e) + 28*a^2*sin(2*f*x + 2
*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 1) + 2*(28*(f*x + e)*a^2*cos(6*f*x + 6*e) + 70*(f
*x + e)*a^2*cos(4*f*x + 4*e) + 28*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e
)*a^2 - 23*a^2*sin(6*f*x + 6*e) - 66*a^2*sin(4*f*x + 4*e) - 23*a^2*sin(2*f*
x + 2*e))*cos(8*f*x + 8*e) + 28*(140*(f*x + e)*a^2*cos(4*f*x + 4*e) + 56*(f
*x + e)*a^2*cos(2*f*x + 2*e) + 2*(f*x + e)*a^2 - 17*a^2*sin(4*f*x + 4*e))*c
os(6*f*x + 6*e) + 28*(140*(f*x + e)*a^2*cos(2*f*x + 2*e) + 5*(f*x + e)*a^2
+ 17*a^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(4*(f*x + e)*a^2*cos(8*f*x
+ 8*e) + 112*(f*x + e)*a^2*cos(6*f*x + 6*e) + 280*(f*x + e)*a^2*cos(4*f*x +
 4*e) + 112*(f*x + e)*a^2*cos(2*f*x + 2*e) - 224*(f*x + e)*a^2*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 224*(f*x + e)*a^2*cos(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*(f*x + e)*a^2*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(f*x + e)*a^2 + 3*a^2*sin(8*f*x + 8
*e) - 8*a^2*sin(6*f*x + 6*e) - 54*a^2*sin(4*f*x + 4*e) - 8*a^2*sin(2*f*x +
2*e) + 48*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 48*a^2
*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*cos(8*f*x + 8*e) + 7
84*(f*x + e)*a^2*cos(6*f*x + 6*e) + 1960*(f*x + e)*a^2*cos(4*f*x + 4*e) + 7
84*(f*x + e)*a^2*cos(2*f*x + 2*e) - 1568*(f*x + e)*a^2*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 224*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 28*(f*x + e)*a^2 + 27*a^2*sin(8*f*x + 8*e)
 + 112*a^2*sin(6*f*x + 6*e) + 42*a^2*sin(4*f*x + 4*e) + 112*a^2*sin(2*f*x +
 2*e) - 48*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*cos(8*
f*x + 8*e) + 784*(f*x + e)*a^2*cos(6*f*x + 6*e) + 1960*(f*x + e)*a^2*cos(4*
f*x + 4*e) + 784*(f*x + e)*a^2*cos(2*f*x + 2*e) - 224*(f*x + e)*a^2*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 28*(f*x + e)*a^2 + 27*a^2*s
in(8*f*x + 8*e) + 112*a^2*sin(6*f*x + 6*e) + 42*a^2*sin(4*f*x + 4*e) + 112*
a^2*sin(2*f*x + 2*e) - 48*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(4*(f*x +
 e)*a^2*cos(8*f*x + 8*e) + 112*(f*x + e)*a^2*cos(6*f*x + 6*e) + 280*(f*x +
e)*a^2*cos(4*f*x + 4*e) + 112*(f*x + e)*a^2*cos(2*f*x + 2*e) + 4*(f*x + e)*
a^2 + 3*a^2*sin(8*f*x + 8*e) - 8*a^2*sin(6*f*x + 6*e) - 54*a^2*sin(4*f*x +
4*e) - 8*a^2*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 2*(28*(f*x + e)*a^2*sin(6*f*x + 6*e) + 70*(f*x + e)*a^2*sin(4*f*
x + 4*e) + 28*(f*x + e)*a^2*sin(2*f*x + 2*e) + 23*a^2*cos(6*f*x + 6*e) + 66
*a^2*cos(4*f*x + 4*e) + 23*a^2*cos(2*f*x + 2*e))*sin(8*f*x + 8*e) + 2*(1960
*(f*x + e)*a^2*sin(4*f*x + 4*e) + 784*(f*x + e)*a^2*sin(2*f*x + 2*e) + 238*
a^2*cos(4*f*x + 4*e) - 23*a^2)*sin(6*f*x + 6*e) + 4*(980*(f*x + e)*a^2*sin(
2*f*x + 2*e) - 119*a^2*cos(2*f*x + 2*e) - 33*a^2)*sin(4*f*x + 4*e) - 4*(4*(
f*x + e)*a^2*sin(8*f*x + 8*e) + 112*(f*x + e)*a^2*sin(6*f*x + 6*e) + 280*(f
*x + e)*a^2*sin(4*f*x + 4*e) + 112*(f*x + e)*a^2*sin(2*f*x + 2*e) - 224*(f*

x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 224*(f*x
+ e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*(f*x + e
)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*a^2*cos(8*f*
x + 8*e) + 8*a^2*cos(6*f*x + 6*e) + 54*a^2*cos(4*f*x + 4*e) + 8*a^2*cos(2*f
*x + 2*e) - 48*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4
8*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*a^2)*sin(7/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*sin(8*f
*x + 8*e) + 784*(f*x + e)*a^2*sin(6*f*x + 6*e) + 1960*(f*x + e)*a^2*sin(4*f
*x + 4*e) + 784*(f*x + e)*a^2*sin(2*f*x + 2*e) - 1568*(f*x + e)*a^2*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 224*(f*x + e)*a^2*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 27*a^2*cos(8*f*x + 8*e) - 112
*a^2*cos(6*f*x + 6*e) - 42*a^2*cos(4*f*x + 4*e) - 112*a^2*cos(2*f*x + 2*e)
+ 48*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 27*a^2)*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*sin
(8*f*x + 8*e) + 784*(f*x + e)*a^2*sin(6*f*x + 6*e) + 1960*(f*x + e)*a^2*sin
(4*f*x + 4*e) + 784*(f*x + e)*a^2*sin(2*f*x + 2*e) - 224*(f*x + e)*a^2*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 27*a^2*cos(8*f*x + 8*e)
- 112*a^2*cos(6*f*x + 6*e) - 42*a^2*cos(4*f*x + 4*e) - 112*a^2*cos(2*f*x +
2*e) + 48*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 27*a^2
)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(4*(f*x + e)*a^2
*sin(8*f*x + 8*e) + 112*(f*x + e)*a^2*sin(6*f*x + 6*e) + 280*(f*x + e)*a^2*
sin(4*f*x + 4*e) + 112*(f*x + e)*a^2*sin(2*f*x + 2*e) - 3*a^2*cos(8*f*x + 8
*e) + 8*a^2*cos(6*f*x + 6*e) + 54*a^2*cos(4*f*x + 4*e) + 8*a^2*cos(2*f*x +
2*e) - 3*a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)
*sqrt(c)/((c^5*cos(8*f*x + 8*e)^2 + 784*c^5*cos(6*f*x + 6*e)^2 + 4900*c^5*c
os(4*f*x + 4*e)^2 + 784*c^5*cos(2*f*x + 2*e)^2 + 64*c^5*cos(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(5/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 64*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + c^5*sin(8*f*x + 8*e)^2 + 784*c^5*sin(6*f*x + 6*e)^2 + 4900*c^5*sin
(4*f*x + 4*e)^2 + 3920*c^5*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*c^5*sin(
2*f*x + 2*e)^2 + 64*c^5*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 3136*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 313
6*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*c^5*cos(2*f*x + 2*e)
 + c^5 + 2*(28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(
2*f*x + 2*e) + c^5)*cos(8*f*x + 8*e) + 56*(70*c^5*cos(4*f*x + 4*e) + 28*c^5
*cos(2*f*x + 2*e) + c^5)*cos(6*f*x + 6*e) + 140*(28*c^5*cos(2*f*x + 2*e) +
c^5)*cos(4*f*x + 4*e) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e)
+ 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(5/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*cos(3/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + c^5)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
12*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e
) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(

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2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + c^5)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos
(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*
cos(2*f*x + 2*e) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + c^5)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*co
s(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5
*cos(2*f*x + 2*e) + c^5)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 28*(2*c^5*sin(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x
+ 2*e))*sin(8*f*x + 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x +
2*e))*sin(6*f*x + 6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e)
 + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(5/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c
^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 2
8*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*
e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x +
 2*e) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*sin(8*f*x + 8*e) +
28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e)
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(9/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(9/2), x)
```

3.109 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal result 757
Rubi [A] (verified) 758
Mathematica [A] (verified) 760
Maple [A] (warning: unable to verify) 761
Fricas [F] 761
Sympy [F(-1)] 762
Maxima [B] (verification not implemented) 762
Giac [F] 768
Mupad [F(-1)] 768

Optimal result

Integrand size = 30, antiderivative size = 244

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx &= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}} \\ &\quad - \frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} \\ &\quad - \frac{a^3\tan(e+fx)}{2c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} \\ &\quad - \frac{a^3\tan(e+fx)}{c^4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} \\ &\quad + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^5f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

```
[Out] -4/5*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)/(a+a*sec(f*x+e))^(1/2)-1/3*a^
3*tan(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a^3*ta
n(f*x+e)/c^3/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^3*tan(f*x+e)
/c^4/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-cos(f*x+e))*t
an(f*x+e)/c^5/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3995, 3992, 3996, 31}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^5 f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$
$$-\frac{a^3\tan(e+fx)}{c^4 f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$
$$-\frac{a^3\tan(e+fx)}{2c^3 f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$
$$-\frac{a^3\tan(e+fx)}{3c^2 f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$
$$-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2),x]
```

```
[Out] (-4*a^3*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(1
1/2)) - (a^3*Tan[e + f*x])/(3*c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e +
 f*x])^(7/2)) - (a^3*Tan[e + f*x])/(2*c^3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c
*Sec[e + f*x])^(5/2)) - (a^3*Tan[e + f*x])/(c^4*f*Sqrt[a + a*Sec[e + f*x]]*
(c - c*Sec[e + f*x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^5
*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}}+\frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx}{c^2}\\
&= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}+\frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx}{c^3}\\
&= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{2c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}+\frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c^4}\\
&= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{2c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{a^3\tan(e+fx)}{c^4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{a^2\int\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c^5}
\end{aligned}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a^3\tan(e+fx)}{2c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a^3\tan(e+fx)}{c^4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{(a^3\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{c^4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a^3\tan(e+fx)}{2c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a^3\tan(e+fx)}{c^4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^5f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.49

$$\int\frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{a^3\left(-30\log(\cos(e+fx))-30\log(1-\sec(e+fx))+\frac{-24-10(-1+\sec(e+fx))^2+15(-1+\sec(e+fx))^3-30(-1+\sec(e+fx))^4}{(-1+\sec(e+fx))^5}\right.}{30c^5f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2),x]
```

```
[Out] -1/30*(a^3*(-30*Log[Cos[e + f*x]] - 30*Log[1 - Sec[e + f*x]] + (-24 - 10*(-
1 + Sec[e + f*x])^2 + 15*(-1 + Sec[e + f*x])^3 - 30*(-1 + Sec[e + f*x])^4)/
(-1 + Sec[e + f*x])^5)*Tan[e + f*x])/(c^5*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt
[c - c*Sec[e + f*x]])
```

Maple [A] (warning: unable to verify)

Time = 2.43 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.25

method	result
default	$\frac{\sqrt{2}\, a^{2} \sqrt{-\frac{2 a}{\left(1-\cos\left(f x+e\right)\right)^{2} \csc\left(f x+e\right)^{2}-1}}\, \left(1-\cos\left(f x+e\right)\right) \left(240 \ln\left(-\cot\left(f x+e\right)+\csc\left(f x+e\right)\right) \left(1-\cos\left(f x+e\right)\right)^{10} \csc\left(f x+e\right)^{10}-120 \ln\left(\left(1-\right.\right.\right.}{}$
risch	$\frac{a^{2} \sqrt{\frac{a \left(e^{i \left(f x+e\right)}+1\right)^{2}}{1+e^{2 i \left(f x+e\right)}}}\, \left(e^{i \left(f x+e\right)}-1\right) x}{c^{5} \left(e^{i \left(f x+e\right)}+1\right) \sqrt{\frac{c \left(e^{i \left(f x+e\right)}-1\right)^{2}}{1+e^{2 i \left(f x+e\right)}}}}-\frac{2 a^{2} \sqrt{\frac{a \left(e^{i \left(f x+e\right)}+1\right)^{2}}{1+e^{2 i \left(f x+e\right)}}}\, \left(e^{i \left(f x+e\right)}-1\right) \left(f x+e\right)}{c^{5} \left(e^{i \left(f x+e\right)}+1\right) \sqrt{\frac{c \left(e^{i \left(f x+e\right)}-1\right)^{2}}{1+e^{2 i \left(f x+e\right)}}}\, f}+\frac{2 i a^{2} \sqrt{\frac{a \left(e^{i \left(f x+e\right)}+1\right)^{2}}{1+e^{2 i \left(f x+e\right)}}}\, \left(105\, e^{9 i \left(f x+e\right)}-555\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240/f*2^(1/2)*a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^5/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e
)^2-1)*csc(f*x+e)^2)^(11/2)*(1-cos(f*x+e))*(240*ln(-cot(f*x+e)+csc(f*x+e))*
(1-cos(f*x+e))^10*csc(f*x+e)^10-120*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-
cos(f*x+e))^10*csc(f*x+e)^10+120*(1-cos(f*x+e))^8*csc(f*x+e)^8-60*(1-cos(f*
x+e))^6*csc(f*x+e)^6+35*(1-cos(f*x+e))^4*csc(f*x+e)^4-15*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+3)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*sqrt(-c*sec(f*x + e) + c)/(c^6*sec(f*x + e)^6 - 6*c^6*sec(f*x + e)^5
 + 15*c^6*sec(f*x + e)^4 - 20*c^6*sec(f*x + e)^3 + 15*c^6*sec(f*x + e)^2 -
6*c^6*sec(f*x + e) + c^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9150 vs. $2(218) = 436$.

Time = 70.10 (sec) , antiderivative size = 9150, normalized size of antiderivative = 37.50

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="maxi
ma")
[Out] -1/15*(15*(f*x + e)*a^2*cos(10*f*x + 10*e)^2 + 30375*(f*x + e)*a^2*cos(8*f*
x + 8*e)^2 + 661500*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 661500*(f*x + e)*a^2
*cos(4*f*x + 4*e)^2 + 30375*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 1500*(f*x +
e)*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*(f*x
 + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 952560*(
f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 21600
0*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 15
00*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1
5*(f*x + e)*a^2*sin(10*f*x + 10*e)^2 + 30375*(f*x + e)*a^2*sin(8*f*x + 8*e)
^2 + 661500*(f*x + e)*a^2*sin(6*f*x + 6*e)^2 + 661500*(f*x + e)*a^2*sin(4*f
*x + 4*e)^2 + 30375*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 1500*(f*x + e)*a^2*s
in(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*(f*x + e)*a^
2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 952560*(f*x + e)
*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*(f*x +
 e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1500*(f*x
+ e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1350*(f*x
 + e)*a^2*cos(2*f*x + 2*e) + 15*(f*x + e)*a^2 - 1110*a^2*sin(2*f*x + 2*e) -
 30*(a^2*cos(10*f*x + 10*e)^2 + 2025*a^2*cos(8*f*x + 8*e)^2 + 44100*a^2*cos
(6*f*x + 6*e)^2 + 44100*a^2*cos(4*f*x + 4*e)^2 + 2025*a^2*cos(2*f*x + 2*e)^
2 + 100*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*
a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*a^2*cos(
5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*a^2*cos(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*a^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(10*f*x + 10*e)^2 + 2025*a^2*sin(
```

8*f*x + 8*e)^2 + 44100*a^2*sin(6*f*x + 6*e)^2 + 44100*a^2*sin(4*f*x + 4*e)^
2 + 18900*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 2025*a^2*sin(2*f*x + 2*e)
^2 + 100*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400
*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*a^2*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*a^2*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*a^2*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*a^2*cos(2*f*x + 2*e) + a^2 + 2*(45*a
^2*cos(8*f*x + 8*e) + 210*a^2*cos(6*f*x + 6*e) + 210*a^2*cos(4*f*x + 4*e) +
45*a^2*cos(2*f*x + 2*e) + a^2)*cos(10*f*x + 10*e) + 90*(210*a^2*cos(6*f*x
+ 6*e) + 210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) + a^2)*cos(8*f*
x + 8*e) + 420*(210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) + a^2)*c
os(6*f*x + 6*e) + 420*(45*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) - 20
*(a^2*cos(10*f*x + 10*e) + 45*a^2*cos(8*f*x + 8*e) + 210*a^2*cos(6*f*x + 6*
e) + 210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) - 120*a^2*cos(7/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 252*a^2*cos(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 120*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 10*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + a^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(a
^2*cos(10*f*x + 10*e) + 45*a^2*cos(8*f*x + 8*e) + 210*a^2*cos(6*f*x + 6*e)
+ 210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) - 252*a^2*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*a^2*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 10*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + a^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 504*(a^2*cos(10*f*x + 10*e) + 45*a^2*cos(8*f*x + 8*e) + 210*a^2*cos(6*f*
x + 6*e) + 210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) - 120*a^2*cos
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*a^2*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2)*cos(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 240*(a^2*cos(10*f*x + 10*e) + 45*a^2*cos(8*f*x + 8*
e) + 210*a^2*cos(6*f*x + 6*e) + 210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x
+ 2*e) - 10*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2
)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(a^2*cos(10*f*x
+ 10*e) + 45*a^2*cos(8*f*x + 8*e) + 210*a^2*cos(6*f*x + 6*e) + 210*a^2*cos
(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 30*(3*a^2*sin(8*f*x + 8*e) + 14*a^2*sin(6*f*x +
6*e) + 14*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e))*sin(10*f*x + 10*e)
+ 1350*(14*a^2*sin(6*f*x + 6*e) + 14*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*
x + 2*e))*sin(8*f*x + 8*e) + 6300*(14*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) - 20*(a^2*sin(10*f*x + 10*e) + 45*a^2*sin(8*f*x
+ 8*e) + 210*a^2*sin(6*f*x + 6*e) + 210*a^2*sin(4*f*x + 4*e) + 45*a^2*sin(2
*f*x + 2*e) - 120*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 252*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*a^2*si
n(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*a^2*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(9/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - 240*(a^2*sin(10*f*x + 10*e) + 45*a^2*sin(8*f*x + 8*e) +
210*a^2*sin(6*f*x + 6*e) + 210*a^2*sin(4*f*x + 4*e) + 45*a^2*sin(2*f*x + 2*

e) - 252*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*a^2
*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*a^2*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) - 504*(a^2*sin(10*f*x + 10*e) + 45*a^2*sin(8*f*x + 8*e)
 + 210*a^2*sin(6*f*x + 6*e) + 210*a^2*sin(4*f*x + 4*e) + 45*a^2*sin(2*f*x +
 2*e) - 120*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*a
^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(a^2*sin(10*f*x + 10*e) + 45*a^2*s
in(8*f*x + 8*e) + 210*a^2*sin(6*f*x + 6*e) + 210*a^2*sin(4*f*x + 4*e) + 45*
a^2*sin(2*f*x + 2*e) - 10*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(a^2*sin
(10*f*x + 10*e) + 45*a^2*sin(8*f*x + 8*e) + 210*a^2*sin(6*f*x + 6*e) + 210*
a^2*sin(4*f*x + 4*e) + 45*a^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 1
0*(135*(f*x + e)*a^2*cos(8*f*x + 8*e) + 630*(f*x + e)*a^2*cos(6*f*x + 6*e)
+ 630*(f*x + e)*a^2*cos(4*f*x + 4*e) + 135*(f*x + e)*a^2*cos(2*f*x + 2*e) +
 3*(f*x + e)*a^2 - 111*a^2*sin(8*f*x + 8*e) - 625*a^2*sin(6*f*x + 6*e) - 62
5*a^2*sin(4*f*x + 4*e) - 111*a^2*sin(2*f*x + 2*e))*cos(10*f*x + 10*e) + 450
(630(f*x + e)*a^2*cos(6*f*x + 6*e) + 630*(f*x + e)*a^2*cos(4*f*x + 4*e) +
 135*(f*x + e)*a^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2 - 107*a^2*sin(6*f*x +
 6*e) - 107*a^2*sin(4*f*x + 4*e))*cos(8*f*x + 8*e) + 450*(2940*(f*x + e)*a^
2*cos(4*f*x + 4*e) + 630*(f*x + e)*a^2*cos(2*f*x + 2*e) + 14*(f*x + e)*a^2
+ 107*a^2*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 450*(630*(f*x + e)*a^2*cos(2
*f*x + 2*e) + 14*(f*x + e)*a^2 + 107*a^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e)
 - 10*(30*(f*x + e)*a^2*cos(10*f*x + 10*e) + 1350*(f*x + e)*a^2*cos(8*f*x +
 8*e) + 6300*(f*x + e)*a^2*cos(6*f*x + 6*e) + 6300*(f*x + e)*a^2*cos(4*f*x
+ 4*e) + 1350*(f*x + e)*a^2*cos(2*f*x + 2*e) - 3600*(f*x + e)*a^2*cos(7/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 7560*(f*x + e)*a^2*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3600*(f*x + e)*a^2*cos(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 300*(f*x + e)*a^2*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(f*x + e)*a^2 + 21*a^2*sin(10*
f*x + 10*e) - 165*a^2*sin(8*f*x + 8*e) - 1840*a^2*sin(6*f*x + 6*e) - 1840*a
^2*sin(4*f*x + 4*e) - 165*a^2*sin(2*f*x + 2*e) + 940*a^2*sin(7/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2472*a^2*sin(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 940*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(180
*(f*x + e)*a^2*cos(10*f*x + 10*e) + 8100*(f*x + e)*a^2*cos(8*f*x + 8*e) + 3
7800*(f*x + e)*a^2*cos(6*f*x + 6*e) + 37800*(f*x + e)*a^2*cos(4*f*x + 4*e)
+ 8100*(f*x + e)*a^2*cos(2*f*x + 2*e) - 45360*(f*x + e)*a^2*cos(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 21600*(f*x + e)*a^2*cos(3/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1800*(f*x + e)*a^2*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 180*(f*x + e)*a^2 + 173*a^2*sin(10*f
*x + 10*e) + 1125*a^2*sin(8*f*x + 8*e) - 1170*a^2*sin(6*f*x + 6*e) - 1170*a
^2*sin(4*f*x + 4*e) + 1125*a^2*sin(2*f*x + 2*e) + 2988*a^2*sin(5/2*arctan2(

sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 470*a^2*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) - 12*(630*(f*x + e)*a^2*cos(10*f*x + 10*e) + 28350*(f*x + e)*a^2*cos(8*
f*x + 8*e) + 132300*(f*x + e)*a^2*cos(6*f*x + 6*e) + 132300*(f*x + e)*a^2*c
os(4*f*x + 4*e) + 28350*(f*x + e)*a^2*cos(2*f*x + 2*e) - 75600*(f*x + e)*a^
2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6300*(f*x + e)*a^2
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 630*(f*x + e)*a^2 +
 647*a^2*sin(10*f*x + 10*e) + 5805*a^2*sin(8*f*x + 8*e) + 4620*a^2*sin(6*f*
x + 6*e) + 4620*a^2*sin(4*f*x + 4*e) + 5805*a^2*sin(2*f*x + 2*e) - 4980*a^2
*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2060*a^2*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 20*(180*(f*x + e)*a^2*cos(10*f*x + 10*e) + 8100*(f*
x + e)*a^2*cos(8*f*x + 8*e) + 37800*(f*x + e)*a^2*cos(6*f*x + 6*e) + 37800*
(f*x + e)*a^2*cos(4*f*x + 4*e) + 8100*(f*x + e)*a^2*cos(2*f*x + 2*e) - 1800
*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 180*(
f*x + e)*a^2 + 173*a^2*sin(10*f*x + 10*e) + 1125*a^2*sin(8*f*x + 8*e) - 117
0*a^2*sin(6*f*x + 6*e) - 1170*a^2*sin(4*f*x + 4*e) + 1125*a^2*sin(2*f*x + 2
*e) - 470*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*(30*(f*x + e)*a^2*cos(10
*f*x + 10*e) + 1350*(f*x + e)*a^2*cos(8*f*x + 8*e) + 6300*(f*x + e)*a^2*cos
(6*f*x + 6*e) + 6300*(f*x + e)*a^2*cos(4*f*x + 4*e) + 1350*(f*x + e)*a^2*co
s(2*f*x + 2*e) + 30*(f*x + e)*a^2 + 21*a^2*sin(10*f*x + 10*e) - 165*a^2*sin
(8*f*x + 8*e) - 1840*a^2*sin(6*f*x + 6*e) - 1840*a^2*sin(4*f*x + 4*e) - 165
*a^2*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 10*(135*(f*x + e)*a^2*sin(8*f*x + 8*e) + 630*(f*x + e)*a^2*sin(6*f*x + 6
e) + 630(f*x + e)*a^2*sin(4*f*x + 4*e) + 135*(f*x + e)*a^2*sin(2*f*x + 2*
e) + 111*a^2*cos(8*f*x + 8*e) + 625*a^2*cos(6*f*x + 6*e) + 625*a^2*cos(4*f*
x + 4*e) + 111*a^2*cos(2*f*x + 2*e))*sin(10*f*x + 10*e) + 30*(9450*(f*x + e
)*a^2*sin(6*f*x + 6*e) + 9450*(f*x + e)*a^2*sin(4*f*x + 4*e) + 2025*(f*x +
e)*a^2*sin(2*f*x + 2*e) + 1605*a^2*cos(6*f*x + 6*e) + 1605*a^2*cos(4*f*x +
4*e) - 37*a^2)*sin(8*f*x + 8*e) + 50*(26460*(f*x + e)*a^2*sin(4*f*x + 4*e)
+ 5670*(f*x + e)*a^2*sin(2*f*x + 2*e) - 963*a^2*cos(2*f*x + 2*e) - 125*a^2)
*sin(6*f*x + 6*e) + 50*(5670*(f*x + e)*a^2*sin(2*f*x + 2*e) - 963*a^2*cos(2
*f*x + 2*e) - 125*a^2)*sin(4*f*x + 4*e) - 10*(30*(f*x + e)*a^2*sin(10*f*x +
 10*e) + 1350*(f*x + e)*a^2*sin(8*f*x + 8*e) + 6300*(f*x + e)*a^2*sin(6*f*x
 + 6*e) + 6300*(f*x + e)*a^2*sin(4*f*x + 4*e) + 1350*(f*x + e)*a^2*sin(2*f*
x + 2*e) - 3600*(f*x + e)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) - 7560*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 3600*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
e))) - 300(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) - 21*a^2*cos(10*f*x + 10*e) + 165*a^2*cos(8*f*x + 8*e) + 1840*a^2*cos(6
*f*x + 6*e) + 1840*a^2*cos(4*f*x + 4*e) + 165*a^2*cos(2*f*x + 2*e) - 940*a^
2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2472*a^2*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 940*a^2*cos(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 21*a^2)*sin(9/2*arctan2(sin(2*f*x + 2*e),

cos(2*f*x + 2*e))) - 20*(180*(f*x + e)*a^2*sin(10*f*x + 10*e) + 8100*(f*x + e)*a^2*sin(8*f*x + 8*e) + 37800*(f*x + e)*a^2*sin(6*f*x + 6*e) + 37800*(f*x + e)*a^2*sin(4*f*x + 4*e) + 8100*(f*x + e)*a^2*sin(2*f*x + 2*e) - 45360*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 21600*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1800*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 173*a^2*cos(10*f*x + 10*e) - 1125*a^2*cos(8*f*x + 8*e) + 1170*a^2*cos(6*f*x + 6*e) + 1170*a^2*cos(4*f*x + 4*e) - 1125*a^2*cos(2*f*x + 2*e) - 2988*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 470*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 173*a^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(630*(f*x + e)*a^2*sin(10*f*x + 10*e) + 28350*(f*x + e)*a^2*sin(8*f*x + 8*e) + 132300*(f*x + e)*a^2*sin(6*f*x + 6*e) + 132300*(f*x + e)*a^2*sin(4*f*x + 4*e) + 28350*(f*x + e)*a^2*sin(2*f*x + 2*e) - 75600*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6300*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 647*a^2*cos(10*f*x + 10*e) - 5805*a^2*cos(8*f*x + 8*e) - 4620*a^2*cos(6*f*x + 6*e) - 4620*a^2*cos(4*f*x + 4*e) - 5805*a^2*cos(2*f*x + 2*e) + 4980*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2060*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 647*a^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(180*(f*x + e)*a^2*sin(10*f*x + 10*e) + 8100*(f*x + e)*a^2*sin(8*f*x + 8*e) + 37800*(f*x + e)*a^2*sin(6*f*x + 6*e) + 37800*(f*x + e)*a^2*sin(4*f*x + 4*e) + 8100*(f*x + e)*a^2*sin(2*f*x + 2*e) - 1800*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 173*a^2*cos(10*f*x + 10*e) - 1125*a^2*cos(8*f*x + 8*e) + 1170*a^2*cos(6*f*x + 6*e) + 1170*a^2*cos(4*f*x + 4*e) - 1125*a^2*cos(2*f*x + 2*e) + 470*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 173*a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*(30*(f*x + e)*a^2*sin(10*f*x + 10*e) + 1350*(f*x + e)*a^2*sin(8*f*x + 8*e) + 6300*(f*x + e)*a^2*sin(6*f*x + 6*e) + 6300*(f*x + e)*a^2*sin(4*f*x + 4*e) + 1350*(f*x + e)*a^2*sin(2*f*x + 2*e) - 21*a^2*cos(10*f*x + 10*e) + 165*a^2*cos(8*f*x + 8*e) + 1840*a^2*cos(6*f*x + 6*e) + 1840*a^2*cos(4*f*x + 4*e) + 165*a^2*cos(2*f*x + 2*e) - 21*a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^6*cos(10*f*x + 10*e)^2 + 2025*c^6*cos(8*f*x + 8*e)^2 + 44100*c^6*cos(6*f*x + 6*e)^2 + 44100*c^6*cos(4*f*x + 4*e)^2 + 2025*c^6*cos(2*f*x + 2*e)^2 + 100*c^6*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^6*sin(10*f*x + 10*e)^2 + 2025*c^6*sin(8*f*x + 8*e)^2 + 44100*c^6*sin(6*f*x + 6*e)^2 + 44100*c^6*sin(4*f*x + 4*e)^2 + 18900*c^6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 2025*c^6*sin(2*f*x + 2*e)^2 + 100*c^6*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*sin(1/2*arctan2(s

in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*c^6*cos(2*f*x + 2*e) + c^6 + 2*(
45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*
e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(10*f*x + 10*e) + 90*(210*c^6*cos(6*
f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(
8*f*x + 8*e) + 420*(210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^
6)*cos(6*f*x + 6*e) + 420*(45*c^6*cos(2*f*x + 2*e) + c^6)*cos(4*f*x + 4*e)
- 20*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x
+ 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120*c^6*cos(7
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 252*c^6*cos(5/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + c^6)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 24
0*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6
*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 252*c^6*cos(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + c^6)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 504*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(
6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120*c^6
*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(5/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - 240*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x
+ 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2
*f*x + 2*e) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 c^6)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(c^6*cos(10
*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6
*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 30*(3*c^6*sin(8*f*x + 8*e) + 14*c^6*sin(6*f*
x + 6*e) + 14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(10*f*x + 1
0*e) + 1350*(14*c^6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(
2*f*x + 2*e))*sin(8*f*x + 8*e) + 6300*(14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(
2*f*x + 2*e))*sin(6*f*x + 6*e) - 20*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*
f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*s
in(2*f*x + 2*e) - 120*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) - 252*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^
6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(9/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e
) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x
+ 2*e) - 252*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120
*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 504*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x +
8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f
*x + 2*e) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -

```
10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10*e) + 45*c
^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) +
 45*c^6*sin(2*f*x + 2*e) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(c^6
*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) +
210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\int \frac{(a\sec\left(fx+e\right)+a)^{\frac{5}{2}}}{(-c\sec\left(fx+e\right)+c)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{11/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(11/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(11/2), x)
```

3.110 $\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result 769
Rubi [A] (verified) 769
Mathematica [A] (verified) 770
Maple [A] (verified) 771
Fricas [F] 771
Sympy [F(-1)] 771
Maxima [F(-2)] 772
Giac [F(-2)] 772
Mupad [F(-1)] 772

Optimal result

Integrand size = 30, antiderivative size = 204

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{8c^4\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{4c^4\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{c^4\sec^2(e+fx)\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] c^4*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+8*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)-4*c^4*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x
+e))^(1/2)+1/2*c^4*sec(f*x+e)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*se
c(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^4\tan(e+fx)\sec^2(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{4c^4\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{8c^4\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{c^4\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (8*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (4*c^4*Sec[e + f*x]*Tan[e + f*x])
/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (c^4*Sec[e + f*x]^
2*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(\frac{4c^3}{a}+\frac{c^3}{ax}-\frac{c^3x}{a}-\frac{8c^3}{a(1+x)}\right)\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{8c^4\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad -\frac{4c^4\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{c^4\sec^2(e+fx)\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.41

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^4(2(\log(\cos(e+fx))+8\log(1+\sec(e+fx)))-8\sec(e+fx)+\sec^2(e+fx}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^4*(2*(Log[Cos[e + f*x]] + 8*Log[1 + Sec[e + f*x]]) - 8*Sec[e + f*x] + Se
c[e + f*x]^2)*Tan[e + f*x])/(2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[
e + f*x]])
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81

method	result
default	$-\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^3 c^3 \sqrt{a(\sec(fx+e)+1)}\,\left(14\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)+1)+14\cos(fx+e)^2\ln(-\cdots\right.}{2fa(\cos(fx+e)-\cdots}$
risch	$\frac{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)}-\frac{2c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)f}+\frac{2ic^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,\left(4e^{2i(fx+e)}-e^{i(fx+\cdots}\right.}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)\cdots}$

```
[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f/a*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^3*c^3*(a*(sec(f*x+e)+1))^
(1/2)*(14*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+14*cos(f*x+e)^2*ln(-cot
(f*x+e)+csc(f*x+e)-1)+2*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+9*cos(f*x+e)^2+8*
cos(f*x+e)-1)/(cos(f*x+e)-1)^3*cos(f*x+e)*cot(f*x+e)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{7}{2}}}{\sqrt{a\sec(fx+e)+a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) -
 c^3)*sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{\sqrt{a + a\sec(e + fx)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{\sqrt{a + a\sec(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.111 $\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result 773
Rubi [A] (verified) 773
Mathematica [A] (verified) 774
Maple [A] (verified) 775
Fricas [F] 775
Sympy [F(-1)] 775
Maxima [F(-2)] 776
Giac [F(-2)] 776
Mupad [F(-1)] 776

Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{c^3\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] c^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+4*c^3*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)-c^3*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e
))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c^3\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{c^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
```

```
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (c^3*Sec[e + f*x]*Tan[e + f*x])/(
f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int\frac{(c-cx)^2}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int\left(\frac{c^2}{a}+\frac{c^2}{ax}-\frac{4c^2}{a(1+x)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{c^3\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.46

$$\int\frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^3(\log(\cos(e+fx))+4\log(1+\sec(e+fx))-\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^3*(Log[Cos[e + f*x]] + 4*Log[1 + Sec[e + f*x]] - Sec[e + f*x])*Tan[e + f
*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

method	result
default	$\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^2 c^2 \sqrt{a(\sec(fx+e)+1)}\,\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+3\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)-\right.}{fa(\cos(fx+e)-1)^2}$
risch	$-\frac{c^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,\left(-3ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)-2ie^{i(fx+e)}+e^{3i(fx+e)}fx-3ie^{i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+8ie^{2i(fx+e)}\ln\left(e^{i(fx\right.\right.}}{}$

```
[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^2*c^2*(a*(sec(f*x+e)+1))^(1/
2)*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))+3*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)-
1)+3*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+cos(f*x+e)+1)/(cos(f*x+e)-1)^2
*cos(f*x+e)*cot(f*x+e)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{5}{2}}}{\sqrt{a\sec(fx+e)+a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(-c*sec(f*x +
e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{\sqrt{a + a\sec(e + fx)}}\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{\sqrt{a + a\sec(e + fx)}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{\sqrt{a + a\sec(e + fx)}}\, dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\, dx$$

```
[In] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.112 $\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result 777
Rubi [A] (verified) 777
Mathematica [A] (verified) 778
Maple [A] (verified) 779
Fricas [F] 779
Sympy [F] 779
Maxima [A] (verification not implemented) 780
Giac [F(-2)] 780
Mupad [F(-1)] 780

Optimal result

Integrand size = 30, antiderivative size = 102

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+2*c^2*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3997, 78}

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{c-cx}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(\frac{c}{ax}-\frac{2c}{a(1+x)}\right)\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^2(\log(\cos(e+fx))+2\log(1+\sec(e+fx)))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
[Out] (c^2*(Log[Cos[e + f*x]] + 2*Log[1 + Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt[a*
(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

method	result
default	$-\frac{\sqrt{-c(\sec(fx+e)-1)}\, c\sqrt{a(\sec(fx+e)+1)}\, \ln\left(-\frac{4\cos(fx+e)}{(\cos(fx+e)+1)^2}\right)(\cot(fx+e)-\cos(fx+e)\cot(fx+e))}{fa(\cos(fx+e)-1)}$
risch	$\frac{c(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\, x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)} - \frac{2c(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)f} - \frac{4ic(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,\ln(e^{i(fx+e}}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)f}$

```
[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/f/a*(-c*(sec(f*x+e)-1))^(1/2)*c*(a*(sec(f*x+e)+1))^(1/2)*ln(-4*cos(f*x+e
)/(cos(f*x+e)+1)^2)/(cos(f*x+e)-1)*(cot(f*x+e)-cos(f*x+e)*cot(f*x+e))
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-c\sec(fx+e)+c)^{\frac{3}{2}}}{\sqrt{a\sec(fx+e)+a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
[Out] integral((-c*sec(f*x + e) + c)^(3/2)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{(c - c\sec(e + fx))^{3/2}}{\sqrt{a + a\sec(e + fx)}}\,dx = -\frac{((fx + e)c + c\arctan(\sin(2\,fx + 2\,e), \cos(2\,fx + 2\,e) + 1) - 4\,c\arctan(\sin(fx + e), \cos(fx + e) + 1))\sqrt{}}{\sqrt{a}f}$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c + c*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*c*arc
tan2(sin(f*x + e), cos(f*x + e) + 1))*sqrt(c)/(sqrt(a)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{3/2}}{\sqrt{a + a\sec(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{3/2}}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.113 $\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result . . . 781
Rubi [A] (verified) . . . 781
Mathematica [A] (verified) . . . 782
Maple [A] (verified) . . . 782
Fricas [F] . . . 783
Sympy [F] . . . 783
Maxima [A] (verification not implemented) . . . 783
Giac [A] (verification not implemented) . . . 783
Mupad [F(-1)] . . . 784

Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{c\log(1+\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] c*ln(1+cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3996, 31}

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
```

```
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cos(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &= \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c(\log(\cos(e+fx))+\log(1+\sec(e+fx)))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c*(Log[Cos[e + f*x]] + Log[1 + Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt[a*(1 +
 Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\,\ln\left(\frac{2}{\cos(fx+e)+1}\right)\cot(fx+e)}{fa}$
risch	$\frac{(\mathrm{e}^{i(fx+e)}+1)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,x}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,(\mathrm{e}^{i(fx+e)}-1)} - \frac{2(\mathrm{e}^{i(fx+e)}+1)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,(fx+e)}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,(\mathrm{e}^{i(fx+e)}-1)f} - \frac{2i(\mathrm{e}^{i(fx+e)}+1)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,\ln(\mathrm{e}^{i(fx+e)}+1)}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,(\mathrm{e}^{i(fx+e)}-1)f}$

```
[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*ln(2/(cos(f*x+e)+1
))*cot(f*x+e)
```

Fricas [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{-c\sec\left(fx+e\right)+c}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
[Out] integral(sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{\sqrt{c}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{-a}f}$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] sqrt(c)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*f)
```

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=-\frac{\sqrt{-ac}c\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{af|c|}$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] -sqrt(-a*c)*c*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a*f*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{\sqrt{a + a\sec(e + fx)}}\, dx = \int \frac{\sqrt{c - \frac{c}{\cos(e+f\,x)}}}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\, dx$$

```
[In] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.114 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result 785
Rubi [A] (verified) 785
Mathematica [A] (verified) 786
Maple [A] (verified) 786
Fricas [B] (verification not implemented) 787
Sympy [F] 787
Maxima [A] (verification not implemented) 788
Giac [A] (verification not implemented) 788
Mupad [F(-1)] 788

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] ln(sin(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3990, 3556}

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\tan(e+fx)\log(\sin(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
```

```
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\tan(e+fx)\int\cot(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &= \frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{(\log(\cos(e+fx))+\log(\tan(e+fx)))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
[Out] ((Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec[e
 + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(\ln(-\cot(fx+e)+\csc(fx+e))-\ln\left(\frac{2}{\cos(fx+e)+1}\right)\right)(\cot(fx+e)-\csc(fx+e))}{fa\sqrt{-c(\sec(fx+e)-1)}}$
risch	$\frac{(\mathrm{e}^{i(fx+e)}+1)(\mathrm{e}^{i(fx+e)}-1)x}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}(1+\mathrm{e}^{2i(fx+e)})\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}}-\frac{2(\mathrm{e}^{i(fx+e)}+1)(\mathrm{e}^{i(fx+e)}-1)(fx+e)}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}(1+\mathrm{e}^{2i(fx+e)})\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}f}-\frac{i(\mathrm{e}^{i(fx+e)}+1)(}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}}$

```
[In] int(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
[Out] -1/f/a*(a*(sec(f*x+e)+1))^(1/2)*(ln(-cot(f*x+e)+csc(f*x+e))-ln(2/(cos(f*x+e
)+1)))/(-c*(sec(f*x+e)-1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(42) = 84.

Time = 0.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.91

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \left[-\frac{\sqrt{-ac}\log\left(-\frac{8\left(\left(256\cos(fx+e)^5-512\cos(fx+e)^3+175\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}-\left(256\,ac\cos(fx+e)^4-51\right.}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right.}{2\,acf}, -\frac{\sqrt{ac}\arctan\left(\frac{\left(16\cos(fx+e)^3-7\cos(fx+e)\right)\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{\left(16\,ac\cos(fx+e)^2-25\,ac\right)\sin(fx+e)}\right)}{acf}\right]$$

```
[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-8*((256*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*cos
(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f
*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x + e)
^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f), -
sqrt(a*c)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(a*c)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((16*a
*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))/(a*c*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

```
[In] integrate(1/(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{fx+e-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)-1\right)}{\sqrt{a}\sqrt{c}f}$$

```
[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="max
ima")
[Out] -(f*x + e - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) - 1))/(sqrt(a)*sqrt(
c)*f)
```

Giac [A] (verification not implemented)

none

Time = 1.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\frac{\sqrt{-ac}\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{a|c|} - \frac{2\sqrt{-ac}\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c\right|\right)}{a|c|}}{2\,f}$$

```
[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gia
c")
[Out] 1/2*(sqrt(-a*c)*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(a*abs(c)) - 2*sqrt(-a*c
)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a*abs(c)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.115 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result 789
Rubi [A] (verified) 789
Mathematica [A] (verified) 791
Maple [A] (verified) 791
Fricas [F] 791
Sympy [F] 792
Maxima [B] (verification not implemented) 792
Giac [A] (verification not implemented) 793
Mupad [F(-1)] 793

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{2cf(1-\cos(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+}} + \frac{3\log(1-\cos(e+fx))\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1+\cos(e+fx))\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*tan(f*x+e)/c/f/(1-cos(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)+3/4*ln(1-cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)+1/4*ln(1+cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c
*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3997, 84}

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)}{2cf(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{3\tan(e+fx)\log(1-\sec(e+fx))}{4cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\log(\sec(e+fx)+1)}{4cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\log(\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*S
ec[e + f*x]]) + (3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(4*c*f*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[1 + Sec[e + f*x]]*Tan[e + f*x]
)/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/
(2*c*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{x(a+ax)(c-cx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(\frac{1}{2ac^2(-1+x)^2} - \frac{3}{4ac^2(-1+x)} + \frac{1}{ac^2x} - \frac{1}{4ac^2(1+x)}\right) dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= \frac{\log(\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{3\log(1-\sec(e+fx))\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{\log(1+\sec(e+fx))\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{\tan(e+fx)}{2cf(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\big(4\log(\cos(e+fx))+3\log(1-\sec(e+fx))+\log(1+\mathrm{se}}{4cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-}}$$

[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]

[Out] ((4*Log[Cos[e + f*x]] + 3*Log[1 - Sec[e + f*x]] + Log[1 + Sec[e + f*x]] + 2/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(4*c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\left(6\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))-4\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-\cos(fx+e)-6\ln(-\cot(fx+e)+\csc(fx+e))+4\ln\left(\frac{}{\cos}\right.\right.}{4fa\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx+e)-1)(\cos(fx+e)+1)}$
risch	$\frac{2ie^{i(fx+e)}-3ie^{3i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-2e^{3i(fx+e)}fx+3ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+3ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-4e^{3i(fx+e)}e+2e^{2}}{2c}$

[In] int(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4/f/a*(6*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e))-4*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-cos(f*x+e)-6*ln(-cot(f*x+e)+csc(f*x+e))+4*ln(2/(cos(f*x+e)+1))-1)*(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f*x+e)-1)/(cos(f*x+e)+1)*tan(f*x+e)

Fricas [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^2*sec(f*x + e)^3 - a*c^2*sec(f*x + e)^2 - a*c^2*sec(f*x + e) + a*c^2), x)

Sympy [F]

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}\left(-c\left(\sec\left(e + fx\right) - 1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(148) = 296$.

Time = 0.40 (sec) , antiderivative size = 818, normalized size of antiderivative = 4.87

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="max
ima")
[Out] -1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x +
e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - (cos(2*
f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1
) - 3*(cos(2*f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(4*f*x + 4*(f*x + e)*cos(
2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e) - 1)*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*e)/((c*cos(2*f*x +
 2*e)^2 + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*si
n(2*f*x + 2*e)^2 - 4*c*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 2*c*cos(2*f*x + 2*e) - 4*(c*cos(2*f*x + 2*e) + c)*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*sqrt(a)*sqrt(c)*f)
```

Giac [A] (verification not implemented)

none

Time = 1.88 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\frac{3\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}|c|} - \frac{4\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}|c|} - \frac{3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}c|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{4f}$$

```
[In] integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gia
c")
[Out] -1/4*(3*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*abs(c)) - 4*log(abs(
c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*abs(c)) - (3*c*tan(1/2*f*x + 1/2
*e)^2 - c)/(sqrt(-a*c)*c*abs(c)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+fx)}}\left(c-\frac{c}{\cos(e+fx)}\right)^{3/2}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.116 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result . . . 794
Rubi [A] (verified) . . . 794
Mathematica [A] (verified) . . . 796
Maple [A] (verified) . . . 796
Fricas [F] . . . 797
Sympy [F] . . . 797
Maxima [B] (verification not implemented) . . . 797
Giac [A] (verification not implemented) . . . 799
Mupad [F(-1)] . . . 799

Optimal result

Integrand size = 30, antiderivative size = 274

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx=\frac{\log(\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{7\log(1-\sec(e+fx))\tan(e+fx)}{8c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\log(1+\sec(e+fx))\tan(e+fx)}{8c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4c^2f(1-\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)}{4c^2f(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] ln(cos(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+7/8*ln(1-sec(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)+1/8*ln(1+sec(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c
-c*sec(f*x+e))^(1/2)-1/4*tan(f*x+e)/c^2/f/(1-sec(f*x+e))^2/(a+a*sec(f*x+e))
^(1/2)/(c-c*sec(f*x+e))^(1/2)-3/4*tan(f*x+e)/c^2/f/(1-sec(f*x+e))/(a+a*sec(
f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

= {3997, 84}

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{3\tan(e+fx)}{4c^2f(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4c^2f(1-\sec(e+fx))^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{7\tan(e+fx)\log(1-\sec(e+fx))}{8c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\log(\sec(e+fx)+1)}{8c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\log(\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (7*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(8*c^2*f*Sqrt[a +
a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[1 + Sec[e + f*x]]*Tan[e +
f*x])/(8*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e +
 f*x]/(4*c^2*f*(1 - Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]]) - (3*Tan[e + f*x])/(4*c^2*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{x(a+ax)(c-cx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int\left(-\frac{1}{2ac^3(-1+x)^3}+\frac{3}{4ac^3(-1+x)^2}-\frac{7}{8ac^3(-1+x)}+\frac{1}{ac^3x}-\frac{1}{8ac^3(1+x)}\right)dx,x,\sec(e\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{7\log(1-\sec(e+fx))\tan(e+fx)}{8c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1+\sec(e+fx))\tan(e+fx)}{8c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4c^2f(1-\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{3\tan(e+fx)}{4c^2f(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\left(8\log(\cos(e+fx))+7\log(1-\sec(e+fx))+\log(1+\sec\right.}{8c^2f\sqrt{a(1+\sec(e+fx)}}$$

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
[Out] ((8*Log[Cos[e + f*x]] + 7*Log[1 - Sec[e + f*x]] + Log[1 + Sec[e + f*x]] - 2
/(-1 + Sec[e + f*x])^2 + 6/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(8*c^2*f*Sqrt
[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\sqrt{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(16\ln\left((1-\cos(fx+e))^2\csc(fx+e)^2+1\right)(1-\cos(fx+e))^4\csc(fx+e)^4-28\ln(-\right.}{32fa\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^2\left(\frac{c(1-\cos(f}{(1-\cos(fx+}\right.}$
risch	$\frac{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)x}{c^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}(1+e^{2i(fx+e)})} - \frac{2(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)(fx+e)}{c^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}(1+e^{2i(fx+e)})f} + \frac{i(5e}{2c^2\sqrt{\frac{a(e^{i(fx+}}{1+e^{2i(f}}}}$

```
[In] int(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/32/f*2^(1/2)/a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(16*ln((1-cos(f*x+e))^2*csc(f*x+e)^
2+1)*(1-cos(f*x+e))^4*csc(f*x+e)^4-28*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*x
+e))^4*csc(f*x+e)^4-8*(1-cos(f*x+e))^2*csc(f*x+e)^2+1)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}(-c\sec\left(fx+e\right)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^3*sec(f*x
 + e)^4 - 2*a*c^3*sec(f*x + e)^3 + 2*a*c^3*sec(f*x + e) - a*c^3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(5/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(242) = 484$.

Time = 0.52 (sec) , antiderivative size = 2206, normalized size of antiderivative = 8.05

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 + 6
4*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x + e)
*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1/2*ar
```

ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*f*x - (2*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 - 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*sin(2*f*x + 2*e)^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) - 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 7*(2*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 - 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*sin(2*f*x + 2*e)^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) - 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + e - 2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 48*(f*x + e)*cos(2*f*x + 2*e) - 2*(16*f*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos(2*f*x + 2*e) - 64*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(16*f*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos(2*f*x + 2*e) + 16*e + 5*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*(3*(f*x + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) - 2*(16*(f*x + e)*sin(4*f*x + 4*e) + 96*(f*x + e)*sin(2*f*x + 2*e) - 64*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e) - 5)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(16*(f*x + e)*sin(4*f*x + 4*e) + 96*(f*x + e)*sin(2*f*x + 2*e) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e) - 5)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*e - 16*sin(2*f*x + 2*e))/((c^2*cos(4*f*x + 4*e)^2 + 36*c^2*cos(2*f*x + 2*e)^2 + 16*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c^2*cos(

```
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2
 + 12*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c^2*sin(2*f*x + 2*e)^2 + 1
6*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c^2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*c^2*cos(2*f*x + 2*e)
 + c^2 + 2*(6*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e) - 8*(c^2*cos(4*f
*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) - 4*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + c^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 8*(c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^2*sin(4*f*x + 4*e) + 6*c^2
*sin(2*f*x + 2*e) - 4*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^2*sin(4*f*
x + 4*e) + 6*c^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*sqrt(a)*sqrt(c)*f)
```

Giac [A] (verification not implemented)

none

Time = 1.70 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{14\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}c|c|}-\frac{16\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}c|c|}-\frac{21\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+34\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c+14c^2}{\sqrt{-ac}c^3|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}}{16f}$$

```
[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/16*(14*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*c*abs(c)) - 16*log
(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*c*abs(c)) - (21*(c*tan(1/2*
f*x + 1/2*e)^2 - c)^2 + 34*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 14*c^2)/(sqrt
(-a*c)*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+fx)}}\left(c-\frac{c}{\cos(e+fx)}\right)^{5/2}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.117 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result 800
Rubi [A] (verified) 800
Mathematica [A] (verified) 802
Maple [A] (verified) 802
Fricas [F] 803
Sympy [F(-1)] 803
Maxima [B] (verification not implemented) 803
Giac [F(-2)] 805
Mupad [F(-1)] 805

Optimal result

Integrand size = 30, antiderivative size = 215

$$\begin{aligned}\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx &= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &- \frac{4c^4\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &+ \frac{c^4\sec(e+fx)\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &- \frac{8c^4\tan(e+fx)}{af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

```
[Out] c^4*ln(cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-4*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(
f*x+e))^(1/2)+c^4*sec(f*x+e)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec
(f*x+e))^(1/2)-8*c^4*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(
c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used

$= \{3997, 90\}$

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c^4\tan(e+fx)\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{8c^4\tan(e+fx)}{af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{4c^4\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{c^4\tan(e+fx)\log(\cos(e+fx))}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (4*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a
+ a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (c^4*Sec[e + f*x]*Tan[e + f*x
])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (8*c^4*Tan[e +
 f*x])/(a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e +
f*x]])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(-\frac{c^3}{a^2}+\frac{c^3}{a^2x}-\frac{8c^3}{a^2(1+x)^2}+\frac{4c^3}{a^2(1+x)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$
\begin{aligned}
&= \frac{c^4 \log(\cos(e+fx)) \tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{4c^4 \log(1+\sec(e+fx)) \tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{c^4 \sec(e+fx) \tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{8c^4 \tan(e+fx)}{af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.45

$$
\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-c^3\log(\cos(e+fx)) + 4c^3\log(1+\sec(e+fx)) - c^3\sec(e+fx) + \frac{8c^3}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((c*(-(c^3*Log[Cos[e + f*x]]) + 4*c^3*Log[1 + Sec[e + f*x]] - c^3*Sec[e +
f*x] + (8*c^3)/(1 + Sec[e + f*x]))*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e + f
*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.08

method	result
default	$-\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^3 c^3 \sqrt{a(\sec(fx+e)+1)}\,\left(5\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)-1)+5\cos(fx+e)^2\ln(-\cot(\right.}{}$
risch	$\frac{c^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(8i\ln(e^{i(fx+e)}+1)e^{4i(fx+e)}-18ie^{i(fx+e)}-fx-2e^{3i(fx+e)}fx-2e-2e^{4i(fx+e)}e-4e^{3i(fx+e)}e-4e^{2i(fx+e)}e-2e^{2}}{}$

```
[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/a^2*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^3*c^3*(a*(sec(f*x+e)+1))^
(1/2)*(5*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)-1)+5*cos(f*x+e)^2*ln(-cot(f
*x+e)+csc(f*x+e)+1)-cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-3*cos(f*x+e)^2+5*cos(
f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)-1)+5*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+
```

```
1)-cos(f*x+e)*ln(2/(cos(f*x+e)+1))+6*cos(f*x+e)+1)/(cos(f*x+e)-1)^2*cot(f*x
+e)^3
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{7}{2}}}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) -
 c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^
2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2393 vs. $2(199) = 398$.

Time = 0.53 (sec) , antiderivative size = 2393, normalized size of antiderivative = 11.13

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c^3*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*cos(2*f*x + 2*e)^2 + (
f*x + e)*c^3*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 - 4*c^3*sin(2*f*x + 2*e) + 4*(
(f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c
^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*
```

```
x + 2*e), cos(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 5*(c
^3*cos(4*f*x + 4*e)^2 + 4*c^3*cos(2*f*x + 2*e)^2 + c^3*sin(4*f*x + 4*e)^2 +
 4*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^3*sin(2*f*x + 2*e)^2 + 4*c^3
*cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x + 4*e)
)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 8*(c^3*cos(4*f*x + 4*e)
^2 + 4*c^3*cos(2*f*x + 2*e)^2 + 4*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 4*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + c^3*sin(4*f*x + 4*e)^2 + 4*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
4*c^3*sin(2*f*x + 2*e)^2 + 4*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 4*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 4*c^3*cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*
x + 4*e) + 4*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x + 2*e) + 2*c^3*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x
+ 2*e) + c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^3
*sin(4*f*x + 4*e) + 2*c^3*sin(2*f*x + 2*e) + 2*c^3*sin(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 4*(c^3*sin(4*f*x + 4*e) + 2*c^3*sin(2*f*x + 2*e))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*arctan2(sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 1) + 2*(2*(f*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 - 2*c^3*sin(
2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(2*(f*x + e)*c^3*cos(4*f*x + 4*e) + 4*(f
*x + e)*c^3*cos(2*f*x + 2*e) + 2*(f*x + e)*c^3 + 9*c^3*sin(4*f*x + 4*e) + 1
4*c^3*sin(2*f*x + 2*e) - 10*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x + 2*e)
+ c^3)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 4*((f*x + e)*c^3 -
 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 2*(2*(f*x + e)*c^3*cos(4*f*x + 4*e) + 4*(f*x + e)*c^3*cos(2*
f*x + 2*e) + 2*(f*x + e)*c^3 + 9*c^3*sin(4*f*x + 4*e) + 14*c^3*sin(2*f*x +
2*e) - 10*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x + 2*e) + c^3)*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 4*((f*x + e)*c^3*sin(2*f*x + 2*e) + c^3*cos(2*f*x + 2*e))
*sin(4*f*x + 4*e) + 2*(2*(f*x + e)*c^3*sin(4*f*x + 4*e) + 4*(f*x + e)*c^3*s
in(2*f*x + 2*e) - 9*c^3*cos(4*f*x + 4*e) - 14*c^3*cos(2*f*x + 2*e) - 9*c^3
- 10*(c^3*sin(4*f*x + 4*e) + 2*c^3*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1) + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)
*c^3*sin(4*f*x + 4*e) + 4*(f*x + e)*c^3*sin(2*f*x + 2*e) - 9*c^3*cos(4*f*x
+ 4*e) - 14*c^3*cos(2*f*x + 2*e) - 9*c^3 - 10*(c^3*sin(4*f*x + 4*e) + 2*c^3
*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(c)/((a*cos(4*f*x + 4*e)
^2 + 4*a*cos(2*f*x + 2*e)^2 + 4*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
```

```
*x + 2*e)))^2 + 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ a*sin(4*f*x + 4*e)^2 + 4*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a*sin(2*
f*x + 2*e)^2 + 4*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 4*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(2*a*cos(2*
f*x + 2*e) + a)*cos(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + 4*(a*cos(4*f*x +
4*e) + 2*a*cos(2*f*x + 2*e) + 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + a)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(
a*cos(4*f*x + 4*e) + 2*a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 4*(a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e) + 2
*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x
+ 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sqrt(a)*f
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.118 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result 806
Rubi [A] (verified) 806
Mathematica [A] (verified) 807
Maple [B] (verified) 808
Fricas [B] (verification not implemented) 808
Sympy [F(-1)] 809
Maxima [F(-2)] 809
Giac [F(-2)] 809
Mupad [F(-1)] 810

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^3\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -4*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^3*ln(co
s(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3995, 3990, 3556}

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c^3\tan(e+fx)\log(\cos(e+fx))}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-4*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} \\
&\quad+\frac{c^2\int\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx}{a^2} \\
&= -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{(c^3\tan(e+fx))\int\tan(e+fx)\,dx}{a\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^3\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int\frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{c^3(-4+\log(\cos(e+fx))+\log(\cos(e+fx))\sec(e+fx))\tan(e+fx)}{f(a(1+\sec(e+fx)))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2),x]
[Out] (c^3*(-4 + Log[Cos[e + f*x]] + Log[Cos[e + f*x]]*Sec[e + f*x])*Tan[e + f*x]
)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(88) = 176$.

Time = 1.96 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.28

method	result
default	$\frac{\sqrt{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^3\left(\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)^{\frac{5}{2}}\sin(fx+e)^5\left(2(1-\cos(fx+e\right.}{2f\,a^2(1-\cos(fx-}$
risch	$-\frac{c^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(ie^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+e^{2i(fx+e)}fx+2\,e^{2i(fx+e)}e+2ie^{i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+2\,e^{i(fx+e)}fx+8ie^{i(fx+e}\right.}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f}$

```
[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*2^(1/2)/a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^3*(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)*csc(f*x+e)^2)^(5/2)/(1-cos(f*x+e))^5*sin(f*x+e)^5*(2*(1-cos(f*x+e))^2*
csc(f*x+e)^2+ln(-cot(f*x+e)+csc(f*x+e)+1)-ln((1-cos(f*x+e))^2*csc(f*x+e)^2+
1)+ln(-cot(f*x+e)+csc(f*x+e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(88) = 176$.

Time = 0.34 (sec) , antiderivative size = 453, normalized size of antiderivative = 4.72

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\left(ac^2\cos(fx+e\right.}{2\left(a^2\right.}\right.$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
 c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a*c
^2*cos(f*x + e) + a*c^2)*sqrt(-c/a)*log(1/2*(c*cos(f*x + e)^4 - (cos(f*x +
e)^3 + cos(f*x + e))*sqrt(-c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
t((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + c)/cos(f*x + e)^2))/(a^
2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*c^2*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqr
t(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(c*cos(f*x + e)^2 +
c)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.119 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result 811
Rubi [A] (verified) 811
Mathematica [A] (verified) 812
Maple [A] (verified) 813
Fricas [F] 813
Sympy [F] 813
Maxima [A] (verification not implemented) 814
Giac [A] (verification not implemented) 814
Mupad [F(-1)] 814

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^2*ln(1+
cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3993, 3996, 31}

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c^2\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^2*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3993

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c\int\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}\\
&= -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{(c^2\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{a+ax}\,dx,x,\cos(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^2\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$
\int\frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=
-\frac{c\left(-c\log(\cos(e+fx))-c\log(1+\sec(e+fx))+\frac{2c}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((c*(-(c*Log[Cos[e + f*x]]) - c*Log[1 + Sec[e + f*x]] + (2*c)/(1 + Sec[e +
 f*x]))*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*
x]]))
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

method	result
default	$\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)c\sqrt{a(\sec(fx+e)+1)}\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos(fx+e)+\ln\left(\frac{2}{\cos(fx+e)+1}\right)-1\right)\cot(fx+e}{f\,a^2}$
risch	$-\frac{c\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(2ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+e^{2i(fx+e)}fx+2\,e^{2i(fx+e)}e+4ie^{i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+2\,e^{i(fx+e)}fx+4ie^{i(fx+e}\right.}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f}$

```
[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/f/a^2*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)*c*(a*(sec(f*x+e)+1))^(1/2)
*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))+cos(f*x+e)+ln(2/(cos(f*x+e)+1))-1)*cot(f*
x+e)^2*csc(f*x+e)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec\left(fx+e\right)+c)^{\frac{3}{2}}}{(a\sec\left(fx+e\right)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")

[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^2*sec(f*x
 + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c(\sec\left(e+fx\right)-1))^{\frac{3}{2}}}{(a\left(\sec\left(e+fx\right)+1\right))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)

[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{(c - c\sec(e + fx))^{3/2}}{(a + a\sec(e + fx))^{3/2}}\,dx = \frac{\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{-a}a} - \frac{c^{\frac{3}{2}}\sin(fx+e)^2}{\sqrt{-a}a(\cos(fx+e)+1)^2}}{f}$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
[Out] (c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a) - c^(3/2
)*sin(f*x + e)^2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f
```

Giac [A] (verification not implemented)

none

Time = 1.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{(c - c\sec(e + fx))^{3/2}}{(a + a\sec(e + fx))^{3/2}}\,dx = -\frac{\frac{\sqrt{-ac}c^2\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^2|c|} - \frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}c}{a^2|c|}}{f}$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
[Out] -(sqrt(-a*c)*c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^2*abs(c)) - (c*t
an(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*c/(a^2*abs(c)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{3/2}}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.120 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result 815
Rubi [A] (verified) 815
Mathematica [A] (verified) 816
Maple [A] (verified) 817
Fricas [F] 817
Sympy [F] 817
Maxima [B] (verification not implemented) 818
Giac [A] (verification not implemented) 818
Mupad [F(-1)] 819

Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c*ln(1+cos(f*
x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3992, 3996, 31}

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]))
 + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\int\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}\\ &= -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\\ &\quad+\frac{(c\tan(e+fx))\text{Subst}\left(\int\frac{1}{a+ax}\,dx,x,\cos(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int\frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-\log(\cos(e+fx))-\log(1+\sec(e+fx))+\frac{1}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2),x]
[Out] -((c*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + (1 + Sec[e + f*x])^(-1))
*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result
default	$\frac{\left(2\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos(fx+e)-1\right)\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,\cot(fx+e)}{2f\,a^{2}(\cos(fx+e)+1)}$
risch	$-\frac{\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,\left(2i\mathrm{e}^{2i(fx+e)}\ln\left(\mathrm{e}^{i(fx+e)}+1\right)+\mathrm{e}^{2i(fx+e)}fx+4i\mathrm{e}^{i(fx+e)}\ln\left(\mathrm{e}^{i(fx+e)}+1\right)+2\,\mathrm{e}^{2i(fx+e)}e+2\,\mathrm{e}^{i(fx+e)}fx+2i\ln\left(\mathrm{e}^{i(fx}\cdots\right.\right.}{a\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,\left(\mathrm{e}^{i(fx+e)}-1\right)f}$

```
[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/a^2*(2*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+2*ln(2/(cos(f*x+e)+1))+cos(f*x
+e)-1)*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)/(cos(f*x+e)+1)*co
t(f*x+e)
```

Fricas [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-c\sec\left(fx+e\right)+c}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x +
e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(86) = 172$.

Time = 0.37 (sec) , antiderivative size = 395, normalized size of antiderivative = 4.20

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{3/2}}\,dx = -\frac{\left((fx+e)\cos\left(2\,fx+2\,e\right)^2+4\,(fx+e)\cos\left(fx+e\right)^2+(fx+e)\sin\left(2\,fx+2\,e\right)^2+4\,(fx+e)\sin\left(fx+e\right)^2\right.}{}$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin
(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(2*cos(f*x + e) +
 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f*x + e)^2 + sin(2*f*x +
2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f*x + e)^2 + 4*cos(f*x + e
) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 2*(f*x + e)*cos(f
*x + e) + e - sin(f*x + e))*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + 2
*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e - 2*sin(f*x
 + e))*sqrt(a)*sqrt(c)/((a^2*cos(2*f*x + 2*e)^2 + 4*a^2*cos(f*x + e)^2 + a^
2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a^2*sin(f*x
+ e)^2 + 4*a^2*cos(f*x + e) + a^2 + 2*(2*a^2*cos(f*x + e) + a^2)*cos(2*f*x
+ 2*e))*f)
```

Giac [A] (verification not implemented)

none

Time = 1.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{2\sqrt{2}\sqrt{-ac}c\log\left(\left|-2\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,c\right|\right)}{a^2|c|}-\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\sqrt{-ac}}{a^2|c|}\right)}{4\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -1/4*sqrt(2)*(2*sqrt(2)*sqrt(-a*c)*c*log(abs(-2*c*tan(1/2*f*x + 1/2*e)^2 -
2*c))/(a^2*abs(c)) - sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)/(a^2
*abs(c)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.121 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result 820
Rubi [A] (verified) 821
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Optimal result

Integrand size = 30, antiderivative size = 215

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1-\sec(e+fx))\tan(e+fx)}{4af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{3\log(1+\sec(e+fx))\tan(e+fx)}{4af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] ln(cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
+1/4*ln(1-sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+3/4*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec
(f*x+e))^(1/2)-1/2*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-
c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$\int \frac{1}{(a + a\sec(e + fx))^{3/2}\sqrt{c - c\sec(e + fx)}}\,dx = -\frac{\tan(e + fx)}{2af(\sec(e + fx) + 1)\sqrt{a\sec(e + fx) + a}\sqrt{c - c\sec(e + fx)}} + \frac{\tan(e + fx)\log(1 - \sec(e + fx))}{4af\sqrt{a\sec(e + fx) + a}\sqrt{c - c\sec(e + fx)}} + \frac{3\tan(e + fx)\log(\sec(e + fx) + 1)}{4af\sqrt{a\sec(e + fx) + a}\sqrt{c - c\sec(e + fx)}} + \frac{\tan(e + fx)\log(\cos(e + fx))}{af\sqrt{a\sec(e + fx) + a}\sqrt{c - c\sec(e + fx)}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*S
ec[e + f*x]]) + (Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(4*a*f*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (3*Log[1 + Sec[e + f*x]]*Tan[e + f*x]
)/(4*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/
(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e + fx))\text{Subst}\left(\int \frac{1}{x(a+ax)^2(c-cx)}\,dx, x, \sec(e + fx)\right)}{f\sqrt{a + a\sec(e + fx)}\sqrt{c - c\sec(e + fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int\left(-\frac{1}{4a^2c(-1+x)}+\frac{1}{a^2cx}-\frac{1}{2a^2c(1+x)^2}-\frac{3}{4a^2c(1+x)}\right)dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1-\sec(e+fx))\tan(e+fx)}{4af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{3\log(1+\sec(e+fx))\tan(e+fx)}{4af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{c\left(-\frac{\log(\cos(e+fx))}{c}-\frac{\log(1-\sec(e+fx))}{4c}-\frac{3\log(1+\sec(e+fx))}{4c}+\frac{1}{2c(1+\sec(e+fx))}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -((c*(-(Log[Cos[e + f*x]]/c) - Log[1 - Sec[e + f*x]]/(4*c) - (3*Log[1 + Sec
[e + f*x]])/(4*c) + 1/(2*c*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a*f*Sqrt[a*(
1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.63

method	result
default	$-\frac{\sin(fx+e)\left(4\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))+4\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\ln(-\cot(fx+e)+\csc(f\right.}{4fa^2(\cos(fx+e)+1)^2\sqrt{-c(\sec(fx+e)-1)}}$
risch	$-\frac{-ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-2ie^{i(fx+e)}+2\,e^{3i(fx+e)}fx+4\,e^{3i(fx+e)}e+3i\ln\left(e^{i(fx+e)}+1\right)e^{3i(fx+e)}+2ie^{2i(fx+e)}+2\,e^{2i(fx+e)}fx+4\,e^{}}{2}$

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f/a^2*sin(f*x+e)*(4*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)*ln(-c
ot(f*x+e)+csc(f*x+e))+4*ln(2/(cos(f*x+e)+1))-2*ln(-cot(f*x+e)+csc(f*x+e))+c
```

```
os(f*x+e)-1)*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)^2/(-c*(sec(f*x+e)-1))^
(1/2)
```

Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c*sec(f*x
 + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e) - a^2*c), x)
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{1}{(a(\sec(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(193) = 386$.

Time = 0.39 (sec) , antiderivative size = 818, normalized size of antiderivative = 3.80

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="max
ima")
[Out] -1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x +
e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - 3*(cos(
2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e),
```

```
cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 1) - (cos(2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) + 2*(4*f*x + 4*(f*x + e)*cos(
2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e) - 1)*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*e)/((a*cos(2*f*x +
 2*e)^2 + 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*si
n(2*f*x + 2*e)^2 + 4*a*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 2*a*cos(2*f*x + 2*e) + 4*(a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sqrt(a)*sqrt(c)*f)
```

Giac [A] (verification not implemented)

none

Time = 1.86 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.46

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}\,a|c|} - \frac{\sqrt{-ac}\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{a^2|c|} + \frac{4\sqrt{-ac}\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^2|c|}}{4f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/4*((c*tan(1/2*f*x + 1/2*e)^2 - c)/(sqrt(-a*c)*a*abs(c)) - sqrt(-a*c)*log
(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(a^2*abs(c)) + 4*sqrt(-a*c)*log(abs(c*tan(1
/2*f*x + 1/2*e)^2 + c))/(a^2*abs(c)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.122 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}} \, dx$

Optimal result 825
Rubi [A] (verified) 825
Mathematica [A] (verified) 826
Maple [A] (verified) 827
Fricas [B] (verification not implemented) 827
Sympy [F] 828
Maxima [B] (verification not implemented) 828
Giac [A] (verification not implemented) 829
Mupad [F(-1)] 829

Optimal result

Integrand size = 30, antiderivative size = 101

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}} \, dx = \frac{\cot(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(\sin(e+fx))\tan(e+fx)}{acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*cot(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+ln(sin(f
*x+e))*tan(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3990, 3554, 3556}

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}} \, dx = \frac{\cot(e+fx)}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\log(\sin(e+fx))}{acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] Cot[e + f*x]/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) +
(Log[Sin[e + f*x]]*Tan[e + f*x])/(a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\tan(e+fx)\int\cot^3(e+fx)\,dx}{ac\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{\cot(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\int\cot(e+fx)\,dx}{ac\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{\cot(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\log(\sin(e+fx))\tan(e+fx)}{acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$
\int\frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\cot(e+fx)+2(\log(\cos(e+fx))+\log(\tan(e+fx)))\,\text{ta}}{2acf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
[Out] (Cot[e + f*x] + 2*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Tan[e + f*x])/(2*
a*c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.55

method	result
default	$-\frac{\left(4\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e))-4\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx+e)+1}\right)-\cos(fx+e)^2-4\ln(-\cot(fx+e)+\csc(fx+e))+4\ln\left(\right.\right.}{4f\,a^2\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx+e)-1)(\cos(fx+e)+1)^2}$
risch	$-\frac{i\mathrm{e}^{4i(fx+e)}\ln\left(\mathrm{e}^{2i(fx+e)}-1\right)+\mathrm{e}^{4i(fx+e)}fx+2\,\mathrm{e}^{4i(fx+e)}e-2i\mathrm{e}^{2i(fx+e)}\ln\left(\mathrm{e}^{2i(fx+e)}-1\right)-2\,\mathrm{e}^{2i(fx+e)}fx-4\,\mathrm{e}^{2i(fx+e)}e-2i\mathrm{e}^{2i(fx+e)}+}{ac\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\left(1+\mathrm{e}^{2i(fx+e)}\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,f}$

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f/a^2*(4*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e))-4*cos(f*x+e)^2*ln(2/(
cos(f*x+e)+1))-cos(f*x+e)^2-4*ln(-cot(f*x+e)+csc(f*x+e))+4*ln(2/(cos(f*x+e)
+1))-1)*(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f*x+e)-1)
/(cos(f*x+e)+1)^2*tan(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(91) = 182.

Time = 0.49 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.87

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{9\sqrt{-ac}\left(\cos(fx+e)^2-1\right)\log\left(-\frac{8\left(\left(256\cos(fx+e)^5-5\right.\right.}{}\right.}{}\right.$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/18*(9*sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-8*((256*cos(f*x + e)^5 - 512
*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x + e)
^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*
sin(f*x + e)))*sin(f*x + e) + (16*cos(f*x + e)^3 - 25*cos(f*x + e))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(
(a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e)), -1/18*(18*sqrt(a*c)*(
cos(f*x + e)^2 - 1)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(a*c)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))*sin(f*x + e) + (16*cos
(f*x + e)^3 - 25*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f
)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{(a\left(\sec\left(e+fx\right)+1\right))^{\frac{3}{2}}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1))**(3/2)),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(91) = 182$.

Time = 0.38 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.81

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\left((fx+e)\cos\left(4\,fx+4\,e\right)^2+4\,(fx+e)\cos\left(2\,fx+2\,e\right)^2+(fx+e)\sin\left(4\,fx+4\,e\right)^2+4\,(fx+e)\sin\left(2\,fx\right.\right.}{}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="max
ima")
[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e)
*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(2*cos(2*f*
x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*cos(2*f*x + 2*e)^2
- sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - 4*sin(2*f*x +
2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 - 1) + 2*(f*x - 2*(f*x + e)*cos(2*f*x + 2*e) + e + sin(2*f*x + 2*e))*cos(4
*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(2*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + e + 2*sin(2*f*x + 2*e))*sqrt(a)*sq
rt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2
*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*c
^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2 - 2*(2*a^2*c^2
*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x + 4*e))*f)
```

Giac [A] (verification not implemented)

none

Time = 2.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\frac{4\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}a|c|} - \frac{8\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}a|c|} + \frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}ac|c|} - \frac{4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}ac|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{8f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] -1/8*(4*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a*abs(c)) - 8*log(ab
s(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*a*abs(c)) + (c*tan(1/2*f*x + 1
/2*e)^2 - c)/(sqrt(-a*c)*a*c*abs(c)) - (4*c*tan(1/2*f*x + 1/2*e)^2 - c)/(sq
rt(-a*c)*a*c*abs(c)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+fx)}\right)^{3/2}\left(c-\frac{c}{\cos(e+fx)}\right)^{3/2}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.123 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result 830
Rubi [A] (verified) 831
Mathematica [A] (verified) 832
Maple [A] (warning: unable to verify) 833
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Maxima [B] (verification not implemented) 834
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Optimal result

Integrand size = 30, antiderivative size = 347

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\log(\cos(e+fx))\tan(e+fx)}{ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{11\log(1-\sec(e+fx))\tan(e+fx)}{16ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{5\log(1+\sec(e+fx))\tan(e+fx)}{16ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(1-\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2ac^2f(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] ln(cos(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)+11/16*ln(1-sec(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*
sec(f*x+e))^(1/2)+5/16*ln(1+sec(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))
^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a/c^2/f/(1-sec(f*x+e))^2/(a+a*
sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*tan(f*x+e)/a/c^2/f/(1-sec(f*x+
e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a/c^2/f/(1
+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3997, 90}

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)}{2ac^2f(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(1-\sec(e+fx))^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{11\tan(e+fx)\log(1-\sec(e+fx))}{16ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{5\tan(e+fx)\log(\sec(e+fx)+1)}{16ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\log(\cos(e+fx))}{ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) + (11*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(16*a*c^2*f*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (5*Log[1 + Sec[e + f*x]]*
Tan[e + f*x])/(16*a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Tan[e + f*x]/(8*a*c^2*f*(1 - Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(2*a*c^2*f*(1 - Sec[e + f*x])*Sqrt[
a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(8*a*c^2*f*(1
+ Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{x(a+ax)^2(c-cx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(-\frac{1}{4a^2c^3(-1+x)^3}+\frac{1}{2a^2c^3(-1+x)^2}-\frac{11}{16a^2c^3(-1+x)}+\frac{1}{a^2c^3x}-\frac{1}{8a^2c^3(1+x)^2}-\frac{}{16a^2 \cdots}\right.\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{11\log(1-\sec(e+fx))\tan(e+fx)}{16ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{5\log(1+\sec(e+fx))\tan(e+fx)}{16ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(1-\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2ac^2f(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\left(16\log(\cos(e+fx))+11\log(1-\sec(e+fx))+5\log(1 \cdots\right.}{16ac^2f\sqrt{a(1+\cdots}}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
[Out] ((16*Log[Cos[e + f*x]] + 11*Log[1 - Sec[e + f*x]] + 5*Log[1 + Sec[e + f*x]]
 - 2/(-1 + Sec[e + f*x])^2 + 8/(-1 + Sec[e + f*x]) - 2/(1 + Sec[e + f*x]))*
Tan[e + f*x])/(16*a*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x
]])
```

Maple [A] (warning: unable to verify)

Time = 2.00 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\sqrt{2}\,\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(-2(1-\cos(fx+e))^6\csc(fx+e)^6+32\ln\left((1-\cos(fx+e))^2\csc(fx+e)^2+1\right)(1-\right.}{64 f a^2\left((1-\cos(fx+e))^2\csc(fx+e\right.}$
risch	$\frac{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)x}{a c^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}}-\frac{2(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)(fx+e)}{a c^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}\,f}+\frac{}{4a c^2 (e^{i(f}}$

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/64/f*2^(1/2)/a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e
)^2-1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(-2*(1-cos(f*x+e))^6*csc(f*x+e)^6
+32*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*x+e))^4*csc(f*x+e)^4-44*ln
(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*x+e))^4*csc(f*x+e)^4-10*(1-cos(f*x+e))^2*
csc(f*x+e)^2+1)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c^3*sec(f
*x + e)^5 - a^2*c^3*sec(f*x + e)^4 - 2*a^2*c^3*sec(f*x + e)^3 + 2*a^2*c^3*s
ec(f*x + e)^2 + a^2*c^3*sec(f*x + e) - a^2*c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4272 vs. $2(309) = 618$.

Time = 1.88 (sec) , antiderivative size = 4272, normalized size of antiderivative = 12.31

$$\int \frac{1}{(a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{5/2}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] -1/8*(8*(f*x + e)*cos(6*f*x + 6*e)^2 + 8*(f*x + e)*cos(4*f*x + 4*e)^2 + 8*(
f*x + e)*cos(2*f*x + 2*e)^2 + 32*(f*x + e)*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 32*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 8*(f*x + e)*sin(6*f*x + 6*e)^2 + 8*(f*x + e)*sin(4*f*x + 4*
e)^2 + 8*(f*x + e)*sin(2*f*x + 2*e)^2 + 32*(f*x + e)*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 8*f*x + 5*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
 - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos(
4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 + 4*(cos(6*f*x + 6*e
) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 4*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(6*f*x + 6*e) - cos(4
*f*x + 4*e) - cos(2*f*x + 2*e) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16
*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(cos(6*f*x + 6*
e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6*
f*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - s
in(2*f*x + 2*e)^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*
e) + 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 - 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) - 2*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*sin
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 1) + 11*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
```

- 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos(4
*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 + 4*(cos(6*f*x + 6*e)
 - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 4*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(6*f*x + 6*e) - cos(4*
f*x + 4*e) - cos(2*f*x + 2*e) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(cos(6*f*x + 6*e
) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6*f
*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - si
n(2*f*x + 2*e)^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e
) + 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 - 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) - 2*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 1) + 4*(4*f*x - 4*(f*x + e)*cos(4*f*x + 4*e) - 4*
(f*x + e)*cos(2*f*x + 2*e) + 4*e + 3*sin(4*f*x + 4*e) + 3*sin(2*f*x + 2*e))
*cos(6*f*x + 6*e) - 16*(f*x - (f*x + e)*cos(2*f*x + 2*e) + e)*cos(4*f*x + 4
e) - 16(f*x + e)*cos(2*f*x + 2*e) - 2*(16*f*x + 16*(f*x + e)*cos(6*f*x +
6*e) - 16*(f*x + e)*cos(4*f*x + 4*e) - 16*(f*x + e)*cos(2*f*x + 2*e) + 64*(
f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*(f*x + e
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(6*f*x
 + 6*e) + 7*sin(4*f*x + 4*e) + 7*sin(2*f*x + 2*e) - 8*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 4*(16*f*x + 16*(f*x + e)*cos(6*f*x + 6*e) - 16*(f*x + e)*cos(4
*f*x + 4*e) - 16*(f*x + e)*cos(2*f*x + 2*e) - 32*(f*x + e)*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 7*sin(6*f*x + 6*e) + 5*sin(4*
f*x + 4*e) + 5*sin(2*f*x + 2*e) - 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(16
*f*x + 16*(f*x + e)*cos(6*f*x + 6*e) - 16*(f*x + e)*cos(4*f*x + 4*e) - 16*(
f*x + e)*cos(2*f*x + 2*e) + 16*e + 5*sin(6*f*x + 6*e) + 7*sin(4*f*x + 4*e)
+ 7*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 4*(4*(f*x + e)*sin(4*f*x + 4*e) + 4*(f*x + e)*sin(2*f*x + 2*e) + 3*cos(4*
f*x + 4*e) + 3*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(4*(f*x + e)*sin(2*f*
x + 2*e) + 3)*sin(4*f*x + 4*e) - 2*(16*(f*x + e)*sin(6*f*x + 6*e) - 16*(f*x
 + e)*sin(4*f*x + 4*e) - 16*(f*x + e)*sin(2*f*x + 2*e) + 64*(f*x + e)*sin(3

```
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*(f*x + e)*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*cos(6*f*x + 6*e) - 7*cos(4*f*x
 + 4*e) - 7*cos(2*f*x + 2*e) + 8*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 5)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(1
6*(f*x + e)*sin(6*f*x + 6*e) - 16*(f*x + e)*sin(4*f*x + 4*e) - 16*(f*x + e)
*sin(2*f*x + 2*e) - 32*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 7*cos(6*f*x + 6*e) - 5*cos(4*f*x + 4*e) - 5*cos(2*f*x + 2*e) +
 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 7)*sin(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(16*(f*x + e)*sin(6*f*x + 6*e) -
 16*(f*x + e)*sin(4*f*x + 4*e) - 16*(f*x + e)*sin(2*f*x + 2*e) - 5*cos(6*f*
x + 6*e) - 7*cos(4*f*x + 4*e) - 7*cos(2*f*x + 2*e) - 5)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*e + 12*sin(2*f*x + 2*e))/((a*c^2*cos(
6*f*x + 6*e)^2 + a*c^2*cos(4*f*x + 4*e)^2 + a*c^2*cos(2*f*x + 2*e)^2 + 4*a*
c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a*c^2*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a*c^2*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*c^2*sin(6*f*x + 6*e)^2 + a*c^2*
sin(4*f*x + 4*e)^2 + 2*a*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + a*c^2*sin(
2*f*x + 2*e)^2 + 4*a*c^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 16*a*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*
a*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*a*c^2*cos(
2*f*x + 2*e) + a*c^2 - 2*(a*c^2*cos(4*f*x + 4*e) + a*c^2*cos(2*f*x + 2*e) -
 a*c^2)*cos(6*f*x + 6*e) + 2*(a*c^2*cos(2*f*x + 2*e) - a*c^2)*cos(4*f*x + 4
*e) - 4*(a*c^2*cos(6*f*x + 6*e) - a*c^2*cos(4*f*x + 4*e) - a*c^2*cos(2*f*x
+ 2*e) + 4*a*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*a
*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a*c^2)*cos(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a*c^2*cos(6*f*x + 6*e) -
a*c^2*cos(4*f*x + 4*e) - a*c^2*cos(2*f*x + 2*e) - 2*a*c^2*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + a*c^2)*cos(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) - 4*(a*c^2*cos(6*f*x + 6*e) - a*c^2*cos(4*f*x + 4*e)
 - a*c^2*cos(2*f*x + 2*e) + a*c^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 2*(a*c^2*sin(4*f*x + 4*e) + a*c^2*sin(2*f*x + 2*e))*sin(6*f*
x + 6*e) - 4*(a*c^2*sin(6*f*x + 6*e) - a*c^2*sin(4*f*x + 4*e) - a*c^2*sin(2
*f*x + 2*e) + 4*a*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 2*a*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a*c^2*sin(6*f*x + 6*e) - a*c
^2*sin(4*f*x + 4*e) - a*c^2*sin(2*f*x + 2*e) - 2*a*c^2*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 4*(a*c^2*sin(6*f*x + 6*e) - a*c^2*sin(4*f*x + 4*e) - a*c^2*si
n(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(
a)*sqrt(c)*f)
```

Giac [A] (verification not implemented)

none

Time = 1.99 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{22\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}ac|c|}-\frac{32\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}ac|c|}+\frac{2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{\sqrt{-ac}ac^2|c|}-\frac{33\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+56\left(c\tan\left(\frac{1}{2}fx+\right.\right.}{\sqrt{-ac}ac^3|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}}{32f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] -1/32*(22*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a*c*abs(c)) - 32*l
og(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*a*c*abs(c)) + 2*(c*tan(1/
2*f*x + 1/2*e)^2 - c)/(sqrt(-a*c)*a*c^2*abs(c)) - (33*(c*tan(1/2*f*x + 1/2*
e)^2 - c)^2 + 56*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 24*c^2)/(sqrt(-a*c)*a*c
^3*abs(c)*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.124 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result 838
Rubi [A] (verified) 838
Mathematica [A] (verified) 840
Maple [A] (warning: unable to verify) 840
Fricas [F] 841
Sympy [F(-1)] 841
Maxima [F(-2)] 841
Giac [A] (verification not implemented) 842
Mupad [F(-1)] 842

Optimal result

Integrand size = 30, antiderivative size = 220

$$\begin{aligned}\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx &= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &+ \frac{2c^4\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &- \frac{4c^4\tan(e+fx)}{a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &+ \frac{4c^4\tan(e+fx)}{a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

```
[Out] c^4*ln(cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+2*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*
sec(f*x+e))^(1/2)-4*c^4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^
(1/2)/(c-c*sec(f*x+e))^(1/2)+4*c^4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec
(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used

$= \{3997, 90\}$

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{4c^4\tan(e+fx)}{a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{4c^4\tan(e+fx)}{a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{2c^4\tan(e+fx)\log(\sec(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{c^4\tan(e+fx)\log(\cos(e+fx))}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (2*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (4*c^4*Tan[e + f*x])/(a^2
*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
+ (4*c^4*Tan[e + f*x])/(a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(\frac{c^3}{a^3x} - \frac{8c^3}{a^3(1+x)^3} + \frac{4c^3}{a^3(1+x)^2} - \frac{2c^3}{a^3(1+x)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$
\begin{aligned}
&= \frac{c^4 \log(\cos(e+fx)) \tan(e+fx)}{a^2 f \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{2c^4 \log(1+\sec(e+fx)) \tan(e+fx)}{a^2 f \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{4c^4 \tan(e+fx)}{a^2 f (1+\sec(e+fx))^2 \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{4c^4 \tan(e+fx)}{a^2 f (1+\sec(e+fx)) \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.39

$$
\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^4\left(-\log(\cos(e+fx)) - 2\log(1+\sec(e+fx)) - \frac{4\sec(e+fx)}{(1+\sec(e+fx))^2}\right)\tan(e+fx)}{a^2 f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((c^4*(-Log[Cos[e + f*x]] - 2*Log[1 + Sec[e + f*x]] - (4*Sec[e + f*x])/(1
+ Sec[e + f*x])^2)*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
 c*Sec[e + f*x]]))
```

Maple [A] (warning: unable to verify)

Time = 2.36 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\sqrt{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^4\left(\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)^{\frac{7}{2}}\sin(fx+e)^7\left((1-\cos(fx+\right.}{2f\,a^3(1-\cos(f}$
risch	$-\frac{c^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(-4i\ln\left(1+e^{2i(fx+e)}\right)e^{3i(fx+e)}+8ie^{i(fx+e)}+e^{4i(fx+e)}fx+2\,e^{4i(fx+e)}e-4ie^{i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)+16i\ln(e\right.}{}$

```
[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*2^(1/2)/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^4*(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)*csc(f*x+e)^2)^(7/2)/(1-cos(f*x+e))^7*sin(f*x+e)^7*((1-cos(f*x+e))^4*c
```

```
sc(f*x+e)^4+ln(-cot(f*x+e)+csc(f*x+e)+1)+ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1
)+ln(-cot(f*x+e)+csc(f*x+e)-1))
```

Fricas [F]

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{(-c\sec(fx + e) + c)^{\frac{7}{2}}}{(a\sec(fx + e) + a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) -
 c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^
3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [A] (verification not implemented)

none

Time = 1.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.42

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\left(\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2\sqrt{-ac}a^2|c|+2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\sqrt{-ac}a^2c|c|\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\right.}{a^5f}$$

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*abs(c) + 2*(c*tan(1/2*f*x
 + 1/2*e)^2 - c)*sqrt(-a*c)*a^2*c*abs(c))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(
1/2*f*x + 1/2*e))/(a^5*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.125 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result 843
Rubi [A] (verified) 843
Mathematica [A] (verified) 844
Maple [A] (verified) 845
Fricas [F] 845
Sympy [F(-1)] 846
Maxima [A] (verification not implemented) 846
Giac [A] (verification not implemented) 846
Mupad [F(-1)] 847

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^3\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)+c^3*ln(1+
cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3995, 3996, 31}

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c^3\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^3*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^2\int\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a^2}\\
&= -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{\left(c^3\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{a+ax}\,dx,x,\cos(e+fx)\right)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^3\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int\frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^3\left(-\log(\cos(e+fx))-\log(1+\sec(e+fx))+\frac{2}{(1+\sec(e+fx))^2}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

[Out] -((c^3*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + 2/(1 + Sec[e + f*x])^2
)*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
)

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.45

method	result
default	$\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^2c^2\sqrt{a(\sec(fx+e)+1)}\left(2\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+3\cos(fx+e)^2+4\cos(fx+e)\ln\left(\frac{2}{\cos(fx+}\right.\right.}{2f\,a^3}$
risch	$-\frac{c^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(8ie^{i(fx+e)}+e^{4i(fx+e)}fx+8ie^{i(fx+e)}\ln(e^{i(fx+e)}+1)+2\,e^{4i(fx+e)}e+4\,e^{3i(fx+e)}fx+8ie^{2i(fx+e)}+8\,e^{3i(fx+e}}{}$

[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/2/f/a^3*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^2*c^2*(a*(sec(f*x+e)+1))
^(1/2)*(2*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+3*cos(f*x+e)^2+4*cos(f*x+e)*ln(
2/(cos(f*x+e)+1))-2*cos(f*x+e)+2*ln(2/(cos(f*x+e)+1))-1)*cot(f*x+e)^3*csc(f
*x+e)^2

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(-c\sec(fx+e)+c)^{\frac{5}{2}}}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")

[Out] integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(a*sec(f*x + e
) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2
 + 3*a^3*sec(f*x + e) + a^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e+fx))^{5/2}}{(a + a\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \frac{(c - c\sec(e+fx))^{5/2}}{(a + a\sec(e+fx))^{5/2}}\,dx = \frac{\frac{2\,c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{-a}a^2} + \frac{\frac{2\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}{a^3}}{2\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/2*(2*c^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a^2)
+ (2*sqrt(-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - sqrt(-a)*c^(5/2
)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 1.66 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{(c - c\sec(e+fx))^{5/2}}{(a + a\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{2\sqrt{-ac}c^3\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c\right|\right)}{a^3|c|} + \frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2\sqrt{-ac}|c|}{a^3c}}{2\,f}$$

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/2*(2*sqrt(-a*c)*c^3*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^3*abs(c))
+ (c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*abs(c)/(a^3*c))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.126 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result 848
Rubi [A] (verified) 848
Mathematica [A] (verified) 850
Maple [A] (verified) 850
Fricas [F] 851
Sympy [F] 851
Maxima [B] (verification not implemented) 851
Giac [A] (verification not implemented) 853
Mupad [F(-1)] 853

Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)-c^2*tan(f*x
+e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^2*ln(1+cos(f*x+e))*
tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3993, 3992, 3996, 31}

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c^2\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\tan(e+fx)}{af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]
)) - (c^2*Tan[e + f*x])/(a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e +
f*x]]) + (c^2*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3993

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}+\frac{c\int\frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx}{a}\\
&= -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\\
&\quad-\frac{c^2\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c\int\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{c^2 \tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{c^2 \tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{(c^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cos(e+fx)\right)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{c^2 \tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{c^2 \tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{c^2 \log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$
\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^2\left(-\log(\cos(e+fx)) - \log(1+\sec(e+fx)) + \frac{2+\sec(e+fx)}{(1+\sec(e+fx))^2}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((c^2*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + (2 + Sec[e + f*x])/(1
+ Sec[e + f*x])^2)*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
 c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07

method	result
default	$-\frac{(\sec(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}\,c\sqrt{a(\sec(fx+e)+1)}\left(4\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+8\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+5\cos(fx\right)}{4f\,a^3(\cos(fx+e)+1)^2(\cos(fx+e)-1)}$
risch	$-\frac{c\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(6ie^{i(fx+e)}+e^{4i(fx+e)}fx+2i\ln\left(e^{i(fx+e)}+1\right)e^{4i(fx+e)}+2\,e^{4i(fx+e)}e+4\,e^{3i(fx+e)}fx+8ie^{2i(fx+e)}+8\,e^{3i(fx+e)}e\right)}{}$

```
[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/a^3*(sec(f*x+e)-1)*(-c*(sec(f*x+e)-1))^(1/2)*c*(a*(sec(f*x+e)+1))^(1
/2)*(4*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+8*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+
5*cos(f*x+e)^2+4*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)-3)/(cos(f*x+e)+1)^2/(cos
(f*x+e)-1)*cos(f*x+e)*cot(f*x+e)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{3}{2}}}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^3*sec(f*x
+ e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}{(a(\sec(e+fx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. $2(132) = 264$.

Time = 0.51 (sec) , antiderivative size = 1786, normalized size of antiderivative = 12.40

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*c*cos(2*f*x + 2*e)^2 + 16*(
f*x + e)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x
 + e)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x + e)*
c*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*c*sin(2*f*x + 2*e)^2 + 16*(f*x + e)*c*s
in(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*c*sin(
```

1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*c*cos(2*f
*x + 2*e) + (f*x + e)*c - 2*(c*cos(4*f*x + 4*e)^2 + 36*c*cos(2*f*x + 2*e)^2
 + 16*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(4*f*x + 4*e)^2 +
12*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c*sin(2*f*x + 2*e)^2 + 16*c*sin
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*c*cos(2*f*x + 2*e) + c)*cos(4
*f*x + 4*e) + 12*c*cos(2*f*x + 2*e) + 8*(c*cos(4*f*x + 4*e) + 6*c*cos(2*f*x
 + 2*e) + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*cos
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*cos(4*f*x + 4*e) +
 6*c*cos(2*f*x + 2*e) + c)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x + 2*e) + 4*c*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x + 2*e))*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*arctan2(sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 1) + 2*(6*(f*x + e)*c*cos(2*f*x + 2*e) + (f*x + e)*c - 4*c*sin
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(4*(f*x + e)*c*cos(4*f*x + 4*e) + 24*(f
*x + e)*c*cos(2*f*x + 2*e) + 16*(f*x + e)*c*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*(f*x + e)*c + 3*c*sin(4*f*x + 4*e) + 2*c*sin(2*f*
x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(4*(f*x
+ e)*c*cos(4*f*x + 4*e) + 24*(f*x + e)*c*cos(2*f*x + 2*e) + 4*(f*x + e)*c +
 3*c*sin(4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(3*(f*x + e)*c*sin(2*f*x + 2*e) + 2*c*cos(2*f*x
 + 2*e))*sin(4*f*x + 4*e) - 8*c*sin(2*f*x + 2*e) + 2*(4*(f*x + e)*c*sin(4*f
*x + 4*e) + 24*(f*x + e)*c*sin(2*f*x + 2*e) + 16*(f*x + e)*c*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*c*cos(4*f*x + 4*e) - 2*c*cos(2*f
*x + 2*e) - 3*c)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(
4*(f*x + e)*c*sin(4*f*x + 4*e) + 24*(f*x + e)*c*sin(2*f*x + 2*e) - 3*c*cos(
4*f*x + 4*e) - 2*c*cos(2*f*x + 2*e) - 3*c)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a^3*cos(4*f*x + 4*e)^2 + 36*a^3*cos
(2*f*x + 2*e)^2 + 16*a^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 16*a^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^3*
sin(4*f*x + 4*e)^2 + 12*a^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a^3*sin(
2*f*x + 2*e)^2 + 16*a^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 16*a^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*a^
3*cos(2*f*x + 2*e) + a^3 + 2*(6*a^3*cos(2*f*x + 2*e) + a^3)*cos(4*f*x + 4*e
) + 8*(a^3*cos(4*f*x + 4*e) + 6*a^3*cos(2*f*x + 2*e) + 4*a^3*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^3)*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 8*(a^3*cos(4*f*x + 4*e) + 6*a^3*cos(2*f*x + 2*e)
+ a^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a^3*sin(4*
f*x + 4*e) + 6*a^3*sin(2*f*x + 2*e) + 4*a^3*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 8*(a^3*sin(4*f*x + 4*e) + 6*a^3*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*f)

Giac [A] (verification not implemented)

none

Time = 1.61 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{4\sqrt{-ac}c^2\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^3|c|}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}a^3|c|-2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}a^3c|c|}{a^6c^2}}{4f}$$

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/4*(4*sqrt(-a*c)*c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^3*abs(c))
+ ((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3*abs(c) - 2*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*sqrt(-a*c)*a^3*c*abs(c))/(a^6*c^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.127 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result 854
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Optimal result

Integrand size = 30, antiderivative size = 140

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)-c*tan(f*x
+e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c*ln(1+cos(f*x+e))*ta
n(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3992, 3996, 31}

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+fx)}{af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]
]) - (c*Tan[e + f*x])/(a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*
x]]) + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}+\frac{\int\frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx}{a}\\
&= -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\\
&\quad-\frac{c\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\int\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a^2}\\
&= -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\\
&\quad-\frac{c\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{(c\tan(e+fx))\text{Subst}\left(\int\frac{1}{a+ax}\,dx,x,\cos(e+fx)\right)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c\left(-\log(\cos(e+fx)) - \log(1+\sec(e+fx)) + \frac{1}{2(1+\sec(e+fx))^2} + \frac{1}{1+\sec(e+fx)}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]
[Out] -((c*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + 1/(2*(1 + Sec[e + f*x])^
2) + (1 + Sec[e + f*x])^(-1))*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x]
)]*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

method	result
default	$\frac{\left(8\cos(fx+e)^2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+16\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+7\cos(fx+e)^2+8\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)-5\right)\sqrt{a(\sec(fx+e)}}{8f\,a^3(\cos(fx+e)+1)^2}$
risch	$-\frac{\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(2i\ln(e^{i(fx+e)}+1)e^{4i(fx+e)}+e^{4i(fx+e)}fx+8ie^{i(fx+e)}\ln(e^{i(fx+e)}+1)+2e^{4i(fx+e)}e+4e^{3i(fx+e)}fx+4ie^{i(fx+e)}}{}$

```
[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/8/f/a^3*(8*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+16*cos(f*x+e)*ln(2/(cos(f*x+
e)+1))+7*cos(f*x+e)^2+8*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)-5)*(a*(sec(f*x+e)
+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)/(cos(f*x+e)+1)^2*cot(f*x+e)
```

Fricas [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{-c\sec\left(fx+e\right)+c}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x +
e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2),x)
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(126) = 252$.

Time = 0.49 (sec) , antiderivative size = 1165, normalized size of antiderivative = 8.32

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x
+ e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f*x
 + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2*e)
^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(4*cos(3*f*x + 3*e) + 6*cos(2
*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8
*(6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x
+ 3*e)^2 + 12*(4*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2
 + 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f
*x + e))*sin(4*f*x + 4*e) + sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2
*sin(f*x + e))*sin(3*f*x + 3*e) + 16*sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*
e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e
```

```
) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 4*(f*x + e)*cos(3
*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + e -
 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*cos(4*f*x + 4*e)
 + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + e)*co
s(3*f*x + 3*e) + 12*(f*x + 4*(f*x + e)*cos(f*x + e) + e)*cos(2*f*x + 2*e) +
 8*(f*x + e)*cos(f*x + e) + 2*(4*(f*x + e)*sin(3*f*x + 3*e) + 6*(f*x + e)*s
in(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x + 3*e) + 3*cos(2*f
*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) + 4*(12*(f*x + e)*sin(2*f*x +
2*e) + 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e) + 6*(8*(f*x + e)*sin(
f*x + e) - 1)*sin(2*f*x + 2*e) + e - 4*sin(f*x + e))*sqrt(a)*sqrt(c)/((a^3*
cos(4*f*x + 4*e)^2 + 16*a^3*cos(3*f*x + 3*e)^2 + 36*a^3*cos(2*f*x + 2*e)^2
+ 16*a^3*cos(f*x + e)^2 + a^3*sin(4*f*x + 4*e)^2 + 16*a^3*sin(3*f*x + 3*e)^
2 + 36*a^3*sin(2*f*x + 2*e)^2 + 48*a^3*sin(2*f*x + 2*e)*sin(f*x + e) + 16*a
^3*sin(f*x + e)^2 + 8*a^3*cos(f*x + e) + a^3 + 2*(4*a^3*cos(3*f*x + 3*e) +
6*a^3*cos(2*f*x + 2*e) + 4*a^3*cos(f*x + e) + a^3)*cos(4*f*x + 4*e) + 8*(6*
a^3*cos(2*f*x + 2*e) + 4*a^3*cos(f*x + e) + a^3)*cos(3*f*x + 3*e) + 12*(4*a
^3*cos(f*x + e) + a^3)*cos(2*f*x + 2*e) + 4*(2*a^3*sin(3*f*x + 3*e) + 3*a^3
*sin(2*f*x + 2*e) + 2*a^3*sin(f*x + e))*sin(4*f*x + 4*e) + 16*(3*a^3*sin(2*
f*x + 2*e) + 2*a^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

Giac [A] (verification not implemented)

none

Time = 1.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{5/2}}\,dx = -\frac{\sqrt{2}\left(\frac{8\sqrt{2}\sqrt{-ac}c\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^3|c|} + \frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}a^3c|c| - 4\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}a^3c^2|c|}{a^6c^4}\right)}{16f}$$

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/16*sqrt(2)*(8*sqrt(2)*sqrt(-a*c)*c*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c)
)/(a^3*abs(c)) + (sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3*c
*abs(c) - 4*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^3*c^2*abs(c
))/(a^6*c^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.128 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result . . . 860
Rubi [A] (verified) . . . 860
Mathematica [A] (verified) . . . 862
Maple [A] (verified) . . . 862
Fricas [F] . . . 863
Sympy [F] . . . 863
Maxima [B] (verification not implemented) . . . 863
Giac [A] (verification not implemented) . . . 865
Mupad [F(-1)] . . . 865

Optimal result

Integrand size = 30, antiderivative size = 270

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1-\sec(e+fx))\tan(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{7\log(1+\sec(e+fx))\tan(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{3\tan(e+fx)}{4a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] ln(cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+1/8*ln(1-sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)+7/8*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c
-c*sec(f*x+e))^(1/2)-1/4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))
^(1/2)/(c-c*sec(f*x+e))^(1/2)-3/4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(
f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

= {3997, 84}

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{3\tan(e+fx)}{4a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\log(1-\sec(e+fx))}{8a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{7\tan(e+fx)\log(\sec(e+fx)+1)}{8a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\log(\cos(e+fx))}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(8*a^2*f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (7*Log[1 + Sec[e + f*x]]*Tan[e +
f*x])/(8*a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e +
 f*x]/(4*a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]]) - (3*Tan[e + f*x])/(4*a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{x(a+ax)^3(c-cx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int\left(-\frac{1}{8a^3c(-1+x)}+\frac{1}{a^3cx}-\frac{1}{2a^3c(1+x)^3}-\frac{3}{4a^3c(1+x)^2}-\frac{7}{8a^3c(1+x)}\right)dx,x,\sec(e+\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1-\sec(e+fx))\tan(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{7\log(1+\sec(e+fx))\tan(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{3\tan(e+fx)}{4a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\left(8\log(\cos(e+fx))+\log(1-\sec(e+fx))+7\log(1+\sec\right.}{8a^2f\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] ((8*Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + 7*Log[1 + Sec[e + f*x]] - 2
/(1 + Sec[e + f*x])^2 - 6/(1 + Sec[e + f*x]))*Tan[e + f*x])/(8*a^2*f*Sqrt[a
*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\sqrt{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left((1-\cos(fx+e))^4\csc(fx+e)^4-8(1-\cos(fx+e))^2\csc(fx+e)^2+16\ln\left((1-\cos(f\right.\right.}{32fa^3\sqrt{\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}}$
risch	$\frac{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)x}{a^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}} - \frac{2(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)(fx+e)}{a^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\,(1+e^{2i(fx+e)})\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}\,f} - \frac{i(5e}{2a^2(e^{i(fx+e)}+}$

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/32/f*2^(1/2)/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/(c*(1-co
s(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)^2)^(1/2)*(1-cos(f*
x+e))*((1-cos(f*x+e))^4*csc(f*x+e)^4-8*(1-cos(f*x+e))^2*csc(f*x+e)^2+16*ln(
(1-cos(f*x+e))^2*csc(f*x+e)^2+1)-4*ln(-cot(f*x+e)+csc(f*x+e)))*csc(f*x+e)
```

Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*c*sec(f*x
 + e)^4 + 2*a^3*c*sec(f*x + e)^3 - 2*a^3*c*sec(f*x + e) - a^3*c), x)
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{5}{2}}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(242) = 484$.

Time = 0.51 (sec) , antiderivative size = 2206, normalized size of antiderivative = 8.17

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="max
ima")
[Out] -1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 + 6
4*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x + e)
*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*f*x - 7*(2*(6*cos(2*f*x +
```

```
2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 + 8
*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(4
*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*sin(2*f*x
 + 2*e)^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) + 4*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
1) - (2*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36
*cos(2*f*x + 2*e)^2 + 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 4*cos(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + 36*sin(2*f*x + 2*e)^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e)
 + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 1) + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + e - 2*s
in(2*f*x + 2*e))*cos(4*f*x + 4*e) + 48*(f*x + e)*cos(2*f*x + 2*e) + 2*(16*f
*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos(2*f*x + 2*e) + 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(
4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 2*(16*f*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos
(2*f*x + 2*e) + 16*e + 5*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*(3*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + 2*(16*(f*x + e)*sin(4*f*x + 4*e) +
 96*(f*x + e)*sin(2*f*x + 2*e) + 64*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e) - 5)*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(16*(f*x + e)*sin(4*f*
x + 4*e) + 96*(f*x + e)*sin(2*f*x + 2*e) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x
 + 2*e) - 5)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*e - 1
6*sin(2*f*x + 2*e))/((a^2*cos(4*f*x + 4*e)^2 + 36*a^2*cos(2*f*x + 2*e)^2 +
16*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a^2*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2
```

```
 + 12*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a^2*sin(2*f*x + 2*e)^2 + 1
6*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a^2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*a^2*cos(2*f*x + 2*e)
 + a^2 + 2*(6*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) + 8*(a^2*cos(4*f
*x + 4*e) + 6*a^2*cos(2*f*x + 2*e) + 4*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 8*(a^2*cos(4*f*x + 4*e) + 6*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a^2*sin(4*f*x + 4*e) + 6*a^2
*sin(2*f*x + 2*e) + 4*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a^2*sin(4*f*
x + 4*e) + 6*a^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*sqrt(a)*sqrt(c)*f)
```

Giac [A] (verification not implemented)

none

Time = 1.68 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}a^2c|c|-6\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}a^2c^2|c|}{16\,a^5c^5f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gia
c")
[Out] -1/16*((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*c*abs(c) - 6*(c*tan(
1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^2*c^2*abs(c))/(a^5*c^5*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.129 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result . . . 866
Rubi [A] (verified) . . . 867
Mathematica [A] (verified) . . . 868
Maple [A] (warning: unable to verify) . . . 869
Fricas [F] . . . 869
Sympy [F(-1)] . . . 869
Maxima [B] (verification not implemented) . . . 870
Giac [A] (verification not implemented) . . . 873
Mupad [F(-1)] . . . 873

Optimal result

Integrand size = 30, antiderivative size = 345

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx &= \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&+\frac{5\log(1-\sec(e+fx))\tan(e+fx)}{16a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&+\frac{11\log(1+\sec(e+fx))\tan(e+fx)}{16a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&-\frac{\tan(e+fx)}{8a^2cf(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&-\frac{\tan(e+fx)}{8a^2cf(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&-\frac{\tan(e+fx)}{2a^2cf(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

```
[Out] ln(cos(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)+5/16*ln(1-sec(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*s
ec(f*x+e))^(1/2)+11/16*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))
^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a^2/c/f/(1-sec(f*x+e))/(a+a*se
c(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a^2/c/f/(1+sec(f*x+e)
)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*tan(f*x+e)/a^2/c/f/(1
+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3997, 90}

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)}{8a^2cf(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2a^2cf(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8a^2cf(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{5\tan(e+fx)\log(1-\sec(e+fx))}{16a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{11\tan(e+fx)\log(\sec(e+fx)+1)}{16a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\log(\cos(e+fx))}{a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) + (5*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(16*a^2*c*f*Sqrt
[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (11*Log[1 + Sec[e + f*x]]*
Tan[e + f*x])/(16*a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Tan[e + f*x]/(8*a^2*c*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(8*a^2*c*f*(1 + Sec[e + f*x])^2*Sqrt[
a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(2*a^2*c*f*(1
+ Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{x(a+ax)^3(c-cx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(\frac{1}{8a^3c^2(-1+x)^2} - \frac{5}{16a^3c^2(-1+x)} + \frac{1}{a^3c^2x} - \frac{1}{4a^3c^2(1+x)^3} - \frac{1}{2a^3c^2(1+x)^2} - \frac{11}{16a^3c^2(1\cdots}\right.\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{5\log(1-\sec(e+fx))\tan(e+fx)}{16a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{11\log(1+\sec(e+fx))\tan(e+fx)}{16a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8a^2cf(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8a^2cf(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2a^2cf(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\left(16\log(\cos(e+fx)) + 5\log(1-\sec(e+fx)) + 11\log(1\cdots\right.}{16a^2cf\sqrt{a(1+\cdots}}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
[Out] ((16*Log[Cos[e + f*x]] + 5*Log[1 - Sec[e + f*x]] + 11*Log[1 + Sec[e + f*x]]
 + 2/(-1 + Sec[e + f*x]) - 2/(1 + Sec[e + f*x])^2 - 8/(1 + Sec[e + f*x]))*T
an[e + f*x])/(16*a^2*c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]
])
```

Maple [A] (warning: unable to verify)

Time = 2.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.76

method	result
default	$\frac{\sqrt{2}\,\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(-(1-\cos(fx+e))^6\csc(fx+e)^6+10(1-\cos(fx+e))^4\csc(fx+e)^4+20\ln(-\cot(f\right.}{64f\,a^3\left((1-\cos(fx+e))^2\csc(fx+e)^2\right.}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{a^2c\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{a^2c\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}-\frac{}{4a^2c(1+e^{2}}$

[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)

[Out] 1/64/f*2^(1/2)/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)*csc(f*x+e)^2)^(3/2)*(1-cos(f*x+e))*(-(1-cos(f*x+e))^6*csc(f*x+e)^6+10*(
1-cos(f*x+e))^4*csc(f*x+e)^4+20*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*x+e))^2
*csc(f*x+e)^2-32*ln((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(1-cos(f*x+e))^2*csc(f
*x+e)^2+2)*csc(f*x+e)

Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fri
cas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*c^2*sec(f*
x + e)^5 + a^3*c^2*sec(f*x + e)^4 - 2*a^3*c^2*sec(f*x + e)^3 - 2*a^3*c^2*se
c(f*x + e)^2 + a^3*c^2*sec(f*x + e) + a^3*c^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4272 vs. 2(309) = 618.

Time = 1.90 (sec) , antiderivative size = 4272, normalized size of antiderivative = 12.38

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/8*(8*(f*x + e)*cos(6*f*x + 6*e)^2 + 8*(f*x + e)*cos(4*f*x + 4*e)^2 + 8*(
f*x + e)*cos(2*f*x + 2*e)^2 + 32*(f*x + e)*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 32*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 8*(f*x + e)*sin(6*f*x + 6*e)^2 + 8*(f*x + e)*sin(4*f*x + 4*
e)^2 + 8*(f*x + e)*sin(2*f*x + 2*e)^2 + 32*(f*x + e)*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 8*f*x + 11*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e
) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos
(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 - 4*(cos(6*f*x + 6*
e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) - 4*cos(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(6*f*x + 6*e) - cos(
4*f*x + 4*e) - cos(2*f*x + 2*e) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 4*(cos(6*f*x + 6
*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6
*f*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) -
sin(2*f*x + 2*e)^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2
*e) - 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) + 2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 1) + 5*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
```

- 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos(4
*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 - 4*(cos(6*f*x + 6*e)
 - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) - 4*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(6*f*x + 6*e) - cos(4*
f*x + 4*e) - cos(2*f*x + 2*e) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 4*(cos(6*f*x + 6*e
) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6*f
*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - si
n(2*f*x + 2*e)^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e
) - 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) + 2*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 1) + 4*(4*f*x - 4*(f*x + e)*cos(4*f*x + 4*e) - 4*
(f*x + e)*cos(2*f*x + 2*e) + 4*e + 3*sin(4*f*x + 4*e) + 3*sin(2*f*x + 2*e))
*cos(6*f*x + 6*e) - 16*(f*x - (f*x + e)*cos(2*f*x + 2*e) + e)*cos(4*f*x + 4
e) - 16(f*x + e)*cos(2*f*x + 2*e) + 2*(16*f*x + 16*(f*x + e)*cos(6*f*x +
6*e) - 16*(f*x + e)*cos(4*f*x + 4*e) - 16*(f*x + e)*cos(2*f*x + 2*e) - 64*(
f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 32*(f*x + e
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(6*f*x
 + 6*e) + 7*sin(4*f*x + 4*e) + 7*sin(2*f*x + 2*e) + 8*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 4*(16*f*x + 16*(f*x + e)*cos(6*f*x + 6*e) - 16*(f*x + e)*cos(4
*f*x + 4*e) - 16*(f*x + e)*cos(2*f*x + 2*e) + 32*(f*x + e)*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 7*sin(6*f*x + 6*e) + 5*sin(4*
f*x + 4*e) + 5*sin(2*f*x + 2*e) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(16
*f*x + 16*(f*x + e)*cos(6*f*x + 6*e) - 16*(f*x + e)*cos(4*f*x + 4*e) - 16*(
f*x + e)*cos(2*f*x + 2*e) + 16*e + 5*sin(6*f*x + 6*e) + 7*sin(4*f*x + 4*e)
+ 7*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 4*(4*(f*x + e)*sin(4*f*x + 4*e) + 4*(f*x + e)*sin(2*f*x + 2*e) + 3*cos(4*
f*x + 4*e) + 3*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(4*(f*x + e)*sin(2*f*
x + 2*e) + 3)*sin(4*f*x + 4*e) + 2*(16*(f*x + e)*sin(6*f*x + 6*e) - 16*(f*x
 + e)*sin(4*f*x + 4*e) - 16*(f*x + e)*sin(2*f*x + 2*e) - 64*(f*x + e)*sin(3

```
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 32*(f*x + e)*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*cos(6*f*x + 6*e) - 7*cos(4*f*x
 + 4*e) - 7*cos(2*f*x + 2*e) - 8*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 5)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(1
6*(f*x + e)*sin(6*f*x + 6*e) - 16*(f*x + e)*sin(4*f*x + 4*e) - 16*(f*x + e)
*sin(2*f*x + 2*e) + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 7*cos(6*f*x + 6*e) - 5*cos(4*f*x + 4*e) - 5*cos(2*f*x + 2*e) -
 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 7)*sin(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(16*(f*x + e)*sin(6*f*x + 6*e) -
 16*(f*x + e)*sin(4*f*x + 4*e) - 16*(f*x + e)*sin(2*f*x + 2*e) - 5*cos(6*f*
x + 6*e) - 7*cos(4*f*x + 4*e) - 7*cos(2*f*x + 2*e) - 5)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*e + 12*sin(2*f*x + 2*e))/((a^2*c*cos(
6*f*x + 6*e)^2 + a^2*c*cos(4*f*x + 4*e)^2 + a^2*c*cos(2*f*x + 2*e)^2 + 4*a^
2*c*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a^2*c*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*c*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*c*sin(6*f*x + 6*e)^2 + a^2*c*
sin(4*f*x + 4*e)^2 + 2*a^2*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + a^2*c*sin(
2*f*x + 2*e)^2 + 4*a^2*c*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 16*a^2*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*
a^2*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*a^2*c*cos(
2*f*x + 2*e) + a^2*c - 2*(a^2*c*cos(4*f*x + 4*e) + a^2*c*cos(2*f*x + 2*e) -
 a^2*c)*cos(6*f*x + 6*e) + 2*(a^2*c*cos(2*f*x + 2*e) - a^2*c)*cos(4*f*x + 4
*e) + 4*(a^2*c*cos(6*f*x + 6*e) - a^2*c*cos(4*f*x + 4*e) - a^2*c*cos(2*f*x
+ 2*e) - 4*a^2*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*a
^2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2*c)*cos(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a^2*c*cos(6*f*x + 6*e) -
a^2*c*cos(4*f*x + 4*e) - a^2*c*cos(2*f*x + 2*e) + 2*a^2*c*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2*c)*cos(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 4*(a^2*c*cos(6*f*x + 6*e) - a^2*c*cos(4*f*x + 4*e)
 - a^2*c*cos(2*f*x + 2*e) + a^2*c)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 2*(a^2*c*sin(4*f*x + 4*e) + a^2*c*sin(2*f*x + 2*e))*sin(6*f*
x + 6*e) + 4*(a^2*c*sin(6*f*x + 6*e) - a^2*c*sin(4*f*x + 4*e) - a^2*c*sin(2
*f*x + 2*e) - 4*a^2*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 2*a^2*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a^2*c*sin(6*f*x + 6*e) - a^2
*c*sin(4*f*x + 4*e) - a^2*c*sin(2*f*x + 2*e) + 2*a^2*c*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 4*(a^2*c*sin(6*f*x + 6*e) - a^2*c*sin(4*f*x + 4*e) - a^2*c*si
n(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(
a)*sqrt(c)*f)
```

Giac [A] (verification not implemented)

none

Time = 2.00 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{\frac{10\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}a^2|c|}-\frac{32\log\left(\left|-c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right|\right)}{\sqrt{-ac}a^2|c|}-\frac{2\left(5c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{\sqrt{-ac}a^2c|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}a^2c^2|c|-8}{a^5c^7}}{32f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] -1/32*(10*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a^2*abs(c)) - 32*l
og(abs(-c*tan(1/2*f*x + 1/2*e)^2 - c))/(sqrt(-a*c)*a^2*abs(c)) - 2*(5*c*tan
(1/2*f*x + 1/2*e)^2 - c)/(sqrt(-a*c)*a^2*c*abs(c)*tan(1/2*f*x + 1/2*e)^2) +
 ((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*c^2*abs(c) - 8*(c*tan(1/2
*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^2*c^3*abs(c))/(a^5*c^7))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.130 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result 874
Rubi [A] (verified) 874
Mathematica [A] (verified) 876
Maple [A] (verified) 876
Fricas [A] (verification not implemented) 877
Sympy [F(-1)] 877
Maxima [B] (verification not implemented) 878
Giac [A] (verification not implemented) 879
Mupad [F(-1)] 879

Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(\sin(e+fx))\tan(e+fx)}{a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*cot(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4*
cot(f*x+e)^3/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+ln(sin
(f*x+e))*tan(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3990, 3554, 3556}

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\log(\sin(e+fx))}{a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] Cot[e + f*x]/(2*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Cot[e + f*x]^3/(4*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e +
 f*x]]) + (Log[Sin[e + f*x]]*Tan[e + f*x])/(a^2*c^2*f*Sqrt[a + a*Sec[e + f*
x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)\int\cot^5(e+fx)\,dx}{a^2c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad-\frac{\tan(e+fx)\int\cot^3(e+fx)\,dx}{a^2c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad-\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{\tan(e+fx)\int\cot(e+fx)\,dx}{a^2c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

$$= \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(\sin(e+fx))\tan(e+fx)}{a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{2\cot(e+fx)-\cot^3(e+fx)+4(\log(\cos(e+fx))+\log}{4a^2c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec}}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (2*Cot[e + f*x] - Cot[e + f*x]^3 + 4*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]]
)*Tan[e + f*x])/(4*a^2*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e +
f*x]])
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.47

method	result
default	$\frac{\left(32\cos(fx+e)^4\ln(-\cot(fx+e)+\csc(fx+e))-32\cos(fx+e)^4\ln\left(\frac{2}{\cos(fx+e)+1}\right)-13\cos(fx+e)^4-64\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(\right.}{32f\,a^3\sqrt{-c(\sec(fx}}$
risch	$-\frac{-4i e^{2i(fx+e)}\ln\left(e^{2i(fx+e)}-1\right)+e^{8i(fx+e)}fx+2e^{8i(fx+e)}e+4ie^{4i(fx+e)}-4e^{6i(fx+e)}fx-8e^{6i(fx+e)}e+ie^{8i(fx+e)}\ln\left(e^{2i(fx+e)}-1\right)+}{a^2c^2\left(1+e^{2i(fx+e)}\right)\left(e^{i(\right.}}$

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/32/f/a^3*(32*cos(f*x+e)^4*ln(-cot(f*x+e)+csc(f*x+e))-32*cos(f*x+e)^4*ln(2
/(cos(f*x+e)+1))-13*cos(f*x+e)^4-64*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e))
+64*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e)^2+32*ln(-cot(f*x+e)+csc(
f*x+e))-32*ln(2/(cos(f*x+e)+1))+11)*(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e
)-1))^(1/2)/(sec(f*x+e)-1)^2/c^2/(cos(f*x+e)+1)^3*tan(f*x+e)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 564, normalized size of antiderivative = 3.74

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{162\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{-ac}\log\right.}{}\right.$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [-1/324*(162*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*c)*log(-8*((25
6*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) -
 (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x + e))/
((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) + (832*cos(f*x + e)^5 - 1
988*cos(f*x + e)^3 + 1075*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 -
2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e)), -1/324*(324*(cos(f*x
 + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(a*c)*arctan((16*cos(f*x + e)^3 - 7*cos
(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))*
sin(f*x + e) + (832*cos(f*x + e)^5 - 1988*cos(f*x + e)^3 + 1075*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*
f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(135) = 270$.

Time = 0.50 (sec) , antiderivative size = 1386, normalized size of antiderivative = 9.18

$$\int \frac{1}{(a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] -((f*x + e)*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*cos(6*f*x + 6*e)^2 + 36*(f*x
+ e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e)*sin(8
*f*x + 8*e)^2 + 16*(f*x + e)*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*sin(4*f*x +
4*e)^2 + 16*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(4*cos(6*f*x + 6*e) - 6
*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) - cos(8*f*x +
8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) -
 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - 36*
cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) - 3*sin(
4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8*f*x + 8*e)^2 +
16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*sin(6*f*
x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) -
 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) - 1) + 2*(f*x - 4*(f*x + e)*cos(6*f*x + 6*e) + 6*(f*x + e)
*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e + 2*sin(6*f*x + 6*e) -
 2*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) - 8*(f*x + 6*(f*
x + e)*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e + sin(4*f*x + 4*
e))*cos(6*f*x + 6*e) + 4*(3*f*x - 12*(f*x + e)*cos(2*f*x + 2*e) + 3*e + 2*s
in(2*f*x + 2*e))*cos(4*f*x + 4*e) - 8*(f*x + e)*cos(2*f*x + 2*e) - 4*(2*(f*
x + e)*sin(6*f*x + 6*e) - 3*(f*x + e)*sin(4*f*x + 4*e) + 2*(f*x + e)*sin(2*
f*x + 2*e) + cos(6*f*x + 6*e) - cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*sin(8*
f*x + 8*e) - 4*(12*(f*x + e)*sin(4*f*x + 4*e) - 8*(f*x + e)*sin(2*f*x + 2*e
) - 2*cos(4*f*x + 4*e) - 1)*sin(6*f*x + 6*e) - 4*(12*(f*x + e)*sin(2*f*x +
2*e) + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + e + 4*sin(2*f*x + 2*e))*s
qrt(a)*sqrt(c)/((a^3*c^3*cos(8*f*x + 8*e)^2 + 16*a^3*c^3*cos(6*f*x + 6*e)^2
 + 36*a^3*c^3*cos(4*f*x + 4*e)^2 + 16*a^3*c^3*cos(2*f*x + 2*e)^2 + a^3*c^3*
sin(8*f*x + 8*e)^2 + 16*a^3*c^3*sin(6*f*x + 6*e)^2 + 36*a^3*c^3*sin(4*f*x +
 4*e)^2 - 48*a^3*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*a^3*c^3*sin(2*f
*x + 2*e)^2 - 8*a^3*c^3*cos(2*f*x + 2*e) + a^3*c^3 - 2*(4*a^3*c^3*cos(6*f*x
 + 6*e) - 6*a^3*c^3*cos(4*f*x + 4*e) + 4*a^3*c^3*cos(2*f*x + 2*e) - a^3*c^3
)*cos(8*f*x + 8*e) - 8*(6*a^3*c^3*cos(4*f*x + 4*e) - 4*a^3*c^3*cos(2*f*x +
2*e) + a^3*c^3)*cos(6*f*x + 6*e) - 12*(4*a^3*c^3*cos(2*f*x + 2*e) - a^3*c^3
)*cos(4*f*x + 4*e) - 4*(2*a^3*c^3*sin(6*f*x + 6*e) - 3*a^3*c^3*sin(4*f*x +
4*e) + 2*a^3*c^3*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - 16*(3*a^3*c^3*sin(4*f
*x + 4*e) - 2*a^3*c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

Giac [A] (verification not implemented)

none

Time = 1.86 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{32\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}a^2c|c|} - \frac{64\log\left(\left|-c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right|\right)}{\sqrt{-ac}a^2c|c|} - \frac{48\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+84\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c+37c^2}{\sqrt{-ac}a^2c^3|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4} + \frac{\left(c\tan \cdots\right.}{\cdots}}{64f}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] -1/64*(32*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a^2*c*abs(c)) - 64
*log(abs(-c*tan(1/2*f*x + 1/2*e)^2 - c))/(sqrt(-a*c)*a^2*c*abs(c)) - (48*(c
*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 84*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 37*c
^2)/(sqrt(-a*c)*a^2*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4) + ((c*tan(1/2*f*x +
1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*c^3*abs(c) - 10*(c*tan(1/2*f*x + 1/2*e)^2 -
c)*sqrt(-a*c)*a^2*c^4*abs(c))/(a^5*c^9))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.131 $\int (1+\sec(e+fx))^m (c-c\sec(e+fx))^n \, dx$

Optimal result 880
Rubi [A] (verified) 880
Mathematica [F] 881
Maple [F] 881
Fricas [F] 882
Sympy [F] 882
Maxima [F] 882
Giac [F] 882
Mupad [F(-1)] 883

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int (1+\sec(e+fx))^m (c-c\sec(e+fx))^n \, dx$$
$$= \frac{2^{\frac{1}{2}+m} \operatorname{AppellF1}\left(\frac{1}{2}+n, \frac{1}{2}-m, 1, \frac{3}{2}+n, \frac{1}{2}(1-\sec(e+fx)), 1-\sec(e+fx)\right)(c-c\sec(e+fx))^n \tan(e+}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2+n,1,1/2-m,3/2+n,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(c-
c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3997, 141}

$$\int (1+\sec(e+fx))^m (c-c\sec(e+fx))^n \, dx$$
$$= \frac{2^{m+\frac{1}{2}} \tan(e+fx)(c-c\sec(e+fx))^n \operatorname{AppellF1}\left(n+\frac{1}{2}, \frac{1}{2}-m, 1, n+\frac{3}{2}, \frac{1}{2}(1-\sec(e+fx)), 1-\sec(e+f}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

```
[In] Int[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2 + n, 1/2 - m, 1, 3/2 + n, (1 - Sec[e + f*x])/2, 1
 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 +
 Sec[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(c\tan(e+fx))\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1+\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= \frac{2^{\frac{1}{2}+m}\,\text{AppellF1}\left(\frac{1}{2}+n, \frac{1}{2}-m, 1, \frac{3}{2}+n, \frac{1}{2}(1-\sec(e+fx)), 1-\sec(e+fx)\right)(c-c\sec(e+fx))^n}{f(1+2n)\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

```
[In] Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]

$$\int (\sec(fx+e)+1)^m\,(c-c\sec(fx+e))^n\,dx$$

```
[In] int((sec(f*x+e)+1)^m*(c-c*sec(f*x+e))^n,x)
[Out] int((sec(f*x+e)+1)^m*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (-c\sec(fx+e)+c)^n(\sec(fx+e)+1)^m\,dx$$

```
[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Sympy [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (-c(\sec(e+fx)-1))^n\,(\sec(e+fx)+1)^m\,dx$$

```
[In] integrate((1+sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
[Out] Integral((-c*(sec(e + f*x) - 1))**n*(sec(e + f*x) + 1)**m, x)
```

Maxima [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (-c\sec(fx+e)+c)^n(\sec(fx+e)+1)^m\,dx$$

```
[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Giac [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (-c\sec(fx+e)+c)^n(\sec(fx+e)+1)^m\,dx$$

```
[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int \left(\frac{1}{\cos(e+f\,x)}+1\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

```
[In] int((1/cos(e + f*x) + 1)^m*(c - c/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^m*(c - c/cos(e + f*x))^n, x)
```

3.132 $\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

Optimal result . . . 884
Rubi [A] (verified) . . . 884
Mathematica [F] . . . 886
Maple [F] . . . 886
Fricas [F] . . . 886
Sympy [F] . . . 886
Maxima [F] . . . 887
Giac [F] . . . 887
Mupad [F(-1)] . . . 887

Optimal result

Integrand size = 26, antiderivative size = 109

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2}-n,1,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+fx))^m \ldots}{f(1+2m)}$$

```
[Out] 2^(1/2+n)*c*AppellF1(1/2+m,1,1/2-n,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(
1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)
/f/(1+2*m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3997, 142, 141}

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}\,\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2}-n \ldots\right.}{f(2m+1)}$$

```
[In] Int[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*AppellF1[1/2 + m, 1/2 - n, 1, 3/2 + m, (1 + Sec[e + f*x])/2,
 1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c -
 c*Sec[e + f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}-\frac{x}{2}\right)^{-\frac{1}{2}+n}(a+ax)^{-\frac{1}{2}+}}{x}\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2}-n,1,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}}}{f(1+2m)}$$

Mathematica [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx=\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

```
[In] Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]

$$\int \left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{n}dx$$

```
[In] int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int \left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{n}dx$$

```
[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{n}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**n, x)
```

Maxima [F]

$$\int (a + a\sec(e + fx))^m (c - c\sec(e + fx))^n \, dx$$
$$= \int (a\sec(fx + e) + a)^m (-c\sec(fx + e) + c)^n \, dx$$

```
[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a + a\sec(e + fx))^m (c - c\sec(e + fx))^n \, dx$$
$$= \int (a\sec(fx + e) + a)^m (-c\sec(fx + e) + c)^n \, dx$$

```
[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^m (c - c\sec(e + fx))^n \, dx$$
$$= \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^m \left(c - \frac{c}{\cos(e + f\,x)}\right)^n \, dx$$

```
[In] int((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n, x)
```

3.133 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$

Optimal result 888
Rubi [A] (verified) 888
Mathematica [F] 890
Maple [F] 890
Fricas [F] 890
Sympy [F] 890
Maxima [F] 891
Giac [F] 891
Mupad [F(-1)] 891

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{7}{2},\frac{1}{2}-n,1,\frac{9}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+fx}{7f}$$

```
[Out] 1/7*2^(1/2+n)*c*AppellF1(7/2,1,1/2-n,9/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(
1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)
/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3997, 142, 141}

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = \frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^3(1-\sec(e+fx))^{\frac{1}{2}-n}\,\text{AppellF1}\left(\frac{7}{2},\frac{1}{2}-n,1,\frac{9}{2},\frac{1}{2}(\sec(e+fx)+1),\text{se}\right.}{7f}$$

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*AppellF1[7/2, 1/2 - n, 1, 9/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^3*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(7*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{5/2}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}-\frac{x}{2}\right)^{-\frac{1}{2}+n}(a+ax)^{5/2}}{x}\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{7}{2},\frac{1}{2}-n,1,\frac{9}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\,\text{s}}{7f}$$

Mathematica [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx=\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$$

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]

$$\int \left(a+a\sec\left(fx+e\right)\right)^{3}\left(c-c\sec\left(fx+e\right)\right)^{n}dx$$

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx=\int \left(a\sec\left(fx+e\right)+a\right)^{3}\left(-c\sec\left(fx+e\right)+c\right)^{n}dx$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx=a^{3}\left(\int 3(-c\sec\left(e+fx\right)+c)^{n}\sec\left(e+fx\right)dx+\int 3(-c\sec\left(e+fx\right)+c)^{n}\sec^{2}\left(e+fx\right)dx+\int \left(-c\sec\left(e+fx\right)+c\right)^{n}\sec^{3}\left(e+fx\right)dx+\int \left(-c\sec\left(e+fx\right)+c\right)^{n}\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**n,x)
[Out] a**3*(Integral(3*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral(3*(-c
*sec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c)*
*n*sec(e + f*x)**3, x) + Integral((-c*sec(e + f*x) + c)**n, x))
```

Maxima [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^3(-c\sec(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^3(-c\sec(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned}&\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx\\&=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^3\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx\end{aligned}$$

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^n, x)
```

3.134 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$

Optimal result 892
Rubi [A] (verified) 892
Mathematica [F] 894
Maple [F] 894
Fricas [F] 894
Sympy [F] 894
Maxima [F] 895
Giac [F] 895
Mupad [F(-1)] 895

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{5}{2},\frac{1}{2}-n,1,\frac{7}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+fx}{5f}$$

```
[Out] 1/5*2^(1/2+n)*c*AppellF1(5/2,1,1/2-n,7/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(
1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)
/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3997, 142, 141}

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^2(1-\sec(e+fx))^{\frac{1}{2}-n}\,\text{AppellF1}\left(\frac{5}{2},\frac{1}{2}-n,1,\frac{7}{2},\frac{1}{2}(\sec(e+fx)+1),\text{se}}{5f}$$

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*AppellF1[5/2, 1/2 - n, 1, 7/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^2*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(5*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int\frac{(a+ax)^{3/2}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\left(\int\frac{\left(\frac{1}{2}-\frac{x}{2}\right)^{-\frac{1}{2}+n}(a+ax)^{3/2}}{x}\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{5}{2},\frac{1}{2}-n,1,\frac{7}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\,\text{s}}{5f}$$

Mathematica [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx=\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$$

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]

$$\int (a+a\sec(fx+e))^2(c-c\sec(fx+e))^n\,dx$$

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx=\int (a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(-c*sec(f*x + e) +
 c)^n, x)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx=a^2\left(\int 2(-c\sec(e+fx)+c)^n\sec(e+fx)\,dx+\int(-c\sec(e+fx)+c)^n\sec^2(e+fx)\,dx+\int(-c\sec(e+fx)+c)^n\,dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**n,x)
[Out] a**2*(Integral(2*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*s
ec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c)**n
, x))
```

Maxima [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^2\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^n, x)
```

3.135 $\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$

Optimal result . . . 896
Rubi [A] (verified) . . . 896
Mathematica [F] . . . 897
Maple [F] . . . 898
Fricas [F] . . . 898
Sympy [F] . . . 898
Maxima [F] . . . 898
Giac [F] . . . 899
Mupad [F(-1)] . . . 899

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{2}-n,1,\frac{5}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+fx}{3f}$$

```
[Out] 1/3*2^(1/2+n)*c*AppellF1(3/2,1,1/2-n,5/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(
1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3997, 142, 141}

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)(1-\sec(e+fx))^{\frac{1}{2}-n}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{2}-n,1,\frac{5}{2},\frac{1}{2}(\sec(e+fx)+1),\sec}{3f}$$

```
[In] Int[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*AppellF1[3/2, 1/2 - n, 1, 5/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])*(c - c*Sec[e +
 f*x])^(-1 + n)*Tan[e + f*x])/(3*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
```

```
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{a+ax}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}-\frac{x}{2}\right)^{-\frac{1}{2}+n}\sqrt{a+ax}}{x}\,dx}{f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{2}-n,1,\frac{5}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\,\text{s}}{3f}$$

Mathematica [F]

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$$

```
[In] Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]

$$\int (a + a\sec(fx + e))(c - c\sec(fx + e))^n\, dx$$

```
[In] int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a + a\sec(e + fx))(c - c\sec(e + fx))^n\, dx = \int (a\sec(fx + e) + a)(-c\sec(fx + e) + c)^n\, dx$$

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]

$$\int (a + a\sec(e + fx))(c - c\sec(e + fx))^n\, dx = a\left(\int (-c\sec(e + fx) + c)^n \sec(e + fx)\, dx + \int (-c\sec(e + fx) + c)^n\, dx\right)$$

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))**n,x)
[Out] a*(Integral((-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*sec(e
+ f*x) + c)**n, x))
```

Maxima [F]

$$\int (a + a\sec(e + fx))(c - c\sec(e + fx))^n\, dx = \int (a\sec(fx + e) + a)(-c\sec(fx + e) + c)^n\, dx$$

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)(-c\sec(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

```
[In] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^n, x)
```

3.136 $\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)} \, dx$

Optimal result 900
Rubi [A] (verified) 900
Mathematica [F] 902
Maple [F] 902
Fricas [F] 902
Sympy [F] 902
Maxima [F] 903
Giac [F] 903
Mupad [F(-1)] 903

Optimal result

Integrand size = 26, antiderivative size = 99

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)} \, dx = -\frac{2^{\frac{1}{2}+n} c \operatorname{AppellF1}\left(-\frac{1}{2},\frac{1}{2}-n,1,\frac{1}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e-}{f(a+a\sec(e+fx))}$$

```
[Out] -2^(1/2+n)*c*AppellF1(-1/2,1,1/2-n,1/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1-
sec(f*x+e))^(1/2-n)*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3997, 142, 141}

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)} \, dx = -\frac{c 2^{n+\frac{1}{2}} \tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n} \operatorname{AppellF1}\left(-\frac{1}{2},\frac{1}{2}-n,1,\frac{1}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(a\sec(e+fx)+a)}$$

```
[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((2^(1/2 + n)*c*AppellF1[-1/2, 1/2 - n, 1, 1/2, (1 + Sec[e + f*x])/2, 1 +
Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 + n)*Ta
n[e + f*x])/(f*(a + a*Sec[e + f*x])))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}-\frac{x}{2}\right)^{-\frac{1}{2}+n}}{x(a+ax)^{3/2}}\,dx, x, \text{se}\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(-\frac{1}{2},\frac{1}{2}-n,1,\frac{1}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-}{f(a+a\sec(e+fx))}$$

Mathematica [F]

$$\int \frac{(c - c\sec(e + fx))^n}{a + a\sec(e + fx)}\,dx = \int \frac{(c - c\sec(e + fx))^n}{a + a\sec(e + fx)}\,dx$$

```
[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]
[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]), x]
```

Maple [F]

$$\int \frac{(c - c\sec(fx + e))^n}{a + a\sec(fx + e)}dx$$

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(c - c\sec(e + fx))^n}{a + a\sec(e + fx)}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{a\sec(fx + e) + a}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^n}{a + a\sec(e + fx)}\,dx = \frac{\int \frac{(-c\sec(e+fx)+c)^n}{\sec(e+fx)+1}\,dx}{a}$$

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e)),x)
[Out] Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(c - c\sec(e + fx))^n}{a + a\sec(e + fx)}\, dx = \int \frac{(-c\sec(fx + e) + c)^n}{a\sec(fx + e) + a}\, dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c - c\sec(e + fx))^n}{a + a\sec(e + fx)}\, dx = \int \frac{(-c\sec(fx + e) + c)^n}{a\sec(fx + e) + a}\, dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^n}{a + a\sec(e + fx)}\, dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^n}{a + \frac{a}{\cos(e+f\,x)}}\, dx$$

```
[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

3.137 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal result 904
Rubi [A] (verified) 904
Mathematica [F] 906
Maple [F] 906
Fricas [F] 906
Sympy [F] 906
Maxima [F] 907
Giac [F] 907
Mupad [F(-1)] 907

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = -\frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(-\frac{3}{2},\frac{1}{2}-n,1,-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e}{3f(a+a\sec(e+fx))^2}$$

```
[Out] -1/3*2^(1/2+n)*c*AppellF1(-3/2,1,1/2-n,-1/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e)
)*(1-sec(f*x+e))^(1/2-n)*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(a+a*sec(f*x+
e))^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3997, 142, 141}

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = -\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}\,\text{AppellF1}\left(-\frac{3}{2},\frac{1}{2}-n,1,-\frac{1}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(2^(1/2 + n)*c*AppellF1[-3/2, 1/2 - n, 1, -1/2, (1 + Sec[e + f*x])/2,
1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 + n
)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^2)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int\frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^{5/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\left(\int\frac{\left(\frac{1}{2}-\frac{x}{2}\right)^{-\frac{1}{2}+n}}{x(a+ax)^{5/2}}\,dx,x,\text{se}\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(-\frac{3}{2},\frac{1}{2}-n,1,-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c}{3f(a+a\sec(e+fx))^2}$$

Mathematica [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx=\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$$

```
[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2, x]
```

Maple [F]

$$\int \frac{(c-c\sec(fx+e))^n}{(a+a\sec(fx+e))^2}dx$$

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx=\int \frac{(-c\sec(fx+e)+c)^n}{(a\sec(fx+e)+a)^2}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
[Out] integral((-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) +
 a^2), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx=\frac{\int \frac{(-c\sec(e+fx)+c)^n}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**2,x)
[Out] Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x
)/a**2
```

Maxima [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2}\, dx = \int \frac{(-c\sec(fx + e) + c)^n}{(a\sec(fx + e) + a)^2}\, dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2}\, dx = \int \frac{(-c\sec(fx + e) + c)^n}{(a\sec(fx + e) + a)^2}\, dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2}\, dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^n}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^2}\, dx$$

```
[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

3.138 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx$

Optimal result 908
Rubi [A] (verified) 908
Mathematica [A] (verified) 910
Maple [F] 910
Fricas [F] 910
Sympy [F(-1)] 911
Maxima [F] 911
Giac [F] 911
Mupad [F(-1)] 911

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = \frac{6a^3(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{2a^3(c-c\sec(e+fx))^{1+n}\tan(e+fx)}{cf(3+2n)\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 6*a^3*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^3*
hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/
(1+2*n)/(a+a*sec(f*x+e))^(1/2)-2*a^3*(c-c*sec(f*x+e))^(1+n)*tan(f*x+e)/c/f/
(3+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3997, 90, 67}

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = \frac{2a^3\tan(e+fx)(c-c\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+f\right.}{f(2n+1)\sqrt{a\sec(e+fx)+a}} + \frac{6a^3\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}} - \frac{2a^3\tan(e+fx)(c-c\sec(e+fx))^{n+1}}{cf(2n+3)\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (6*a^3*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e +
 f*x]]) + (2*a^3*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(
c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
- (2*a^3*(c - c*Sec[e + f*x])^(1 + n)*Tan[e + f*x])/(c*f*(3 + 2*n)*Sqrt[a +
 a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^2(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(3a^2(c-cx)^{-\frac{1}{2}+n}+\frac{a^2(c-cx)^{-\frac{1}{2}+n}}{x}-\frac{a^2(c-cx)^{\frac{1}{2}+n}}{c}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= \frac{6a^3(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{2a^3(c-c\sec(e+fx))^{1+n}\tan(e+fx)}{cf(3+2n)\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(a^3c\tan(e+fx))\,\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

$$= \frac{6a^3(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{2a^3(c-c\sec(e+fx))^{1+n}\tan(e+fx)}{cf(3+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.59

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = \frac{2a^3(c-c\sec(e+fx))^n\left(4(2+n)+(3+2n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\right.\right.}{f(1+2n)(3+2n)\sqrt{a(1+\sec(e+fx)}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]
[Out] (2*a^3*(c - c*Sec[e + f*x])^n*(4*(2 + n) + (3 + 2*n)*Hypergeometric2F1[1, 1
/2 + n, 3/2 + n, 1 - Sec[e + f*x]] + (1 + 2*n)*Sec[e + f*x])*Tan[e + f*x])/
(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [F]

$$\int (a+a\sec(fx+e))^{\frac{5}{2}}(c-c\sec(fx+e))^n\,dx$$

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^(5/2)*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^n, x)
```

3.139 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx$

Optimal result 912
Rubi [A] (verified) 912
Mathematica [A] (verified) 914
Maple [F] 914
Fricas [F] 914
Sympy [F(-1)] 914
Maxima [F] 915
Giac [F] 915
Mupad [F(-1)] 915

Optimal result

Integrand size = 28, antiderivative size = 119

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx=\frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a^2*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*
hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/
(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3994, 3997, 67}

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx=\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+f\right.}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2*a^2*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e +
 f*x]]) + (2*a^2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(
c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3994

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a, Int[Sqrt[a + b
*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

integral

$$= \frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + a\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$

$$= \frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{(a^2c\tan(e+fx))\,\text{Subst}\left(\int\frac{(c-cx)^{-\frac{1}{2}+n}}{x}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx = -\frac{2a^2\left(-6-4n+(1+2n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+n,\frac{5}{2}+n,1-\sec(e+fx)\right)(-1+\sec(e+fx))\right)(c-c}{f(1+2n)(3+2n)\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (-2*a^2*(-6 - 4*n + (1 + 2*n)*Hypergeometric2F1[1, 3/2 + n, 5/2 + n, 1 - Se
c[e + f*x]]*(-1 + Sec[e + f*x]))*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1
 + 2*n)*(3 + 2*n)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [F]

$$\int (a+a\sec(fx+e))^{\frac{3}{2}}(c-c\sec(fx+e))^n\,dx$$

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^n\,dx = \int (a\sec(fx + e) + a)^{\frac{3}{2}}(-c\sec(fx + e) + c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^n\,dx = \int (a\sec(fx + e) + a)^{\frac{3}{2}}(-c\sec(fx + e) + c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^n\,dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{3/2}\left(c - \frac{c}{\cos(e + f\,x)}\right)^n\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^n, x)
```

3.140 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$

Optimal result 916
Rubi [A] (verified) 916
Mathematica [A] (verified) 917
Maple [F] 917
Fricas [F] 918
Sympy [F] 918
Maxima [F] 918
Giac [F] 918
Mupad [F(-1)] 919

Optimal result

Integrand size = 28, antiderivative size = 68

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e
)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3997, 67}

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \frac{2a\tan(e+fx)(c-c\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2*a*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e
+ f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{2a\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}+n, \frac{3}{2}+n, 1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$
$$= \frac{2\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}+n, \frac{3}{2}+n, 1-\sec(e+fx)\right)\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))^n\tan\left(\frac{1}{2}(\right.}{f+2fn}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*Sqrt[a*(1 + Sec
[e + f*x])]*(c - c*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f + 2*f*n)
```

Maple [F]

$$\int \sqrt{a+a\sec(fx+e)}\,(c-c\sec(fx+e))^n\,dx$$

```
[In] int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$
$$=\int \sqrt{a\sec\left(fx+e\right)+a}(-c\sec\left(fx+e\right)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx=\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}(-c(\sec\left(e+fx\right)-1))^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**n,x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**n, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$
$$=\int \sqrt{a\sec\left(fx+e\right)+a}(-c\sec\left(fx+e\right)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$
$$=\int \sqrt{a\sec\left(fx+e\right)+a}(-c\sec\left(fx+e\right)+c)^n\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$
$$=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^n, x)
```

3.141 $\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result . . . 920
Rubi [A] (verified) . . . 920
Mathematica [A] (verified) . . . 922
Maple [F] . . . 922
Fricas [F] . . . 922
Sympy [F] . . . 923
Maxima [F] . . . 923
Giac [F] . . . 923
Mupad [F(-1)] . . . 923

Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -hypergeom([1, 1/2+n],[3/2+n],1/2-1/2*sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*
x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*hypergeom([1, 1/2+n],[3/2+n],1-sec(
f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3997, 88, 67, 70}

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\tan(e+fx)(c-c\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}} - \frac{\tan(e+fx)(c-c\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c - c*Sec[
e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])) + (2*Hype
rgeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n
*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(c\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{a+ax}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}+n, \frac{3}{2}+n, \frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}+n, \frac{3}{2}+n, 1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\left(\text{Hypergeometric2F1}\left(1, \frac{1}{2}+n, \frac{3}{2}+n, \frac{1}{2}(1-\sec(e+fx))\right) - 2\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}+n, \frac{3}{2}+n, 1-\right.\right.}{(f+2fn)\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
[Out] -(((Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2] - 2*Hyperg
eometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]])*(c - c*Sec[e + f*x])^n*
Tan[e + f*x])/((f + 2*f*n)*Sqrt[a*(1 + Sec[e + f*x])]))
```

Maple [F]

$$\int \frac{(c-c\sec(fx+e))^n}{\sqrt{a+a\sec(fx+e)}}dx$$

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-c\sec(fx+e)+c)^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c(\sec{(e+fx)}-1))^n}{\sqrt{a\left(\sec{(e+fx)}+1\right)}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral((-c*(sec(e + f*x) - 1))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec{(fx+e)}+c)^n}{\sqrt{a\sec{(fx+e)}+a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec{(fx+e)}+c)^n}{\sqrt{a\sec{(fx+e)}+a}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

3.142 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result 924
Rubi [A] (verified) 924
Mathematica [A] (verified) 927
Maple [F] 927
Fricas [F] 927
Sympy [F] 927
Maxima [F] 928
Giac [F] 928
Mupad [F(-1)] 928

Optimal result

Integrand size = 28, antiderivative size = 205

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(5-2n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{4af(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{af(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{(c-c\sec(e+fx))^n\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -1/4*(5-2*n)*hypergeom([1, 1/2+n],[3/2+n],1/2-1/2*sec(f*x+e))*(c-c*sec(f*x+
e))^n*tan(f*x+e)/a/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*hypergeom([1, 1/2+n],
[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/a/f/(1+2*n)/(a+a*sec(f*
x+e))^(1/2)-1/2*(c-c*sec(f*x+e))^n*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f
*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

$= \{3997, 105, 162, 67, 70\}$

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(5-2n)\tan(e+fx)(c-c\sec(e+fx))^n \operatorname{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{4af(2n+1)\sqrt{a\sec(e+fx)+a}} + \frac{2\tan(e+fx)(c-c\sec(e+fx))^n \operatorname{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{af(2n+1)\sqrt{a\sec(e+fx)+a}} - \frac{\tan(e+fx)(c-c\sec(e+fx))^n}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/4*((5 - 2*n)*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2
]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(a*f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*
x]]) + (2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*S
ec[e + f*x])^n*Tan[e + f*x])/(a*f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]]) - ((c
 - c*Sec[e + f*x])^n*Tan[e + f*x])/(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec
[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(c-c\sec(e+fx))^n\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}\left(2ac-\frac{1}{2}ac(1-2n)x\right)}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{2af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(c-c\sec(e+fx))^n\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{(c\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{(c(5-2n)\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{a+ax}\,dx, x, \sec(e+fx)\right)}{4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(5-2n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+}{4af(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{af(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{(c-c\sec(e+fx))^n\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{(c-c\sec(e+fx))^n\left(-2-4n+(-5+2n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}\right.\right.}{}$$

```
[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((c - c*Sec[e + f*x])^n*(-2 - 4*n + (-5 + 2*n)*Hypergeometric2F1[1, 1/2 + n
, 3/2 + n, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x]) + 8*Hypergeometric2F1[1
, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(1 + Sec[e + f*x]))*Tan[e + f*x])/(4*
(f + 2*f*n)*(a*(1 + Sec[e + f*x]))^(3/2))
```

Maple [F]

$$\int \frac{(c-c\sec(fx+e))^n}{(a+a\sec(fx+e))^{\frac{3}{2}}}dx$$

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-c\sec(fx+e)+c)^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e)
^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-c(\sec(e+fx)-1))^n}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{(a\sec(fx + e) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{(a\sec(fx + e) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^n}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

3.143 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal result 929
Rubi [A] (verified) 929
Mathematica [C] (verified) 931
Maple [A] (verified) 931
Fricas [A] (verification not implemented) 931
Sympy [F] 932
Maxima [F] 932
Giac [F] 932
Mupad [F(-1)] 933

Optimal result

Integrand size = 27, antiderivative size = 91

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{cf}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c/f-arctan(1/2*
a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)*a^(1/2)/c/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {21, 3861, 3859, 209, 3880}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf} - \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) -
 (Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(c*f)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \frac{1}{\sqrt{a+a\sec(e+fx)}}\,dx}{c}\\
&= \frac{\int \sqrt{a+a\sec(e+fx)}\,dx}{c} - \frac{a\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c}\\
&= -\frac{(2a)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{(2a)\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\
&= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{cf}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = -\frac{i\sqrt{1+e^{2i(e+fx)}}\left(\operatorname{arcsinh}\left(e^{i(e+fx)}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{-1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)-\operatorname{arctanh}\left(\sqrt{1+e^{2i(e+fx)}}\right)\right)\sqrt{a(1+s}}{c\left(1+e^{i(e+fx)}\right)f}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]
```

```
[Out] ((-I)*Sqrt[1 + E^((2*I)*(e + f*x))]*(ArcSinh[E^(I*(e + f*x))] - Sqrt[2]*Arc
Tanh[(-1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])] - ArcT
anh[Sqrt[1 + E^((2*I)*(e + f*x))]])*Sqrt[a*(1 + Sec[e + f*x])])/(c*(1 + E^(
I*(e + f*x)))*f)
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.49

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^{2}-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^{2}-1}\right)-\right.}{cf}$

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/f*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2^(1/2)
*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^
2-1)^(1/2))-2*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \left[\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3\,a\cos(fx+e)^{2}+2\,a\cos(fx+e)-a}{\cos(fx+e)^{2}+2\,\cos(fx+e)+1}\right)+2\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^{2}}{\right.}}{2\,cf}\right.$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x +
e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a)*log((2*a*cos(f*
x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*
sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), (sqrt(2)*sqr
t(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - 2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + c\sec(e + fx)}\,dx = \frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)+1}\,dx}{c}$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) + 1), x)/c
```

Maxima [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + c\sec(e + fx)}\,dx = \int \frac{\sqrt{a\sec(fx+e) + a}}{c\sec(fx+e) + c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + c\sec(e + fx)}\,dx = \int \frac{\sqrt{a\sec(fx+e) + a}}{c\sec(fx+e) + c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{c+\frac{c}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + c/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + c/cos(e + f*x)), x)
```

3.144 $\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$

Optimal result . . . 934
Rubi [A] (verified) . . . 934
Mathematica [B] (verified) . . . 936
Maple [A] (verified) . . . 937
Fricas [F] . . . 937
Sympy [F] . . . 937
Maxima [F] . . . 938
Giac [F] . . . 938
Mupad [F(-1)] . . . 938

Optimal result

Integrand size = 27, antiderivative size = 231

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = -\frac{2c\cot(e+fx)\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}(c+d\sec(e+f}{a\sqrt{c+d}f} - \frac{(c-d)E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

```
[Out] -2*c*cot(f*x+e)*EllipticPi((c+d)^(1/2)/(c+d*sec(f*x+e))^(1/2),c/(c+d),((c-d
)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(-d*(1-sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)
*(d*(1+sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)/a/f/(c+d)^(1/2)-(c-d)*EllipticE(
tan(f*x+e)/(1+sec(f*x+e)),((c-d)/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(c+
d*sec(f*x+e))^(1/2)/a/f/((c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used

$= \{4012, 3865, 4053\}$

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = -\frac{2c\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\right.}{af\sqrt{c+d}} - \frac{(c-d)\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

```
[In] Int[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-2*c*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec[
e + f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e + f
*x]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e + f*x
]))/(a*Sqrt[c + d]*f) - ((c - d)*EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e +
 f*x])], (c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e +
f*x]])/(a*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] +
 Dist[(b*c - a*d)/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x])))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{c\int\sqrt{c+d\sec(e+fx)}\,dx}{a} + (-c+d)\int\frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$$

$$= -\frac{2c\cot(e+fx)\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}(c+}{a\sqrt{c+d}f}$$

$$-\frac{(c-d)E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 810 vs. 2(231) = 462.

Time = 34.07 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.51

$$\int\frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)(c+d\sec(e+fx))^{3/2}\left(2\sec\left(\frac{1}{2}(e+fx)\right)\left(-c\sin\left(\frac{1}{2}(e+fx)\right)+c\right.\right.}{f(d+c\cos(e+fx))(a+a\sec(e+f}$$

$$+\frac{2\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)(c+d\sec(e+fx))^{3/2}\left(c^2\tan\left(\frac{1}{2}(e+fx)\right)-d^2\tan\left(\frac{1}{2}(e+fx)\right)-2c^2\tan^3\left(\frac{1}{2}(e+fx)\right)+2c\right.}{}$$

```
[In] Integrate[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Cos[e/2 + (f*x)/2]^2*(c + d*Sec[e + f*x])^(3/2)*(2*Sec[(e + f*x)/2]*(-(c*S
in[(e + f*x)/2]) + d*Sin[(e + f*x)/2]) - 2*(-c + d)*Sin[e + f*x]))/(f*(d +
c*Cos[e + f*x])*(a + a*Sec[e + f*x])) + (2*Cos[e/2 + (f*x)/2]^2*(c + d*Sec[
e + f*x])^(3/2)*(c^2*Tan[(e + f*x)/2] - d^2*Tan[(e + f*x)/2] - 2*c^2*Tan[(e
 + f*x)/2]^3 + 2*c*d*Tan[(e + f*x)/2]^3 + c^2*Tan[(e + f*x)/2]^5 - 2*c*d*Ta
n[(e + f*x)/2]^5 + d^2*Tan[(e + f*x)/2]^5 - 4*c^2*EllipticPi[-1, ArcSin[Tan
[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(c + d -
 c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] - 4*c^2*EllipticPi[-
1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Tan[(e + f*x)/2]^2*Sqrt[1 - T
an[(e + f*x)/2]^2]*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^
2)/(c + d)] + (c^2 - d^2)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c +
d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(c + d - c*T
an[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] + 2*c*(c - d)*EllipticF[
ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1
+ Tan[(e + f*x)/2]^2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/
2]^2)/(c + d)]))/(f*(d + c*Cos[e + f*x])^(3/2)*Sqrt[Sec[e + f*x]]*(a + a*Se
c[e + f*x])*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[(e + f*x)/2]^2)*(
1 + Tan[(e + f*x)/2]^2)^(3/2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e
 + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

Maple [A] (verified)

Time = 8.25 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.11

method	result
default	$\frac{(\cos(fx+e)+1)\left(2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)c^{2}-2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)cd+\mathrm{EllipticE}\left(\cot(fx+e\right.\right.}{}$

[In] int((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/a/f*(cos(f*x+e)+1)*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2)
)*c^2-2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*c*d+EllipticE(
cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*c^2-EllipticE(cot(f*x+e)-csc(f*x
+e),((c-d)/(c+d))^(1/2))*d^2-4*c^2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-
d)/(c+d))^(1/2)))*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(d+c*cos(f*x+e))

Fricas [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\int \frac{(d\sec\left(fx+e\right)+c)^{\frac{3}{2}}}{a\sec\left(fx+e\right)+a}\,dx$$

[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")

[Out] integral((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\frac{\int \frac{c\sqrt{c+d\sec\left(e+fx\right)}}{\sec\left(e+fx\right)+1}\,dx+\int \frac{d\sqrt{c+d\sec\left(e+fx\right)}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

[In] integrate((c+d*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)

[Out] (Integral(c*sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x) + Integral(d*sq
rt(c + d*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x))/a

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{a\sec(fx+e)+a}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{a\sec(fx+e)+a}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x)),x)
[Out] int((c + d/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x)), x)
```

3.145 $\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$

Optimal result 939
Rubi [A] (verified) 939
Mathematica [A] (verified) 941
Maple [A] (verified) 941
Fricas [F] 942
Sympy [F] 942
Maxima [F] 942
Giac [F] 942
Mupad [F(-1)] 943

Optimal result

Integrand size = 27, antiderivative size = 225

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}(c+d\sec(e}{a\sqrt{c+d}f} - \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((c+d)^(1/2)/(c+d*sec(f*x+e))^(1/2),c/(c+d),((c-d)/
(c+d))^(1/2))*(c+d*sec(f*x+e))*(-d*(1-sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)*(
d*(1+sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)/a/f/(c+d)^(1/2)-EllipticE(tan(f*x+
e)/(1+sec(f*x+e)),((c-d)/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(c+d*sec(f*
x+e))^(1/2)/a/f/((c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{4010, 3865, 4053\}$

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),}{af\sqrt{c+d}} - \frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

```
[In] Int[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-2*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec[e
+ f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e + f*x
]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e + f*x])
)/(a*Sqrt[c + d]*f) - (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (
c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*
f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4010

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[d/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b
^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x])))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \sqrt{c+d\sec(e+fx)}\,dx}{a} - \int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$$

$$= -\frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}(c+}{a\sqrt{c+d}f}$$

$$-\frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

Mathematica [A] (verified)

Time = 7.53 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\left((c+d)E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)+2(c-d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+f\right.\right.\right.}{a(c+d)f(1+\cos(e+fx))}$$

```
[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^4*((c + d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)
/(c + d)] + 2*(c - d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]
- 4*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)])*Sqrt[(1 +
Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(c + d)*f*(1 + Cos[e + f*x
])^2*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))])
```

Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.10

method	result
default	$\frac{(\cos(fx+e)+1)\left(2\,\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)c-2\,\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)d+c\,\text{EllipticE}\left(\cot(fx+e\right.\right.}{}$

```
[In] int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/f*(cos(f*x+e)+1)*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2)
)*c-2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*d+c*EllipticE(co
t(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE(cot(f*x+e)-csc(f*x+e),
```

```
((c-d)/(c+d))^(1/2))-4*c*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-d)/(c+d))^
(1/2)))*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(d+c*cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")
[Out] integral(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{\sqrt{c+d\sec\left(e+fx\right)}}{\sec\left(e+fx\right)+1}\,dx}{a}$$

```
[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)
[Out] Integral(sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x)),x)
[Out] int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x)), x)
```

3.146 $\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result 944
Rubi [A] (verified) 945
Mathematica [A] (verified) 947
Maple [A] (verified) 947
Fricas [F(-1)] 948
Sympy [F] 948
Maxima [F] 948
Giac [F] 948
Mupad [F(-1)] 949

Optimal result

Integrand size = 27, antiderivative size = 319

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sec(e+fx))}{c-d}}}{a(c-d)f} - \frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{c+d}{c},\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sec(e+fx))}{c-d}}}{acf} - \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{a(c-d)f\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

```
[Out] 2*cot(f*x+e)*EllipticF((c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2),((c+d)/(c-d))^(1/
2))*(c+d)^(1/2)*(d*(1-sec(f*x+e))/(c+d))^(1/2)*(-d*(1+sec(f*x+e))/(c-d))^(1
/2)/a/(c-d)/f-2*cot(f*x+e)*EllipticPi((c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2),(c
+d)/c,((c+d)/(c-d))^(1/2))*(c+d)^(1/2)*(d*(1-sec(f*x+e))/(c+d))^(1/2)*(-d*(
1+sec(f*x+e))/(c-d))^(1/2)/a/c/f-EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((c-d)
/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))^(1/2)/a/(c-d)/f/((
c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4014, 4006, 3869, 3917, 4053}

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \frac{2\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)}{af(c-d)}$$
$$-\frac{2\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}\,\text{EllipticPi}\left(\frac{c+d}{c},\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)}{acf}$$
$$-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af(c-d)\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Sqrt[c + d]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[
c + d]], (c + d)/(c - d)]*Sqrt[(d*(1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1
 + Sec[e + f*x]))/(c - d))])/(a*(c - d)*f) - (2*Sqrt[c + d]*Cot[e + f*x]*El
lipticPi[(c + d)/c, ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[c + d]], (c + d)/(
c - d)]*Sqrt[(d*(1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1 + Sec[e + f*x]))/
(c - d))])/(a*c*f) - (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (c
 - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(
c - d)*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4014

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Dist[1/(c*(b*c - a*d)), Int[(b*c - a*d - b*d*C
sc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d^2/(c*(b*c - a*d)), I
nt[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] ||
EqQ[c^2 - d^2, 0])
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))]]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{-ac+ad-ad\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}\,dx}{a^2(c-d)} + \frac{a\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx}{-ac+ad} \\
&= -\frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{a(c-d)f\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}} \\
&\quad + \frac{\int \frac{1}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} + \frac{d\int \frac{\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}\,dx}{a(c-d)} \\
&= \frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sec(e+fx))}{c-d}}}{a(c-d)f} \\
&\quad - \frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{c+d}{c},\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sec(e+fx))}{c-d}}}{acf} \\
&\quad - \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{a(c-d)f\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 8.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{d+c\cos(e+fx)}{(c+d)(1+\cos(e+fx))}}\left((c+d)E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)+2(c-2d)\,\text{EllipticF}\left(a\right.\right.}{a(c-}$$

[In] Integrate[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]

[Out] (-4*Cos[(e + f*x)/2]^4*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]
))]*((c + d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)] + 2*(c -
2*d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)] + 4*(-c + d)*Elli
pticPi[-1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)])*Sec[e + f*x]^2*((1 +
 Sec[e + f*x])^(-1))^(3/2))/(a*(c - d)*f*Sqrt[c + d*Sec[e + f*x]])

Maple [A] (verified)

Time = 7.30 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.90

method	result
default	$\frac{(\cos(fx+e)+1)\left(2\,\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)c-4\,\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)d+c\,\text{EllipticE}\left(\cot(fx+e\right.\right.}{}$

[In] int(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/a/f/(c-d)*(cos(f*x+e)+1)*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))
^(1/2))*c-4*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*d+c*Ellipt
icE(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE(cot(f*x+e)-csc(f
*x+e),((c-d)/(c+d))^(1/2))-4*c*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-d)/(
c+d))^(1/2))+4*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-d)/(c+d))^(1/2))*d)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))
^(1/2)*(c+d*sec(f*x+e))^(1/2)/(d+c*cos(f*x+e))

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{1}{\sqrt{c+d\sec(e+fx)}\sec(e+fx)+\sqrt{c+d\sec(e+fx)}}\,dx}{a}$$

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(c + d*sec(e + f*x))*sec(e + f*x) + sqrt(c + d*sec(e + f*x)
)), x)/a
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{(a\sec(fx+e)+a)\sqrt{d\sec(fx+e)+c}}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{(a\sec(fx+e)+a)\sqrt{d\sec(fx+e)+c}}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(e + fx))\sqrt{c + d\sec(e + fx)}}\,dx = \int \frac{1}{\left(a + \frac{a}{\cos(e+f\,x)}\right)\,\sqrt{c + \frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)),x)
[Out] int(1/((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)), x)
```

3.147 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$

Optimal result 950
Rubi [A] (verified) 950
Mathematica [A] (warning: unable to verify) 953
Maple [A] (verified) 953
Fricas [A] (verification not implemented) 954
Sympy [F] 955
Maxima [F] 955
Giac [F] 961
Mupad [F(-1)] 962

Optimal result

Integrand size = 27, antiderivative size = 271

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx &= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{3/2}c^4\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &\quad-\frac{2d^2\left(6c^2+8cd+3d^2\right)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}\\ &\quad+\frac{2d^3(4c+3d)(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}}-\frac{2d^4(a-a\sec(e+fx))^3\tan(e+fx)}{7a^2f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

```
[Out] 2*a*d*(2*c+d)*(2*c^2+2*c*d+d^2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*d^2
*(6*c^2+8*c*d+3*d^2)*(a-a*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2
/5*d^3*(4*c+3*d)*(a-a*sec(f*x+e))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-2
/7*d^4*(a-a*sec(f*x+e))^3*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)
*c^4*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used

$= \{4025, 90, 65, 212\}$

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$
$$= \frac{2a^{3/2}c^4\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{2d^4\tan(e+fx)(a-a\sec(e+fx))^3}{7a^2f\sqrt{a\sec(e+fx)+a}}$$
$$- \frac{2d^2(6c^2+8cd+3d^2)\tan(e+fx)(a-a\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}}$$
$$+ \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$
$$+ \frac{2d^3(4c+3d)\tan(e+fx)(a-a\sec(e+fx))^2}{5af\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (2*a*d*(2*c + d)*(2*c^2 + 2*c*d + d^2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e +
f*x]]) + (2*a^(3/2)*c^4*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f
*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(6*c^2
+ 8*c*d + 3*d^2)*(a - a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e +
 f*x]]) + (2*d^3*(4*c + 3*d)*(a - a*Sec[e + f*x])^2*Tan[e + f*x])/(5*a*f*Sq
rt[a + a*Sec[e + f*x]]) - (2*d^4*(a - a*Sec[e + f*x])^3*Tan[e + f*x])/(7*a^
2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^4}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{d(2c+d)\left(2c^2+2cd+d^2\right)}{\sqrt{a-ax}}+\frac{c^4}{x\sqrt{a-ax}}-\frac{d^2\left(6c^2+8cd+3d^2\right)\sqrt{a-ax}}{a}+\frac{d^3(4c+3d)(a-ax)^{3/2}}{a^2}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{2d^2\left(6c^2+8cd+3d^2\right)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2d^3(4c+3d)(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{2d^4(a-a\sec(e+fx))^3\tan(e+fx)}{7a^2f\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\left(a^2c^4\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{2d^2\left(6c^2+8cd+3d^2\right)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2d^3(4c+3d)(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{2d^4(a-a\sec(e+fx))^3\tan(e+fx)}{7a^2f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\left(2ac^4\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^4\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^2\left(6c^2+8cd+3d^2\right)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3(4c+3d)(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}} - \frac{2d^4(a-a\sec(e+fx))^3\tan(e+fx)}{7a^2f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (warning: unable to verify)

Time = 8.41 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.85

$$\int\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$

$$= \frac{2\sqrt{a(1+\sec(e+fx))}(c+d\sec(e+fx))^4\left(105c^4\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\sqrt{\frac{\sec(e+fx)}{(1+\sec(e+fx))^2}}\sqrt{1+\sec(e+fx)}\right.}{}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^4*(105*c^4*ArcTan[Tan[(e
 + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]]*Sqrt[Sec[e + f*x]/(1 + Se
c[e + f*x])^2]*Sqrt[1 + Sec[e + f*x]] + d*Sqrt[Sec[e + f*x]]*(420*c^3 + 420
*c^2*d + 224*c*d^2 + 48*d^3 + 2*d*(105*c^2 + 56*c*d + 12*d^2)*Sec[e + f*x]
+ 6*d^2*(14*c + 3*d)*Sec[e + f*x]^2 + 15*d^3*Sec[e + f*x]^3)*Tan[(e + f*x)/
2]))/(105*f*(d + c*Cos[e + f*x])^4*Sec[e + f*x]^(9/2))
```

Maple [A] (verified)

Time = 6.83 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.11

method	result
parts	$\frac{2c^4\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f} + \frac{2d^4\left(16\cos(fx+e)^3+8\cos(fx+e)^2+6\cos(fx\right.}{35f(co}$
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(105\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}c^4\cos(fx+e)+105\operatorname{arctanh}\left(\frac{\sin(fx-}{(\cos(fx+e)+1)\sqrt{-}}\right.\right.}{}$

```
[In] int((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^4/f*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh
(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+2/35*d^4/f*(
16*cos(f*x+e)^3+8*cos(f*x+e)^2+6*cos(f*x+e)+5)*(a*(sec(f*x+e)+1))^(1/2)/(co
s(f*x+e)+1)*tan(f*x+e)*sec(f*x+e)^2-8*c^3*d/f*(a*(sec(f*x+e)+1))^(1/2)*(cot
(f*x+e)-csc(f*x+e))+4*c^2*d^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(2*
sin(f*x+e)+tan(f*x+e))+8/15*c*d^3/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)
*(8*sin(f*x+e)+4*tan(f*x+e)+3*sec(f*x+e)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.74

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$

$$= \left[\frac{105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e}{\cos(fx+e)+1}\right.}{} \right.$$

$$-\frac{2\left(105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(15\,d^4+4\left(105\,c^3d+1\right.\right.}{}$$

```
[In] integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*d^4 + 4*(
105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^3 + 2*(105*c^2*d^
2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^2 + 6*(14*c*d^3 + 3*d^4)*cos(f*x + e))*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f
*cos(f*x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqr
t(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s
in(f*x + e))) - (15*d^4 + 4*(105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4)*c
os(f*x + e)^3 + 2*(105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^2 + 6*(14*
c*d^3 + 3*d^4)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*
x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx = \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^4\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**4*(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**4, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx = \int \sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^4\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -1/210*(16*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)^(1/4)*(7*(15*c^3*d*sin(6*f*x + 6*e) + 5*(9*c^3*d + 3*c^2*d^2 + 4*c*d^3)*
sin(4*f*x + 4*e) + (45*c^3*d + 30*c^2*d^2 + 28*c*d^3 + 6*d^4)*sin(2*f*x + 2
*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (105*c^3*d*
cos(6*f*x + 6*e) + 105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4 + 35*(9*c^3*
d + 3*c^2*d^2 + 4*c*d^3)*cos(4*f*x + 4*e) + 7*(45*c^3*d + 30*c^2*d^2 + 28*c
*d^3 + 6*d^4)*cos(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)))*sqrt(a) + 105*((c^4*cos(2*f*x + 2*e)^4 + c^4*sin(2*f*x + 2*e
)^4 + 4*c^4*cos(2*f*x + 2*e)^3 + 6*c^4*cos(2*f*x + 2*e)^2 + 4*c^4*cos(2*f*x
 + 2*e) + c^4 + 2*(c^4*cos(2*f*x + 2*e)^2 + 2*c^4*cos(2*f*x + 2*e) + c^4)*s
in(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c^4*
cos(2*f*x + 2*e)^4 + c^4*sin(2*f*x + 2*e)^4 + 4*c^4*cos(2*f*x + 2*e)^3 + 6*
c^4*cos(2*f*x + 2*e)^2 + 4*c^4*cos(2*f*x + 2*e) + c^4 + 2*(c^4*cos(2*f*x +
2*e)^2 + 2*c^4*cos(2*f*x + 2*e) + c^4)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^4*f*cos(2*f*x + 2*e)^4 + c^4*f*sin(
2*f*x + 2*e)^4 + 4*c^4*f*cos(2*f*x + 2*e)^3 + 6*c^4*f*cos(2*f*x + 2*e)^2 +
4*c^4*f*cos(2*f*x + 2*e) + c^4*f + 2*(c^4*f*cos(2*f*x + 2*e)^2 + 2*c^4*f*co
s(2*f*x + 2*e) + c^4*f)*sin(2*f*x + 2*e)^2)*integrate((((cos(10*f*x + 10*e)
*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e
```

)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)
^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2
*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x
+ 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x
 + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*
x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f
*x + 2*e))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*
f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(
6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin
(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*si
n(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*
cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4
*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e
)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*
e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e
)^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x + 8*e) + 6*cos(
6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) +
cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f
*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e
) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f
*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*
(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x +
 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) +
4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*
e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin
(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e)
 + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6
*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*
cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x +
 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*
cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x
 + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(1
0*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x +
2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + s
in(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x +
4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 8*((c^4 + 2*c^3*d

```
+ 6*c^2*d^2)*f*cos(2*f*x + 2*e)^4 + (c^4 + 2*c^3*d + 6*c^2*d^2)*f*sin(2*f*x
 + 2*e)^4 + 4*(c^4 + 2*c^3*d + 6*c^2*d^2)*f*cos(2*f*x + 2*e)^3 + 6*(c^4 + 2
*c^3*d + 6*c^2*d^2)*f*cos(2*f*x + 2*e)^2 + 4*(c^4 + 2*c^3*d + 6*c^2*d^2)*f*
cos(2*f*x + 2*e) + 2*((c^4 + 2*c^3*d + 6*c^2*d^2)*f*cos(2*f*x + 2*e)^2 + 2*
(c^4 + 2*c^3*d + 6*c^2*d^2)*f*cos(2*f*x + 2*e) + (c^4 + 2*c^3*d + 6*c^2*d^2
)*f)*sin(2*f*x + 2*e)^2 + (c^4 + 2*c^3*d + 6*c^2*d^2)*f)*integrate((((cos(1
0*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*co
s(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos
(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*
sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)
*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*
e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2
*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x +
8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x +
 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x +
 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*
x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*
x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*
f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*
x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f
*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(
10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*s
in(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + si
n(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x +
8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f
*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4
*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*co
s(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2
 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
+ 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e)
+ sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*
f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*s
in(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x +
6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos
(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2
 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x
 + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
)*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(
4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e)
```

```
+ 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x +
10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) +
 sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f
*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16
*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e
)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 4*((3
*c^4 + 12*c^3*d + 24*c^2*d^2 + 16*c*d^3 + 8*d^4)*f*cos(2*f*x + 2*e)^4 + (3*
c^4 + 12*c^3*d + 24*c^2*d^2 + 16*c*d^3 + 8*d^4)*f*sin(2*f*x + 2*e)^4 + 4*(3
*c^4 + 12*c^3*d + 24*c^2*d^2 + 16*c*d^3 + 8*d^4)*f*cos(2*f*x + 2*e)^3 + 6*(
3*c^4 + 12*c^3*d + 24*c^2*d^2 + 16*c*d^3 + 8*d^4)*f*cos(2*f*x + 2*e)^2 + 4*
(3*c^4 + 12*c^3*d + 24*c^2*d^2 + 16*c*d^3 + 8*d^4)*f*cos(2*f*x + 2*e) + 2*(
(3*c^4 + 12*c^3*d + 24*c^2*d^2 + 16*c*d^3 + 8*d^4)*f*cos(2*f*x + 2*e)^2 + 2
*(3*c^4 + 12*c^3*d + 24*c^2*d^2 + 16*c*d^3 + 8*d^4)*f*cos(2*f*x + 2*e) + (3
*c^4 + 12*c^3*d + 24*c^2*d^2 + 16*c*d^3 + 8*d^4)*f)*sin(2*f*x + 2*e)^2 + (3
*c^4 + 12*c^3*d + 24*c^2*d^2 + 16*c*d^3 + 8*d^4)*f)*integrate((((cos(10*f*x
 + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f
*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*
x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2
*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(
2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*si
n(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*s
in(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*
sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)
*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)
*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2
*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 1
0*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x +
 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2
*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x +
2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*
x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*
f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f
*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x + 8*e)
+ 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x +
10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) +
 cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f
*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16
*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e
)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin
(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x +
```

```
 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*
f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e)
 + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*
f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*
x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*
(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*
e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos
(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x
 + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*s
in(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e)
 + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(
2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x +
4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(
4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 8*((c^4 +
6*c^3*d + 6*c^2*d^2 + 8*c*d^3)*f*cos(2*f*x + 2*e)^4 + (c^4 + 6*c^3*d + 6*c^
2*d^2 + 8*c*d^3)*f*sin(2*f*x + 2*e)^4 + 4*(c^4 + 6*c^3*d + 6*c^2*d^2 + 8*c*
d^3)*f*cos(2*f*x + 2*e)^3 + 6*(c^4 + 6*c^3*d + 6*c^2*d^2 + 8*c*d^3)*f*cos(2
*f*x + 2*e)^2 + 4*(c^4 + 6*c^3*d + 6*c^2*d^2 + 8*c*d^3)*f*cos(2*f*x + 2*e)
 + 2*((c^4 + 6*c^3*d + 6*c^2*d^2 + 8*c*d^3)*f*cos(2*f*x + 2*e)^2 + 2*(c^4 +
6*c^3*d + 6*c^2*d^2 + 8*c*d^3)*f*cos(2*f*x + 2*e) + (c^4 + 6*c^3*d + 6*c^2*
d^2 + 8*c*d^3)*f)*sin(2*f*x + 2*e)^2 + (c^4 + 6*c^3*d + 6*c^2*d^2 + 8*c*d^3
)*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*
cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)
*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e
) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*
e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 1
0*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x +
 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x
 + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x
 + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*
f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4
*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(
2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin
(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(
10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*c
os(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + co
s(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)
*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e),
```

```
cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
))/(((2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos
(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x +
 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*
f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e)
+ 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*
f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e
) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x
+ 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*
sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2
 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*c
os(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e)
 + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(
2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x +
4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(
4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 +
 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e)
 + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x +
 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*
sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x +
 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/4)), x) - 2*((c^4 + 8*c^3*d)*f*cos(2*f*x + 2*e)^4 + (c^4 + 8*c^3*d)
*f*sin(2*f*x + 2*e)^4 + 4*(c^4 + 8*c^3*d)*f*cos(2*f*x + 2*e)^3 + 6*(c^4 + 8
*c^3*d)*f*cos(2*f*x + 2*e)^2 + 4*(c^4 + 8*c^3*d)*f*cos(2*f*x + 2*e) + 2*((c
^4 + 8*c^3*d)*f*cos(2*f*x + 2*e)^2 + 2*(c^4 + 8*c^3*d)*f*cos(2*f*x + 2*e) +
 (c^4 + 8*c^3*d)*f)*sin(2*f*x + 2*e)^2 + (c^4 + 8*c^3*d)*f)*integrate((((co
s(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6
*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*
e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x +
 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x
+ 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x
 + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*
x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*
x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2
*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10
*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos
(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1
```

```
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2
*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + s
in(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) +
6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x
 + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(1
0*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x
+ 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4
*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e
)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f
*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*
e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin
(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 1
6*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x
 + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*
cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e
)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*
f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2
*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*c
os(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*
e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x
 + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e
) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(
4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 +
 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x +
2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x))*sqr
t(a))/(f*cos(2*f*x + 2*e)^4 + f*sin(2*f*x + 2*e)^4 + 4*f*cos(2*f*x + 2*e)^3
 + 6*f*cos(2*f*x + 2*e)^2 + 2*(f*cos(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e)
+ f)*sin(2*f*x + 2*e)^2 + 4*f*cos(2*f*x + 2*e) + f)
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx=\int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)^4\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(e + fx)}(c+d\sec(e+fx))^4\,dx = \int \sqrt{a + \frac{a}{\cos(e + f\,x)}}\left(c + \frac{d}{\cos(e + f\,x)}\right)^4 dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^4,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^4, x)
```

3.148 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$

Optimal result 963
Rubi [A] (verified) 963
Mathematica [A] (verified) 965
Maple [A] (verified) 966
Fricas [A] (verification not implemented) 966
Sympy [F] 967
Maxima [F] 967
Giac [F] 971
Mupad [F(-1)] 971

Optimal result

Integrand size = 27, antiderivative size = 205

$$\begin{aligned}&\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx\\&=\frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{3/2}c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\&\quad-\frac{2d^2(3c+2d)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}+\frac{2d^3(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

```
[Out] 2*a*d*(3*c^2+3*c*d+d^2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*d^2*(3*c+2*
d)*(a-a*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/5*d^3*(a-a*sec(f*
x+e))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)*c^3*arctanh((a-a*se
c(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e
))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4025, 90, 65, 212}

$$\begin{aligned}&\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx\\&=\frac{2a^{3/2}c^3\tan(e+fx)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}\\&\quad-\frac{2d^2(3c+2d)\tan(e+fx)(a-a\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2d^3\tan(e+fx)(a-a\sec(e+fx))^2}{5af\sqrt{a\sec(e+fx)+a}}\end{aligned}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a*d*(3*c^2 + 3*c*d + d^2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (
2*a^(3/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(3*c + 2*d)*(a -
a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*(a -
a*Sec[e + f*x])^2*Tan[e + f*x])/(5*a*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(c+dx)^3}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{d(3c^2+3cd+d^2)}{\sqrt{a-ax}}+\frac{c^3}{x\sqrt{a-ax}}-\frac{d^2(3c+2d)\sqrt{a-ax}}{a}+\frac{d^3(a-ax)^{3/2}}{a^2}\right)\,dx,x,\sec(e+f\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(3c+2d)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}} - \frac{(a^2c^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(3c+2d)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}} + \frac{(2ac^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(3c+2d)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 9.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.94

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$$

$$= \frac{2\sqrt{a(1+\sec(e+fx))}(c+d\sec(e+fx))^3\left(15c^3\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\sqrt{\frac{\sec(e+fx)}{(1+\sec(e+fx))^2}}\sqrt{1+\sec(e+fx)}\right.}{15f(d+c\cos(e+fx}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^3*(15*c^3*ArcTan[Tan[(e
+ f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]]*Sqrt[Sec[e + f*x]/(1 + Sec
[e + f*x])^2]*Sqrt[1 + Sec[e + f*x]] + d*Sqrt[Sec[e + f*x]]*(45*c^2 + 30*c*
d + 8*d^2 + d*(15*c + 4*d)*Sec[e + f*x] + 3*d^2*Sec[e + f*x]^2)*Tan[(e + f*
x)/2]))/(15*f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(7/2))
```

Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.09

method	result
parts	$\frac{2c^3\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{2d^3\sqrt{a(\sec(fx+e)+1)}\,(8\sin(fx+e)+4\tan(fx+\ldots}{15f(\cos(fx+e)+1)\ldots}$
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(15\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,c^3\cos(fx+e)+15\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\ldots}}\ldots\right.\right.}{15\ldots}$

[In] int((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*c^3/f*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh
(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+2/15*d^3/f*(
a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(8*sin(f*x+e)+4*tan(f*x+e)+3*sec(f*x
+e)*tan(f*x+e))-6*c^2*d/f*(a*(sec(f*x+e)+1))^(1/2)*(cot(f*x+e)-csc(f*x+e))+
2*c*d^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(2*sin(f*x+e)+tan(f*x+e))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.91

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$$

$$=\left[\frac{15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)\ldots}{\cos(fx+e)+1}\right.\ldots}{15\left(f\cos(fx+\ldots\right.}\right.$$

$$-\frac{2\left(15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3d^3+(45c^2d+30cd^2\ldots\right.\right.}{15\left(f\cos(fx+e)^3+f\cos(fx\ldots\right.}$$

[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*d^3 + (45*c^
2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2 + (15*c*d^2 + 4*d^3)*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*
cos(f*x + e)^2), -2/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a

```
)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(
f*x + e))) - (3*d^3 + (45*c^2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2 + (15*c*
d^2 + 4*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$$
$$=\int\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^3\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**3*(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx=\int\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^3\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -1/30*(15*((c^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f
*x + 2*e) + c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c^3*co
s(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f*x + 2*e) + c^3)*a
rctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^3*f*cos(2*f*x + 2*e)
^2 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f)*integrate
((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e)
 + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8
*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x +
2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8
*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x +
4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x +
```

```
2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x +
 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x +
6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e)
 + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e
) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x +
6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2
*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*co
s(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 +
6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x +
6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2
*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*
e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 +
 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(3*cos(6*f*x + 6*e) + 3*cos
(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6
*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6
*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*
f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e
))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 +
6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 6*((c^3 + 2*c^2*d + 4*c*d^2
)*f*cos(2*f*x + 2*e)^2 + (c^3 + 2*c^2*d + 4*c*d^2)*f*sin(2*f*x + 2*e)^2 + 2
*(c^3 + 2*c^2*d + 4*c*d^2)*f*cos(2*f*x + 2*e) + (c^3 + 2*c^2*d + 4*c*d^2)*f
)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2
*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + si
n(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*s
in(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*c
os(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - co
s(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*c
os(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((c
os(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*
cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*c
os(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2
*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(
2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*s
in(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + si
n(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin
```

(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 2*((3*c^3 + 12*c^2*d + 12*c*d^2 + 8*d^3)*f*cos(2*f*x + 2*e)^2 + (3*c^3 + 12*c^2*d + 12*c*d^2 + 8*d^3)*f*sin(2*f*x + 2*e)^2 + 2*(3*c^3 + 12*c^2*d + 12*c*d^2 + 8*d^3)*f*cos(2*f*x + 2*e) + (3*c^3 + 12*c^2*d + 12*c*d^2 + 8*d^3)*f)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*si

```
n(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*
x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*c
os(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2
 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x +
 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*si
n(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x + 2
*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin
(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 2*((c^3 + 6*c^2*d)*f*cos(2*f*x +
 2*e)^2 + (c^3 + 6*c^2*d)*f*sin(2*f*x + 2*e)^2 + 2*(c^3 + 6*c^2*d)*f*cos(2*
f*x + 2*e) + (c^3 + 6*c^2*d)*f)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2
*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x +
2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x
 + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x
+ 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x
 + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*
x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x
 + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*
x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x
 + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*
x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x +
2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x
 + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*
x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))
*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x
+ 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*
cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6
*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x +
 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*si
n(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))^2 + (2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*
cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x +
 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*c
os(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*
e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x +
8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin
(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
```

```
) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(1/4)), x))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/4)*sqrt(a) + 4*(5*(9*c^2*d*sin(4*f*x + 4*e) + 2*(9*c^2*d + 3*c*d^2
+ 2*d^3)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) - (45*c^2*d*cos(4*f*x + 4*e) + 45*c^2*d + 30*c*d^2 + 8*d^3 + 10*(9
*c^2*d + 3*c*d^2 + 2*d^3)*cos(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)))*sqrt(a))/((f*cos(2*f*x + 2*e)^2 + f*sin(2*f*x +
2*e)^2 + 2*f*cos(2*f*x + 2*e) + f)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)^(1/4))
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx=\int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)^3\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c+\frac{d}{\cos(e+f\,x)}\right)^3\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3, x)
```

3.149 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$

Optimal result 972
Rubi [A] (verified) 972
Mathematica [C] (warning: unable to verify) 974
Maple [A] (verified) 975
Fricas [A] (verification not implemented) 975
Sympy [F] 976
Maxima [F] 976
Giac [F] 978
Mupad [F(-1)] 979

Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a*d*(2*c+d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*d^2*(a-a*sec(f*x+e))*
tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)*c^2*arctanh((a-a*sec(f*x+e))^
(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4025, 90, 65, 212}

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = \frac{2a^{3/2}c^2\tan(e+fx)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}} - \frac{2d^2\tan(e+fx)(a-a\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a*d*(2*c + d)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^(3/2)*c^
2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec
[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(a - a*Sec[e + f*x])*Tan[e +
f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(c+dx)^2}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\left(\frac{d(2c+d)}{\sqrt{a-ax}}+\frac{c^2}{x\sqrt{a-ax}}-\frac{d^2\sqrt{a-ax}}{a}\right)\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

$$= \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} - \frac{(a^2c^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{(2ac^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.98 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.08

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$$

$$= \frac{\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}(c+d\sec(e+fx))^2\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}\right.}}{}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[
e + f*x])^2*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/
2]^2]*(256*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(e + f*x)/2]^2]
*Sin[(e + f*x)/2]^6*(c + d - 2*c*Sin[(e + f*x)/2]^2)^2 + 1024*Hypergeometri
c2F1[3/2, 7/2, 9/2, 2*Sin[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^6*(d^2 + c*d*(2
- 3*Sin[(e + f*x)/2]^2) + c^2*(1 - 3*Sin[(e + f*x)/2]^2 + 2*Sin[(e + f*x)/2
]^4)) - (7*Sqrt[2]*(-3*ArcSin[Sqrt[2]*Sqrt[Sin[(e + f*x)/2]^2]] + Sqrt[2]*S
qrt[Sin[(e + f*x)/2]^2]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*(3 + 4*Sin[(e + f*x)
/2]^2))*(15*d^2 + 10*c*d*(3 - 2*Sin[(e + f*x)/2]^2) + c^2*(15 - 20*Sin[(e +
 f*x)/2]^2 + 12*Sin[(e + f*x)/2]^4)))/Sqrt[Sin[(e + f*x)/2]^2]))/(672*f*(d
+ c*Cos[e + f*x])^2*Sec[e + f*x]^(5/2))
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10

method	result
parts	$\frac{2c^2\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}+\frac{2d^2\sqrt{a(\sec(fx+e)+1)}\,(2\sin(fx+e)+\tan(fx-}{3f(\cos(fx+e)+1)}$
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(3\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)c^2\cos(fx+e)+3\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos(}\right.\right.}{3f(\cos(fx+e)+1)}$

```
[In] int((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^2/f*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh
(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+2/3*d^2/f*(a
*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(2*sin(f*x+e)+tan(f*x+e))-4*c*d/f*(a*
(sec(f*x+e)+1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.22

$$\int\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$$
$$=\left[\frac{3\left(c^2\cos(fx+e)^2+c^2\cos(fx+e)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-}{\cos(fx+e)+1}\right.}{3\left(f\cos(fx+e)^2+f\cos(fx+e)\right)}\right.$$
$$-\frac{2\left(3\left(c^2\cos(fx+e)^2+c^2\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-(d^2+2\left(3\,cd+d^2\right)\cos(f\right.}{3\left(f\cos(fx+e)^2+f\cos(fx+e)\right)}$$

```
[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(d^2 + 2*(3*c*d + d
^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*
cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x
 + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/
(sqrt(a)*sin(f*x + e))) - (d^2 + 2*(3*c*d + d^2)*cos(f*x + e))*sqrt((a*cos(
```

```
f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e
))]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$$
$$= \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^2\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**2*(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = \int \sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^2\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -1/6*(8*(3*c*d*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin
(2*f*x + 2*e) - (3*c*d*cos(2*f*x + 2*e) + 3*c*d + d^2)*sin(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 3*((c^2*cos(2*f*x + 2*e)^2 +
c^2*sin(2*f*x + 2*e)^2 + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2((cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)) + 1) - (c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2*f*x + 2*
e)^2 + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) - 1) - 2*(c^2*f*cos(2*f*x + 2*e)^2 + c^2*f*sin(2*f*x + 2*e)^2 + 2*c^2
*f*cos(2*f*x + 2*e) + c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e)
 + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6
*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*
e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2
*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*
e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2
```

```
*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*
e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
 cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*
x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + c
os(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6
*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*
f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6
*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*
f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))
*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 4*((c^2 + 2*c*d + 2*d^2)*f*cos(2*f*x +
 2*e)^2 + (c^2 + 2*c*d + 2*d^2)*f*sin(2*f*x + 2*e)^2 + 2*(c^2 + 2*c*d + 2*d
^2)*f*cos(2*f*x + 2*e) + (c^2 + 2*c*d + 2*d^2)*f)*integrate((((cos(6*f*x +
6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2
*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x +
4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2
*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6
*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*
e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*
e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e
) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*
f*x + 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)
*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x
+ 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*si
n(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x
+ 2*e))*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*co
s(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e
) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x +
 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 2*((c^2 + 4*c*d)*
```

```
f*cos(2*f*x + 2*e)^2 + (c^2 + 4*c*d)*f*sin(2*f*x + 2*e)^2 + 2*(c^2 + 4*c*d)
*f*cos(2*f*x + 2*e) + (c^2 + 4*c*d)*f)*integrate((((cos(6*f*x + 6*e)*cos(2*
f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin
(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(
2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos
(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(
6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos
(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(
4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*
x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1)))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e)
+ cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x
+ 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin
(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos
(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4
*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4
*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x))*sqrt(a))/(f*cos(2*f*x + 2*e)^
2 + f*sin(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e) + f)
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx=\int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)^2\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2, x)
```

3.150 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$

Optimal result 980
Rubi [A] (verified) 980
Mathematica [A] (verified) 981
Maple [A] (verified) 982
Fricas [A] (verification not implemented) 982
Sympy [F] 983
Maxima [B] (verification not implemented) 983
Giac [F] 983
Mupad [F(-1)] 984

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx=\frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{2ad\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f+2*a*d*tan(f
*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4000, 3859, 209, 3877}

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx=\frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2ad\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a*d*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c\int \sqrt{a+a\sec(e+fx)}\,dx + d\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,dx \\
&= \frac{2ad\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(2ac)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} \\
&= \frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2ad\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$= \frac{\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\left(\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{\cos(e+fx)}+2d\sin\left(\frac{1}{2}(e+fx\right.\right.}{f}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
[Out] (Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[
(e + f*x)/2]]*Sqrt[Cos[e + f*x]] + 2*d*Sin[(e + f*x)/2]))/f
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.45

method	result
default	$\frac{2\sqrt{a(\sec(fx+e)+1)}\left(\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)c-d\cot(fx+e)+d\csc(fx+e)\right)}{f}$
parts	$\frac{2c\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f}-\frac{2d\sqrt{a(\sec(fx+e)+1)}\,(\cot(fx+e)-\csc(fx+e))}{f}$

```
[In] int((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(a*(sec(f*x+e)+1))^(1/2)*((-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(si
n(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*c-d*cot(f*x+e)+
d*csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.56

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$$

$$=\left[\frac{(c\cos(fx+e)+c)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\,d\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{f\cos(fx+e)+f},\right.$$
$$\left.-\frac{2\left((c\cos(fx+e)+c)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-d\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{f\cos(fx+e)+f}\right]$$

```
[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e
) - a)/(cos(f*x + e) + 1)) + 2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arctan(s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))
 - d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)
+ f)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx=\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}(c+d\sec\left(e+fx\right))\;dx$$

```
[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(58) = 116$.

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.23

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$$

$$=\frac{\sqrt{a}c\arctan\left(\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right.\right.\right.\right.}{}$$

```
[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] sqrt(a)*c*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(
f*x + e))/f
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)\,dx$$

```
[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx=\int \sqrt{a+\frac{a}{\cos{(e+f\,x)}}}\,\left(c+\frac{d}{\cos{(e+f\,x)}}\right)\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)), x)
```

3.151 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result 985
Rubi [A] (verified) 985
Mathematica [A] (verified) 987
Maple [B] (warning: unable to verify) 987
Fricas [A] (verification not implemented) 988
Sympy [F] 989
Maxima [F] 989
Giac [F] 989
Mupad [F(-1)] 989

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{2\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{c+d}f}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c/f-2*arctan(a^
(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)*d^(1/2
)/c/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4010, 3859, 209, 4052, 211}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf} - \frac{2\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{cf\sqrt{c+d}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) -
 (2*Sqrt[a]*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sec[e + f*x]])])/(c*Sqrt[c + d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4010

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[d/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b
^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \sqrt{a+a\sec(e+fx)}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{c} \\
&= -\frac{(2a)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{(2ad)\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} \\
&= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{2\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{c+d}f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.89 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\left(\sqrt{c+d}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)-\sqrt{d}\arctan\left(\frac{\sqrt{d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{a(1+\sec(e+fx))}}{c\sqrt{c+d}f}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*(Sqrt[c + d]*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x]
)]] - Sqrt[d]*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f
*x]/(1 + Cos[e + f*x])])])*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[a*(1
+ Sec[e + f*x])])/(c*Sqrt[c + d]*f)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(85) = 170$.

Time = 15.15 (sec) , antiderivative size = 499, normalized size of antiderivative = 4.75

method	result
default	$\frac{\sqrt{2}\left(2\sqrt{(c+d)(c-d)}\sqrt{\frac{d}{c-d}}\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)-d\ln\left(-\frac{2\left(\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\sqrt{2}\sqrt{\frac{d}{c-d}}c-\sqrt{2}\sqrt{\frac{c}{c-}}\right.}{-c(-\cot(fx+e)+\csc(}\right.\right.}{}$

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*2^(1/2)/(d/(c-d))^(1/2)/((c+d)*(c-d))^(1/2)/c*(2*((c+d)*(c-d))^(1/2)*
(d/(c-d))^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-c
ot(f*x+e)+csc(f*x+e)))-d*ln(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(
1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f
*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))+d*ln(2*(
((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(
d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d-((c+d)*(c-d))^(1/2
)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+cs
c(f*x+e))*d+((c+d)*(c-d))^(1/2))))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*
(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 669, normalized size of antiderivative = 6.37

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{-\frac{ad}{c+d}}\log\left(\frac{2\,(c+d)\sqrt{-\frac{ad}{c+d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac+2\,ad)\cos(fx+e)^2-ad+(ac+ad)\cos(fx+e)}{c\cos(fx+e)^2+(c+d)\cos(fx+e)+d}\right)+\sqrt{-a}\log\left(\right.}{cf}\right.$$

$$-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\sqrt{-\frac{ad}{c+d}}\log\left(\frac{2\,(c+d)\sqrt{-\frac{ad}{c+d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac+2\,ad)\cos(\right.}{c\cos(fx+e)^2+(c+d)\cos(fx+e)+d}}{cf}$$

$$\left.-\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\sqrt{\frac{ad}{c+d}}\arctan\left(\frac{(c+d)\sqrt{\frac{ad}{c+d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{ad\sin(fx+e)}\right)\right)}{cf}\right]$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^
2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e
) + d)) + sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f
*x + e) + 1)))/(c*f), -(2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(-a*d/(c + d))*log((2*(c +
 d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x +
e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*f), (2*sqrt(a*d/(c +
 d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)/(a*d*sin(f*x + e))) + sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*
sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -2*(sqrt(a)*arctan(sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sq
rt(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))))/(c*f)]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{c+d\sec\left(e+fx\right)}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{d\sec\left(fx+e\right)+c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{d\sec\left(fx+e\right)+c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)), x)
```

3.152 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$

Optimal result 990
Rubi [A] (verified) 990
Mathematica [A] (warning: unable to verify) 993
Maple [B] (warning: unable to verify) 994
Fricas [A] (verification not implemented) 994
Sympy [F] 995
Maxima [F] 995
Giac [F] 995
Mupad [F(-1)] 996

Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx = \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{3/2}\sqrt{d}(3c+2d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

```
[Out] -a*d*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)
*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)-a^(3/2)*(3*c+2*d)*arctanh(d^(1/2)*(a-a*sec(f*x
+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*d^(1/2)*tan(f*x+e)/c^2/(c+d)^(3/2)/f/(a-a*s
ec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used

= {4025, 105, 162, 65, 212, 214}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx = -\frac{a^{3/2}\sqrt{d}(3c+2d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^{3/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{ad\tan(e+fx)}{cf(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(3/2)*Sqrt[d]*(3*c +
 2*d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan
[e + f*x])/(c^2*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]]) - (a*d*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*S
ec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad - \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{a(c+d)-\frac{adx}{2}}{x\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{c(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad - \frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\left(a\left(\frac{acd}{2}+ad(c+d)\right)\tan(e+fx)\right)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{c^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= -\frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$
$$+\frac{(2a\tan(e+fx))\text{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{\left(2\left(\frac{acd}{2}+ad(c+d)\right)\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{c^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{a^{3/2}\sqrt{d}(3c+2d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [A] (warning: unable to verify)

Time = 5.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.11

$$\int\frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{(d+c\cos(e+fx))^2\sec^{\frac{3}{2}}(e+fx)\sqrt{a(1+\sec(e+fx))}\left(\frac{2\left(2(c+d)^{3/2}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)-\sqrt{d}(3c+2d)\arctan\left(\frac{\sqrt{d}\cdots}{\sqrt{c+\cdots}}\right)\cdots\right.}{(c+d)^{3/2}}\cdots\right.}{2c^2f(c+d\sec(e+fx))^2}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((d + c*Cos[e + f*x])^2*Sec[e + f*x]^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*((2*(
2*(c + d)^(3/2)*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x]
)]] - Sqrt[d]*(3*c + 2*d)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sq
rt[Cos[e + f*x]/(1 + Cos[e + f*x])])])*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])
^2]*Sqrt[1 + Sec[e + f*x]])/(c + d)^(3/2) - (2*c*d*Tan[(e + f*x)/2])/((c +
d)*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]])))/(2*c^2*f*(c + d*Sec[e + f*x])
^2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 38051 vs. $2(189) = 378$.

Time = 15.52 (sec) , antiderivative size = 38052, normalized size of antiderivative = 173.75

method	result	size
default	Expression too large to display	38052

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 1.57 (sec) , antiderivative size = 1413, normalized size of antiderivative = 6.45

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^2}\, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
[Out] [-1/2*(2*c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
 e) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d + 2*
d^2)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a
*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c
 + d)*cos(f*x + e) + d)) - 2*((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2
 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(
f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 +
 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -1/2*(2*c*d*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 4*((c^2 + c
*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(a)*
arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*
x + e))) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d
 + 2*d^2)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d)
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c
+ 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2
 + (c + d)*cos(f*x + e) + d)))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c
^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -(c*d*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - ((3*c^2 + 2*c*d)*cos(
f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d + 2*d^2)*cos(f*x + e))*sqrt(a*d
/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)/(a*d*sin(f*x + e))) - ((c^2 + c*d)*cos(f*x + e)^2 + c*
```

```
d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^
2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x +
 e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -(
c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 2*(
(c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*
sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a
)*sin(f*x + e))) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2
 + 5*c*d + 2*d^2)*cos(f*x + e))*sqrt(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(
c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x +
 e))))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*
x + e) + (c^3*d + c^2*d^2)*f)]
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^2}\,dx = \int \frac{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}}{\left(c + d\sec\left(e + fx\right)\right)^2}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**2,x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^2}\,dx = \int \frac{\sqrt{a\sec\left(fx + e\right) + a}}{\left(d\sec\left(fx + e\right) + c\right)^2}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^2}\,dx = \int \frac{\sqrt{a\sec\left(fx + e\right) + a}}{\left(d\sec\left(fx + e\right) + c\right)^2}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c + d \sec(e + fx))^2} \, dx = \int \frac{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}{\left(c + \frac{d}{\cos(e+f\,x)}\right)^2} \, dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^2, x)
```

3.153 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$

Optimal result 997
Rubi [A] (verified) 998
Mathematica [A] (warning: unable to verify) 1001
Maple [B] (warning: unable to verify) 1001
Fricas [B] (verification not implemented) 1002
Sympy [F] 1003
Maxima [F(-1)] 1003
Giac [F] 1004
Mupad [F(-1)] 1004

Optimal result

Integrand size = 27, antiderivative size = 287

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{3/2}\sqrt{d}(15c^2+20cd+8d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{ad(7c+4d)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

```
[Out] -1/2*a*d*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/4
*a*d*(7*c+4*d)*tan(f*x+e)/c^2/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(
1/2)+2*a^(3/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/(a-
a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*a^(3/2)*(15*c^2+20*c*d+8*d^2
)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*d^(1/2)*tan(f
*x+e)/c^3/(c+d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {4025, 105, 156, 162, 65, 212, 214}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{2a^{3/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$- \frac{a^{3/2}\sqrt{d}(15c^2+20cd+8d^2)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4c^3f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$- \frac{ad(7c+4d)\tan(e+fx)}{4c^2f(c+d)^2\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}$$
$$- \frac{ad\tan(e+fx)}{2cf(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(3/2)*Sqrt[d]*(15*c^
2 + 20*c*d + 8*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqr
t[c + d])]*Tan[e + f*x])/(4*c^3*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - (a*d*Tan[e + f*x])/(2*c*(c + d)*f*Sqrt[a + a*Sec[
e + f*x]]*(c + d*Sec[e + f*x])^2) - (a*d*(7*c + 4*d)*Tan[e + f*x])/(4*c^2*(
c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{2a(c+d)-\frac{3adx}{2}}{x\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{2c(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{ad(7c+4d)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{2a^2(c+d)^2-\frac{1}{4}a^2d(7c+4d)x}{x\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{2c^2(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{ad(7c+4d)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{(a^2\tan(e+fx))\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a^2d(15c^2+20cd+8d^2)\tan(e+fx))\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{8c^3(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{ad(7c+4d)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(2a\tan(e+fx))\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(ad(15c^2+20cd+8d^2)\tan(e+fx))\text{Subst}\left(\int \frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{4c^3(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{3/2}\sqrt{d}(15c^2+20cd+8d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{ad(7c+4d)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [A] (warning: unable to verify)

Time = 7.57 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{(d+c\cos(e+fx))^3\sec\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{5}{2}}(e+fx)\sqrt{a(1+\sec(e+fx))}\left(\frac{\left(8(c+d)^{5/2}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)-\sqrt{d}(\right.}{}\right.}{8c^3}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]*Sec[e + f*x]^(5/2)*Sqrt[a*(1 + Sec
[e + f*x])]*(((8*(c + d)^(5/2)*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1
 + Cos[e + f*x])]] - Sqrt[d]*(15*c^2 + 20*c*d + 8*d^2)*ArcTan[(Sqrt[d]*Tan[
(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])])*Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])]*Sec[(e + f*x)/2]*Sqrt[1 + Sec[e + f*x]])/((c
 + d)^(5/2)*Sqrt[(1 + Cos[e + f*x])^(-1)]) - (2*c*d*(d*(7*c + 4*d) + 3*c*(3
*c + 2*d)*Cos[e + f*x])*Sec[e + f*x]^(3/2)*Sin[(e + f*x)/2])/((c + d)^2*(c
+ d*Sec[e + f*x])^2)))/(8*c^3*f*(c + d*Sec[e + f*x])^3)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 76268 vs. 2(249) = 498.

Time = 17.66 (sec) , antiderivative size = 76269, normalized size of antiderivative = 265.75

method	result	size
default	Expression too large to display	76269

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(249) = 498$.

Time = 4.36 (sec) , antiderivative size = 2368, normalized size of antiderivative = 8.25

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/8*((15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(
f*x + e)^3 + (15*c^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (
30*c^3*d + 55*c^2*d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(-a*d/(c + d))*
log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)
*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 8*(c^2*d^2
+ 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)^3 + (c^4 + 4*c^3*d
 + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d
^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a
)/(cos(f*x + e) + 1)) - 2*(3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*
d^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*
x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^
5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^
3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/8*(16*(c^2*d
^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)^3 + (c^4 + 4*c^
3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3
+ d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (1
5*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*c^4 + 50*c^3*d + 48*c^2*
d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2*d^2 + 36*c*d^3 + 8*d^4)
*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*
cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d
)*cos(f*x + e) + d)) + 2*(3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*d
^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
 + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5
*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3
*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), 1/4*((15*c^2*d^2
 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*
c^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2
*d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(a*d/(c + d))*arctan((c + d)*sqr
```

```
t(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*si
n(f*x + e))) + 4*(c^2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f
*x + e)^3 + (c^4 + 4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d
 + 5*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^
2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - (3*(3*c^3*d + 2*c^2*d^2)*
cos(f*x + e)^2 + (7*c^2*d^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3
 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*
c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*
d^4)*f), -1/4*(8*(c^2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f
*x + e)^3 + (c^4 + 4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d
 + 5*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*c^2*d^2
+ 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*c
^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2*
d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(a*d/(c + d))*arctan((c + d)*sqrt
(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin
(f*x + e))) + (3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*d^2 + 4*c*d^
3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^
7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^
4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos
(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f)]
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^3}\,dx = \int \frac{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}}{\left(c + d\sec\left(e + fx\right)\right)^3}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**3,x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^3}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{{\left(c+\frac{d}{\cos(e+f\,x)}\right)}^3}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^3, x)
```

3.154 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx$

Optimal result 1005
Rubi [A] (verified) 1005
Mathematica [A] (verified) 1008
Maple [A] (verified) 1008
Fricas [A] (verification not implemented) 1009
Sympy [F] 1010
Maxima [F] 1010
Giac [F] 1019
Mupad [F(-1)] 1020

Optimal result

Integrand size = 27, antiderivative size = 241

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \frac{2a^{5/2}c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(2(36c^3+243c^2d+189cd^2+52d^3)+d(24c^2+111cd+52d^2)\sec(e+fx))\tan(e+fx)}{105f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2/35*a^2*(6*c+13*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+
2/7*a^2*(c+d*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/105*a^2*(7
2*c^3+486*c^2*d+378*c*d^2+104*d^3+d*(24*c^2+111*c*d+52*d^2)*sec(f*x+e))*tan
(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(5/2)*c^3*arctanh((a-a*sec(f*x+e))^(1/
2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4025, 158, 152, 65, 212}

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \frac{2a^{5/2}c^3\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^2\tan(e+fx)\left(d(24c^2+111cd+52d^2)\sec(e+fx)+2(36c^3+243c^2d+189cd^2+52d^3)\right)}{105f\sqrt{a\sec(e+fx)+a}} + \frac{2a^2\tan(e+fx)(c+d\sec(e+fx))^3}{7f\sqrt{a\sec(e+fx)+a}} + \frac{2a^2(6c+13d)\tan(e+fx)(c+d\sec(e+fx))^2}{35f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(5/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(6*c + 13*d)*(c
+ d*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*
(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^
2*(2*(36*c^3 + 243*c^2*d + 189*c*d^2 + 52*d^3) + d*(24*c^2 + 111*c*d + 52*d
^2)*Sec[e + f*x])*Tan[e + f*x])/(105*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)(c+dx)^3}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^2(c+d\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{(2a\tan(e+fx))\text{Subst}\left(\int \frac{(c+dx)^2\left(-\frac{7a^2c}{2}-\frac{1}{2}a^2(6c+13d)x\right)}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{7f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} - \frac{(4\tan(e+fx))\text{Subst}\left(\int \frac{(c+dx)\left(\frac{35a^3c^2}{4}+\frac{1}{4}a^3(24c^2+111cd+52d^2)x\right)}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{35f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\left(2(36c^3+243c^2d+189cd^2+52d^3)+d(24c^2+111cd+52d^2)\sec(e+fx)\right)\tan(e+fx)}{105f\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^3c^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\left(2(36c^3+243c^2d+189cd^2+52d^3)+d(24c^2+111cd+52d^2)\sec(e+fx)\right)\tan(e+fx)}{105f\sqrt{a+a\sec(e+fx)}} + \frac{\left(2a^2c^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^{5/2}c^3 \operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(2(36c^3+243c^2d+189cd^2+52d^3)+d(24c^2+111cd+52d^2)\sec(e+fx))\tan(e+fx)}{105f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 3.63 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.91

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\left(420\sqrt{2}c^3\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[2]*
c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 2*(210*c^2*d + 37
8*c*d^2 + 164*d^3 + 9*(35*c^3 + 175*c^2*d + 154*c*d^2 + 52*d^3)*Cos[e + f*x
] + 2*d*(105*c^2 + 189*c*d + 52*d^2)*Cos[2*(e + f*x)] + 105*c^3*Cos[3*(e +
f*x)] + 525*c^2*d*Cos[3*(e + f*x)] + 378*c*d^2*Cos[3*(e + f*x)] + 104*d^3*C
os[3*(e + f*x)])*Sin[(e + f*x)/2]))/(420*f)
```

Maple [A] (verified)

Time = 6.62 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.23

method	result
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(105\ \operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,c^3\cos(fx+e)+105\ \operatorname{arctanh}\left(\frac{\sin(fx+}{(\cos(fx+e)+1)\sqrt{-}}\right.\right.}{}$
parts	$\frac{2c^3a\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+}{\cos(fx+e}}}\right.\right.}{f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*a/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(105*arctanh(sin(f*x+e)/(
cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*c^3*cos(f*x+e)+105*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)
/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c^3+105*sin(f*x+
e)*c^3+525*sin(f*x+e)*c^2*d+378*sin(f*x+e)*c*d^2+104*sin(f*x+e)*d^3+105*c^2
*d*tan(f*x+e)+189*c*d^2*tan(f*x+e)+52*d^3*tan(f*x+e)+63*c*d^2*tan(f*x+e)*se
c(f*x+e)+39*d^3*tan(f*x+e)*sec(f*x+e)+15*d^3*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.00

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \left[\frac{105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\mathrm{co}}{\cos(fx+e}\right.}{}\right.$$

$$-\frac{2\left(105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(15\,ad^3+\left(105\,ac^3+5\right.\right.\right.}{}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e)^3)*sqrt(-a)*log((2*a
*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*a*d^3
 + (105*a*c^3 + 525*a*c^2*d + 378*a*c*d^2 + 104*a*d^3)*cos(f*x + e)^3 + (10
5*a*c^2*d + 189*a*c*d^2 + 52*a*d^3)*cos(f*x + e)^2 + 3*(21*a*c*d^2 + 13*a*d
^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*
cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a*c^3*cos(f*x + e)^4 + a*c
^3*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a*d^3 + (105*a*c^3 + 525*a*c^2*d
+ 378*a*c*d^2 + 104*a*d^3)*cos(f*x + e)^3 + (105*a*c^2*d + 189*a*c*d^2 + 52
*a*d^3)*cos(f*x + e)^2 + 3*(21*a*c*d^2 + 13*a*d^3)*cos(f*x + e))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x +
 e)^3)]
```

Sympy [F]

$$\int (a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^3\,dx = \int \left(a(\sec{(e + fx)} + 1)\right)^{\frac{3}{2}} (c + d\sec{(e + fx)})^3\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**3,x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**3, x)
```

Maxima [F]

$$\int (a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^3\,dx = \int (a\sec{(fx + e)} + a)^{\frac{3}{2}}(d\sec{(fx + e)} + c)^3\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
[Out] -1/210*(4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*(7*(15*(a*c^3 + 3*a*c^2*d)*sin(6*f*x + 6*e) + 5*(9*a*c^3 + 33*a*c^2
*d + 18*a*c*d^2 + 4*a*d^3)*sin(4*f*x + 4*e) + (45*a*c^3 + 195*a*c^2*d + 144
*a*c*d^2 + 52*a*d^3)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)) - (105*a*c^3 + 525*a*c^2*d + 378*a*c*d^2 + 104*a*d^3 +
 105*(a*c^3 + 3*a*c^2*d)*cos(6*f*x + 6*e) + 35*(9*a*c^3 + 33*a*c^2*d + 18*a
*c*d^2 + 4*a*d^3)*cos(4*f*x + 4*e) + 7*(45*a*c^3 + 195*a*c^2*d + 144*a*c*d^
2 + 52*a*d^3)*cos(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)))*sqrt(a) + 105*((a*c^3*cos(2*f*x + 2*e)^4 + a*c^3*sin(2*f*x +
 2*e)^4 + 4*a*c^3*cos(2*f*x + 2*e)^3 + 6*a*c^3*cos(2*f*x + 2*e)^2 + 4*a*c^3
*cos(2*f*x + 2*e) + a*c^3 + 2*(a*c^3*cos(2*f*x + 2*e)^2 + 2*a*c^3*cos(2*f*x
 + 2*e) + a*c^3)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) + 1) - (a*c^3*cos(2*f*x + 2*e)^4 + a*c^3*sin(2*f*x + 2*e)^4 + 4*a*c^3
*cos(2*f*x + 2*e)^3 + 6*a*c^3*cos(2*f*x + 2*e)^2 + 4*a*c^3*cos(2*f*x + 2*e)
 + a*c^3 + 2*(a*c^3*cos(2*f*x + 2*e)^2 + 2*a*c^3*cos(2*f*x + 2*e) + a*c^3)*
sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(a
*c^3*f*cos(2*f*x + 2*e)^4 + a*c^3*f*sin(2*f*x + 2*e)^4 + 4*a*c^3*f*cos(2*f*
```

```
x + 2*e)^3 + 6*a*c^3*f*cos(2*f*x + 2*e)^2 + 4*a*c^3*f*cos(2*f*x + 2*e) + a*
c^3*f + 2*(a*c^3*f*cos(2*f*x + 2*e)^2 + 2*a*c^3*f*cos(2*f*x + 2*e) + a*c^3*
f)*sin(2*f*x + 2*e)^2)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos
(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(
2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin
(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*c
os(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin
(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*si
n(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin
(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*si
n(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin
(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*si
n(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos
(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*co
s(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin
(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*si
n(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(
(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
+ 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2
+ (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(c
os(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
+ 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos
(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(
2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3
 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x +
 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x
+ 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
```

6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2), x) - 12*((a*c^3 + 3*a*c^2*d + 2*a*c*d^2)*f*cos(2*f*x + 2*e)^4 + (a*c^3 + 3*a*c^2*d + 2*a*c*d^2)*f*sin(2*f*x + 2*e)^4 + 4*(a*c^3 + 3*a*c^2*d + 2*a*c*d^2)*f*cos(2*f*x + 2*e)^3 + 6*(a*c^3 + 3*a*c^2*d + 2*a*c*d^2)*f*cos(2*f*x + 2*e)^2 + 4*(a*c^3 + 3*a*c^2*d + 2*a*c*d^2)*f*cos(2*f*x + 2*e) + 2*((a*c^3 + 3*a*c^2*d + 2*a*c*d^2)*f*cos(2*f*x + 2*e)^2 + 2*(a*c^3 + 3*a*c^2*d + 2*a*c*d^2)*f*cos(2*f*x + 2*e) + (a*c^3 + 3*a*c^2*d + 2*a*c*d^2)*f)*sin(2*f*x + 2*e)^2 + (a*c^3 + 3*a*c^2*d + 2*a*c*d^2)*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(

```
2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos
(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(
6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*
x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f
*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(
6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2
*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 +
sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x
 + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + co
s(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x +
 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(
2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2
*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e
) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) +
(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x +
 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3
+ (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x
 + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(
2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e
)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*
cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*
f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2
```

+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2), x) - 12*((2*a*c^3 + 7*a*c^2*d + 8*a*c*d^2 + 4*a*d^3)*f*cos(2*f*x + 2*e)^4 + (2*a*c^3 + 7*a*c^2*d + 8*a*c*d^2 + 4*a*d^3)*f*sin(2*f*x + 2*e)^4 + 4*(2*a*c^3 + 7*a*c^2*d + 8*a*c*d^2 + 4*a*d^3)*f*cos(2*f*x + 2*e)^3 + 6*(2*a*c^3 + 7*a*c^2*d + 8*a*c*d^2 + 4*a*d^3)*f*cos(2*f*x + 2*e)^2 + 4*(2*a*c^3 + 7*a*c^2*d + 8*a*c*d^2 + 4*a*d^3)*f*cos(2*f*x + 2*e) + 2*((2*a*c^3 + 7*a*c^2*d + 8*a*c*d^2 + 4*a*d^3)*f*cos(2*f*x + 2*e)^2 + 2*(2*a*c^3 + 7*a*c^2*d + 8*a*c*d^2 + 4*a*d^3)*f*cos(2*f*x + 2*e) + (2*a*c^3 + 7*a*c^2*d + 8*a*c*d^2 + 4*a*d^3)*f)*sin(2*f*x + 2*e)^2 + (2*a*c^3 + 7*a*c^2*d + 8*a*c*d^2 + 4*a*d^3)*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*

```
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 +
2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x +
2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*
cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*
e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e
)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*
x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*
sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*
x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)
)*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
)^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2
+ 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(
4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4
*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2
+ 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)
+ 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(
4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) +
6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) +
2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x +
2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2
*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3
+ 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(
6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
```

```
in(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (c
os(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6
*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1))^2), x) - 4*((5*a*c^3 + 15*a*c^2*d + 18*a*c*d^2 + 4*a*d^3
)*f*cos(2*f*x + 2*e)^4 + (5*a*c^3 + 15*a*c^2*d + 18*a*c*d^2 + 4*a*d^3)*f*si
n(2*f*x + 2*e)^4 + 4*(5*a*c^3 + 15*a*c^2*d + 18*a*c*d^2 + 4*a*d^3)*f*cos(2*
f*x + 2*e)^3 + 6*(5*a*c^3 + 15*a*c^2*d + 18*a*c*d^2 + 4*a*d^3)*f*cos(2*f*x
+ 2*e)^2 + 4*(5*a*c^3 + 15*a*c^2*d + 18*a*c*d^2 + 4*a*d^3)*f*cos(2*f*x + 2*
e) + 2*((5*a*c^3 + 15*a*c^2*d + 18*a*c*d^2 + 4*a*d^3)*f*cos(2*f*x + 2*e)^2
+ 2*(5*a*c^3 + 15*a*c^2*d + 18*a*c*d^2 + 4*a*d^3)*f*cos(2*f*x + 2*e) + (5*a
*c^3 + 15*a*c^2*d + 18*a*c*d^2 + 4*a*d^3)*f)*sin(2*f*x + 2*e)^2 + (5*a*c^3
+ 15*a*c^2*d + 18*a*c*d^2 + 4*a*d^3)*f)*integrate((cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f
*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*
f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin
(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(
2*f*x + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos
(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*co
s(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos
(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*co
s(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos
(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*co
s(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*
f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin
(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*si
n(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*
x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^
2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*si
n(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*
x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*
f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*
```

x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*
x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(c
os(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*
x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x
+ 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*si
n(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4
+ (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*
f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e
)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(
2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
+ 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*
f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*
f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e
)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*
x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2), x) - 6*((a*c^3 + 2*a*c^2
*d)*f*cos(2*f*x + 2*e)^4 + (a*c^3 + 2*a*c^2*d)*f*sin(2*f*x + 2*e)^4 + 4*(a*
c^3 + 2*a*c^2*d)*f*cos(2*f*x + 2*e)^3 + 6*(a*c^3 + 2*a*c^2*d)*f*cos(2*f*x +
2*e)^2 + 4*(a*c^3 + 2*a*c^2*d)*f*cos(2*f*x + 2*e) + 2*((a*c^3 + 2*a*c^2*d)
*f*cos(2*f*x + 2*e)^2 + 2*(a*c^3 + 2*a*c^2*d)*f*cos(2*f*x + 2*e) + (a*c^3 +
2*a*c^2*d)*f)*sin(2*f*x + 2*e)^2 + (a*c^3 + 2*a*c^2*d)*f)*integrate((cos(2

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*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8
*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(
4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2
*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(
2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(
6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2
*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(
2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(
8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin
(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(
2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(
6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2
*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(
2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^
4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(
8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8
*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*si
n(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f
*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(
2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(
6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2
*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*
f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 +
```

```
sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x
 + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + co
s(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x +
 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(
2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2
*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e
) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) +
(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x +
 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3
+ (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x
 + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2), x))*s
qrt(a))/(f*cos(2*f*x + 2*e)^4 + f*sin(2*f*x + 2*e)^4 + 4*f*cos(2*f*x + 2*e)
^3 + 6*f*cos(2*f*x + 2*e)^2 + 2*(f*cos(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e
) + f)*sin(2*f*x + 2*e)^2 + 4*f*cos(2*f*x + 2*e) + f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)^3\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e+fx))^{3/2}(c + d\sec(e+fx))^3\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^3\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3, x)
```

3.155 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx$

Optimal result 1021
Rubi [A] (verified) 1021
Mathematica [A] (verified) 1024
Maple [A] (verified) 1024
Fricas [A] (verification not implemented) 1025
Sympy [F] 1026
Maxima [F] 1026
Giac [F] 1031
Mupad [F(-1)] 1031

Optimal result

Integrand size = 27, antiderivative size = 176

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \frac{2a^{5/2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2/5*a^2*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/15*a^2*(12
*c^2+50*c*d+18*d^2+d*(4*c+9*d)*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1
/2)+2*a^(5/2)*c^2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a
*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

$= \{4025, 158, 152, 65, 212\}$

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \frac{2a^{5/2}c^2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^2\tan(e+fx)\left(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx)\right)}{15f\sqrt{a\sec(e+fx)+a}} + \frac{2a^2\tan(e+fx)(c+d\sec(e+fx))^2}{5f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(c + d*Sec[e + f
*x])^2*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(2*(6*c^2 + 25
*c*d + 9*d^2) + d*(4*c + 9*d)*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[a + a*
Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)(c+dx)^2}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}}+\frac{(2a\tan(e+fx))\text{Subst}\left(\int\frac{(c+dx)\left(-\frac{5a^2c}{2}-\frac{1}{2}a^2(4c+9d)x\right)}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}}-\frac{(a^3c^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$
\begin{aligned}
&= \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2a^2(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{(2a^2c^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= \frac{2a^{5/2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2a^2(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.82

$$
\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\left(30\sqrt{2}c^2\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}
$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2]*c
^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(15*c^2 + 50*c*d
 + 24*d^2 + 2*d*(10*c + 9*d)*Cos[e + f*x] + (15*c^2 + 50*c*d + 18*d^2)*Cos[
2*(e + f*x)])*Sin[(e + f*x)/2]))/(30*f)
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.32

method	result
default	$\frac{2a\sqrt{a(\sec(fx+e)+1)}\left(15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)c^2\cos(fx+e)+15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\right.\right.}{15f}$
parts	$\frac{2c^2a\sqrt{a(\sec(fx+e)+1)}\left(\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)}}}\right.\right.}{f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*a/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(15*(-cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))
^(1/2))*c^2*cos(f*x+e)+15*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(sin(f*
x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*c^2+15*sin(f*x+e)*c
^2+50*sin(f*x+e)*c*d+18*sin(f*x+e)*d^2+10*c*d*tan(f*x+e)+9*d^2*tan(f*x+e)+3
*d^2*tan(f*x+e)*sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.26

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \left[\frac{15\left(ac^2\cos\left(fx+e\right)^3+ac^2\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(\cdots}{\cos(fx+e)+\cdots}\right.}{\cdots}\right.$$

$$-\frac{2\left(15\left(ac^2\cos\left(fx+e\right)^3+ac^2\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3\,ad^2+\left(15\,ac^2+50\,a\cdots\right.\right.}{15\left(f\cos\left(fx+e\right)^3+f\cos\left(\cdots\right.\right.}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*c
os(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a*d^2 +
(15*a*c^2 + 50*a*c*d + 18*a*d^2)*cos(f*x + e)^2 + (10*a*c*d + 9*a*d^2)*cos(
f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
+ e)^3 + f*cos(f*x + e)^2), -2/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x
 + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e))) - (3*a*d^2 + (15*a*c^2 + 50*a*c*d + 18*a*d^2)*cos
(f*x + e)^2 + (10*a*c*d + 9*a*d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int (a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^2\,dx = \int \left(a(\sec(e + fx) + 1)\right)^{\frac{3}{2}}(c + d\sec(e + fx))^2\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**2,x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int (a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^2\,dx = \int (a\sec(fx + e) + a)^{\frac{3}{2}}(d\sec(fx + e) + c)^2\,dx$$

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[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
[Out] -1/30*(15*((a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*c
os(2*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) -
 (a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*cos(2*f*x +
 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(a*c^2*
f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*cos(2*f*x + 2
*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x +
 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x +
2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x +
6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2
*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x +
4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2
*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*
e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e
```

```
)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x +
 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*
e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2
*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6
*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*
e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 +
 (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x
+ 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2
*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*
x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)
*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))
*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*
x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*
e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(
4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2),
x) - 2*((5*a*c^2 + 12*a*c*d + 4*a*d^2)*f*cos(2*f*x + 2*e)^2 + (5*a*c^2 + 12
*a*c*d + 4*a*d^2)*f*sin(2*f*x + 2*e)^2 + 2*(5*a*c^2 + 12*a*c*d + 4*a*d^2)*f
*cos(2*f*x + 2*e) + (5*a*c^2 + 12*a*c*d + 4*a*d^2)*f)*integrate((cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x
+ 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x +
 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x
+ 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x +
 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan
```

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2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x +
 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x +
2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x +
4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2
*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6
*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (
2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(
2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2
*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3
+ cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x +
2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x +
 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin
(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 +
 (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f
*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)
^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2
*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2
*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2
*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*
f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 +
 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4
*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1))^2), x) - 2*((7*a*c^2 + 16*a*c*d + 12*a*d
^2)*f*cos(2*f*x + 2*e)^2 + (7*a*c^2 + 16*a*c*d + 12*a*d^2)*f*sin(2*f*x + 2*
e)^2 + 2*(7*a*c^2 + 16*a*c*d + 12*a*d^2)*f*cos(2*f*x + 2*e) + (7*a*c^2 + 16
*a*c*d + 12*a*d^2)*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(
4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2
```

```
*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6
*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*
f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4
*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*
f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f
*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*
x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(
2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f
*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e
)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e)
)*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f
*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2
*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin
(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 +
 (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*c
os(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x
 + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
+ 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + c
os(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f
*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
))^2), x) - 2*((3*a*c^2 + 4*a*c*d)*f*cos(2*f*x + 2*e)^2 + (3*a*c^2 + 4*a*c*
```

```
d)*f*sin(2*f*x + 2*e)^2 + 2*(3*a*c^2 + 4*a*c*d)*f*cos(2*f*x + 2*e) + (3*a*c
^2 + 4*a*c*d)*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x
 + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x
+ 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x
+ 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x +
 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x
+ 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x +
 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x +
6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2
*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x
 + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x +
2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin
(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos
(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x +
2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3
 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*
x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos
(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 +
 sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*
f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*
e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e
))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*
f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2
```

```
*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x +
2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*si
n(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)
, x))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1
/4)*sqrt(a) + 4*(5*(3*(a*c^2 + 2*a*c*d)*sin(4*f*x + 4*e) + 2*(3*a*c^2 + 8*a
*c*d + 3*a*d^2)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)) - (15*a*c^2 + 50*a*c*d + 18*a*d^2 + 15*(a*c^2 + 2*a*c*d)*co
s(4*f*x + 4*e) + 10*(3*a*c^2 + 8*a*c*d + 3*a*d^2)*cos(2*f*x + 2*e))*sin(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a))/((f*cos(2*f*x +
 2*e)^2 + f*sin(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e) + f)*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4))
```

Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)^2\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2, x)
```

3.156 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

Optimal result 1032
Rubi [A] (verified) 1032
Mathematica [A] (verified) 1034
Maple [B] (verified) 1034
Fricas [A] (verification not implemented) 1035
Sympy [F] 1035
Maxima [B] (verification not implemented) 1036
Giac [F] 1037
Mupad [F(-1)] 1037

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\frac{2a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}+\frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f}$$

```
[Out] 2*a^(3/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f+2/3*a^2*(3*
c+4*d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a*d*(a+a*sec(f*x+e))^(1/2)*t
an(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4002, 4000, 3859, 209, 3877}

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\frac{2a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a^2*(3*c + 4*d)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*Sqr
t[a + a*Sec[e + f*x]]*Tan[e + f*x])/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4002

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
(b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*
m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f} \\ &\quad + \frac{2}{3}\int \sqrt{a+a\sec(e+fx)}\left(\frac{3ac}{2}+\frac{1}{2}a(3c+4d)\sec(e+fx)\right)dx \\ &= \frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f}+(ac)\int \sqrt{a+a\sec(e+fx)}\,dx \\ &\quad + \frac{1}{3}(a(3c+4d))\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,dx\end{aligned}$$

$$= \frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f} - \frac{(2a^2c)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\left(3\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\cos\right.}{3f}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*(3*Sqrt[2]*c*Ar
cSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(3/2) + 2*(d + (3*c + 5*d)*Cos[
e + f*x])*Sin[(e + f*x)/2]))/(3*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(91) = 182$.

Time = 1.46 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.96

method	result
default	$\frac{2ca\sqrt{a(\sec(fx+e)+1)}\left(\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e}{\cos(fx+e)-}}}\right.\right.}{f(\cos(fx+e)+1)}$
parts	$\frac{2ca\sqrt{a(\sec(fx+e)+1)}\left(\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+\text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e}{\cos(fx+e)-}}}\right.\right.}{f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c/f*a*(a*(sec(f*x+e)+1))^(1/2)*(arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f
*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+
```

```
arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e))/(cos(f*x+e)+1)+2/3*d/f*a*(a*(sec(
f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(5*sin(f*x+e)+tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.01

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \left[\frac{3\left(ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\cdots}{\cos(fx+e)+1}\cdots\right.}{3\left(f\cos\left(fx+e\right)^2\cdots\right.}\cdots\right.$$

$$-\frac{2\left(3\left(ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(ad+\left(3\,ac+5\,ad\right)\cos\left(fx\cdots\right.\right.\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(a*d + (3*a*c + 5*a
*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*
cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x
 + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/
(sqrt(a)*sin(f*x + e))) - (a*d + (3*a*c + 5*a*d)*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e
))]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(91) = 182$.

Time = 0.39 (sec) , antiderivative size = 998, normalized size of antiderivative = 9.50

$$\int (a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 + 1) + a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - (a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))*sqrt(a))*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)), x)
```

3.157 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

Optimal result 1038
Rubi [A] (verified) 1038
Mathematica [A] (verified) 1040
Maple [B] (warning: unable to verify) 1040
Fricas [A] (verification not implemented) 1041
Sympy [F] 1042
Maxima [F] 1042
Giac [F] 1042
Mupad [F(-1)] 1042

Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{2a^{3/2}(c-d)\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{d}\sqrt{c+d}f}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+2*a^(3/2)*(
c-d)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/2))/
c/f/d^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4012, 3859, 209, 4052, 211}

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{2a^{3/2}(c-d)\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{c\sqrt{d}f\sqrt{c+d}} + \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (2*a^(3/2)*(c - d)*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sec[e + f*x]])])/(c*Sqrt[d]*Sqrt[c + d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] +
 Dist[(b*c - a*d)/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a\int\sqrt{a+a\sec(e+fx)}\,dx}{c}+\frac{(ac-ad)\int\frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{c}\\
&= -\frac{(2a^2)\,\text{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\
&\quad-\frac{(2a^2(c-d))\,\text{Subst}\left(\int\frac{1}{ac+ad+dx^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\
&= \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{2a^{3/2}(c-d)\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{d}\sqrt{c+d}f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{\sqrt{2}a\left(\sqrt{d}\sqrt{c+d}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)+(c-d)\arctan\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right.\right.}{c\sqrt{d}\sqrt{c+d}f}$$

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]

[Out] (Sqrt[2]*a*(Sqrt[d]*Sqrt[c + d]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (c - d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]])])*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(c*Sqrt[d]*Sqrt[c + d]*f)

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. 2(90) = 180.

Time = 14.28 (sec) , antiderivative size = 840, normalized size of antiderivative = 7.64

method	result
default	$\frac{\sqrt{2}\,a\left(2\sqrt{(c+d)(c-d)}\,\sqrt{\frac{d}{c-d}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\ln\left(-\frac{2\left(\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,c-\sqrt{2}\,\sqrt{\frac{d}{c-d}}\right.}{-c(-\cot(fx+e)+\csc(fx+e)}\right.\right.}{}$

[In] int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/2/f*2^(1/2)*a/((c+d)*(c-d))^(1/2)/c/(d/(c-d))^(1/2)*(2*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+ln(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*c-d*ln(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))-ln(2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*c+d*ln(2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2))))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.91 (sec) , antiderivative size = 731, normalized size of antiderivative = 6.65

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \left[-\frac{(ac-ad)\sqrt{-\frac{a}{cd+d^2}}\log\left(\frac{2\,(cd+d^2)\sqrt{-\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac+2\cdots}{c\cos(fx+e)^2+(c+d)\cos(fx+\cdots}\right.}{\cdots}\right.$$

$$-\frac{2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+(ac-ad)\sqrt{-\frac{a}{cd+d^2}}\log\left(\frac{2\,(cd+d^2)\sqrt{-\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e\cdots}{c\cos(fx+e)^2+(c+d)\cdots}\right.}{cf}$$

$$-\frac{2\,(ac-ad)\sqrt{\frac{a}{cd+d^2}}\arctan\left(\frac{(c+d)\sqrt{\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{a\sin(fx+e)}\right)-\sqrt{-a}a\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cdots}{\cos(fx+\cdots}\right.}{cf}$$

$$\left.-\frac{2\left((ac-ad)\sqrt{\frac{a}{cd+d^2}}\arctan\left(\frac{(c+d)\sqrt{\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{a\sin(fx+e)}\right)+a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)\right)}{cf}\right]$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-((a*c - a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c +
 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2
+ (c + d)*cos(f*x + e) + d)) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*sqrt(
-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*c
os(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -(2*a^(3/2)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a*c -
a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*co
s(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*
cos(f*x + e) + d)))/(c*f), -(2*(a*c - a*d)*sqrt(a/(c*d + d^2))*arctan((c +
d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
/(a*sin(f*x + e))) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e
) - a)/(cos(f*x + e) + 1)))/(c*f), -2*((a*c - a*d)*sqrt(a/(c*d + d^2))*arct
an((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)/(a*sin(f*x + e))) + a^(3/2)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{c+d\sec(e+fx)}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{d\sec(fx+e)+c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{d\sec(fx+e)+c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)), x)
```

3.158 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx$

Optimal result 1043
Rubi [A] (verified) 1043
Mathematica [A] (warning: unable to verify) 1046
Maple [B] (warning: unable to verify) 1047
Fricas [A] (verification not implemented) 1047
Sympy [F] 1048
Maxima [F(-1)] 1048
Giac [F] 1049
Mupad [F(-1)] 1049

Optimal result

Integrand size = 27, antiderivative size = 229

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}(c^2-3cd-2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2\sqrt{d}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

```
[Out] a^2*(c-d)*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*a^
(5/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*sec(f*x
+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+a^(5/2)*(c^2-3*c*d-2*d^2)*arctanh(d^(1/2)
*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(c+d)^(3/2)/f/d
^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used

$= \{4025, 156, 162, 65, 212, 214\}$

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{a^{5/2}(c^2-3cd-2d^2)\tan(e+fx)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2\sqrt{d}f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^{5/2}\tan(e+fx)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{a^2(c-d)\tan(e+fx)}{cf(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^(5/2)*(c^2 - 3*c*d -
 2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*T
an[e + f*x])/(c^2*Sqrt[d]*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) + (a^2*(c - d)*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + a*Sec[
e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{a+ax}{x\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\\
&\quad -\frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{a^2(c+d)+\frac{1}{2}a^2(c-d)x}{x\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{c(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\\
&\quad -\frac{\left(a^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad -\frac{\left(a^3\left(c^2-3cd-2d^2\right)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{2c^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(2a^2\tan(e+fx))\operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a^2(c^2-3cd-2d^2)\tan(e+fx))\operatorname{Subst}\left(\int \frac{1}{c+d-\frac{dx^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{c^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}(c^2-3cd-2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2\sqrt{d}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [A] (warning: unable to verify)

Time = 4.74 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.28

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{(d+c\cos(e+fx))^2\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}(a(1+\sec(e+fx)))^{3/2}\Bigg(}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^3*Sqrt[Sec[e + f*x]]*(a*(1 + Sec[e
 + f*x]))^(3/2)*(((2*Sqrt[d]*(c + d)^(3/2)*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos
[e + f*x]/(1 + Cos[e + f*x])]] + (c^2 - 3*c*d - 2*d^2)*ArcTan[(Sqrt[d]*Tan[
(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])])*Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])]*Sec[(e + f*x)/2]*Sqrt[1 + Sec[e + f*x]])/(Sq
rt[d]*(c + d)^(3/2)*Sqrt[(1 + Cos[e + f*x])^(-1)]) + (2*c*(c - d)*Sqrt[Sec[
e + f*x]]*Sin[(e + f*x)/2])/((c + d)*(c + d*Sec[e + f*x]))))/(4*c^2*f*(c +
d*Sec[e + f*x])^2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 25447 vs. 2(199) = 398.

Time = 15.93 (sec) , antiderivative size = 25448, normalized size of antiderivative = 111.13

method	result	size
default	Expression too large to display	25448

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 4.13 (sec) , antiderivative size = 1640, normalized size of antiderivative = 7.16

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
[Out] [1/2*(2*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*a*
c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*x +
 e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*co
s(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*
cos(f*x + e) + d)) + 2*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a
*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*
sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x + e)^2
+ (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), 1/2*(2*(
a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) - 4*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a*c^2 + 2*a*c*
d + a*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3
+ (a*c^3 - 3*a*c^2*d - 2*a*c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a
*c*d^2 - 2*a*d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqr
t(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(
c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((c^4 + c^3*d)*f*cos(f*x + e
)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), ((a*
c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
 e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*a*c*d^2)*cos(
```

```
f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*x + e))*sqrt(a
/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + (a*c*d + a*d^2 + (a*c^2 + a*
c*d)*cos(f*x + e)^2 + (a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 +
c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d
 + c^2*d^2)*f), ((a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2
*d - 2*a*c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*
cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) - 2*(a*c*d
 + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a*c^2 + 2*a*c*d + a*d^2)*cos(f
*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e))))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d
 + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f)]
```

Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^2}\,dx = \int \frac{(a(\sec(e + fx) + 1))^{\frac{3}{2}}}{(c + d\sec(e + fx))^2}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a\sec\left(fx+e\right)+a)^{\frac{3}{2}}}{(d\sec\left(fx+e\right)+c)^2}\,dx$$

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")

[Out] sage0*x

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

[In] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^2,x)

[Out] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^2, x)

3.159 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx$

Optimal result 1050
Rubi [A] (verified) 1051
Mathematica [A] (warning: unable to verify) 1053
Maple [B] (warning: unable to verify) 1054
Fricas [B] (verification not implemented) 1054
Sympy [F] 1056
Maxima [F(-1)] 1056
Giac [F] 1056
Mupad [F(-1)] 1057

Optimal result

Integrand size = 27, antiderivative size = 310

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}(3c^3-15c^2d-20cd^2-8d^3)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c^3\sqrt{d}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^2(3c^2-7cd-4d^2)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

```
[Out] 1/2*a^2*(c-d)*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2
)+1/4*a^2*(3*c^2-7*c*d-4*d^2)*tan(f*x+e)/c^2/(c+d)^2/f/(c+d*sec(f*x+e))/(a+
a*sec(f*x+e))^(1/2)+2*a^(5/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f
*x+e)/c^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+1/4*a^(5/2)*(3*c^
3-15*c^2*d-20*c*d^2-8*d^3)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(
c+d)^(1/2))*tan(f*x+e)/c^3/(c+d)^(5/2)/f/d^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+
a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4025, 156, 162, 65, 212, 214}

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \frac{2a^{5/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{a^{5/2}(3c^3-15c^2d-20cd^2-8d^3)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4c^3\sqrt{d}f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{a^2(3c^2-7cd-4d^2)\tan(e+fx)}{4c^2f(c+d)^2\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))} + \frac{a^2(c-d)\tan(e+fx)}{2cf(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}$$

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^(5/2)*(3*c^3 - 15*c^
2*d - 20*c*d^2 - 8*d^3)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]
*Sqrt[c + d])]*Tan[e + f*x])/(4*c^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a - a*Sec[
e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^2*(c - d)*Tan[e + f*x])/(2*c*(c +
d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (a^2*(3*c^2 - 7*c*d
 - 4*d^2)*Tan[e + f*x])/(4*c^2*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*
Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{a+ax}{x\sqrt{a-ax}(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\\
&\quad -\frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{2a^2(c+d)+\frac{3}{2}a^2(c-d)x}{x\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{2c(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\\
&\quad +\frac{a^2(3c^2-7cd-4d^2)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\\
&\quad -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{2a^3(c+d)^2+\frac{1}{4}a^3(3c^2-7cd-4d^2)x}{x\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{2c^2(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^2(3c^2-7cd-4d^2)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{(a^3\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(a^3(3c^3-15c^2d-20cd^2-8d^3)\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{8c^3(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^2(3c^2-7cd-4d^2)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(2a^2\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a^2(3c^3-15c^2d-20cd^2-8d^3)\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{4c^3(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}(3c^3-15c^2d-20cd^2-8d^3)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c^3\sqrt{d}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^2(3c^2-7cd-4d^2)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [A] (warning: unable to verify)

Time = 5.86 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.16

$$\int\frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \frac{(d+c\cos(e+fx))^3\sec^3\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)(a(1+\sec(e+fx)))^{3/2}\Bigg(}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^3*Sec[e + f*x]^(3/2)*(a*(1 + Sec[e
 + f*x]))^(3/2)*((Sqrt[2]*(8*Sqrt[d]*(c + d)^(5/2)*ArcTan[Tan[(e + f*x)/2]/
Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] + (3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d
^3)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + C
os[e + f*x])])]))*Sec[(e + f*x)/2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sqr
t[Cos[(e + f*x)/2]^2*Sec[e + f*x]])/(Sqrt[d]*(c + d)^(5/2)*Sqrt[Sec[(e + f*
x)/2]^2]) + (2*c*Sqrt[Sec[e + f*x]]*(c*(5*c^2 - 7*c*d - 6*d^2) + d*(3*c^2 -
 7*c*d - 4*d^2)*Sec[e + f*x])*Sin[(e + f*x)/2])/((c + d)^2*(c + d*Sec[e + f
*x])^2)))/(16*c^3*f*(c + d*Sec[e + f*x])^3)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 55535 vs. $2(272) = 544$.

Time = 16.91 (sec) , antiderivative size = 55536, normalized size of antiderivative = 179.15

method	result	size
default	Expression too large to display	55536

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(272) = 544$.

Time = 11.67 (sec) , antiderivative size = 2729, normalized size of antiderivative = 8.80

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a
*c^4*d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d
- 50*a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 2
7*a*c^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(-a/(c
*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 -
a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) +
d)) - 8*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*co
s(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e)^2
 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
```

```
os(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*((5*
a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2
- 4*a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*
d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*
d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/8*(16*(a*c^2*d
^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e)^3 + (
a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e)^2 + (2*a*c^3*d +
5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (3*a*c^3*
d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*d - 20*a*c^
3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*a*c^3*d^2 -
 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c^3*d^2 - 55
*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((
2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d
)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) - 2*((5*a*c^
4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2 - 4*
a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)
)/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2
+ 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)
*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/4*((3*a*c^3*d^2 -
15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*d - 20*a*c^3*d^2
- 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*a*c^3*d^2 - 48*a*
c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c^3*d^2 - 55*a*c^2
*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c +
d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
/(a*sin(f*x + e))) - 4*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d
+ a*c^2*d^2)*cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)
*cos(f*x + e)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e
))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e)
 + 1)) - ((5*a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d -
 7*a*c^2*d^2 - 4*a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c
^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c
^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/4
*((3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*
d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*
a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c
^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(a/(c*d + d
^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)/(a*sin(f*x + e))) + 8*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a
*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2
*d^2 + 2*a*c*d^3)*cos(f*x + e)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a
```

```
*d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - ((5*a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)
*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2 - 4*a*c*d^3)*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)
*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^
2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 +
 2*c^4*d^3 + c^3*d^4)*f)]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}}{(c+d\sec{(e+fx)})^3}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a\sec{(fx+e)}+a)^{\frac{3}{2}}}{(d\sec{(fx+e)}+c)^3}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^3, x)
```

3.160 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx$

Optimal result 1058
Rubi [A] (verified) 1059
Mathematica [A] (verified) 1061
Maple [A] (verified) 1062
Fricas [A] (verification not implemented) 1063
Sympy [F] 1063
Maxima [F(-1)] 1064
Giac [F] 1064
Mupad [F(-1)] 1064

Optimal result

Integrand size = 27, antiderivative size = 336

$$\begin{aligned}
&\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \frac{2a^3(3c^3+12c^2d+12cd^2+4d^3)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} \\
&+\frac{2a^{7/2}c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&+\frac{2ad(3c^2+15cd+13d^2)\,(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} \\
&-\frac{6d^2(c+2d)(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} \\
&+\frac{2d^3(a-a\sec(e+fx))^4\tan(e+fx)}{9af\sqrt{a+a\sec(e+fx)}} \\
&-\frac{2(c^3+12c^2d+24cd^2+12d^3)\,(a^3-a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

```
[Out] 2*a^3*(3*c^3+12*c^2*d+12*c*d^2+4*d^3)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2
/5*a*d*(3*c^2+15*c*d+13*d^2)*(a-a*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e
))^(1/2)-6/7*d^2*(c+2*d)*(a-a*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(
1/2)+2/9*d^3*(a-a*sec(f*x+e))^4*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-2/3*(
c^3+12*c^2*d+24*c*d^2+12*d^3)*(a^3-a^3*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*
x+e))^(1/2)+2*a^(7/2)*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e
)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4025, 186, 65, 212}

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \frac{2a^{7/2}c^3\tan(e+fx)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{a-a\sec(e+fx)}} - \frac{2(c^3+12c^2d+24cd^2+12d^3)\tan(e+fx)\,(a^3-a^3\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}} + \frac{2a^3(3c^3+12c^2d+12cd^2+4d^3)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}} + \frac{2ad(3c^2+15cd+13d^2)\tan(e+fx)(a-a\sec(e+fx))^2}{5f\sqrt{a\sec(e+fx)+a}} - \frac{6d^2(c+2d)\tan(e+fx)(a-a\sec(e+fx))^3}{7f\sqrt{a\sec(e+fx)+a}} + \frac{2d^3\tan(e+fx)(a-a\sec(e+fx))^4}{9af\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^3*(3*c^3 + 12*c^2*d + 12*c*d^2 + 4*d^3)*Tan[e + f*x])/(f*Sqrt[a + a*Se
c[e + f*x]]) + (2*a^(7/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan
[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*(
3*c^2 + 15*c*d + 13*d^2)*(a - a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[a +
 a*Sec[e + f*x]]) - (6*d^2*(c + 2*d)*(a - a*Sec[e + f*x])^3*Tan[e + f*x])/(
7*f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*(a - a*Sec[e + f*x])^4*Tan[e + f*x])
/(9*a*f*Sqrt[a + a*Sec[e + f*x]]) - (2*(c^3 + 12*c^2*d + 24*c*d^2 + 12*d^3)
*(a^3 - a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
```

```
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)^2(c+dx)^3}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\left(\frac{a^2(3c^3+12c^2d+12cd^2+4d^3)}{\sqrt{a-ax}}+\frac{a^2c^3}{x\sqrt{a-ax}}-a(c^3+12c^2d+24cd^2+12d^3)\sqrt{a-\cdots}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{\cdots}}$$

$$= \frac{2a^3(3c^3+12c^2d+12cd^2+4d^3)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2ad(3c^2+15cd+13d^2)\,(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} - \frac{6d^2(c+2d)(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3(a-a\sec(e+fx))^4\tan(e+fx)}{9af\sqrt{a+a\sec(e+fx)}} - \frac{2(c^3+12c^2d+24cd^2+12d^3)\,(a^3-a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} - \frac{(a^4c^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3(3c^3+12c^2d+12cd^2+4d^3)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2ad(3c^2+15cd+13d^2)(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} - \frac{6d^2(c+2d)(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3(a-a\sec(e+fx))^4\tan(e+fx)}{9af\sqrt{a+a\sec(e+fx)}} - \frac{2(c^3+12c^2d+24cd^2+12d^3)(a^3-a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{(2a^3c^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3(3c^3+12c^2d+12cd^2+4d^3)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2ad(3c^2+15cd+13d^2)(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} - \frac{6d^2(c+2d)(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3(a-a\sec(e+fx))^4\tan(e+fx)}{9af\sqrt{a+a\sec(e+fx)}} - \frac{2(c^3+12c^2d+24cd^2+12d^3)(a^3-a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 4.73 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.85

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\left(2520\sqrt{2}c^3\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*(2520*Sqrt[
2]*c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(9/2) + 2*(2520*c^3 +
```

8883*c^2*d + 8370*c*d^2 + 2908*d^3 + (630*c^3 + 5292*c^2*d + 7290*c*d^2 + 2
792*d^3)*Cos[e + f*x] + 4*(840*c^3 + 2898*c^2*d + 2610*c*d^2 + 803*d^3)*Cos
[2*(e + f*x)] + 210*c^3*Cos[3*(e + f*x)] + 1764*c^2*d*Cos[3*(e + f*x)] + 20
70*c*d^2*Cos[3*(e + f*x)] + 584*d^3*Cos[3*(e + f*x)] + 840*c^3*Cos[4*(e + f
*x)] + 2709*c^2*d*Cos[4*(e + f*x)] + 2070*c*d^2*Cos[4*(e + f*x)] + 584*d^3*
Cos[4*(e + f*x)])*Sin[(e + f*x)/2]))/(2520*f)

Maple [A] (verified)

Time = 90.86 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.09

method	result
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(315\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,c^3\cos(fx+e)+315\,\operatorname{arctanh}\left(\frac{\sin(fx}{(\cos(fx+e)+1)\sqrt{-}}\right.\right.}{}$
parts	$\frac{2c^3a^2\sqrt{a(\sec(fx+e)+1)}\left(3\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+3\,\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos}{\cos(}}}\right.\right.}{3f(\cos(fx+e)+1)}$

[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 2/315*a^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(315*arctanh(sin(f*x+e)
/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*c^3*cos(f*x+e)+315*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+
e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c^3+840*sin(f*
x+e)*c^3+2709*sin(f*x+e)*c^2*d+2070*sin(f*x+e)*c*d^2+584*sin(f*x+e)*d^3+105
*c^3*tan(f*x+e)+882*c^2*d*tan(f*x+e)+1035*c*d^2*tan(f*x+e)+292*d^3*tan(f*x+
e)+189*c^2*d*tan(f*x+e)*sec(f*x+e)+540*c*d^2*tan(f*x+e)*sec(f*x+e)+219*d^3*
tan(f*x+e)*sec(f*x+e)+135*c*d^2*tan(f*x+e)*sec(f*x+e)^2+130*d^3*tan(f*x+e)*
sec(f*x+e)^2+35*d^3*tan(f*x+e)*sec(f*x+e)^3)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.85

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \left[\frac{315\left(a^2c^3\cos(fx+e)^5+a^2c^3\cos(fx+e)^4\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cdots}{\cos(fx\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{2\left(315\left(a^2c^3\cos(fx+e)^5+a^2c^3\cos(fx+e)^4\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(35\,a^2d^3+\left(840\,a^2c^3\cdots\right.\right.\right.}{\cdots}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
[Out] [1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(35*a
^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*a^2*d^3)*cos(
f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 + 292*a^2*d^3)*c
os(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3)*cos(f*x + e)^
2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -2/315*(3
15*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arctan(sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (35
*a^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*a^2*d^3)*co
s(f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 + 292*a^2*d^3)
*cos(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3)*cos(f*x + e
)^2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(c+d\sec\left(e+fx\right)\right)^3\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**3,x)
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)^3\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^3\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3, x)
```

3.161 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx$

Optimal result 1065
Rubi [A] (verified) 1066
Mathematica [A] (verified) 1068
Maple [A] (verified) 1068
Fricas [A] (verification not implemented) 1069
Sympy [F] 1070
Maxima [F] 1070
Giac [F] 1076
Mupad [F(-1)] 1076

Optimal result

Integrand size = 27, antiderivative size = 258

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2ad(2c+5d)(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} - \frac{2(c^2+8cd+8d^2)(a^3-a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a^3*(c+2*d)*(3*c+2*d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/5*a*d*(2*c+5*
d)*(a-a*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/7*d^2*(a-a*sec(
f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*(c^2+8*c*d+8*d^2)*(a^3-a^
3*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*c^2*arctanh((a-
a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f
*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4025, 186, 65, 212}

$$\int (a + a\sec(e + fx))^{5/2}(c + d\sec(e + fx))^2 \, dx = \frac{2a^{7/2}c^2\tan(e + fx)\text{arctanh}\left(\frac{\sqrt{a - a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a - a\sec(e + fx)}\sqrt{a\sec(e + fx) + a}} - \frac{2(c^2 + 8cd + 8d^2)\tan(e + fx)\,(a^3 - a^3\sec(e + fx))}{3f\sqrt{a\sec(e + fx) + a}} + \frac{2a^3(c + 2d)(3c + 2d)\tan(e + fx)}{f\sqrt{a\sec(e + fx) + a}} + \frac{2ad(2c + 5d)\tan(e + fx)(a - a\sec(e + fx))^2}{5f\sqrt{a\sec(e + fx) + a}} - \frac{2d^2\tan(e + fx)(a - a\sec(e + fx))^3}{7f\sqrt{a\sec(e + fx) + a}}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*(c + 2*d)*(3*c + 2*d)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (
2*a^(7/2)*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*(2*c + 5*d)*(a -
a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(a
- a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[a + a*Sec[e + f*x]]) - (2*(c^2
+ 8*c*d + 8*d^2)*(a^3 - a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec
[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^2(c+dx)^2}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{a^2(c+2d)(3c+2d)}{\sqrt{a-ax}}+\frac{a^2c^2}{x\sqrt{a-ax}}-a\left(c^2+8cd+8d^2\right)\sqrt{a-ax}+d(2c+5d)(\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2ad(2c+5d)(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} - \frac{2\left(c^2+8cd+8d^2\right)\left(a^3-a^3\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^4c^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2ad(2c+5d)(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} - \frac{2\left(c^2+8cd+8d^2\right)\left(a^3-a^3\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{\left(2a^3c^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+ \frac{2ad(2c+5d)(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}}$$
$$- \frac{2d^2(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}}$$
$$- \frac{2(c^2+8cd+8d^2)(a^3-a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.74

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = \frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\left(420\sqrt{2}c^2\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[2
]*c^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 4*(35*c^2 + 196
*c*d + 145*d^2 + (420*c^2 + 987*c*d + 465*d^2)*Cos[e + f*x] + (35*c^2 + 196
*c*d + 115*d^2)*Cos[2*(e + f*x)] + 140*c^2*Cos[3*(e + f*x)] + 301*c*d*Cos[3
*(e + f*x)] + 115*d^2*Cos[3*(e + f*x)])*Sin[(e + f*x)/2]))/(420*f)
```

Maple [A] (verified)

Time = 21.68 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.09

method	result
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(105\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)c^2\cos(fx+e)+105\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \text{arctanh}\left(\right.\right.}{}$
parts	$\frac{2c^2a^2\sqrt{a(\sec(fx+e)+1)}\left(3\ \text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \cos(fx+e)+3\ \text{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos}{\cos(}}}\right.\right.}{3f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*a^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(105*(-cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)
```

```
+1))^(1/2))*c^2*cos(f*x+e)+105*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(s
in(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*c^2+280*sin(f*
x+e)*c^2+602*sin(f*x+e)*c*d+230*sin(f*x+e)*d^2+35*c^2*tan(f*x+e)+196*c*d*ta
n(f*x+e)+115*d^2*tan(f*x+e)+42*c*d*tan(f*x+e)*sec(f*x+e)+60*d^2*tan(f*x+e)*
sec(f*x+e)+15*d^2*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.94

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = \left[\frac{105\left(a^2c^2\cos\left(fx+e\right)^4+a^2c^2\cos\left(fx+e\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{\cos(fx}\right.}{}\right.$$

$$-\frac{2\left(105\left(a^2c^2\cos\left(fx+e\right)^4+a^2c^2\cos\left(fx+e\right)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(15\,a^2d^2+2\,(140\,a^2\right.\right.}{}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*a
^2*d^2 + 2*(140*a^2*c^2 + 301*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (35*a
^2*c^2 + 196*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10*a^2*
d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f
*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x + e)^4 +
a^2*c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a^2*d^2 + 2*(140*a^2*c^2 + 3
01*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (35*a^2*c^2 + 196*a^2*c*d + 115*
a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10*a^2*d^2)*cos(f*x + e))*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x
 + e)^3)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\int \left(a(\sec{(e+fx)}+1)\right)^{\frac{5}{2}}(c+d\sec{(e+fx)})^2\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**2,x)
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\int (a\sec{(fx+e)}+a)^{\frac{5}{2}}(d\sec{(fx+e)}+c)^2\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
[Out] -1/210*(105*((a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*a
^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) + 1) - (a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*a^2
*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
) - 1) - 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*f*sin(2*f*x + 2*e)^2 + 2
*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + s
in(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + si
n(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (c
os(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - co
s(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(
9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*c
os(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*co
s(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*
f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin
```

(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 8*((3*a^2*c^2 + 5*a^2*c*d + a^2*d^2)*f*cos(2*f*x + 2*e)^2 + (3*a^2*c^2 + 5*a^2*c*d + a^2*d^2)*f*sin(2*f*x + 2*e)^2 + 2*(3*a^2*c^2 + 5*a^2*c*d + a^2*d^2)*f*cos(2*f*x + 2*e) + (3*a^2*c^2 + 5*a^2*c*d + a^2*d^2)*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2

*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*
f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f
*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*
x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
 + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(
2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f
*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e
)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e)
)*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f
*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2
*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin
(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 +
 (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*c
os(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x
 + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
+ 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + c
os(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f
*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1
/4)), x) - 4*((13*a^2*c^2 + 30*a^2*c*d + 20*a^2*d^2)*f*cos(2*f*x + 2*e)^2 +
 (13*a^2*c^2 + 30*a^2*c*d + 20*a^2*d^2)*f*sin(2*f*x + 2*e)^2 + 2*(13*a^2*c^
2 + 30*a^2*c*d + 20*a^2*d^2)*f*cos(2*f*x + 2*e) + (13*a^2*c^2 + 30*a^2*c*d
+ 20*a^2*d^2)*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f
*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*

x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*
x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x
 + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*
x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x
 + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x
+ 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x +
 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*
f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x
 + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*
sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*
cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x
 + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e
)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4
*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (
cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos
(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x +
 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x +
2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos
(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x
 + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))
*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4

```
)), x) - 8*((5*a^2*c^2 + 11*a^2*c*d + 5*a^2*d^2)*f*cos(2*f*x + 2*e)^2 + (5*
a^2*c^2 + 11*a^2*c*d + 5*a^2*d^2)*f*sin(2*f*x + 2*e)^2 + 2*(5*a^2*c^2 + 11*
a^2*c*d + 5*a^2*d^2)*f*cos(2*f*x + 2*e) + (5*a^2*c^2 + 11*a^2*c*d + 5*a^2*d
^2)*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*
cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) +
 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) +
 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) -
2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) -
 cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*c
os(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos
(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 +
sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + s
in(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*si
n(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)
^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3
+ (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*
f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x
+ 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x
+ 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2
+ 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e
))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x
+ 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2
*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(
2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(
6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2
*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
```

+ 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 2*((5*a^2*c^2 + 4*a^2*c*d)*f*cos(2*f*x + 2*e)^2 + (5*a^2*c^2 + 4*a^2*c*d)*f*sin(2*f*x + 2*e)^2 + 2*(5*a^2*c^2 + 4*a^2*c*d)*f*cos(2*f*x + 2*e) + (5*a^2*c^2 + 4*a^2*c*d)*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +

```
2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
 + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) +
cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*
f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*
e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)), x))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)^(3/4)*sqrt(a) + 8*(7*(15*(a^2*c^2 + a^2*c*d)*sin(6*f*x + 6*e) + 5*(10*a^
2*c^2 + 17*a^2*c*d + 5*a^2*d^2)*sin(4*f*x + 4*e) + (55*a^2*c^2 + 113*a^2*c*
d + 50*a^2*d^2)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)) - (140*a^2*c^2 + 301*a^2*c*d + 115*a^2*d^2 + 105*(a^2*c^2 +
 a^2*c*d)*cos(6*f*x + 6*e) + 35*(10*a^2*c^2 + 17*a^2*c*d + 5*a^2*d^2)*cos(4
*f*x + 4*e) + 7*(55*a^2*c^2 + 113*a^2*c*d + 50*a^2*d^2)*cos(2*f*x + 2*e))*s
in(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a))/((f*cos(2
*f*x + 2*e)^2 + f*sin(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e) + f)*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(3/4))
```

Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)^2\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2, x)
```

3.162 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

Optimal result 1077
Rubi [A] (verified) 1077
Mathematica [A] (verified) 1079
Maple [A] (verified) 1080
Fricas [A] (verification not implemented) 1080
Sympy [F] 1081
Maxima [B] (verification not implemented) 1081
Giac [F] 1082
Mupad [F(-1)] 1082

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{2a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2ad(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{5f}$$

```
[Out] 2*a^(5/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f+2/5*a*d*(a+
a*sec(f*x+e))^(3/2)*tan(f*x+e)/f+2/15*a^3*(35*c+32*d)*tan(f*x+e)/f/(a+a*sec
(f*x+e))^(1/2)+2/15*a^2*(5*c+8*d)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4002, 4000, 3859, 209, 3877}

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{2a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f} + \frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a\sec(e+fx)+a}} + \frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{15f} + \frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^(5/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a^3*(35*c + 32*d)*Tan[e + f*x])/(15*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*
(5*c + 8*d)*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(15*f) + (2*a*d*(a + a*S
ec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4002

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
(b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*
m]
```

Rubi steps

$$\text{integral} = \frac{2ad(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{5f} + \frac{2}{5}\int(a+a\sec(e+fx))^{3/2}\left(\frac{5ac}{2}+\frac{1}{2}a(5c+8d)\sec(e+fx)\right)\,dx$$

$$= \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2ad(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{5f} + \frac{4}{15}\int\sqrt{a+a\sec(e+fx)}\left(\frac{15a^2c}{4}+\frac{1}{4}a^2(35c+32d)\sec(e+fx)\right)dx$$

$$= \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2ad(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{5f} + (a^2c)\int\sqrt{a+a\sec(e+fx)}\,dx + \frac{1}{15}(a^2(35c+32d))\int\sec(e+fx)\sqrt{a+a\sec(e+fx)}\,dx$$

$$= \frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2ad(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{5f} - \frac{(2a^3c)\,\text{Subst}\left(\int\frac{1}{a+x^2}dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2ad(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\left(30\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\cdots\right.}{\cdots}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2]
*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(40*c + 49*d + 2
*(5*c + 14*d)*Cos[e + f*x] + (40*c + 43*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/
2]))/(30*f)
```

Maple [A] (verified)

Time = 7.70 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.49

method	result
default	$\frac{2a^2\sqrt{a(\sec(fx+e)+1)}\left(15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)c\cos(fx+e)+15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\right.\right.}{15f(\cos(fx+e)}$
parts	$\frac{2c\,a^2\sqrt{a(\sec(fx+e)+1)}\left(3\ \operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \cos(fx+e)+3\ \operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos}{\cos(}}}\right.\right.}{3f(\cos(fx+e)+1)}$

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*a^2/f*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*(15*(-cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1
))^(1/2))*c*cos(f*x+e)+15*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(sin(f*
x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*c+40*sin(f*x+e)*c+4
3*sin(f*x+e)*d+5*c*tan(f*x+e)+14*d*tan(f*x+e)+3*d*tan(f*x+e)*sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.75

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \left[\frac{15\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx}{\cos(fx+e)+1}\right.}{}\right.$$

$$-\frac{2\left(15\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3a^2d+(40a^2c+43a^2d\right.\right.}{15\left(f\cos(fx+e)^3+f\cos(fx+e\right.}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*c
os(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a^2*d +
(40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 + (5*a^2*c + 14*a^2*d)*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*
cos(f*x + e)^2), -2/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sq
```

```
rt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*
sin(f*x + e))) - (3*a^2*d + (40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 + (5*a^2*c
 + 14*a^2*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int (a + a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (a(\sec(e+fx)+1))^{\frac{5}{2}}(c+d\sec(e+fx))\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. $2(124) = 248$.

Time = 0.43 (sec) , antiderivative size = 1396, normalized size of antiderivative = 9.83

$$\int (a + a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] 1/6*(30*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 2*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e
) - 3*a^2*sin(2*f*x + 2*e) - 4*(3*a^2*cos(2*f*x + 2*e) + 4*a^2)*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) + (12*a^2*sin(2*f*x + 2*e)*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2*cos(2*f*x + 2*e) - a^2 + 4*(3*a^2*cos
(2*f*x + 2*e) + 4*a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a) + 3*((a
^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*f*x + 2*e) + a
^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*a
```

```
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))) + 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(
2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e
)^2 + 2*a^2*cos(2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)) + 1) + (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*
f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*sqrt(a)
)*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx=\int (a\sec{(fx+e)}+a)^{\frac{5}{2}}(d\sec{(fx+e)}+c)\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx=\int \left(a+\frac{a}{\cos{(e+f\,x)}}\right)^{5/2}\left(c+\frac{d}{\cos{(e+f\,x)}}\right)\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)), x)
```

3.163 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx$

Optimal result 1083
Rubi [A] (verified) 1083
Mathematica [C] (warning: unable to verify) 1085
Maple [B] (warning: unable to verify) 1086
Fricas [A] (verification not implemented) 1087
Sympy [F] 1088
Maxima [F] 1088
Giac [F] 1088
Mupad [F(-1)] 1088

Optimal result

Integrand size = 27, antiderivative size = 203

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx &= \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2a^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{2a^{7/2}(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{cd^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

```
[Out] 2*a^3*tan(f*x+e)/d/f/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*arctanh((a-a*sec(f*x+
e))^(1/2)/a^(1/2))*tan(f*x+e)/c/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2)-2*a^(7/2)*(c-d)^2*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)
^(1/2))*tan(f*x+e)/c/d^(3/2)/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(
f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4025, 186, 65, 212, 214}

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx &= -\frac{2a^{7/2}(c-d)^2\tan(e+fx)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{cd^{3/2}f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\ &+ \frac{2a^{7/2}\tan(e+fx)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^3\tan(e+fx)}{df\sqrt{a\sec(e+fx)+a}}\end{aligned}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^3*Tan[e + f*x])/(d*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^(7/2)*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c*f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (2*a^(7/2)*(c - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a -
 a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c*d^(3/2)*Sqrt[c +
d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^2}{x\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\left(\frac{a^2}{d\sqrt{a-ax}}+\frac{a^2}{cx\sqrt{a-ax}}-\frac{a^2(c-d)^2}{cd\sqrt{a-ax}(c+dx)}\right)dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} - \frac{(a^4\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a^4(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{cdf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} + \frac{(2a^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2a^3(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{cdf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^{7/2}(c-d)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{cd^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.71 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.69

$$\int\frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \frac{\cos^{\frac{3}{2}}(e+fx)(d+c\cos(e+fx))\sec^5\left(\frac{1}{2}(e+fx)\right)(a(1+\sec(e+fx)))^{5/2}\left(\frac{1}{-}\right.}{}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
[Out] (Cos[e + f*x]^(3/2)*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e +
 f*x]))^(5/2)*((10*(c - d)^2*(c + 3*d + 2*c*Cos[e + f*x])*Csc[(e + f*x)/2]*
(-ArcTanh[Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]] + Sqrt[-((d*(-1 + Sec[e
 + f*x]))/(c + d))]))/(d*(c + d)*Sqrt[Cos[e + f*x]]*Sqrt[-((d*(-1 + Sec[e +
 f*x]))/(c + d))]) + (20*(3*c - d)*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]] - (
16*(c - d)^2*d*(d + c*Cos[e + f*x])*Hypergeometric2F1[2, 5/2, 7/2, (-2*d*Se
c[e + f*x]*Sin[(e + f*x)/2]^2)/(c + d)]*Sin[(e + f*x)/2]^3)/((c + d)^3*Cos[
e + f*x]^(5/2)) + 10*c*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] - (2*Sin[(
e + f*x)/2])/Sqrt[Cos[e + f*x]])))/(40*c^2*f*(c + d*Sec[e + f*x]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1401 vs. 2(173) = 346.

Time = 23.45 (sec) , antiderivative size = 1402, normalized size of antiderivative = 6.91

method	result	size
default	Expression too large to display	1402

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*a^2/c/((c+d)*(c-d))^(1/2)/d/(d/(c-d))^(1/2)*(2*((c+d)*(c-d))^(1/2)*2^
(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*d-2^(1
/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln(-2*(((1-cos(f*x+e))^2*csc(f*
x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e
))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d
))^(1/2)))*c^2+2*2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln(-2*(((1
-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(
c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(
-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(
f*x+e))*d+((c+d)*(c-d))^(1/2)))*c*d-2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)^(1/2)*ln(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(
1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((
c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e)
)+(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*d^2+2^(1/2)*((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^(1/2)*ln(-2*(-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)
*2^(1/2)*(d/(c-d))^(1/2)*c+2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*
x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+c-d)/(c*(-co
t(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*c^2-2
*2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln(-2*(-((1-cos(f*x+e))^2*
csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c+2^(1/2)*(d/(c-d))^(1/2)*((1
-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc
(f*x+e))+c-d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)
*(c-d))^(1/2)))*c*d+2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln(-2*(
-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c+2^(1/2)*
(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/
2)*(-cot(f*x+e)+csc(f*x+e))+c-d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+c
sc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*d^2+4*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2
)*c*(-cot(f*x+e)+csc(f*x+e)))*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2
)
```

Fricas [A] (verification not implemented)

none

Time = 2.93 (sec) , antiderivative size = 1140, normalized size of antiderivative = 5.62

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{c + d\sec(e + fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [(2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + (a^2*c^2 -
 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(-
a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^
2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e
) + d)) + (a^2*d*cos(f*x + e) + a^2*d)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2
*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
 + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*d*f*cos(f*x + e) + c*d*f), (
2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 2*(a^2*d*cos
(f*x + e) + a^2*d)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2
*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d
+ d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f
*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*d*f*cos(f*x + e
) + c*d*f), (2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) +
 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x
 + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + (a^2*d*cos(f*x
+ e) + a^2*d)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(c
os(f*x + e) + 1)))/(c*d*f*cos(f*x + e) + c*d*f), 2*(a^2*c*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2
*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d
)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/
(a*sin(f*x + e))) - (a^2*d*cos(f*x + e) + a^2*d)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*d*f*c
os(f*x + e) + c*d*f)]
```

Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{c + d\sec(e + fx)}\,dx = \int \frac{(a(\sec(e + fx) + 1))^{\frac{5}{2}}}{c + d\sec(e + fx)}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{c + d\sec(e + fx)}\,dx = \int \frac{(a\sec(fx + e) + a)^{\frac{5}{2}}}{d\sec(fx + e) + c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{c + d\sec(e + fx)}\,dx = \int \frac{(a\sec(fx + e) + a)^{\frac{5}{2}}}{d\sec(fx + e) + c}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{c + d\sec(e + fx)}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}{c + \frac{d}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x)), x)
```

3.164 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx$

Optimal result 1089
Rubi [A] (verified) 1090
Mathematica [A] (verified) 1093
Maple [B] (warning: unable to verify) 1093
Fricas [A] (verification not implemented) 1093
Sympy [F] 1095
Maxima [F(-1)] 1095
Giac [F] 1095
Mupad [F(-1)] 1095

Optimal result

Integrand size = 27, antiderivative size = 329

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{2a^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{7/2}(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{cd^{3/2}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}(c-d)\sqrt{c+d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2d^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^3(c-d)^2\tan(e+fx)}{cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

```
[Out] -a^3*(c-d)^2*tan(f*x+e)/c/d/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)
+2*a^(7/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-a^(7/2)*(c-d)^2*arctanh(d^(1/2)*(a-a
*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/d^(3/2)/(c+d)^(3/2)/f/
(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*(c-d)*arctanh(d^(1/
2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*(c+d)^(1/2)*tan(f*x+e)/c^2/d
^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{2a^{7/2}(c-d)\sqrt{c+d}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^{7/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{a^{7/2}(c-d)^2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{cd^{3/2}f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{a^3(c-d)^2\tan(e+fx)}{cdf(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(7/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(7/2)*(c - d)^2*ArcT
anh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])
/(c*d^(3/2)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]]) + (2*a^(7/2)*(c - d)*Sqrt[c + d]*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*
x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*d^(3/2)*f*Sqrt[a - a*Sec[e +
 f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^3*(c - d)^2*Tan[e + f*x])/(c*d*(c + d
)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)^2}{x\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\left(\frac{a^2}{c^2x\sqrt{a-ax}}-\frac{a^2(c-d)^2}{cd\sqrt{a-ax}(c+dx)^2}+\frac{a^2(c^2-d^2)}{c^2d\sqrt{a-ax}(c+dx)}\right)dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= -\frac{(a^4\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad+\frac{(a^4(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{cdf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad-\frac{(a^4(c^2-d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{c^2df\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= -\frac{a^3(c-d)^2\tan(e+fx)}{cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(2a^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a^4(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{2cd(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(2a^3(c^2-d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{c^2df\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^{7/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}(c-d)\sqrt{c+d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2d^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^3(c-d)^2\tan(e+fx)}{cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{(a^3(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{cd(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^{7/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{7/2}(c-d)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{cd^{3/2}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}(c-d)\sqrt{c+d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2d^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^3(c-d)^2\tan(e+fx)}{cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.85

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{\sqrt{\cos(e+fx)}(d+c\cos(e+fx))^2\sec^5\left(\frac{1}{2}(e+fx)\right)(a(1+\sec(e+fx)))^{5/2}\Bigg(}{}$$

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]

[Out] (Sqrt[Cos[e + f*x]]*(d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e + f*x]))^(5/2)*(2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (4*Sqrt[2]*(c - d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]) - ((c - d)^2*(2*c*Cos[e + f*x] - (2*(c + 2*d)*ArcTanh[Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]]*(d + c*Cos[e + f*x]))/((c + d)*Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]))*Sin[(e + f*x)/2])/(d*(c + d)*Sqrt[Cos[e + f*x]]*(d + c*Cos[e + f*x]))))/(8*c^2*f*(c + d*Sec[e + f*x])^2)

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 20137 vs. 2(281) = 562.

Time = 53.69 (sec) , antiderivative size = 20138, normalized size of antiderivative = 61.21

method	result	size
default	Expression too large to display	20138

[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] result too large to display

Fricas [A] (verification not implemented)

none

Time = 13.17 (sec) , antiderivative size = 2031, normalized size of antiderivative = 6.17

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")

```
[Out] [-1/2*(2*(a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*a^2*c*
d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos
(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*a^2*d^
4)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c
 + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^
2 + (c + d)*cos(f*x + e) + d)) - 2*(a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*
c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*s
qrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1
)))/((c^4*d + c^3*d^2)*f*cos(f*x + e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*c
os(f*x + e) + (c^3*d^2 + c^2*d^3)*f), -1/2*(2*(a^2*c^3 - 2*a^2*c^2*d + a^2*
c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) +
4*(a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*
d + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^3*d + 4*
a^2*c^2*d^2 - 3*a^2*c*d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*
d^2 - 2*a^2*c*d^3)*cos(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 -
5*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d
^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x
+ e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((c^4*d + c^3*d^2)*f*
cos(f*x + e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 +
c^2*d^3)*f), -((a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*
a^2*c*d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^
3)*cos(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*
a^2*d^4)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) -
 (a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d
 + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 -
2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4*d + c^3*d^2)*f*cos(f*x +
 e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 + c^2*d^3)*
f), -((a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*a^2*c*d^3
 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos(f*
x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*a^2*d^4)*
cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + 2*(a^2*c
*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d + 2*a^
2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/((c^4*d + c^3*d^2)*f*cos
(f*x + e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 + c^2
```

```
*d^3)*f)]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}{(c+d\sec{(e+fx)})^2}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a\sec{(fx+e)}+a)^{\frac{5}{2}}}{(d\sec{(fx+e)}+c)^2}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^2, x)
```

3.165 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx$

Optimal result 1096
Rubi [A] (verified) 1097
Mathematica [A] (warning: unable to verify) 1101
Maple [B] (warning: unable to verify) 1102
Fricas [A] (verification not implemented) 1102
Sympy [F] 1104
Maxima [F(-1)] 1104
Giac [F] 1104
Mupad [F(-1)] 1105

Optimal result

Integrand size = 27, antiderivative size = 536

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \frac{2a^{7/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3a^{7/2}(c-d)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4cd^{3/2}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{7/2}(c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2d^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^{7/2}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^3(c-d)^2\tan(e+fx)}{2cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{3a^3(c-d)^2\tan(e+fx)}{4cd(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

```
[Out] -1/2*a^3*(c-d)^2*tan(f*x+e)/c/d/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))
^(1/2)+a^3*(c-d)*tan(f*x+e)/c^2/d/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)
-3/4*a^3*(c-d)^2*tan(f*x+e)/c/d/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))
^(1/2)+2*a^(7/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/(
a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/4*a^(7/2)*(c-d)^2*arctanh(d^
(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/d^(3/2)/(c+d
)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+a^(7/2)*(c-d)*arcta
```

```
nh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/d^(3/
2)/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2*a^(7/2)*ar
ctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*d^(1/2)*tan(f*x+e
)/c^3/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = -\frac{2a^{7/2}\sqrt{d}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^3f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^{7/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{a^{7/2}(c-d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{3a^{7/2}(c-d)^2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4cd^{3/2}f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))} - \frac{3a^3(c-d)^2\tan(e+fx)}{4cdf(c+d)^2\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))} - \frac{a^3(c-d)^2\tan(e+fx)}{2cdf(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}$$

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(7/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*a^(7/2)*(c - d)^2*Ar
cTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x
])/(4*c*d^(3/2)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]]) + (a^(7/2)*(c - d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt
[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*d^(3/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e
 + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*a^(7/2)*Sqrt[d]*ArcTanh[(Sqrt[d]*Sq
rt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^3*Sqrt[c +
d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^3*(c - d)^2*Ta
n[e + f*x])/(2*c*d*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^
```

```
2) + (a^3*(c - d)*Tan[e + f*x])/(c^2*d*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Se
c[e + f*x])) - (3*a^3*(c - d)^2*Tan[e + f*x])/(4*c*d*(c + d)^2*f*Sqrt[a + a
*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^2}{x\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{a^2}{c^3x\sqrt{a-ax}} - \frac{a^2(c-d)^2}{cd\sqrt{a-ax}(c+dx)^3} + \frac{a^2(c^2-d^2)}{c^2d\sqrt{a-ax}(c+dx)^2} - \frac{a^2d}{c^3\sqrt{a-ax}(c+dx)}\right)dx, x, s}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^4\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^4(c-d)^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{cdf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^4d\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^4(c^2-d^2)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{c^2df\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^3(c-d)^2\tan(e+fx)}{2cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{\left(2a^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\left(2a^3d\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{c+d-\frac{dx^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(3a^4(c-d)^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{4cd(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^4(c^2-d^2)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2c^2d(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^{7/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^{7/2}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^3(c-d)^2\tan(e+fx)}{2cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{3a^3(c-d)^2\tan(e+fx)}{4cd(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(3a^4(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{8cd(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a^3(c^2-d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{c^2d(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^{7/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{7/2}(c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2d^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^{7/2}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^3(c-d)^2\tan(e+fx)}{2cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{3a^3(c-d)^2\tan(e+fx)}{4cd(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{(3a^3(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{4cd(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^{7/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3a^{7/2}(c-d)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4cd^{3/2}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{7/2}(c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2d^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^{7/2}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^3(c-d)^2\tan(e+fx)}{2cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{3a^3(c-d)^2\tan(e+fx)}{4cd(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [A] (warning: unable to verify)

Time = 9.97 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.03

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \frac{\left(8d^{3/2}(c+d)^2\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right) - \frac{(c^4+10c^3d-15c^2d^2-20cd^3-8d^4)\text{arctanh}\left(\frac{\cdots}{\sqrt{-\cdots}}\right)}{\sqrt{-c-d}}\right.}{\cdots} + \frac{(d+c\cos(e+fx))^3\sec^5\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(a(1+\sec(e+fx)))^{5/2}\left(-\frac{(c^3-12c^2d+5cd^2+6d^3)\sin\left(\frac{1}{2}(e+fx)\right)}{16c^3d(c+d)^2}+\cdots\right.}{f(c+d\,\text{se}\cdots}$$

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((8*d^(3/2)*(c + d)^2*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e
+ f*x])]] - ((c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*ArcTanh[(Sqrt
[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])]
)/Sqrt[-c - d])*(d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sqrt[Cos[e + f*x]
*Sec[(e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x
]]*(a*(1 + Sec[e + f*x]))^(5/2))/(16*Sqrt[2]*c^3*d^(3/2)*(c + d)^2*f*Sqrt[S
ec[(e + f*x)/2]^2]*(c + d*Sec[e + f*x])^3) + ((d + c*Cos[e + f*x])^3*Sec[(e
 + f*x)/2]^5*Sec[e + f*x]*(a*(1 + Sec[e + f*x]))^(5/2)*(-1/16*((c^3 - 12*c^
2*d + 5*c*d^2 + 6*d^3)*Sin[(e + f*x)/2])/(c^3*d*(c + d)^2) + (-(c^2*d*Sin[(
e + f*x)/2]) + 2*c*d^2*Sin[(e + f*x)/2] - d^3*Sin[(e + f*x)/2])/(8*c^3*(c +
```

```
d)*(d + c*Cos[e + f*x])^2) + (3*c^3*Sin[(e + f*x)/2] - 14*c^2*d*Sin[(e + f
*x)/2] + 3*c*d^2*Sin[(e + f*x)/2] + 8*d^3*Sin[(e + f*x)/2])/(16*c^3*(c + d)
^2*(d + c*Cos[e + f*x]))))/(f*(c + d*Sec[e + f*x])^3)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 42562 vs. $2(460) = 920$.

Time = 235.82 (sec) , antiderivative size = 42563, normalized size of antiderivative = 79.41

method	result	size
default	Expression too large to display	42563

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 22.58 (sec) , antiderivative size = 3351, normalized size of antiderivative = 6.25

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
[Out] [-1/8*((a^2*c^4*d^2 + 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^
2*d^6 + (a^2*c^6 + 10*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c
^2*d^4)*cos(f*x + e)^3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c
^3*d^3 - 48*a^2*c^2*d^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*
a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*c
os(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2
*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 +
(c + d)*cos(f*x + e) + d)) - 8*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*
c^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^
3*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^
2*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*((a^2*c^5 - 12*a^2
*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^2*c^4*d + 10*a^
2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^5*d^3)*f*cos(f*x
+ e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*cos(f*x + e)^2 + (2*
```

```
c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e) + (c^5*d^3 + 2*c^
4*d^4 + c^3*d^5)*f), -1/8*(16*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*c
^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^3
*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^2
*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^4
*d^2 + 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^
6 + 10*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x
 + e)^3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2
*c^2*d^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 2
0*a^2*c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sq
rt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x +
 e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x
 + e) + d)) + 2*((a^2*c^5 - 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*c
os(f*x + e)^2 - (a^2*c^4*d + 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*
cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d
 + 2*c^6*d^2 + c^5*d^3)*f*cos(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 +
 2*c^4*d^4)*f*cos(f*x + e)^2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5
)*f*cos(f*x + e) + (c^5*d^3 + 2*c^4*d^4 + c^3*d^5)*f), -1/4*((a^2*c^4*d^2 +
 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^6 + 10
*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x + e)^
3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2*c^2*d
^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 20*a^2*
c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sqrt(a/(
c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) - 4*(a^2*c^2*d^3 + 2*a^2*c*d^4 +
 a^2*d^5 + (a^2*c^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*
c^4*d + 4*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^
2*c^3*d^2 + 5*a^2*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(-a)*l
og((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + ((a^
2*c^5 - 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^2
*c^4*d + 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^5*d
^3)*f*cos(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*cos(f*
x + e)^2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e) + (
c^5*d^3 + 2*c^4*d^4 + c^3*d^5)*f), -1/4*((a^2*c^4*d^2 + 10*a^2*c^3*d^3 - 15
*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^6 + 10*a^2*c^5*d - 15*a^2*
c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x + e)^3 + (a^2*c^6 + 12*a^
2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2*c^2*d^4 - 16*a^2*c*d^5)*c
os(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 55*a^2*c^2
*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((
c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)/(a*sin(f*x + e))) + 8*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*c^4*
```

```
d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^3*d^
2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^2*c^
2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + ((a^2*c^5 -
 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^2*c^4*d
+ 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^5*d^3)*f*c
os(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*cos(f*x + e)^
2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e) + (c^5*d^3
 + 2*c^4*d^4 + c^3*d^5)*f)]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}{(c+d\sec{(e+fx)})^3}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a\sec{(fx+e)}+a)^{\frac{5}{2}}}{(d\sec{(fx+e)}+c)^3}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^3, x)
```

3.166 $\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result 1106
Rubi [A] (verified) 1107
Mathematica [C] (warning: unable to verify) 1109
Maple [B] (warning: unable to verify) 1110
Fricas [A] (verification not implemented) 1111
Sympy [F] 1112
Maxima [F] 1112
Giac [F(-2)] 1112
Mupad [F(-1)] 1112

Optimal result

Integrand size = 27, antiderivative size = 258

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^3(1-\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}(c-d)^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*(3*c-d)*d^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*d^3*tan(f*x+e)/f/(a+a*s
ec(f*x+e))^(1/2)-2/3*d^3*(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
+2*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c-d)^3*arctanh(1/2*(a-a*sec(f*x+e))
^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)
/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4025, 186, 65, 212, 45}

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\sqrt{a}c^3\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{\sqrt{2}\sqrt{a}(c-d)^3\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2d^2(3c-d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}} - \frac{2d^3\tan(e+fx)(1-\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}} + \frac{2d^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*(3*c - d)*d^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*Tan[e
+ f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*d^3*(1 - Sec[e + f*x])*Tan[e + f*
x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*c^3*ArcTanh[Sqrt[a - a*Sec[
e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec
[e + f*x]]) - (Sqrt[2]*Sqrt[a]*(c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(
Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
```

```
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(c+dx)^3}{x\sqrt{a-ax}(a+ax)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{(3c-d)d^2}{a\sqrt{a-ax}}+\frac{c^3}{ax\sqrt{a-ax}}+\frac{d^3x}{a\sqrt{a-ax}}-\frac{(c-d)^3}{a(1+x)\sqrt{a-ax}}\right)dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{\left(ac^3\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad+\frac{\left(a(c-d)^3\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad-\frac{\left(ad^3\tan(e+fx)\right)\text{Subst}\left(\int\frac{x}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{(2c^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2(c-d)^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(ad^3\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{a-ax}}-\frac{\sqrt{a-ax}}{a}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^3(1-\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.07 (sec) , antiderivative size = 787, normalized size of antiderivative = 3.05

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)(c+d\sec(e+fx))^3\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\left(\frac{2c(c^2+3d^2)\sin\left(\frac{1}{2}(e+fx)\right)}{3\left(1-2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)^{3/2}} - \cdots\right.}{\cdots}$$

```
[In] Integrate[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^
(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((2*c*(c^2 + 3*d^2)*Sin[(e + f*x)/2])/
(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) - (4*c^2*(c + 3*d)*Sin[(e + f*x)/2]^3)
/(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) + (4*c*(c^2 + 3*d^2)*Sin[(e + f*x)/2]
)/(3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) + (c^3*Csc[(e + f*x)/2]*Sqrt[1 - 2*Sin
[(e + f*x)/2]^2]*((4*Sin[(e + f*x)/2]^4)/(1 - 2*Sin[(e + f*x)/2]^2)^2 - (6*
Sin[(e + f*x)/2]^2)/(1 - 2*Sin[(e + f*x)/2]^2) + (3*Sqrt[2]*ArcSin[Sqrt[2]*
Sin[(e + f*x)/2]]*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]))/3 - ((
```

```
c - d)^3*Csc[(e + f*x)/2]^5*(-12*Cos[(e + f*x)/2]^4*HypergeometricPFQ[{2, 2
, 7/2}, {1, 9/2}, -(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e
+ f*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[(e + f*x)/2]^2/(1 - 2
*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2]^8*(4 - 7*Sin[(e + f*x)/2]^2 + 3*Sin
[(e + f*x)/2]^4) + 7*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]
*(1 - 2*Sin[(e + f*x)/2]^2)^3*(15 - 20*Sin[(e + f*x)/2]^2 + 8*Sin[(e + f*x)
/2]^4)*((3 - 7*Sin[(e + f*x)/2]^2)*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e
+ f*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2
]^2))]]*(1 - 2*Sin[(e + f*x)/2]^2))))/(63*(1 - 2*Sin[(e + f*x)/2]^2)^(7/2))
))/(f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(5/2)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(227) = 454$.

Time = 5.60 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.97

method	result
parts	$-\frac{c^3 \sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-\right.}{fa}$
default	$\frac{\left(3\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)c^3-3\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}\ln\left(\csc\right.\right.}{}$

```
[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c^3/f/a*(a*(sec(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2^(1/
2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e
)^2-1)^(1/2))-2*arctanh(sin(f*x+e)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)))+1/3*d^3/f/a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(3*l
n(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(3/2)-4*(1-cos(f*x+e))^3*csc(f*x+e)^3)/((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)+3*c^2*d/f/a*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a*(se
c(f*x+e)+1))^(1/2)*2^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x
+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))-3*c*d^2/f/a*(a*(sec(f*x+e)+1))^(1/2)*
(2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f
*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))+2*cot(f*x+e)-2*csc(f
*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 7.80 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.40

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[-\frac{3\sqrt{2}\left((ac^3-3\,ac^2d+3\,acd^2-ad^3)\cos\left(fx+e\right)^2+(ac^3-3\,ac^2d+3\,acd^2-ad^3)\cos\left(fx+e\right)\right)\sqrt{-\frac{1}{a}}\,\mathrm{lo}}{}\right.$$

$$-\frac{6\left(c^3\cos\left(fx+e\right)^2+c^3\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-2\left(d^3+(9\,cd^2-d^3)\cos\left(fx\right.\right.}{3\left(af\,\mathrm{c}\right.}$$

[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/6*(3*sqrt(2)*((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e)^2 + (a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e))*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 6*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(d^3 + (9*c*d^2 - d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -1/3*(6*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(d^3 + (9*c*d^2 - d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 3*sqrt(2)*((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e)^2 + (a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))]

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c+d\sec(e+fx))^3}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral((c + d*sec(e + f*x))**3/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e)+c)^3}{\sqrt{a\sec(fx+e)+a}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)^3/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)
```

3.167 $\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result 1113
Rubi [A] (verified) 1113
Mathematica [C] (warning: unable to verify) 1115
Maple [B] (warning: unable to verify) 1116
Fricas [A] (verification not implemented) 1116
Sympy [F] 1117
Maxima [F] 1117
Giac [F(-2)] 1117
Mupad [F(-1)] 1118

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*d^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*c^2*arctanh((a-a*sec(f*x+e))^(1
/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2)-(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^
(1/2)*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4025, 186, 65, 212}

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\sqrt{a}c^2\tan(e+fx)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{\sqrt{2}\sqrt{a}(c-d)^2\tan(e+fx)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2d^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*d^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*c^2*ArcTanh[
Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*(c - d)^2*ArcTanh[Sqrt[a - a*
Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(c+dx)^2}{x\sqrt{a-ax}(a+ax)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\left(\frac{d^2}{a\sqrt{a-ax}}+\frac{c^2}{ax\sqrt{a-ax}}-\frac{(c-d)^2}{a(1+x)\sqrt{a-ax}}\right)dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(ac^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{(2c^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}(c-d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.73 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.61

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\cos^{\frac{3}{2}}(e+fx)(c+d\sec(e+fx))^2\left(-\frac{(c-d)^2\sqrt{-1+\cos(e+fx)}(2+\cos(e+fx))\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-2\text{arctan}\right.}{2\sqrt{2}}\right.}{}$$

```
[In] Integrate[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Cos[(e + f*x)/2]*Cos[e + f*x]^(3/2)*(c + d*Sec[e + f*x])^2*(-1/2*((c - d
)^2*Sqrt[-1 + Cos[e + f*x]]*(2 + Cos[e + f*x])*Csc[(e + f*x)/2]^3*(-2*ArcTa
nh[Sqrt[-(Sec[e + f*x]*Sin[(e + f*x)/2]^2)]] + Sqrt[2 - 2*Sec[e + f*x]]))/S
qrt[2] + (4*c*d*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]] + c^2*(Sqrt[2]*ArcSin[
Sqrt[2]*Sin[(e + f*x)/2]] - (2*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]]) - ((c
- d)^2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[e + f*x]*Sin[(e + f*x)/2]^2)]*S
in[(e + f*x)/2]*Sin[e + f*x]^2)/(10*Cos[e + f*x]^(5/2))))/(f*(d + c*Cos[e +
 f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(158) = 316.

Time = 5.11 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.90

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left(\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{2}\,c^2-\sqrt{(1-\cos(fx+}\right.}{}$
parts	$-\frac{c^2\sqrt{a(\sec(fx+e)+1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-\right.}{fa}$

```
[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(((1-cos(f*x+e))^2*csc
(f*x+e)^2-1)^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*
(-cot(f*x+e)+csc(f*x+e)))*2^(1/2)*c^2-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*c^2+2*
((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2))*c*d-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*
ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*d^2+2*d^2
*(-cot(f*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 1.96 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.63

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[\frac{4\,d^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)+\sqrt{2}(ac^2-2\,acd+ad^2+(ac^2-2\,acd+ad^2)\cos(fx+e))\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\cdots}{\cdots}\right.}{}\right.$$

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + sqrt(2)*
(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-1/
a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*
x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^
```

```
2 + 2*cos(f*x + e) + 1)) - 2*(c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f*cos(f*x + e
) + a*f), (2*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 2*(
c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + sqrt(2)*(a*c^2 - 2*a*c*d + a*d^2
 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*c
os(f*x + e) + a*f)]
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c+d\sec(e+fx))^2}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral((c + d*sec(e + f*x))**2/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e)+c)^2}{\sqrt{a\sec(fx+e)+a}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)^2/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2), x)
```

3.168 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result 1119
Rubi [A] (verified) 1119
Mathematica [A] (verified) 1120
Maple [B] (verified) 1121
Fricas [A] (verification not implemented) 1121
Sympy [F] 1122
Maxima [C] (verification not implemented) 1122
Giac [F(-2)] 1123
Mupad [F(-1)] 1123

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{\sqrt{2}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-(c-d)*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4005, 3859, 209, 3880}

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f} - \frac{\sqrt{2}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}$$

```
[In] Int[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f) -
 (Sqrt[2]*(c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c\int\sqrt{a+a\sec(e+fx)}\,dx}{a}-(c-d)\int\frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx\\
&= -\frac{(2c)\text{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{(2(c-d))\text{Subst}\left(\int\frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int\frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\left(\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)+(-c+d)\arctan\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (-c + d)*ArcTan[Sin[(e + f
*x)/2]/Sqrt[Cos[e + f*x]]])*Cos[(e + f*x)/2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[a*
(1 + Sec[e + f*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(76) = 152.

Time = 2.40 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.23

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\left(c\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2\right.\right.}{fa}$
parts	$\frac{c\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(2\operatorname{arctanh}\left(\frac{\sin(fx+e)}{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)-\sqrt{2}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2\right.\right.}{fa}$

```
[In] int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/a*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^2*csc
(f*x+e)^2-1)^(1/2)*(c*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)^(1/2))-d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2))-c*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-c
ot(f*x+e)+csc(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.45

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \left[-\frac{\sqrt{2}(ac-ad)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)-3\cos(fx+e)^2-2\cos(fx+e)+1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+2\sqrt{-ac}\log}{2\,af}\right.$$

$$\left.-\frac{2\sqrt{ac}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\frac{\sqrt{2}(ac-ad)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{\sqrt{a}}}{af}\right]$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(a*c - a*d)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2
- 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a)*c
*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*
f), -(2*sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e))) - sqrt(2)*(a*c - a*d)*arctan(sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/
(a*f)]
```

Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{c+d\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 699, normalized size of antiderivative = 7.68

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\Bigg(\sqrt{2}\sqrt{a}\arctan\Bigg(\frac{\left(\left|2e^{(ifx+ie)}+2\right|^4+16\cos(fx+e)^4+16\sin(fx+e)^4+8\left(\cos(fx+e)^2-\sin(fx+e)^2-2\cos(fx+e)+1\right)\left|2e^{(ifx+ie)}+2\right|^2 \cdots}{\cdots}}{\cdots}$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*f*x + I*e) + 2)^4 + 16*cos(f*x + e)^
4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^2 - 2*cos(f*x + e)
 + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 + 32*(cos(f*x + e)^2
 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)^2 - 64*cos(f*x + e)
 + 16)^(1/4)*sin(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(f*x + e)/abs(2*e^(I*f
*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*cos(f*x + e)^2 - 4*sin(
f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*e) + 2)^2)) + 2*sin(f*x
 + e))/abs(2*e^(I*f*x + I*e) + 2), ((abs(2*e^(I*f*x + I*e) + 2)^4 + 16*cos(
f*x + e)^4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^2 - 2*cos
```

```
(f*x + e) + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 + 32*(cos(f
*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)^2 - 64*cos
(f*x + e) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(f*x + e)/abs
(2*e^(I*f*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*cos(f*x + e)^2
 - 4*sin(f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*e) + 2)^2)) +
2*cos(f*x + e) - 2)/abs(2*e^(I*f*x + I*e) + 2)) - sqrt(a)*arctan2((cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(f*x + e), (cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(f*x + e)))*c/(a*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{c+\frac{d}{\cos(e+f\,x)}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2), x)
```

3.169 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result 1124
Rubi [A] (verified) 1125
Mathematica [A] (verified) 1127
Maple [B] (warning: unable to verify) 1127
Fricas [A] (verification not implemented) 1128
Sympy [F] 1129
Maxima [F] 1129
Giac [F(-2)] 1129
Mupad [F(-1)] 1129

Optimal result

Integrand size = 27, antiderivative size = 166

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2d^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c(c-d)\sqrt{c+d}f}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f/a^(1/2)-arctan(1/2*
a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2)+
2*d^(3/2)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1
/2))/c/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {4014, 4005, 3859, 209, 3880, 4052, 211}

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2d^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf(c-d)\sqrt{c+d}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)} + \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf}$$

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c*f) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/(Sqrt[a]*(c - d)*f) + (2*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(S
qrt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*c*(c - d)*Sqrt[c + d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4014

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Dist[1/(c*(b*c - a*d)), Int[(b*c - a*d - b*d*C
sc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d^2/(c*(b*c - a*d)), I
nt[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] ||
EqQ[c^2 - d^2, 0])
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{ac-ad-ad\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{ac(c-d)} + \frac{d^2\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{ac(c-d)} \\
&= \frac{\int \sqrt{a+a\sec(e+fx)}\,dx}{ac} - \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} \\
&\quad - \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c(c-d)f} \\
&= \frac{2d^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c(c-d)\sqrt{c+d}f} - \frac{2\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} \\
&= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2d^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c(c-d)\sqrt{c+d}f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.44 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\left(-c\sqrt{c+d}\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{2}\left((c-d)\sqrt{c+d}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)+d^{3/2}\arctan\left(\frac{\sqrt{d}\tan(}{\sqrt{c+d}\sqrt{}}\right.\right.\right.}{c(c-d)\sqrt{c+d}f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{a(1+\sec(e+}}$$

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*(-(c*Sqrt[c + d]*ArcSin[Tan[(e + f*x)/2]]) + Sqrt[2]*((c - d)*Sqrt[c + d
]*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] + d^(3/2)*
ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + Cos[e
 + f*x])])]))*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*(d + c*Cos[e + f*x])*Se
c[e + f*x]^(3/2)*Sqrt[1 + Sec[e + f*x]])/(c*(c - d)*Sqrt[c + d]*f*Sqrt[Sec[
(e + f*x)/2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(137) = 274$.

Time = 15.29 (sec) , antiderivative size = 659, normalized size of antiderivative = 3.97

method	result
default	$\frac{\left(2\sqrt{\frac{d}{c-d}}\sqrt{2}\sqrt{(c+d)(c-d)}\,\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)c-2\sqrt{\frac{d}{c-d}}\sqrt{2}\sqrt{(c+d)(c-d)}\,\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(}{\sqrt{(1-\cos(fx+e))^2\csc(f}}\right.\right.}{}$

```
[In] int(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(d/(c-d))^(1/2)/(c-d)/c/((c+d)*(c-d))^(1/2)/a*(2*(d/(c-d))^(1/2)*2^(1
/2)*((c+d)*(c-d))^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2)*(-cot(f*x+e)+csc(f*x+e)))*c-2*(d/(c-d))^(1/2)*2^(1/2)*((c+d)*(c-d))^(1
/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+cs
c(f*x+e)))*d-2*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)
+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*c+2^(1/2)*ln(-2*(((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*(
(1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+c
sc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c
+d)*(c-d))^(1/2)))*d^2-2^(1/2)*ln(2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2
)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f
*x+e)^2-1)^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-c
```

```
ot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*d^2)
*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)
^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 10.60 (sec) , antiderivative size = 1050, normalized size of antiderivative = 6.33

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(
f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*a*d*sqrt(-d/(a*c +
 a*d))*log((2*(c + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(
f*x + e) - d)/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 2*sqrt(-a)*(
c - d)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)
))/((a*c^2 - a*c*d)*f), -1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*
cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
 + 4*a*d*sqrt(d/(a*c + a*d))*arctan((c + d)*sqrt(d/(a*c + a*d))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(d*sin(f*x + e))) + 2*sqrt(-a)*(c
 - d)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))
)/((a*c^2 - a*c*d)*f), -(a*d*sqrt(-d/(a*c + a*d))*log((2*(c + d)*sqrt(-d/(a
*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)/(c*cos(f*x + e)^2
+ (c + d)*cos(f*x + e) + d)) - sqrt(2)*sqrt(a)*c*arctan(sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*sqrt(
a)*(c - d)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqr
t(a)*sin(f*x + e))))/((a*c^2 - a*c*d)*f), -(2*a*d*sqrt(d/(a*c + a*d))*arcta
n((c + d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)/(d*sin(f*x + e))) - sqrt(2)*sqrt(a)*c*arctan(sqrt(2)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*sqrt(a)*
(c - d)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a
)*sin(f*x + e))))/((a*c^2 - a*c*d)*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.170 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx$

Optimal result 1130
Rubi [A] (verified) 1131
Mathematica [A] (warning: unable to verify) 1134
Maple [B] (warning: unable to verify) 1135
Fricas [A] (verification not implemented) 1135
Sympy [F] 1137
Maxima [F] 1137
Giac [F(-2)] 1137
Mupad [F(-1)] 1138

Optimal result

Integrand size = 27, antiderivative size = 416

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx \\
&= \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\sqrt{a}d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c(c-d)(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2\sqrt{a}(2c-d)d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c-d)^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{d^2\tan(e+fx)}{c(c^2-d^2)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}
\end{aligned}$$

```
[Out] d^2*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arct
anh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/c^2/f/(a-a*sec(f*x+e
))^(1/2)/(a+a*sec(f*x+e))^(1/2)+d^(3/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1
/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e)/c/(c-d)/(c+d)^(3/2)/f/(a-a*sec(
f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*2^(1/2)*a^(1/2)*tan(f*x+e)/(c-d)^2/f/(a-a*sec(f*x+e))^(1/2)/(
a+a*sec(f*x+e))^(1/2)+2*(2*c-d)*d^(3/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1
/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e)/c^2/(c-d)^2/f/(c+d)^(1/2)/(a-a*
sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx\\
&=\frac{2\sqrt{a}d^{3/2}(2c-d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2f(c-d)^2\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\
&+\frac{2\sqrt{a}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\
&+\frac{\sqrt{a}d^{3/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{cf(c-d)(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\
&-\frac{\sqrt{2}\sqrt{a}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f(c-d)^2\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\
&+\frac{d^2\tan(e+fx)}{cf(c^2-d^2)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}
\end{aligned}$$

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*ArcTan
h[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/((c - d)^2*f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (Sqrt[a]*d^(3/2)*ArcTanh
[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c
*(c - d)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 + (2*Sqrt[a]*(2*c - d)*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/
(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*(c - d)^2*Sqrt[c + d]*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^2*Tan[e + f*x])/(c*(c^2 - d^
2)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{ac^2x\sqrt{a-ax}} - \frac{1}{a(c-d)^2(1+x)\sqrt{a-ax}} + \frac{d^2}{ac(c-d)\sqrt{a-ax}(c+dx)^2} + \frac{(2c-d)d^2}{ac^2(c-d)^2\sqrt{a-ax}(c+\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(ad^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{c(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(a(2c-d)d^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{c^2(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^2\tan(e+fx)}{c\,(c^2-d^2)\,f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{c^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(2(2c-d)d^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{c+d-\frac{dx^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{c^2(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(ad^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2c(c-d)(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}(2c-d)d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c-d)^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^2\tan(e+fx)}{c\,(c^2-d^2)\,f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(d^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{c+d-\frac{dx^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{c(c-d)(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\sqrt{a}d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c(c-d)(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}(2c-d)d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c-d)^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^2\tan(e+fx)}{c(c^2-d^2)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [A] (warning: unable to verify)

Time = 11.36 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \frac{(d+c\cos(e+fx))^2\sec^{\frac{5}{2}}(e+fx)\left(\frac{c(c-d)d^2\sin(e+fx)}{(c+d)(d+c\cos(e+fx))\sqrt{\sec(e+fx)}} + \frac{\left(\sqrt{2}\left(2(c-d)^2(c+d)^{3/2}\operatorname{arctanh}\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{-\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\right.\right.}{}\right.}{c^2(c-d)^2f\sqrt{a(1+}}$$

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^2*Sec[e + f*x]^(5/2)*((c*(c - d)*d^2*Sin[e + f*x])/((
c + d)*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) + ((Sqrt[2]*(2*(c - d)^2*(c
 + d)^(3/2)*ArcTanh[Tan[(e + f*x)/2]/Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x])
)]] + d^(3/2)*(5*c^2 + c*d - 2*d^2)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqr
t[c + d]*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]])]) - 2*c^2*(c + d)^(3/2)*
ArcTanh[Tan[(e + f*x)/2]/Sqrt[-1 + Tan[(e + f*x)/2]^2]])*Sqrt[Cos[(e + f*x)
/2]^2*Sec[e + f*x]]*Sqrt[-1 + Tan[(e + f*x)/2]^2])/((c + d)^(3/2)*Sqrt[Sec[
(e + f*x)/2]^2])))/(c^2*(c - d)^2*f*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e
 + f*x])^2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 54029 vs. $2(355) = 710$.

Time = 16.58 (sec) , antiderivative size = 54030, normalized size of antiderivative = 129.88

method	result	size
default	Expression too large to display	54030

```
[In] int(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 108.96 (sec) , antiderivative size = 2508, normalized size of antiderivative = 6.03

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
[Out] [1/2*(2*(c^2*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e) + sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a*c^4 + a*c^3*d)*cos(f*x
 + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e))*sqrt(-1/a)*log((2*s
qrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(
f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f
*x + e) + 1)) - (5*a*c^2*d^2 + a*c*d^3 - 2*a*d^4 + (5*a*c^3*d + a*c^2*d^2 -
 2*a*c*d^3)*cos(f*x + e)^2 + (5*a*c^3*d + 6*a*c^2*d^2 - a*c*d^3 - 2*a*d^4)*
cos(f*x + e))*sqrt(-d/(a*c + a*d))*log((2*(c + d)*sqrt(-d/(a*c + a*d))*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (c + 2*d)*c
os(f*x + e)^2 + (c + d)*cos(f*x + e) - d)/(c*cos(f*x + e)^2 + (c + d)*cos(f
*x + e) + d)) - 2*(c^3*d - c^2*d^2 - c*d^3 + d^4 + (c^4 - c^3*d - c^2*d^2 +
 c*d^3)*cos(f*x + e)^2 + (c^4 - 2*c^2*d^2 + d^4)*cos(f*x + e))*sqrt(-a)*log
((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((a*c^6
 - a*c^5*d - a*c^4*d^2 + a*c^3*d^3)*f*cos(f*x + e)^2 + (a*c^6 - 2*a*c^4*d^2
 + a*c^2*d^4)*f*cos(f*x + e) + (a*c^5*d - a*c^4*d^2 - a*c^3*d^3 + a*c^2*d^4
)*f), 1/2*(2*(c^2*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a*c^4 + a*c^3*d)*co
s(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e))*sqrt(-1/a)*log
((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)
*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) - 2*(5*a*c^2*d^2 + a*c*d^3 - 2*a*d^4 + (5*a*c^3*d + a*c^
```

```
2*d^2 - 2*a*c*d^3)*cos(f*x + e)^2 + (5*a*c^3*d + 6*a*c^2*d^2 - a*c*d^3 - 2*
a*d^4)*cos(f*x + e))*sqrt(d/(a*c + a*d))*arctan((c + d)*sqrt(d/(a*c + a*d))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(d*sin(f*x + e))) - 2
*(c^3*d - c^2*d^2 - c*d^3 + d^4 + (c^4 - c^3*d - c^2*d^2 + c*d^3)*cos(f*x +
 e)^2 + (c^4 - 2*c^2*d^2 + d^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e
)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f
*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((a*c^6 - a*c^5*d - a*c^
4*d^2 + a*c^3*d^3)*f*cos(f*x + e)^2 + (a*c^6 - 2*a*c^4*d^2 + a*c^2*d^4)*f*c
os(f*x + e) + (a*c^5*d - a*c^4*d^2 - a*c^3*d^3 + a*c^2*d^4)*f), 1/2*(2*(c^2
*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) - 4*(c^3*d - c^2*d^2 - c*d^3 + d^4 + (c^4 - c^3*d - c^2*d^2 + c*d^3)*c
os(f*x + e)^2 + (c^4 - 2*c^2*d^2 + d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (5
*a*c^2*d^2 + a*c*d^3 - 2*a*d^4 + (5*a*c^3*d + a*c^2*d^2 - 2*a*c*d^3)*cos(f*
x + e)^2 + (5*a*c^3*d + 6*a*c^2*d^2 - a*c*d^3 - 2*a*d^4)*cos(f*x + e))*sqrt
(-d/(a*c + a*d))*log((2*(c + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c
 + d)*cos(f*x + e) - d)/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 2*
sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a*c^4 + a*c^3*d)*cos(f*x + e)^2 + (a*c^4 +
2*a*c^3*d + a*c^2*d^2)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((a*c^6 - a*
c^5*d - a*c^4*d^2 + a*c^3*d^3)*f*cos(f*x + e)^2 + (a*c^6 - 2*a*c^4*d^2 + a*
c^2*d^4)*f*cos(f*x + e) + (a*c^5*d - a*c^4*d^2 - a*c^3*d^3 + a*c^2*d^4)*f),
 ((c^2*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e) - (5*a*c^2*d^2 + a*c*d^3 - 2*a*d^4 + (5*a*c^3*d + a*c^2*d^2 - 2*
a*c*d^3)*cos(f*x + e)^2 + (5*a*c^3*d + 6*a*c^2*d^2 - a*c*d^3 - 2*a*d^4)*cos
(f*x + e))*sqrt(d/(a*c + a*d))*arctan((c + d)*sqrt(d/(a*c + a*d))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(d*sin(f*x + e))) - 2*(c^3*d -
c^2*d^2 - c*d^3 + d^4 + (c^4 - c^3*d - c^2*d^2 + c*d^3)*cos(f*x + e)^2 + (c
^4 - 2*c^2*d^2 + d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + sqrt(2)*(a*c^3*d + a
*c^2*d^2 + (a*c^4 + a*c^3*d)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^
2)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((a*c^6 - a*c^5*d - a*c^4*d^2 +
a*c^3*d^3)*f*cos(f*x + e)^2 + (a*c^6 - 2*a*c^4*d^2 + a*c^2*d^4)*f*cos(f*x +
 e) + (a*c^5*d - a*c^4*d^2 - a*c^3*d^3 + a*c^2*d^4)*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^2}\,dx$$

[In] integrate(1/(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2), x)

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)^2}\,dx$$

[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)

[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^2), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx=\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2), x)
```

3.171 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx$

Optimal result 1139
Rubi [A] (verified) 1140
Mathematica [B] (warning: unable to verify) 1145
Maple [B] (warning: unable to verify) 1146
Fricas [F(-1)] 1147
Sympy [F] 1147
Maxima [F] 1147
Giac [F(-2)] 1148
Mupad [F(-1)] 1148

Optimal result

Integrand size = 27, antiderivative size = 653

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx \\
&= \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{3\sqrt{a}d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c(c-d)(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\sqrt{a}(2c-d)d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c-d)^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2\sqrt{a}d^{3/2}(3c^2-3cd+d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{d^2\tan(e+fx)}{2c(c^2-d^2)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} \\
&\quad + \frac{3d^2\tan(e+fx)}{4c(c-d)(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad + \frac{(2c-d)d^2\tan(e+fx)}{c^2(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}
\end{aligned}$$

```
[Out] 1/2*d^2*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)+
3/4*d^2*tan(f*x+e)/c/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2
```

```
)+(2*c-d)*d^2*tan(f*x+e)/c^2/(c-d)^2/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+
e))^(1/2)+2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/c^3/
f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+3/4*d^(3/2)*arctanh(d^(1/2)
*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e)/c/(c-d)/(c+
d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+(2*c-d)*d^(3/2)*ar
ctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e
)/c^2/(c-d)^2/(c+d)^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-a
rctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)*tan(f*x+
e)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*d^(3/2)*(3*c^2
-3*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*a^(
1/2)*tan(f*x+e)/c^3/(c-d)^3/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f
*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx \\
&= \frac{2\sqrt{a}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\
&+ \frac{\sqrt{a}d^{3/2}(2c-d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2f(c-d)^2(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\
&+ \frac{2\sqrt{a}d^{3/2}(3c^2-3cd+d^2)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^3f(c-d)^3\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\
&+ \frac{3\sqrt{a}d^{3/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4cf(c-d)(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\
&- \frac{\sqrt{2}\sqrt{a}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f(c-d)^3\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\
&+ \frac{d^2(2c-d)\tan(e+fx)}{c^2f(c-d)^2(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))} \\
&+ \frac{d^2\tan(e+fx)}{2cf\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2} \\
&+ \frac{3d^2\tan(e+fx)}{4cf(c-d)(c+d)^2\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}
\end{aligned}$$

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*ArcTan
h[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/((c - d)^3*f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (3*Sqrt[a]*d^(3/2)*ArcTa
nh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/
(4*c*(c - d)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*
x]]) + (Sqrt[a]*(2*c - d)*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]]
)/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*(c - d)^2*(c + d)^(3/2)*f*Sqrt[
a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*d^(3/2)*(3*c^2 -
 3*c*d + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c +
d])]*Tan[e + f*x])/(c^3*(c - d)^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) + (d^2*Tan[e + f*x])/(2*c*(c^2 - d^2)*f*Sqrt[a + a*
Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (3*d^2*Tan[e + f*x])/(4*c*(c - d)*(
c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])) + ((2*c - d)*d^2*
Tan[e + f*x])/(c^2*(c - d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[
e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{ac^3x\sqrt{a-ax}} - \frac{1}{a(c-d)^3(1+x)\sqrt{a-ax}} + \frac{d^2}{ac(c-d)\sqrt{a-ax}(c+dx)^3} + \frac{(2c-d)d^2}{ac^2(c-d)^2\sqrt{a-ax}(c+\cdots}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(ad^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{c(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(a(2c-d)d^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{c^2(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(ad^2(3c^2-3cd+d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{c^3(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$
\begin{aligned}
&= \frac{d^2\tan(e+fx)}{2c\left(c^2-d^2\right)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} \\
&\quad + \frac{(2c-d)d^2\tan(e+fx)}{c^2(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad + \frac{(2\tan(e+fx))\text{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(2\tan(e+fx))\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\left(3ad^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{4c(c-d)(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\left(a(2c-d)d^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{2c^2(c-d)^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\left(2d^2\left(3c^2-3cd+d^2\right)\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{c^3(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2\sqrt{a}d^{3/2}\left(3c^2-3cd+d^2\right)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{d^2\tan(e+fx)}{2c\left(c^2-d^2\right)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} \\
&\quad + \frac{3d^2\tan(e+fx)}{4c(c-d)(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad + \frac{(2c-d)d^2\tan(e+fx)}{c^2(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad - \frac{\left(3ad^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{8c(c-d)(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\left((2c-d)d^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{c^2(c-d)^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\sqrt{a}(2c-d)d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c-d)^2(c+d)^{3/2} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}d^{3/2}(3c^2-3cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3(c-d)^3\sqrt{c+d} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^2\tan(e+fx)}{2c\,(c^2-d^2)\,f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{3d^2\tan(e+fx)}{4c(c-d)(c+d)^2 f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(2c-d)d^2\tan(e+fx)}{c^2(c-d)^2(c+d) f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(3d^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{4c(c-d)(c+d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{3\sqrt{a}d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c(c-d)(c+d)^{5/2} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\sqrt{a}(2c-d)d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c-d)^2(c+d)^{3/2} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}d^{3/2}(3c^2-3cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3(c-d)^3\sqrt{c+d} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^2\tan(e+fx)}{2c\,(c^2-d^2)\,f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{3d^2\tan(e+fx)}{4c(c-d)(c+d)^2 f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(2c-d)d^2\tan(e+fx)}{c^2(c-d)^2(c+d) f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2940 vs. 2(653) = 1306.

Time = 22.45 (sec) , antiderivative size = 2940, normalized size of antiderivative = 4.50

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3} dx = \text{Result too large to show}$$

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] (Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^4*(-1/2*(d^2*(-13*c^2
 - c*d + 6*d^2)*Sin[(e + f*x)/2])/(c^3*(-c + d)^2*(c + d)^2) - (d^4*Sin[(e
+ f*x)/2])/(c^3*(-c + d)*(c + d)*(d + c*Cos[e + f*x])^2) + (-15*c^2*d^3*Sin
[(e + f*x)/2] - c*d^4*Sin[(e + f*x)/2] + 8*d^5*Sin[(e + f*x)/2])/(2*c^3*(-c
 + d)^2*(c + d)^2*(d + c*Cos[e + f*x]))))/(f*Sqrt[a*(1 + Sec[e + f*x])]*(c
+ d*Sec[e + f*x])^3) - (Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])^3*((Sqrt[2]*d
^(3/2)*(35*c^4 + 14*c^3*d - 21*c^2*d^2 - 4*c*d^3 + 8*d^4)*ArcTan[(Sqrt[d]*T
an[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))])]))/
(Sqrt[-c - d]*(c - d)) - 2*Sqrt[2]*(c^2 - d^2)^2*Log[Sec[(e + f*x)/2]^2*(-1
 + 2*Cos[e + f*x] - 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]*Sin[e + f*x]
)] + 2*Sqrt[2]*(c^2 - d^2)^2*Log[Sec[(e + f*x)/2]^2*(-1 + 2*Cos[e + f*x] +
2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]*Sin[e + f*x])] + (8*c^3*(c + d)^
2*Log[Tan[(e + f*x)/2] + Sqrt[-1 + Tan[(e + f*x)/2]^2]])/(c - d))*((-2*c*d*
Sec[(e + f*x)/2])/((-c + d)^2*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f
*x]]) - (13*d^2*Sec[(e + f*x)/2])/(8*(-c + d)^2*(c + d)^2*(d + c*Cos[e + f*
x])*Sqrt[Sec[e + f*x]]) + (d^3*Sec[(e + f*x)/2])/(8*c*(-c + d)^2*(c + d)^2*
(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) + (d^4*Sec[(e + f*x)/2])/(2*c^2*(-
c + d)^2*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) + (c^2*Sec[(e +
 f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(-c + d)^2*(c + d)^2*(d + c*Cos[e + f*x]))
+ (3*d^2*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(8*(-c + d)^2*(c + d)^2*(d +
c*Cos[e + f*x])) + (d^3*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(8*c*(-c + d)^
2*(c + d)^2*(d + c*Cos[e + f*x])) + (c^2*Cos[2*(e + f*x)]*Sec[(e + f*x)/2]*
Sqrt[Sec[e + f*x]])/(2*(-c + d)^2*(c + d)^2*(d + c*Cos[e + f*x])) - (d^2*Co
s[2*(e + f*x)]*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/((-c + d)^2*(c + d)^2*(
d + c*Cos[e + f*x])) + (d^4*Cos[2*(e + f*x)]*Sec[(e + f*x)/2]*Sqrt[Sec[e +
f*x]])/(2*c^2*(-c + d)^2*(c + d)^2*(d + c*Cos[e + f*x])))*Sec[e + f*x]^(7/2
)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[-1 + Tan[(e + f*x)/2]^2])/(4*c
^3*(c - d)^2*(c + d)^2*f*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^3*
(-1/8*(((Sqrt[2]*d^(3/2)*(35*c^4 + 14*c^3*d - 21*c^2*d^2 - 4*c*d^3 + 8*d^4)
*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[-(Cos[e + f*x]/(1 + C
os[e + f*x]))])]))/(Sqrt[-c - d]*(c - d)) - 2*Sqrt[2]*(c^2 - d^2)^2*Log[Sec[
(e + f*x)/2]^2*(-1 + 2*Cos[e + f*x] - 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*
x]))]*Sin[e + f*x])] + 2*Sqrt[2]*(c^2 - d^2)^2*Log[Sec[(e + f*x)/2]^2*(-1 +
 2*Cos[e + f*x] + 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]*Sin[e + f*x])]
 + (8*c^3*(c + d)^2*Log[Tan[(e + f*x)/2] + Sqrt[-1 + Tan[(e + f*x)/2]^2]])/
```

```
(c - d))*Sec[(e + f*x)/2]^2*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Tan[(e +
f*x)/2])/(c^3*(c - d)^2*(c + d)^2*Sqrt[-1 + Tan[(e + f*x)/2]^2]) - (Sqrt[Co
s[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[-1 + Tan[(e + f*x)/2]^2]*((-2*Sqrt[2]*(
c^2 - d^2)^2*Cos[(e + f*x)/2]^2*(Sec[(e + f*x)/2]^2*(-2*Cos[e + f*x]*Sqrt[-
(Cos[e + f*x]/(1 + Cos[e + f*x]))] - 2*Sin[e + f*x] - (Sin[e + f*x]*(-((Cos
[e + f*x]*Sin[e + f*x])/(1 + Cos[e + f*x])^2) + Sin[e + f*x]/(1 + Cos[e + f
*x])))/Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]) + Sec[(e + f*x)/2]^2*(-1 +
 2*Cos[e + f*x] - 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]*Sin[e + f*x])*
Tan[(e + f*x)/2]))/(-1 + 2*Cos[e + f*x] - 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e
+ f*x]))]*Sin[e + f*x]) + (2*Sqrt[2]*(c^2 - d^2)^2*Cos[(e + f*x)/2]^2*(Sec[
(e + f*x)/2]^2*(2*Cos[e + f*x]*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))] - 2
*Sin[e + f*x] + (Sin[e + f*x]*(-((Cos[e + f*x]*Sin[e + f*x])/(1 + Cos[e + f
*x])^2) + Sin[e + f*x]/(1 + Cos[e + f*x])))/Sqrt[-(Cos[e + f*x]/(1 + Cos[e
+ f*x]))]) + Sec[(e + f*x)/2]^2*(-1 + 2*Cos[e + f*x] + 2*Sqrt[-(Cos[e + f*x
]/(1 + Cos[e + f*x]))]*Sin[e + f*x])*Tan[(e + f*x)/2]))/(-1 + 2*Cos[e + f*x
] + 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]*Sin[e + f*x]) + (Sqrt[2]*d^(
3/2)*(35*c^4 + 14*c^3*d - 21*c^2*d^2 - 4*c*d^3 + 8*d^4)*((Sqrt[d]*Sec[(e +
f*x)/2]^2)/(2*Sqrt[-c - d]*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]) - (Sqr
t[d]*(-((Cos[e + f*x]*Sin[e + f*x])/(1 + Cos[e + f*x])^2) + Sin[e + f*x]/(1
 + Cos[e + f*x]))*Tan[(e + f*x)/2])/(2*Sqrt[-c - d]*(-(Cos[e + f*x]/(1 + Co
s[e + f*x])))^(3/2))))/(Sqrt[-c - d]*(c - d)*(1 - (d*(1 + Cos[e + f*x])*Sec
[e + f*x]*Tan[(e + f*x)/2]^2)/(-c - d))) + (8*c^3*(c + d)^2*(Sec[(e + f*x)/
2]^2/2 + (Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2*Sqrt[-1 + Tan[(e + f*x)/2
]^2])))/((c - d)*(Tan[(e + f*x)/2] + Sqrt[-1 + Tan[(e + f*x)/2]^2]))))/(4*c
^3*(c - d)^2*(c + d)^2) - (((Sqrt[2]*d^(3/2)*(35*c^4 + 14*c^3*d - 21*c^2*d^
2 - 4*c*d^3 + 8*d^4)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[-
(Cos[e + f*x]/(1 + Cos[e + f*x]))])])/(Sqrt[-c - d]*(c - d)) - 2*Sqrt[2]*(c
^2 - d^2)^2*Log[Sec[(e + f*x)/2]^2*(-1 + 2*Cos[e + f*x] - 2*Sqrt[-(Cos[e +
f*x]/(1 + Cos[e + f*x]))]*Sin[e + f*x])] + 2*Sqrt[2]*(c^2 - d^2)^2*Log[Sec[
(e + f*x)/2]^2*(-1 + 2*Cos[e + f*x] + 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*
x]))]*Sin[e + f*x])] + (8*c^3*(c + d)^2*Log[Tan[(e + f*x)/2] + Sqrt[-1 + Ta
n[(e + f*x)/2]^2]])/(c - d))*Sqrt[-1 + Tan[(e + f*x)/2]^2]*(-(Cos[(e + f*x)
/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e
+ f*x]))/(8*c^3*(c - d)^2*(c + d)^2*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]))
)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 97276 vs. $2(564) = 1128$.

Time = 18.67 (sec) , antiderivative size = 97277, normalized size of antiderivative = 148.97

method	result	size
default	Expression too large to display	97277

```
[In] int(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^3}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^3}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3), x)
```

3.172 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result 1149
Rubi [A] (verified) 1150
Mathematica [C] (warning: unable to verify) 1153
Maple [B] (warning: unable to verify) 1153
Fricas [A] (verification not implemented) 1154
Sympy [F] 1155
Maxima [F] 1155
Giac [F(-2)] 1155
Mupad [F(-1)] 1156

Optimal result

Integrand size = 27, antiderivative size = 324

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c-d)^2(c+2d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*d^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-1/2*(c-d)^3*tan(f*x+e)/a/f/(1+s
ec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1
/2))*tan(f*x+e)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4
*(c-d)^3*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/f*2
^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c-d)^2*(c+2*d
)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)/f/
a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4025, 186, 65, 212, 44}

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^3\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{(c-d)^3\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{\sqrt{2}(c-d)^2(c+2d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{(c-d)^3\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}} + \frac{2d^3\tan(e+fx)}{af\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*d^3*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^3*Tan[e + f*
x])/(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*c^3*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*f*Sqrt[a - a*Sec[e +
 f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*
x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*f*Sqrt[a - a*Sec[e
+ f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - d)^2*(c + 2*d)*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{d^3}{a^2\sqrt{a-ax}}+\frac{c^3}{a^2x\sqrt{a-ax}}-\frac{(c-d)^3}{a^2(1+x)^2\sqrt{a-ax}}-\frac{(c-d)^2(c+2d)}{a^2(1+x)\sqrt{a-ax}}\right)dx, x, \sec(e+\ldots\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{\left(c^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left((c-d)^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left((c-d)^2(c+2d)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{(2c^3\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-d)^3\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2(c-d)^2(c+2d)\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^3\mathrm{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c-d)^2(c+2d)\mathrm{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{((c-d)^3\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{2af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^3\mathrm{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\mathrm{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c-d)^2(c+2d)\mathrm{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.09 (sec) , antiderivative size = 856, normalized size of antiderivative = 2.64

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2\cos^3\left(\frac{1}{2}(e+fx)\right)(c+d\sec(e+fx))^3\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+\right.}}{}$$

```
[In] Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*Cos[(e + f*x)/2]^3*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2
)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((-3*(c - d)^3*ArcTan[(1 - 2*Sin[(e
+ f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]])/2 + (3*(c - d)^3*ArcTan[(1 + 2*
Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]])/2 - (4*c^2*(c - 3*d)*Sin
[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2] + ((c - d)^3*(1 - 2*Sin[(e +
f*x)/2]))/(4*(1 + Sin[(e + f*x)/2])*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) - ((c -
 d)^3*(1 + 2*Sin[(e + f*x)/2]))/(4*(1 - Sin[(e + f*x)/2])*Sqrt[1 - 2*Sin[(e
 + f*x)/2]^2]) - ((c - d)^3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2])/(1 - Sin[(e + f
*x)/2]) + ((c - d)^3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2])/(1 + Sin[(e + f*x)/2])
 - (2*c^3*(-(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]) + 2*Sqrt[2]*ArcSin[S
qrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*x)/2]^2 + 2*Sin[(e + f*x)/2]*Sqrt[1 - 2
*Sin[(e + f*x)/2]^2]))/(1 - 2*Sin[(e + f*x)/2]^2) - ((c - d)^2*(11*c + d)*S
in[(e + f*x)/2]*((2*Cos[(e + f*x)/2]^2*Hypergeometric2F1[2, 5/2, 7/2, -(Sin
[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2]^2)/(1 - 2*Sin
[(e + f*x)/2]^2) + 5*Csc[(e + f*x)/2]^4*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Si
n[(e + f*x)/2]^2))]*(1 - 2*Sin[(e + f*x)/2]^2)^2*(3 - 2*Sin[(e + f*x)/2]^2)
*(-ArcTanh[Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]] + Sqrt[-
(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))])))/(10*(1 - 2*Sin[(e + f*x
)/2]^2)^(3/2))))/(f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(3/2)*(a*(1 + Sec[e
 + f*x]))^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(280) = 560.

Time = 6.06 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left(4\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\sqrt{2}\,c^3+c^3(1-\cos(fx\cdots\right.}{}$
parts	$\frac{c^3\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(4\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(fx\cdots\right.}{4fa^2}$

```
[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(4*arctanh(2^(1/2)
/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*((1-cos(
f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*c^3+c^3*(1-cos(f*x+e))^3*csc(f*x+e)
^3-3*c^2*d*(1-cos(f*x+e))^3*csc(f*x+e)^3+3*c*d^2*(1-cos(f*x+e))^3*csc(f*x+e
)^3-d^3*(1-cos(f*x+e))^3*csc(f*x+e)^3-5*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c^3+
3*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos
(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c^2*d+9*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c*d
^2-7*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*((1-
cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d^3-c^3*(-cot(f*x+e)+csc(f*x+e))+3*c^2*
d*(-cot(f*x+e)+csc(f*x+e))-3*c*d^2*(-cot(f*x+e)+csc(f*x+e))+9*d^3*(-cot(f*x
+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 14.30 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.16

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{\sqrt{2}\left(5\,c^3-3\,c^2d-9\,cd^2+7\,d^3+\left(5\,c^3-3\,c^2d-9\,cd^2+7\,d^3\right)\cos\left(fx+e\right)^2\cdots\right.}{}\right.$$

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c*
d^2 + 7*d^3)*cos(f*x + e)^2 + 2*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x
 + e))*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a
)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^2 + 2*c^3*co
s(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a
)/(cos(f*x + e) + 1)) - 4*(4*d^3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*
```

```
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*
x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), 1/4*(sqrt(2)*(5*c^3 - 3*c^2*d - 9
*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e)^2 + 2*(5*
c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt(2)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 8
*(c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*(4*d^
3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e)
 + a^2*f)]
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(c+d\sec{(e+fx)})^3}{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)
[Out] Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec{(fx+e)}+c)^3}{(a\sec{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)^3/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2), x)
```

3.173 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result 1157
Rubi [A] (verified) 1158
Mathematica [A] (verified) 1161
Maple [A] (warning: unable to verify) 1161
Fricas [A] (verification not implemented) 1162
Sympy [F] 1162
Maxima [F] 1163
Giac [F(-2)] 1163
Mupad [F(-1)] 1163

Optimal result

Integrand size = 27, antiderivative size = 290

$$\begin{aligned}\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx &= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{\sqrt{2}(c^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

```
[Out] -1/2*(c-d)^2*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^2*arc
tanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/a^(1/2)/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*tan(f*x+e)/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*
sec(f*x+e))^(1/2)-(c^2-d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*2^(1/2)*tan(f*x+e)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4025, 186, 65, 212, 44}

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}(c^2-d^2)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2c^2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{(c-d)^2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{(c-d)^2\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/2*((c - d)^2*Tan[e + f*x])/(a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*
x]]) + (2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt
[a]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^2*ArcTa
nh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqr
t[a]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 -
 d^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sq
rt[a]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
```

```
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{c^2}{a^2x\sqrt{a-ax}} - \frac{(c-d)^2}{a^2(1+x)^2\sqrt{a-ax}} + \frac{-c^2+d^2}{a^2(1+x)\sqrt{a-ax}}\right) dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{(c^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{((c-d)^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{((c^2-d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{(2c^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2(c^2-d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{((c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{2af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.61

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{-\sqrt{2}(5c^2-2cd-3d^2)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}{af(1}$$

```
[In] Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-(Sqrt[2]*(5*c^2 - 2*c*d - 3*d^2)*ArcSin[Tan[(e + f*x)/2]]*Cos[(e + f*x)/2
]^4*Sec[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)]) + 8*c^2*ArcTan[Tan[(e + f*x
)/2]/Sqrt[(1 + Sec[e + f*x])^(-1)]]*Cos[(e + f*x)/2]^4*Sec[e + f*x]*Sqrt[(1
 + Sec[e + f*x])^(-1)] - ((c - d)^2*Sin[e + f*x])/2)/(a*f*(1 + Cos[e + f*x]
)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] (warning: unable to verify)

Time = 3.46 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.34

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-4c^2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)-\sqrt{(1-c}\right.}{}$
parts	$\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(f}\right.}{4fa^2}$

```
[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^2
*csc(f*x+e)^2-1)^(1/2)*(-4*c^2*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)^(1/2)*c^2*(-cot(f*x+e)+csc(f*x+e))+2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*c*d*(-cot(f*x+e)+csc(f*x+e))-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)
*d^2*(-cot(f*x+e)+csc(f*x+e))+5*c^2*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1)^(1/2))-2*c*d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2
*csc(f*x+e)^2-1)^(1/2))-3*d^2*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 6.47 (sec) , antiderivative size = 620, normalized size of antiderivative = 2.14

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\left(c^2-2\,cd+d^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{2}\left(\left(5\,c^2-2\,cd\right.\right.}{}\right.$$

$$-\frac{2\left(c^2-2\,cd+d^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{2}\left(\left(5\,c^2-2\,cd-3\,d^2\right)\cos\left(fx+e\right)^2+5\,c^2-2\right.}{}$$

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(c^2 - 2*c*d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) - sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5*c
^2 - 2*c*d - 3*d^2 + 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(-a)*log((
2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) + 8*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt
(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))
/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/4*(2*(c^2 - 2*c*
d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
- sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5*c^2 - 2*c*d - 3*d^2 +
 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 8*(c^2*
cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a^2*f*cos(f*x
 + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c+d\sec\left(e+fx\right)\right)^2}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e)+c)^2}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2), x)
```

3.174 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result 1164
Rubi [A] (verified) 1164
Mathematica [A] (verified) 1166
Maple [B] (verified) 1166
Fricas [B] (verification not implemented) 1167
Sympy [F] 1168
Maxima [F] 1168
Giac [F(-2)] 1168
Mupad [F(-1)] 1168

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{(5c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-1/4*(5*c-d)
*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f*2^
(1/2)-1/2*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4007, 4005, 3859, 209, 3880}

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(5c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f} - \frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

```
[In] Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f) -
 ((5*c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]
```

```
)])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x]
)^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}} - \frac{\int \frac{-2ac+\frac{1}{2}a(c-d)\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{2a^2} \\ &= -\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}} + \frac{c\int\sqrt{a+a\sec(e+fx)}\,dx}{a^2} - \frac{(5c-d)\int\frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{4a}\end{aligned}$$

$$= -\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}} - \frac{(2c)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af} + \frac{(5c-d)\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{2af}$$

$$= \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{(5c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.81

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(-\left((5c-d)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\cos\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e + f*x]*(-((5*c - d)*ArcSin[Tan[(e + f*x)/2]]*Cos[(e
 + f*x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[1 + Sec[e
+ f*x]]) + Sqrt[2]*(4*c*ArcTan[Tan[(e + f*x)/2]/Sqrt[(1 + Sec[e + f*x])^(-1
)]]*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[
1 + Sec[e + f*x]] - (c - d)*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sin[(e + f*x)/2])
))/(f*Sqrt[Sec[(e + f*x)/2]^2]*(a*(1 + Sec[e + f*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(106) = 212$.

Time = 3.02 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.28

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-4c\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)-c\sqrt{(1-\cos\right.}{}$
parts	$\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(4\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{(1-\cos(fx+\right.}{4f\,a^2}$

```
[In] int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^2
*csc(f*x+e)^2-1)^(1/2)*(-4*c*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(
f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-c*((1-cos(f*x+e))^2*csc(f*x+e)^
```

```
2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))+d*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*(-cot(f*x+e)+csc(f*x+e))+5*c*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*
csc(f*x+e)^2-1)^(1/2))-d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(106) = 212$.

Time = 1.88 (sec) , antiderivative size = 548, normalized size of antiderivative = 4.31

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{3/2}}\,dx = \left[-\frac{4\,(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) - \sqrt{2}\big((5\,c-d)\cos(fx+\right.$$

$$-\frac{2\,(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) - \sqrt{2}\big((5\,c-d)\cos(fx+e)^2 + 2\,(5\,c-d)\cos(fx+e) + }{4\,\big(a^2 f\cos(fx+}$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f
*x + e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e) + 5*
c - d)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)
/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c*cos(f*x + e)^2 + 2*c*cos(f*x
 + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(
f*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/4
*(2*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e) + 5*c - d
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e))) + 8*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sq
rt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*
sin(f*x + e))))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{c+d\sec{(e+fx)}}{(a\,(\sec{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)
[Out] Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{d\sec{(fx+e)}+c}{(a\sec{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{c+\frac{d}{\cos(e+f\,x)}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)
```

3.175 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx$

Optimal result 1169
Rubi [A] (verified) 1170
Mathematica [A] (warning: unable to verify) 1173
Maple [B] (warning: unable to verify) 1173
Fricas [A] (verification not implemented) 1174
Sympy [F] 1176
Maxima [F] 1176
Giac [F(-2)] 1176
Mupad [F(-1)] 1177

Optimal result

Integrand size = 27, antiderivative size = 394

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx &= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{\sqrt{2}(c-2d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{2d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c(c-d)^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

```
[Out] -1/2*tan(f*x+e)/a/(c-d)/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh((
a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*tan(f*x+e)/(c-d)/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*
x+e))^(1/2)-(c-2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(
1/2)*tan(f*x+e)/(c-d)^2/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2)-2*d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*
tan(f*x+e)/c/(c-d)^2/f/a^(1/2)/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(
f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4025, 186, 65, 212, 44, 214}

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = -\frac{2d^{5/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}cf(c-d)^2\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}f(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{\sqrt{2}(c-2d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}f(c-d)^2\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}cf\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{\tan(e+fx)}{2af(c-d)(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] -1/2*Tan[e + f*x]/(a*(c - d)*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]])
 + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c*f*
Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 2*d)*Arc
Tanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*(c
- d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[Sqrt
[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*(c
 - d)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^(5/2)*Arc
Tanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x]
)/(Sqrt[a]*c*(c - d)^2*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Se
c[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^2(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{a^2cx\sqrt{a-ax}} - \frac{1}{a^2(c-d)(1+x)^2\sqrt{a-ax}} + \frac{-c+2d}{a^2(c-d)^2(1+x)\sqrt{a-ax}} - \frac{d^3}{a^2c(c-d)^2\sqrt{a-ax}(\cdots}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-2d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(d^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{c(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{acf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2(c-2d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{a(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{4(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2d^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{c+d-\frac{dx^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{ac(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c-2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c(c-d)^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{2a(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c-2d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c(c-d)^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (warning: unable to verify)

Time = 6.10 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^{\frac{5}{2}}(e+fx)\left(\frac{\left(-c(5\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Cos[(e + f*x)/2]^2*(d + c*Cos[e + f*x])*Sec[e + f*x]^(5/2)*(((-(c*(5*c - 9
*d)*Sqrt[c + d]*ArcSin[Tan[(e + f*x)/2]]) + 4*Sqrt[2]*((c - d)^2*Sqrt[c + d
]*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] - d^(5/2)*
ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + Cos[e
 + f*x])])]))*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[1 + Sec[e + f*x]])
/(c*Sqrt[c + d]*Sqrt[Sec[(e + f*x)/2]^2]) + (c - d)*Sqrt[Sec[e + f*x]]*(-Si
n[e + f*x] + Tan[(e + f*x)/2])))/((c - d)^2*f*(a*(1 + Sec[e + f*x]))^(3/2)*
(c + d*Sec[e + f*x]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1418 vs. 2(334) = 668.

Time = 16.33 (sec) , antiderivative size = 1419, normalized size of antiderivative = 3.60

method	result	size
default	Expression too large to display	1419

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/(d/(c-d))^(1/2)/(c-d)^2/c/((c+d)*(c-d))^(1/2)/a^2*(((1-cos(f*x+e))^2
*csc(f*x+e)^2-1)^(3/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c^2*(-cot(f*x+e)
+csc(f*x+e))-2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*((c+d)*(c-d))^(1/2)*
(d/(c-d))^(1/2)*c*d*(-cot(f*x+e)+csc(f*x+e))+((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)^(3/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*d^2*(-cot(f*x+e)+csc(f*x+e))-
((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)
*c^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2
)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c*d*(1-cos(f*x+e))^3*csc(f*x+e)^3-((1
-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*d^
2*(1-cos(f*x+e))^3*csc(f*x+e)^3+5*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(
(c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c^2*(-cot(f*x+e)+csc(f*x+e))-6*((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^(1/2)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c*d*(-co
t(f*x+e)+csc(f*x+e))+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*((c+d)*(c-d))^
(1/2)*(d/(c-d))^(1/2)*d^2*(-cot(f*x+e)+csc(f*x+e))+16*((c+d)*(c-d))^(1/2)*2
^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)
+csc(f*x+e)))*(d/(c-d))^(1/2)*c^2-32*((c+d)*(c-d))^(1/2)*2^(1/2)*arctanh(2^
(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(d/
(c-d))^(1/2)*c*d+16*((c+d)*(c-d))^(1/2)*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x
+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*d^2-
20*((c+d)*(c-d))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e
)^2-1)^(1/2))*(d/(c-d))^(1/2)*c^2+36*((c+d)*(c-d))^(1/2)*ln(csc(f*x+e)-cot(
f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*(d/(c-d))^(1/2)*c*d-8*2^(1/
2)*ln(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c
-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(
c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-co
t(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*d^3+8*2^(1/2)*ln(2*(((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(
1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*
x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*
d+((c+d)*(c-d))^(1/2)))*d^3)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-2*a/
((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 215.92 (sec) , antiderivative size = 2033, normalized size of antiderivative = 5.16

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(c^2 - c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) - sqrt(2)*((5*c^2 - 9*c*d)*cos(f*x + e)^2 + 5*c^2 - 9*c*d + 2*(
```

5*c^2 - 9*c*d)*cos(f*x + e))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 +
 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 8*(a*d^2*co
s(f*x + e)^2 + 2*a*d^2*cos(f*x + e) + a*d^2)*sqrt(-d/(a*c + a*d))*log((2*(c
 + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)/(c
*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 8*((c^2 - 2*c*d + d^2)*cos(f
*x + e)^2 + c^2 - 2*c*d + d^2 + 2*(c^2 - 2*c*d + d^2)*cos(f*x + e))*sqrt(-a
)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((
a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*x + e)^2 + 2*(a^2*c^3 - 2*a^2*c^
2*d + a^2*c*d^2)*f*cos(f*x + e) + (a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f), -
1/8*(4*(c^2 - c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) - sqrt(2)*((5*c^2 - 9*c*d)*cos(f*x + e)^2 + 5*c^2 - 9*c*d + 2*(5*
c^2 - 9*c*d)*cos(f*x + e))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2
*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 16*(a*d^2*cos
(f*x + e)^2 + 2*a*d^2*cos(f*x + e) + a*d^2)*sqrt(d/(a*c + a*d))*arctan((c +
 d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(d*sin(f*x + e))) + 8*((c^2 - 2*c*d + d^2)*cos(f*x + e)^2 + c^2 - 2*c*d +
 d^2 + 2*(c^2 - 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2
 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((a^2*c^3 - 2*a^2*c^2*d + a
^2*c*d^2)*f*cos(f*x + e)^2 + 2*(a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*
x + e) + (a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f), -1/4*(2*(c^2 - c*d)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - sqrt(2)*((5*c
^2 - 9*c*d)*cos(f*x + e)^2 + 5*c^2 - 9*c*d + 2*(5*c^2 - 9*c*d)*cos(f*x + e)
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e))) + 8*((c^2 - 2*c*d + d^2)*cos(f*x + e)^2 + c^2 -
2*c*d + d^2 + 2*(c^2 - 2*c*d + d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 4*(a*d
^2*cos(f*x + e)^2 + 2*a*d^2*cos(f*x + e) + a*d^2)*sqrt(-d/(a*c + a*d))*log(
(2*(c + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) -
d)/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((a^2*c^3 - 2*a^2*c^2*d
+ a^2*c*d^2)*f*cos(f*x + e)^2 + 2*(a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos
(f*x + e) + (a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f), -1/4*(2*(c^2 - c*d)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - sqrt(2)*((
5*c^2 - 9*c*d)*cos(f*x + e)^2 + 5*c^2 - 9*c*d + 2*(5*c^2 - 9*c*d)*cos(f*x +
 e))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)/(sqrt(a)*sin(f*x + e))) - 8*(a*d^2*cos(f*x + e)^2 + 2*a*d^2*cos(f*x +
 e) + a*d^2)*sqrt(d/(a*c + a*d))*arctan((c + d)*sqrt(d/(a*c + a*d))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(d*sin(f*x + e))) + 8*((c^2 -
 2*c*d + d^2)*cos(f*x + e)^2 + c^2 - 2*c*d + d^2 + 2*(c^2 - 2*c*d + d^2)*co
s(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x

```
+ e)/(sqrt(a)*sin(f*x + e))))/((a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*
x + e)^2 + 2*(a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*f*cos(f*x + e) + (a^2*c^3
- 2*a^2*c^2*d + a^2*c*d^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a\left(\sec\left(e+fx\right)+1\right))^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a\sec\left(fx+e\right)+a)^{\frac{3}{2}}(d\sec\left(fx+e\right)+c)}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))),x)
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))), x)
```

3.176 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx$

Optimal result 1178
Rubi [A] (verified) 1179
Mathematica [A] (warning: unable to verify) 1183
Maple [B] (warning: unable to verify) 1184
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Optimal result

Integrand size = 27, antiderivative size = 560

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = & -\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ & + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & - \frac{\sqrt{2}(c-3d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & - \frac{\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & - \frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c(c-d)^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & - \frac{2(3c-d)d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^2(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & - \frac{d^3\tan(e+fx)}{ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\end{aligned}$$

```
[Out] -1/2*tan(f*x+e)/a/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-d^3*tan(f
*x+e)/a/c/(c-d)^2/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh
((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/a^(1/2)/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)-d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)
/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/(c-d)^2/(c+d)^(3/2)/f/a^(1/2)/(a-a*sec(f
```

```
*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*
2^(1/2)/a^(1/2))*tan(f*x+e)/(c-d)^2/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2)-(c-3*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*2^(1/2)*tan(f*x+e)/(c-d)^3/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*
sec(f*x+e))^(1/2)-2*(3*c-d)*d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/
a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(c-d)^3/f/a^(1/2)/(c+d)^(1/2)/(a-a*sec(
f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4025, 186, 65, 212, 44, 214}

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = -\frac{2d^{5/2}(3c-d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2f(c-d)^3\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{d^{5/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}cf(c-d)^2(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}f(c-d)^2\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{\sqrt{2}(c-3d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}f(c-d)^3\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{d^3\tan(e+fx)}{acf(c-d)^2(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))} - \frac{\tan(e+fx)}{2af(c-d)^2(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] -1/2*Tan[e + f*x]/(a*(c - d)^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]
]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c^
2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 3*d)
*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]
*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[
Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a
```

```
]*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^(5/2)
*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e +
f*x])/(Sqrt[a]*c*(c - d)^2*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a
+ a*Sec[e + f*x]]) - (2*(3*c - d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e
 + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^2*(c - d)^3*Sqrt[
c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^3*Tan[e +
f*x])/(a*c*(c - d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]
))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
```

```
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^2(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{1}{a^2c^2x\sqrt{a-ax}} - \frac{1}{a^2(c-d)^2(1+x)^2\sqrt{a-ax}} + \frac{-c+3d}{a^2(c-d)^3(1+x)\sqrt{a-ax}} - \frac{d^3}{a^2c(c-d)^2\sqrt{a-a}}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-3d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(d^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{c(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left((3c-d)d^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{c^2(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$
\begin{aligned}
&= -\frac{\tan(e+fx)}{2a(c-d)^2 f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{d^3\tan(e+fx)}{ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad + \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{ac^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(2(c-3d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{a(c-d)^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{4(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(2(3c-d)d^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{c+d-\frac{dx^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{ac^2(c-d)^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{(d^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2c(c-d)^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{\tan(e+fx)}{2a(c-d)^2 f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\sqrt{2}(c-3d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{2(3c-d)d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^2(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{d^3\tan(e+fx)}{ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{2a(c-d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(d^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{c+d-\frac{dx^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{ac(c-d)^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c-3d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c(c-d)^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2(3c-d)d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^2(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d^3\tan(e+fx)}{ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [A] (warning: unable to verify)

Time = 11.85 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \frac{\left((-c-d)^{3/2}\left(-c^2(5c-13d)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+\cdots\right.\right.}{\cdots} + \frac{\cos^3\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))^2\sec^4(e+fx)\left(-\frac{2(c^3+c^2d+2d^3)\sin\left(\frac{1}{2}(e+fx)\right)}{c^2(-c+d)^2(c+d)}+\frac{4d^4\sin\left(\frac{1}{2}(e+fx)\right)}{c^2(-c+d)^2(c+d)(d+c\cos(e+fx))}\cdots\right)}{f(a(1+\sec(e+fx)))^{3/2}(c+d\sec(e+fx))^2}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (((-c - d)^(3/2)*(-(c^2*(5*c - 13*d)*ArcSin[Tan[(e + f*x)/2]]) + 4*Sqrt[2]*
(c - d)^3*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]]) +
 2*Sqrt[2]*d^(5/2)*(-7*c^2 - 3*c*d + 2*d^2)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/
2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])])*(d + c*Cos[e + f
*x])^2*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sec[e + f*x]^(7/2)*Sqrt[Cos[(e
 + f*x)/2]^2*Sec[e + f*x]])/(c^2*(-c - d)^(3/2)*(c - d)^3*f*(Sec[(e + f*x)/
2]^2)^(3/2)*(a*(1 + Sec[e + f*x]))^(3/2)*(c + d*Sec[e + f*x])^2) + (Cos[(e
+ f*x)/2]^3*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^4*((-2*(c^3 + c^2*d + 2*d^3
)*Sin[(e + f*x)/2])/(c^2*(-c + d)^2*(c + d)) + (4*d^4*Sin[(e + f*x)/2])/(c^
2*(-c + d)^2*(c + d)*(d + c*Cos[e + f*x])) + (Sec[(e + f*x)/2]*Tan[(e + f*x
)/2])/(-c + d)^2))/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(c + d*Sec[e + f*x])^2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 69594 vs. $2(480) = 960$.

Time = 17.96 (sec) , antiderivative size = 69595, normalized size of antiderivative = 124.28

method	result	size
default	Expression too large to display	69595

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{3}{2}}(c+d\sec(e+fx))^2}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)^2}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima"
)
[Out] integrate(1/((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2), x)
```

3.177 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx$

Optimal result 1187
Rubi [A] (verified) 1188
Mathematica [B] (warning: unable to verify) 1194
Maple [B] (warning: unable to verify) 1196
Fricas [F(-1)] 1196
Sympy [F] 1197
Maxima [F(-1)] 1197
Giac [F(-2)] 1197
Mupad [F(-1)] 1197

Optimal result

Integrand size = 27, antiderivative size = 802

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c-4d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4\sqrt{a}c(c-d)^2(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(3c-d)d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^2(c-d)^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^{5/2}(6c^2-4cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^3(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d^3\tan(e+fx)}{2ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{(3c-d)d^3\tan(e+fx)}{ac^2(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{3d^3\tan(e+fx)}{4ac\left(c^2-d^2\right)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

```
[Out] -1/2*tan(f*x+e)/a/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*d^3*t
an(f*x+e)/a/c/(c-d)^2/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-(3*
c-d)*d^3*tan(f*x+e)/a/c^2/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))
^(1/2)-3/4*d^3*tan(f*x+e)/a/c/(c^2-d^2)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e
))^(1/2)+2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/a^(1/2)
/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/4*d^(5/2)*arctanh(d^(1/2)*
(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/(c-d)^2/(c+d)^(5/2
)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(3*c-d)*d^(5/2)*a
rctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(
c-d)^3/(c+d)^(3/2)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-
1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/(c-d)^3/
```

```
f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c-4*d)*arc
tanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)/(c-d)^4
/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2*d^(5/2)*(6*c^2-4
*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f
*x+e)/c^3/(c-d)^4/f/a^(1/2)/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x
+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 802, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = -\frac{(3c-d)\tan(e+fx)d^3}{ac^2(c-d)^3(c+d)f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))} - \frac{3\tan(e+fx)d^3}{4ac\left(c^2-d^2\right)^2 f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))} - \frac{\tan(e+fx)d^3}{2ac(c-d)^2(c+d)f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))^2} - \frac{2(6c^2-4dc+d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)d^{5/2}}{\sqrt{a}c^3(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}} - \frac{(3c-d)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)d^{5/2}}{\sqrt{a}c^2(c-d)^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}} - \frac{3\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)d^{5/2}}{4\sqrt{a}c(c-d)^2(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}} - \frac{\sqrt{2}(c-4d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}} - \frac{\tan(e+fx)}{2a(c-d)^3f(\sec(e+fx)+1)\sqrt{\sec(e+fx)a+a}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] -1/2*Tan[e + f*x]/(a*(c - d)^3*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]
]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c^
3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 4*d)
*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]
*(c - d)^4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[
Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a
]*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*d^(5/
2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e
+ f*x])/(4*Sqrt[a]*c*(c - d)^2*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqr
t[a + a*Sec[e + f*x]]) - ((3*c - d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec
[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^2*(c - d)^3*(c
+ d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^(5/2
)*(6*c^2 - 4*c*d + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]
*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^3*(c - d)^4*Sqrt[c + d]*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^3*Tan[e + f*x])/(2*a*c*(c -
d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) - ((3*c - d
)*d^3*Tan[e + f*x])/(a*c^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c
+ d*Sec[e + f*x])) - (3*d^3*Tan[e + f*x])/(4*a*c*(c^2 - d^2)^2*f*Sqrt[a + a
*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{x\sqrt{a-ax}(a+ax)^2(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\left(\frac{1}{a^2c^3x\sqrt{a-ax}}-\frac{1}{a^2(c-d)^3(1+x)^2\sqrt{a-ax}}+\frac{-c+4d}{a^2(c-d)^4(1+x)\sqrt{a-ax}}-\frac{d^3}{a^2c(c-d)^2\sqrt{a-ax}(\cdots}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec\cdots}}$$

$$\begin{aligned}
&= -\frac{\tan(e+fx)\text{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad+\frac{((c-4d)\tan(e+fx))\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{1}{(1+x)^2\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad+\frac{(d^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{c(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad+\frac{((3c-d)d^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{c^2(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad+\frac{(d^3(6c^2-4cd+d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{c^3(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{d^3\tan(e+fx)}{2ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}$$
$$-\frac{(3c-d)d^3\tan(e+fx)}{ac^2(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$
$$+\frac{(2\tan(e+fx))\text{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{ac^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{(2(c-4d)\tan(e+fx))\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{4(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{(3d^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{4c(c-d)^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{((3c-d)d^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{2c^2(c-d)^3(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{(2d^3(6c^2-4cd+d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{ac^3(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$
\begin{aligned}
&= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{\sqrt{2}(c-4d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{2d^{5/2}(6c^2-4cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^3(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{d^3\tan(e+fx)}{2ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} \\
&- \frac{(3c-d)d^3\tan(e+fx)}{ac^2(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&- \frac{3d^3\tan(e+fx)}{4ac\left(c^2-d^2\right)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&- \frac{\tan(e+fx)\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{2a(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{(3d^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{8c(c-d)^2(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{((3c-d)d^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{ac^2(c-d)^3(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\sqrt{2}(c-4d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(3c-d)d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^2(c-d)^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{2d^{5/2}(6c^2-4cd+d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^3(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{d^3\tan(e+fx)}{2ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} \\
&\quad - \frac{(3c-d)d^3\tan(e+fx)}{ac^2(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad - \frac{3d^3\tan(e+fx)}{4ac\left(c^2-d^2\right)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad - \frac{(3d^3\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{4ac(c-d)^2(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^3 f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c-4d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^4 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4\sqrt{a}c(c-d)^2(c+d)^{5/2} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(3c-d)d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^2(c-d)^3(c+d)^{3/2} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^{5/2}(6c^2-4cd+d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^3(c-d)^4\sqrt{c+d} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d^3\tan(e+fx)}{2ac(c-d)^2(c+d) f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{(3c-d)d^3\tan(e+fx)}{ac^2(c-d)^3(c+d) f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{3d^3\tan(e+fx)}{4ac\left(c^2-d^2\right)^2 f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2632 vs. $2(802) = 1604$.

Time = 18.76 (sec) , antiderivative size = 2632, normalized size of antiderivative = 3.28

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] (Cos[(e + f*x)/2]^3*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^5*(-(((-2*c^5 - 4*c
^4*d - 2*c^3*d^2 - 17*c^2*d^3 - 5*c*d^4 + 6*d^5)*Sin[(e + f*x)/2])/(c^3*(-c
 + d)^3*(c + d)^2)) - (2*d^5*Sin[(e + f*x)/2])/(c^3*(-c + d)^2*(c + d)*(d +
 c*Cos[e + f*x])^2) + (-19*c^2*d^4*Sin[(e + f*x)/2] - 5*c*d^5*Sin[(e + f*x)
/2] + 8*d^6*Sin[(e + f*x)/2])/(c^3*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x]
)) - (Sec[(e + f*x)/2]*Tan[(e + f*x)/2])/(-c + d)^3))/(f*(a*(1 + Sec[e + f*
x]))^(3/2)*(c + d*Sec[e + f*x])^3) - ((2*c^3*(5*c - 17*d)*(c + d)^2*ArcSin[
```

```
Tan[(e + f*x)/2]] - 8*Sqrt[2]*(c - d)^4*(c + d)^2*ArcTan[Tan[(e + f*x)/2]/S
qrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] - (Sqrt[2]*d^(5/2)*(63*c^4 + 54*c^3*d
 - 17*c^2*d^2 - 12*c*d^3 + 8*d^4)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[
-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])])/Sqrt[-c - d])*Cos[(e + f*x
)/2]^3*(d + c*Cos[e + f*x])^3*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*((c^3*S
ec[(e + f*x)/2])/(2*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e +
f*x]]) - (c^2*d*Sec[(e + f*x)/2])/((-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x]
)*Sqrt[Sec[e + f*x]]) - (19*c*d^2*Sec[(e + f*x)/2])/(2*(-c + d)^3*(c + d)^2
*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (33*d^3*Sec[(e + f*x)/2])/(4*(-
c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (3*d^4*Sec[(e
 + f*x)/2])/(4*c*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x
]]) + (d^5*Sec[(e + f*x)/2])/(c^2*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])
*Sqrt[Sec[e + f*x]]) - (c^3*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/((-c + d)^
3*(c + d)^2*(d + c*Cos[e + f*x])) + (3*c^2*d*Sec[(e + f*x)/2]*Sqrt[Sec[e +
f*x]])/(2*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])) + (3*c*d^2*Sec[(e + f*
x)/2]*Sqrt[Sec[e + f*x]])/((-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])) + (9*
d^3*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(4*(-c + d)^3*(c + d)^2*(d + c*Cos
[e + f*x])) + (d^4*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(4*c*(-c + d)^3*(c
+ d)^2*(d + c*Cos[e + f*x])) - (c^3*Cos[2*(e + f*x)]*Sec[(e + f*x)/2]*Sqrt[
Sec[e + f*x]])/((-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])) + (c^2*d*Cos[2*(
e + f*x)]*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/((-c + d)^3*(c + d)^2*(d + c
*Cos[e + f*x])) + (2*c*d^2*Cos[2*(e + f*x)]*Sec[(e + f*x)/2]*Sqrt[Sec[e + f
*x]])/((-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])) - (2*d^3*Cos[2*(e + f*x)]
*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/((-c + d)^3*(c + d)^2*(d + c*Cos[e +
f*x])) - (d^4*Cos[2*(e + f*x)]*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(c*(-c
+ d)^3*(c + d)^2*(d + c*Cos[e + f*x])) + (d^5*Cos[2*(e + f*x)]*Sec[(e + f*x
)/2]*Sqrt[Sec[e + f*x]])/(c^2*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])))*S
ec[e + f*x]^(9/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]])/(2*c^3*(c - d)^4*(
c + d)^2*f*(a*(1 + Sec[e + f*x]))^(3/2)*(c + d*Sec[e + f*x])^3*(-1/4*((2*c^
3*(5*c - 17*d)*(c + d)^2*ArcSin[Tan[(e + f*x)/2]] - 8*Sqrt[2]*(c - d)^4*(c
+ d)^2*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] - (Sq
rt[2]*d^(5/2)*(63*c^4 + 54*c^3*d - 17*c^2*d^2 - 12*c*d^3 + 8*d^4)*ArcTanh[(
Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x]
)])])/Sqrt[-c - d])*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]
^2*Sec[e + f*x])^(3/2)*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2]))/(c^3*(c - d)^4*(c + d)^2) - (Sqrt[Cos[
e + f*x]*Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*((c^3*(5
*c - 17*d)*(c + d)^2*Sec[(e + f*x)/2]^2)/Sqrt[1 - Tan[(e + f*x)/2]^2] - (8*
Sqrt[2]*(c - d)^4*(c + d)^2*(Sec[(e + f*x)/2]^2/(2*Sqrt[Cos[e + f*x]/(1 + C
os[e + f*x])]) - (((Cos[e + f*x]*Sin[e + f*x])/(1 + Cos[e + f*x])^2 - Sin[e
 + f*x]/(1 + Cos[e + f*x]))*Tan[(e + f*x)/2])/(2*(Cos[e + f*x]/(1 + Cos[e +
 f*x]))^(3/2))))/(1 + (1 + Cos[e + f*x])*Sec[e + f*x]*Tan[(e + f*x)/2]^2) -
 (Sqrt[2]*d^(5/2)*(63*c^4 + 54*c^3*d - 17*c^2*d^2 - 12*c*d^3 + 8*d^4)*((Sqr
t[d]*Sec[(e + f*x)/2]^2)/(2*Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x
])]) - (Sqrt[d]*((Cos[e + f*x]*Sin[e + f*x])/(1 + Cos[e + f*x])^2 - Sin[e +
```

```
f*x]/(1 + Cos[e + f*x]))*Tan[(e + f*x)/2])/(2*Sqrt[-c - d]*(Cos[e + f*x]/(
1 + Cos[e + f*x]))^(3/2))))/(Sqrt[-c - d]*(1 - (d*(1 + Cos[e + f*x])*Sec[e
+ f*x]*Tan[(e + f*x)/2]^2)/(-c - d)))))/(2*c^3*(c - d)^4*(c + d)^2) - ((2*c
^3*(5*c - 17*d)*(c + d)^2*ArcSin[Tan[(e + f*x)/2]] - 8*Sqrt[2]*(c - d)^4*(c
 + d)^2*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] - (S
qrt[2]*d^(5/2)*(63*c^4 + 54*c^3*d - 17*c^2*d^2 - 12*c*d^3 + 8*d^4)*ArcTanh[
(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x
])])])/Sqrt[-c - d])*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*(-(Cos[(e + f*x)
/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e
+ f*x]))/(4*c^3*(c - d)^4*(c + d)^2*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]))
)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 117746 vs. $2(694) = 1388$.

Time = 21.10 (sec) , antiderivative size = 117747, normalized size of antiderivative = 146.82

method	result	size
default	Expression too large to display	117747

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^3} dx = \int \frac{1}{(a(\sec(e + fx) + 1))^{\frac{3}{2}} (c + d\sec(e + fx))^3} dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^3} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima"
)
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^3} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^3} dx = \text{Hanged}$$

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3),x)
[Out] \text{Hanged}
```

3.178 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result 1198
Rubi [A] (verified) 1199
Mathematica [A] (verified) 1203
Maple [A] (warning: unable to verify) 1204
Fricas [A] (verification not implemented) 1204
Sympy [F] 1205
Maxima [F(-1)] 1205
Giac [F(-2)] 1206
Mupad [F(-1)] 1206

Optimal result

Integrand size = 27, antiderivative size = 480

$$
\begin{aligned}
\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx &= -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\
&- \frac{3(c-d)^3\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&- \frac{(c-d)^2(c+2d)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{3(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{(c-d)^2(c+2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{\sqrt{2}(c^3-d^3)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

```
[Out] -1/4*(c-d)^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-3/16*
(c-d)^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*(c-d)^2*
(c+2*d)*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^3*arctan
h((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/
2)/(a+a*sec(f*x+e))^(1/2)-3/32*(c-d)^3*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*se
c(f*x+e))^(1/2)-1/4*(c-d)^2*(c+2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1
```

```
/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f
*x+e))^(1/2)-(c^3-d^3)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
2^(1/2)*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4025, 186, 65, 212, 44}

$$\begin{aligned}\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = &-\frac{\sqrt{2}(c^3-d^3)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\ &+\frac{2c^3\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\ &-\frac{3(c-d)^3\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\ &-\frac{(c-d)^2(c+2d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\ &-\frac{3(c-d)^3\tan(e+fx)}{16a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}} \\ &-\frac{(c-d)^2(c+2d)\tan(e+fx)}{2a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}} \\ &-\frac{(c-d)^3\tan(e+fx)}{4a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}}\end{aligned}$$

```
[In] Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/4*((c - d)^3*Tan[e + f*x])/(a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e
+ f*x]]) - (3*(c - d)^3*Tan[e + f*x])/(16*a^2*f*(1 + Sec[e + f*x])*Sqrt[a +
 a*Sec[e + f*x]]) - ((c - d)^2*(c + 2*d)*Tan[e + f*x])/(2*a^2*f*(1 + Sec[e
+ f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]
/Sqrt[a]]*Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]]) - (3*(c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a]
)]*Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*
Sec[e + f*x]]) - ((c - d)^2*(c + 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqr
t[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^3 - d^3)*ArcTanh[Sqrt[a - a*Sec[e +
 f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(c+dx)^3}{x\sqrt{a-ax}(a+ax)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\left(\frac{c^3}{a^3x\sqrt{a-ax}}-\frac{(c-d)^3}{a^3(1+x)^3\sqrt{a-ax}}-\frac{(c-d)^2(c+2d)}{a^3(1+x)^2\sqrt{a-ax}}+\frac{-c^3+d^3}{a^3(1+x)\sqrt{a-ax}}\right)dx,x,\sec(e\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{((c-d)^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{(1+x)^3\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{((c-d)^2(c+2d)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{(1+x)^2\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{((c^3-d^3)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{(c-d)^2(c+2d)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{(2c^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{(3(c-d)^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{(1+x)^2\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{8af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{((c-d)^2(c+2d)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{4af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{(2(c^3-d^3)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=-\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}-\frac{3(c-d)^3\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}-\frac{(c-d)^2(c+2d)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}+\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{\sqrt{2}(c^3-d^3)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}+\frac{(3(c-d)^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{32af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{((c-d)^2(c+2d)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{2a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=-\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}-\frac{3(c-d)^3\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}-\frac{(c-d)^2(c+2d)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}+\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{(c-d)^2(c+2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{\sqrt{2}(c^3-d^3)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{(3(c-d)^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{16a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$
\begin{aligned}
= &-\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\
&- \frac{3(c-d)^3\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&- \frac{(c-d)^2(c+2d)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{3(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{(c-d)^2(c+2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{\sqrt{2}(c^3-d^3)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 9.68 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.91

$$
\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\left(\left(-43c^3+9c^2d+15cd^2+19d^3\right)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+32\sqrt{2}c^3\arctan\left(\cdots\right.\right.}{4f(d+c\cos(e+fx))^3\sqrt{\sec^2\left(\frac{1}{2}(e+\cdots\right.}}
$$
$$
+\frac{\cos^5\left(\frac{1}{2}(e+fx)\right)(c+d\sec(e+fx))^3\left(-\frac{3}{2}(-c+d)^2(5c+3d)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{1}{2}\sec^4\left(\frac{1}{2}(e+fx)\right)\left(-c^3\sin\cdots\right.\right.}{}
$$

```
[In] Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (((-43*c^3 + 9*c^2*d + 15*c*d^2 + 19*d^3)*ArcSin[Tan[(e + f*x)/2]] + 32*Sqr
t[2]*c^3*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*Co
s[(e + f*x)/2]^4*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[1 + Sec[e + f*x
]]*(c + d*Sec[e + f*x])^3)/(4*f*(d + c*Cos[e + f*x])^3*Sqrt[Sec[(e + f*x)/2
]^2]*Sqrt[Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)) + (Cos[(e + f*x)/2]^5
*(c + d*Sec[e + f*x])^3*((-3*(-c + d)^2*(5*c + 3*d)*Sin[(e + f*x)/2])/2 + (
Sec[(e + f*x)/2]^4*(-(c^3*Sin[(e + f*x)/2]) + 3*c^2*d*Sin[(e + f*x)/2] - 3*
c*d^2*Sin[(e + f*x)/2] + d^3*Sin[(e + f*x)/2]))/2 + (Sec[(e + f*x)/2]^2*(19
*c^3*Sin[(e + f*x)/2] - 33*c^2*d*Sin[(e + f*x)/2] + 9*c*d^2*Sin[(e + f*x)/2
] + 5*d^3*Sin[(e + f*x)/2]))/4))/(f*(d + c*Cos[e + f*x])^3*(a*(1 + Sec[e +
f*x]))^(5/2))
```

Maple [A] (warning: unable to verify)

Time = 4.86 (sec) , antiderivative size = 688, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,c^3(1-\cos(fx+e))^3\csc(fx\right.}{}$
parts	Expression too large to display

```
[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/a^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*(2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c^3*(1-c
os(f*x+e))^3*csc(f*x+e)^3-6*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c^2*d*(
1-cos(f*x+e))^3*csc(f*x+e)^3+6*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c*d^
2*(1-cos(f*x+e))^3*csc(f*x+e)^3-2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d
^3*(1-cos(f*x+e))^3*csc(f*x+e)^3-32*c^3*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x
+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-13*((1-cos(f*x+e))^2
*csc(f*x+e)^2-1)^(1/2)*c^3*(-cot(f*x+e)+csc(f*x+e))+15*((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)*c^2*d*(-cot(f*x+e)+csc(f*x+e))+9*((1-cos(f*x+e))^2*csc(
f*x+e)^2-1)^(1/2)*c*d^2*(-cot(f*x+e)+csc(f*x+e))-11*((1-cos(f*x+e))^2*csc(f
*x+e)^2-1)^(1/2)*d^3*(-cot(f*x+e)+csc(f*x+e))+43*c^3*ln(csc(f*x+e)-cot(f*x+
e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))-9*c^2*d*ln(csc(f*x+e)-cot(f*x+e
)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))-15*c*d^2*ln(csc(f*x+e)-cot(f*x+e
)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))-19*d^3*ln(csc(f*x+e)-cot(f*x+e)+
((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 30.82 (sec) , antiderivative size = 880, normalized size of antiderivative = 1.83

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[\frac{\sqrt{2}\left((43\,c^3-9\,c^2d-15\,cd^2-19\,d^3)\cos\left(fx+e\right)^3+43\,c^3-9\,c^2d-15\,cd^2-\right.}{}\right.$$

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^3 + 43*
c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3
)*cos(f*x + e)^2 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e))*s
qrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*
```

```
x + e)^2 + 2*cos(f*x + e) + 1)) - 64*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x +
e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(
-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*c
os(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(3*(5*c^3 - 7*c^2*d - c*d^2 + 3*d^
3)*cos(f*x + e)^2 + (11*c^3 - 9*c^2*d - 15*c*d^2 + 13*d^3)*cos(f*x + e))*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 +
 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43
*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^3 + 43*c^3 - 9*c^2*d - 15*
c*d^2 - 19*d^3 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^2 +
3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt(
2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
e))) - 64*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) +
 c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(
sqrt(a)*sin(f*x + e))) - 2*(3*(5*c^3 - 7*c^2*d - c*d^2 + 3*d^3)*cos(f*x + e
)^2 + (11*c^3 - 9*c^2*d - 15*c*d^2 + 13*d^3)*cos(f*x + e))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f
*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(c+d\sec(e+fx))^3}{(a(\sec(e+fx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)
[Out] Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)
```

3.179 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result 1207
Rubi [A] (verified) 1208
Mathematica [A] (verified) 1212
Maple [A] (warning: unable to verify) 1213
Fricas [A] (verification not implemented) 1213
Sympy [F] 1214
Maxima [F] 1214
Giac [F(-2)] 1214
Mupad [F(-1)] 1215

Optimal result

Integrand size = 27, antiderivative size = 468

$$\begin{aligned}\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = & -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\ & -\frac{3(c-d)^2\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ & -\frac{(c^2-d^2)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ & +\frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & -\frac{\sqrt{2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & -\frac{3(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ & -\frac{(c^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

```
[Out] -1/4*(c-d)^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-3/16*
(c-d)^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*(c^2-d^2
)*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^2*arctanh((a-a
*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+
a*sec(f*x+e))^(1/2)-3/32*(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+
e))^(1/2)-1/4*(c^2-d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
```

```
*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
-c^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)
/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4025, 186, 65, 212, 44}

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{(c^2-d^2)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2c^2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{\sqrt{2}c^2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{3(c-d)^2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{(c^2-d^2)\tan(e+fx)}{2a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}} - \frac{3(c-d)^2\tan(e+fx)}{16a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}} - \frac{(c-d)^2\tan(e+fx)}{4a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/4*((c - d)^2*Tan[e + f*x])/(a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e
+ f*x]]) - (3*(c - d)^2*Tan[e + f*x])/(16*a^2*f*(1 + Sec[e + f*x])*Sqrt[a +
 a*Sec[e + f*x]]) - ((c^2 - d^2)*Tan[e + f*x])/(2*a^2*f*(1 + Sec[e + f*x])*
Sqrt[a + a*Sec[e + f*x]]) + (2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]
]*Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
]) - (Sqrt[2]*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e
 + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3
*(c - d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x]
)/(16*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
- ((c^2 - d^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e +
f*x])/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}(a+ax)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{c^2}{a^3x\sqrt{a-ax}} - \frac{(c-d)^2}{a^3(1+x)^3\sqrt{a-ax}} + \frac{-c^2+d^2}{a^3(1+x)^2\sqrt{a-ax}} - \frac{c^2}{a^3(1+x)\sqrt{a-ax}}\right) dx, x, \sec\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$
\begin{aligned}
&= -\frac{(c^2\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad+\frac{(c^2\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad+\frac{((c-d)^2\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{(1+x)^3\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad+\frac{((c^2-d^2)\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{(1+x)^2\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\
&\quad-\frac{(c^2-d^2)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad+\frac{(2c^2\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad-\frac{(2c^2\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad+\frac{(3(c-d)^2\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{(1+x)^2\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{8af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad+\frac{((c^2-d^2)\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{4af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^2\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{(c^2-d^2)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(3(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{32af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{((c^2-d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{2a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^2\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{(c^2-d^2)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c^2-d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(3(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{16a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^2\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{(c^2-d^2)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 6.25 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.56

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}(c+d\sec(e+fx))^2\left(\frac{\left((-43c^2+6cd+5d^2)\arcsin(\tan(\right.}{}\right.}{}$$

```
[In] Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (Cos[(e + f*x)/2]^4*Sqrt[Sec[e + f*x]]*(c + d*Sec[e + f*x])^2*((((-43*c^2 +
 6*c*d + 5*d^2)*ArcSin[Tan[(e + f*x)/2]] + 32*Sqrt[2]*c^2*ArcTan[Tan[(e + f
*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*Sqrt[Cos[e + f*x]/(1 + Cos[e
 + f*x])]*Sqrt[1 + Sec[e + f*x]])/Sqrt[Sec[(e + f*x)/2]^2] + ((c - d)*(11*c
 + 5*d + (15*c + d)*Cos[e + f*x])*Sec[(e + f*x)/2]^3*Sqrt[Sec[e + f*x]]*(Si
n[(e + f*x)/2] - Sin[(3*(e + f*x))/2]))/4))/(4*f*(d + c*Cos[e + f*x])^2*(a*
(1 + Sec[e + f*x]))^(5/2))
```

Maple [A] (warning: unable to verify)

Time = 4.48 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.12

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}c^2(-\cot(fx+e)+\csc(f$
parts	$\frac{c^2\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-2(1-\cos(fx+e))^3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\csc(fx$

```
[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/a^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*(2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*c^2*(-co
t(f*x+e)+csc(f*x+e))-4*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*c*d*(-cot(f*
x+e)+csc(f*x+e))+2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*d^2*(-cot(f*x+e)
+csc(f*x+e))-32*c^2*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-11*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*c^2*(-cot(f*x+e)+csc(f*x+e))+6*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*c
*d*(-cot(f*x+e)+csc(f*x+e))+5*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d^2*(
-cot(f*x+e)+csc(f*x+e))+43*c^2*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*c
sc(f*x+e)^2-1)^(1/2))-6*c*d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(
f*x+e)^2-1)^(1/2))-5*d^2*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 13.49 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.67

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[\frac{\sqrt{2}\left((43c^2-6cd-5d^2)\cos(fx+e)^3+3(43c^2-6cd-5d^2)\cos(fx+e)\right.}{}\right.$$

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(43*c^2 - 6*c*d
 - 5*d^2)*cos(f*x + e)^2 + 43*c^2 - 6*c*d - 5*d^2 + 3*(43*c^2 - 6*c*d - 5*d
^2)*cos(f*x + e))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*
x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 64*(c^2*cos(f*x + e)^3
 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f
```

```
*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*((15*c^2 - 14*c
*d - d^2)*cos(f*x + e)^2 + (11*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43*c^2 -
6*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e)^2 +
 43*c^2 - 6*c*d - 5*d^2 + 3*(43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt(a)*
arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a
)*sin(f*x + e))) - 64*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*co
s(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c^2 - 14*c*d - d^2)*cos(f*x + e
)^2 + (11*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*
a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(c+d\sec{(e+fx)})^2}{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)
[Out] Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(d\sec{(fx+e)}+c)^2}{(a\sec{(fx+e)}+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)^2/(a*sec(f*x + e) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)
```

3.180 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result 1216
Rubi [A] (verified) 1216
Mathematica [B] (verified) 1218
Maple [B] (warning: unable to verify) 1219
Fricas [B] (verification not implemented) 1219
Sympy [F] 1220
Maxima [F] 1220
Giac [F(-2)] 1220
Mupad [F(-1)] 1221

Optimal result

Integrand size = 25, antiderivative size = 164

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{(43c-3d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-1/32*(43*c-
3*d)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/
f*2^(1/2)-1/4*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-1/16*(11*c-3*d)*tan
(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4007, 4005, 3859, 209, 3880}

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{(43c-3d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f} + \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f} - \frac{(11c-3d)\tan(e+fx)}{16af(a\sec(e+fx)+a)^{3/2}} - \frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

```
[In] Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f) -
 ((43*c - 3*d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*
x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*Tan[e + f*x])/(4*f*(a + a*Sec[e +
f*x])^(5/2)) - ((11*c - 3*d)*Tan[e + f*x])/(16*a*f*(a + a*Sec[e + f*x])^(3/
2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{\int \frac{-4ac+\frac{3}{2}a(c-d)\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}} + \frac{\int \frac{8a^2c-\frac{1}{4}a^2(11c-3d)\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{8a^4}$$

$$= -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}} + \frac{c\int\sqrt{a+a\sec(e+fx)}\,dx}{a^3} - \frac{(43c-3d)\int\frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{32a^2}$$

$$= -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}} - \frac{(2c)\text{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f} + \frac{(43c-3d)\text{Subst}\left(\int\frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{16a^2f}$$

$$= \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{(43c-3d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(164) = 328$.

Time = 7.47 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.09

$$\int\frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\left((-43c+3d)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+32\sqrt{2}c\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\right)\cos^4}{4f(d+c\cos(e+fx))\sqrt{\sec^2\left(\frac{1}{2}(e\right.}} + \frac{\cos^5\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)(c+d\sec(e+fx))\left(\frac{1}{2}(-15c+7d)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{1}{4}\sec^2\left(\frac{1}{2}(e+fx)\right)(19c\,s\right.}{f(d+c\cos(e+fx))(a(1}$$

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (((-43*c + 3*d)*ArcSin[Tan[(e + f*x)/2]] + 32*Sqrt[2]*c*ArcTan[Tan[(e + f*x
)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*Cos[(e + f*x)/2]^4*Sqrt[Cos[e
+ f*x]/(1 + Cos[e + f*x])]*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e + f*x]]*(c + d
*Sec[e + f*x]))/(4*f*(d + c*Cos[e + f*x])*Sqrt[Sec[(e + f*x)/2]^2]*(a*(1 +
Sec[e + f*x]))^(5/2)) + (Cos[(e + f*x)/2]^5*Sec[e + f*x]^2*(c + d*Sec[e + f
*x])*(((-15*c + 7*d)*Sin[(e + f*x)/2])/2 + (Sec[(e + f*x)/2]^2*(19*c*Sin[(e
 + f*x)/2] - 11*d*Sin[(e + f*x)/2]))/4 + (Sec[(e + f*x)/2]^4*(-(c*Sin[(e +
f*x)/2]) + d*Sin[(e + f*x)/2]))/2))/(f*(d + c*Cos[e + f*x])*(a*(1 + Sec[e +
 f*x]))^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(139) = 278$.

Time = 2.89 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.29

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2c\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}(-\cot(fx+e)+\csc(f$
parts	$\frac{c\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-2(1-\cos(fx+e))^3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\csc(fx$

```
[In] int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/a^3/f*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*(2*c*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-cot(
f*x+e)+csc(f*x+e))-2*d*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-cot(f*x+e)
+csc(f*x+e))-32*c*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-11*c*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*(-cot(f*x+e)+csc(f*x+e))+3*d*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-c
ot(f*x+e)+csc(f*x+e))+43*c*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f
*x+e)^2-1)^(1/2))-3*d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(139) = 278$.

Time = 3.61 (sec) , antiderivative size = 670, normalized size of antiderivative = 4.09

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \left[\frac{\sqrt{2}\left((43\,c-3\,d)\cos\left(fx+e\right)^3+3\,(43\,c-3\,d)\cos\left(fx+e\right)^2+3\,(43\,c-3\,d)\right.}{}\right.$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*c - 3*d)*cos(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^2
 + 3*(43*c - 3*d)*cos(f*x + e) + 43*c - 3*d)*sqrt(-a)*log((2*sqrt(2)*sqrt(-
a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*
cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1
)) - 64*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt
(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))
- 4*((15*c - 7*d)*cos(f*x + e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt((a*cos(f
```

```
*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*co
s(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43*c - 3*d)*c
os(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^2 + 3*(43*c - 3*d)*cos(f*x + e)
 + 43*c - 3*d)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 64*(c*cos(f*x + e)^3 + 3*c*cos(f*
x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c - 7*d)*cos(f*x
 + e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*co
s(f*x + e) + a^3*f)]
```

Sympy [F]

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{c + d\sec(e + fx)}{(a(\sec(e + fx) + 1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)
[Out] Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{d\sec(fx + e) + c}{(a\sec(fx + e) + a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{c+\frac{d}{\cos(e+f\,x)}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2), x)
```

3.181 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))} dx$

Optimal result 1222
Rubi [A] (verified) 1223
Mathematica [A] (warning: unable to verify) 1228
Maple [B] (warning: unable to verify) 1228
Fricas [F(-1)] 1230
Sympy [F] 1230
Maxima [F] 1230
Giac [F(-2)] 1230
Mupad [F(-1)] 1231

Optimal result

Integrand size = 27, antiderivative size = 592

$$\begin{aligned}
\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx &= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\
&- \frac{(c-2d)\tan(e+fx)}{2a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&- \frac{3\tan(e+fx)}{16a^2(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{(c-2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{\sqrt{2}(c^2-3cd+3d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{2d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

```
[Out] -1/4*tan(f*x+e)/a^2/(c-d)/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*(c-
2*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/16*ta
```

```
n(f*x+e)/a^2/(c-d)/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh((a-a*s
ec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/c/f/(a-a*sec(f*x+e))^(1/2)/(a+
a*sec(f*x+e))^(1/2)-1/4*(c-2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^2/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*s
ec(f*x+e))^(1/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*t
an(f*x+e)/a^(3/2)/(c-d)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2)-(c^2-3*c*d+3*d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
2^(1/2)*tan(f*x+e)/a^(3/2)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)
)^(1/2)+2*d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2
))*tan(f*x+e)/a^(3/2)/c/(c-d)^3/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*s
ec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = -\frac{\sqrt{2}(c^2-3cd+3d^2)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f(c-d)^3\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2d^{7/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{a^{3/2}cf(c-d)^3\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{3\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}f(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{(c-2d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}cf\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{3\tan(e+fx)}{16a^2f(c-d)(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}} - \frac{(c-2d)\tan(e+fx)}{2a^2f(c-d)^2(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}} - \frac{\tan(e+fx)}{4a^2f(c-d)(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] -1/4*Tan[e + f*x]/(a^2*(c - d)*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*
x]]) - ((c - 2*d)*Tan[e + f*x])/(2*a^2*(c - d)^2*f*(1 + Sec[e + f*x])*Sqrt[
a + a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)*f*(1 + Sec[e + f*x]
)*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*
Tan[e + f*x])/(a^(3/2)*c*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
]) - ((c - 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e +
 f*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[
e + f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a
*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - 3*c*d + 3*d^2)*ArcTanh[Sqrt[a - a*Sec[e +
 f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec
[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a -
 a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c*(c - d)^3
*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^3(c+dx)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{a^3cx\sqrt{a-ax}} - \frac{1}{a^3(c-d)(1+x)^3\sqrt{a-ax}} + \frac{-c+2d}{a^3(c-d)^2(1+x)^2\sqrt{a-ax}} + \frac{-c^2+3cd-3d^2}{a^3(c-d)^3(1+x)\sqrt{a-ax}}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{acf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-2d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{a(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{(1+x)^3\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{a(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(d^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{ac(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c^2-3cd+3d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{a(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-2d)\tan(e+fx)}{2a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{a^2cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-2d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{4a(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(3\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{8a(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(2d^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{c+d-\frac{dx^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{a^2c(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2(c^2-3cd+3d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{a^2(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-2d)\tan(e+fx)}{2a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{3\tan(e+fx)}{16a^2(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-3cd+3d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{((c-2d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{2a^2(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(3\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{32a(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-2d)\tan(e+fx)}{2a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{3\tan(e+fx)}{16a^2(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-3cd+3d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(3\tan(e+fx))\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{16a^2(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-2d)\tan(e+fx)}{2a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{3\tan(e+fx)}{16a^2(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-3cd+3d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (warning: unable to verify)

Time = 11.41 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \frac{\cos^4\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^{\frac{7}{2}}(e+fx)\left(-\frac{4\left(\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Cos[(e + f*x)/2]^4*(d + c*Cos[e + f*x])*Sec[e + f*x]^(7/2)*((-4*(Sqrt[-c -
 d]*(c*(43*c^2 - 126*c*d + 115*d^2)*ArcSin[Tan[(e + f*x)/2]] - 32*Sqrt[2]*(
c - d)^3*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]]) +
32*Sqrt[2]*d^(7/2)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])])])*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqr
t[1 + Sec[e + f*x]])/(c*Sqrt[-c - d]*Sqrt[Sec[(e + f*x)/2]^2]) + (c - d)*(1
1*c - 19*d + (15*c - 23*d)*Cos[e + f*x])*Sec[(e + f*x)/2]^3*Sqrt[Sec[e + f*
x]]*(Sin[(e + f*x)/2] - Sin[(3*(e + f*x))/2])))/(16*(c - d)^3*f*(a*(1 + Sec
[e + f*x]))^(5/2)*(c + d*Sec[e + f*x]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2344 vs. 2(509) = 1018.

Time = 16.76 (sec) , antiderivative size = 2345, normalized size of antiderivative = 3.96

method	result	size
default	Expression too large to display	2345

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192/f/(d/(c-d))^(1/2)/(c-d)^3/c/((c+d)*(c-d))^(1/2)/a^3*(576*((c+d)*(c-d)
)^(1/2)*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-c
ot(f*x+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*c*d^2+12*((c+d)*(c-d))^(1/2)*((1-cos
(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)*(d/(c-d))^(1/2)*c^2*d*(-cot(f*x+e)+csc(f*x
+e))-576*((c+d)*(c-d))^(1/2)*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(
f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*c^2*d-96*2^(1/2
)*ln(-2*(-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c
+2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(
c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+c-d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot
(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))*d^4+96*2^(1/2)*ln(-2*(((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^
(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f
*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e)
```

)*d+((c+d)*(c-d))^(1/2)))*d^4-12*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(
f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c^2*d*(1-cos(f*x+e))^5*csc(f*x+e)^5+5*((c
+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(d/(c-d))^(1/2)*c^
3*(-cot(f*x+e)+csc(f*x+e))-17*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)^(3/2)*(d/(c-d))^(1/2)*d^3*(-cot(f*x+e)+csc(f*x+e))+87*((c+d)*(c-d)
)^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c^3*(-cot(f
*x+e)+csc(f*x+e))+192*((c+d)*(c-d))^(1/2)*2^(1/2)*arctanh(2^(1/2)/((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*c^
3-192*((c+d)*(c-d))^(1/2)*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*d^3+756*((c+d)*(c-
d))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))
*(d/(c-d))^(1/2)*c^2*d-690*((c+d)*(c-d))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+((1
-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*(d/(c-d))^(1/2)*c*d^2-21*((c+d)*(c-d)
)^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*d^3*(-cot(f
*x+e)+csc(f*x+e))-4*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
5/2)*(d/(c-d))^(1/2)*c^3*(-cot(f*x+e)+csc(f*x+e))+4*((c+d)*(c-d))^(1/2)*((1
-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)*(d/(c-d))^(1/2)*d^3*(-cot(f*x+e)+csc(f
x+e))-12((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)*(d/(c
-d))^(1/2)*c*d^2*(-cot(f*x+e)+csc(f*x+e))-27*((c+d)*(c-d))^(1/2)*((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(3/2)*(d/(c-d))^(1/2)*c^2*d*(-cot(f*x+e)+csc(f*x+e)
)+39*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(d/(c-d))^
(1/2)*c*d^2*(-cot(f*x+e)+csc(f*x+e))-243*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c^2*d*(-cot(f*x+e)+csc(f*x+e))-25
((c+d)(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2
)*c^3*(1-cos(f*x+e))^3*csc(f*x+e)^3+25*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*d^3*(1-cos(f*x+e))^3*csc(f*x+e)^3+1
77*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1
/2)*c*d^2*(-cot(f*x+e)+csc(f*x+e))+4*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*
csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c^3*(1-cos(f*x+e))^5*csc(f*x+e)^5-4*(
(c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*
d^3*(1-cos(f*x+e))^5*csc(f*x+e)^5+12*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*
csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c*d^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+7
5*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/
2)*c^2*d*(1-cos(f*x+e))^3*csc(f*x+e)^3-75*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c*d^2*(1-cos(f*x+e))^3*csc(f*x+e
)^3-258*((c+d)*(c-d))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(
f*x+e)^2-1)^(1/2))*(d/(c-d))^(1/2)*c^3)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a\,(\sec{(e+fx)}+1))^{\frac{5}{2}}\,(c+d\sec{(e+fx)})}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a\sec{(fx+e)}+a)^{\frac{5}{2}}(d\sec{(fx+e)}+c)}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/((a*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))),x)
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))), x)
```

3.182 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx$

Optimal result 1233
Rubi [A] (verified) 1234
Mathematica [A] (warning: unable to verify) 1240
Maple [B] (warning: unable to verify) 1241
Fricas [F(-1)] 1241
Sympy [F] 1241
Maxima [F(-1)] 1242
Giac [F(-2)] 1242
Mupad [F(-1)] 1242

Optimal result

Integrand size = 27, antiderivative size = 756

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-3d)\tan(e+fx)}{2a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{3\tan(e+fx)}{16a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-3d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-4cd+6d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2(4c-d)d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^4\tan(e+fx)}{a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

```
[Out] -1/4*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*(
c-3*d)*tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/16*
tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+d^4*tan(f*x+
e)/a^2/c/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh(
(a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/c^2/f/(a-a*sec(f*x+e))^(
1/2)/(a+a*sec(f*x+e))^(1/2)+d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/
a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c/(c-d)^3/(c+d)^(3/2)/f/(a-a*sec(f*
x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*(c-3*d)*arctanh(1/2*(a-a*sec(f*x+e))
^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^3/f*2^(1/2)/(a-a*sec(f*x+e
))^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^2/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(
```

```
a+a*sec(f*x+e))^(1/2)-(c^2-4*c*d+6*d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*
2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)/a^(3/2)/(c-d)^4/f/(a-a*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2)+2*(4*c-d)*d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))
^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c^2/(c-d)^4/f/(c+d)^(1/2)/(a
-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \frac{2d^{7/2}(4c-d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{a^{3/2}c^2f(c-d)^4\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)}}$$
$$-\frac{\sqrt{2}(c^2-4cd+6d^2)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f(c-d)^4\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+\frac{2\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+\frac{d^{7/2}\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{a^{3/2}cf(c-d)^3(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$-\frac{3\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}f(c-d)^2\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$-\frac{(c-3d)\tan(e+fx)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^3\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+\frac{d^4\tan(e+fx)}{a^2cf(c-d)^3(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}$$
$$-\frac{3\tan(e+fx)}{16a^2f(c-d)^2(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}$$
$$-\frac{(c-3d)\tan(e+fx)}{2a^2f(c-d)^3(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}$$
$$-\frac{\tan(e+fx)}{4a^2f(c-d)^2(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] -1/4*Tan[e + f*x]/(a^2*(c - d)^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e +
f*x]]) - ((c - 3*d)*Tan[e + f*x])/(2*a^2*(c - d)^3*f*(1 + Sec[e + f*x])*Sqr
t[a + a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)^2*f*(1 + Sec[e +
```

```
f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[
a]]*Tan[e + f*x])/(a^(3/2)*c^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
+ f*x]]) - ((c - 3*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*T
an[e + f*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a
 + a*Sec[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])
]*Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - 4*c*d + 6*d^2)*ArcTanh[Sqrt[a - a
*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^4*f*Sqrt[a
 - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^(7/2)*ArcTanh[(Sqrt[d]*Sq
rt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c*(c
- d)^3*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) +
 (2*(4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*S
qrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c^2*(c - d)^4*Sqrt[c + d]*f*Sqrt[a - a*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^4*Tan[e + f*x])/(a^2*c*(c - d)
^3*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^3(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{a^3c^2x\sqrt{a-ax}} - \frac{1}{a^3(c-d)^2(1+x)^3\sqrt{a-ax}} + \frac{-c+3d}{a^3(c-d)^3(1+x)^2\sqrt{a-ax}} + \frac{-c^2+4cd-6d}{a^3(c-d)^4(1+x)\sqrt{}}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(}}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{ac^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+ \frac{((c-3d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{a(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+ \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{(1+x)^3\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{a(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$- \frac{(d^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{ac(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$- \frac{((4c-d)d^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{ac^2(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+ \frac{((c^2-4cd+6d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{a(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$
\begin{aligned}
&= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\
&- \frac{(c-3d)\tan(e+fx)}{2a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{d^4\tan(e+fx)}{a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&+ \frac{(2\tan(e+fx))\mathrm{Subst}\left(\int\frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{((c-3d)\tan(e+fx))\mathrm{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{4a(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{(3\tan(e+fx))\mathrm{Subst}\left(\int\frac{1}{(1+x)^2\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{8a(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{(2(4c-d)d^4\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2c^2(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{(d^4\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{2ac(c-d)^3(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{(2(c^2-4cd+6d^2)\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\
&\quad -\frac{(c-3d)\tan(e+fx)}{2a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad -\frac{3\tan(e+fx)}{16a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad +\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad -\frac{\sqrt{2}(c^2-4cd+6d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad +\frac{2(4c-d)d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad +\frac{d^4\tan(e+fx)}{a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad -\frac{((c-3d)\tan(e+fx))\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{2a^2(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad +\frac{(3\tan(e+fx))\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{32a(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad +\frac{(d^4\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2c(c-d)^3(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(c-3d)\tan(e+fx)}{2a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{3\tan(e+fx)}{16a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(c-3d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\sqrt{2}(c^2-4cd+6d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2(4c-d)d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{d^4\tan(e+fx)}{a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad - \frac{(3\tan(e+fx))\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{16a^2(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-3d)\tan(e+fx)}{2a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{3\tan(e+fx)}{16a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\mathrm{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-3d)\mathrm{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3\mathrm{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-4cd+6d^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^{7/2}\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2(4c-d)d^{7/2}\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^4\tan(e+fx)}{a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [A] (warning: unable to verify)

Time = 13.65 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))^2\sec^{\frac{9}{2}}(e+fx)\left(-\frac{\left(c^2 \cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (Cos[(e + f*x)/2]^4*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^(9/2)*(-(((c^2*(43*
c^3 - 123*c^2*d + 53*c*d^2 + 219*d^3)*ArcSin[Tan[(e + f*x)/2]] - 32*Sqrt[2]
```

```
*(c - d)^4*(c + d)*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f
*x])]] + (16*Sqrt[2]*d^(7/2)*(9*c^2 + 5*c*d - 2*d^2)*ArcTanh[(Sqrt[d]*Tan[(
e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])])/Sqrt[-c
 - d])*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e
+ f*x]])/(c^2*(c - d)^4*(c + d)*Sqrt[Sec[(e + f*x)/2]^2])) + (Cos[(e + f*x)
/2]*Sqrt[Sec[e + f*x]]*((2*(15*c^2 - 16*c*d - 31*d^2 - (16*d^4*Cos[e + f*x]
)/(c*(d + c*Cos[e + f*x])))*Sin[(e + f*x)/2])/(c + d) + 32*(c - d)*Csc[e +
f*x]^4*Sin[(e + f*x)/2]^5 + (-19*c + 35*d)*Sec[(e + f*x)/2]*Tan[(e + f*x)/2
]))/(-c + d)^3))/(4*f*(a*(1 + Sec[e + f*x]))^(5/2)*(c + d*Sec[e + f*x])^2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 85543 vs. $2(653) = 1306$.

Time = 20.18 (sec) , antiderivative size = 85544, normalized size of antiderivative = 113.15

method	result	size
default	Expression too large to display	85544

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx=\int \frac{1}{(a\left(\sec\left(e+fx\right)+1\right))^{\frac{5}{2}}\left(c+d\sec\left(e+fx\right)\right)^2}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima"
)
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Hanged}$$

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2),x)
[Out] \text{Hanged}
```

3.183 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx$

Optimal result 1244
Rubi [A] (verified) 1245
Mathematica [B] (warning: unable to verify) 1253
Maple [B] (warning: unable to verify) 1254
Fricas [F(-1)] 1255
Sympy [F] 1255
Maxima [F(-1)] 1255
Giac [F(-2)] 1255
Mupad [F(-1)] 1256

Optimal result

Integrand size = 27, antiderivative size = 999

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-4d)\tan(e+fx)}{2a^2(c-d)^4f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{3\tan(e+fx)}{16a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-4d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-5cd+10d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{3d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4a^{3/2}c(c-d)^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(4c-d)d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2d^{7/2}(10c^2-5cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^3(c-d)^5\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^4\tan(e+fx)}{2a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{3d^4\tan(e+fx)}{4a^2c(c-d)^3(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(4c-d)d^4\tan(e+fx)}{a^2c^2(c-d)^4(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

```
[Out] -1/4*tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*(
c-4*d)*tan(f*x+e)/a^2/(c-d)^4/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/16*
tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+1/2*d^4*tan(
```

```
f*x+e)/a^2/c/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)+3/4*
d^4*tan(f*x+e)/a^2/c/(c-d)^3/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1
/2)+(4*c-d)*d^4*tan(f*x+e)/a^2/c^2/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))/(a+a*se
c(f*x+e))^(1/2)+2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2
)/c^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+3/4*d^(7/2)*arctanh(d
^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c/(c-
d)^3/(c+d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+(4*c-d)*d^
(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e
)/a^(3/2)/c^2/(c-d)^4/(c+d)^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))
^(1/2)-1/4*(c-4*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(
f*x+e)/a^(3/2)/(c-d)^4/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1
/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(
3/2)/(c-d)^3/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c^2-5
*c*d+10*d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*ta
n(f*x+e)/a^(3/2)/(c-d)^5/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*
d^(7/2)*(10*c^2-5*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(
c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c^3/(c-d)^5/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(
1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 999, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used

$= \{4025, 186, 65, 212, 44, 214\}$

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \frac{(4c-d)\tan(e+fx)d^4}{a^2c^2(c-d)^4(c+d)f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx)}$$
$$+\frac{3\tan(e+fx)d^4}{4a^2c(c-d)^3(c+d)^2f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}$$
$$+\frac{\tan(e+fx)d^4}{2a^2c(c-d)^3(c+d)f\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))^2}$$
$$+\frac{2(10c^2-5dc+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)d^{7/2}}{a^{3/2}c^3(c-d)^5\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}$$
$$+\frac{(4c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)d^{7/2}}{a^{3/2}c^2(c-d)^4(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}$$
$$+\frac{3\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)d^{7/2}}{4a^{3/2}c(c-d)^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}$$
$$+\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}$$
$$-\frac{\sqrt{2}(c^2-5dc+10d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}$$
$$-\frac{3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}$$
$$-\frac{(c-4d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}$$
$$-\frac{3\tan(e+fx)}{16a^2(c-d)^3f(\sec(e+fx)+1)\sqrt{\sec(e+fx)a+a}}$$
$$-\frac{(c-4d)\tan(e+fx)}{2a^2(c-d)^4f(\sec(e+fx)+1)\sqrt{\sec(e+fx)a+a}}$$
$$-\frac{\tan(e+fx)}{4a^2(c-d)^3f(\sec(e+fx)+1)^2\sqrt{\sec(e+fx)a+a}}$$

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] -1/4*Tan[e + f*x]/(a^2*(c - d)^3*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e +
f*x]]) - ((c - 4*d)*Tan[e + f*x])/(2*a^2*(c - d)^4*f*(1 + Sec[e + f*x])*Sqr
t[a + a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)^3*f*(1 + Sec[e +
f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[
a]]*Tan[e + f*x])/(a^(3/2)*c^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
+ f*x]]) - ((c - 4*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*T
```

```
an[e + f*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a
 + a*Sec[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])
]*Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - 5*c*d + 10*d^2)*ArcTanh[Sqrt[a -
a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^5*f*Sqrt[
a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (3*d^(7/2)*ArcTanh[(Sqrt[d]
*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(4*a^(3/2)*
c*(c - d)^3*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]]) + ((4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a
]*Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c^2*(c - d)^4*(c + d)^(3/2)*f*Sqrt[a
 - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*d^(7/2)*(10*c^2 - 5*c*d +
 d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan
[e + f*x])/(a^(3/2)*c^3*(c - d)^5*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) + (d^4*Tan[e + f*x])/(2*a^2*c*(c - d)^3*(c + d)*f*S
qrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (3*d^4*Tan[e + f*x])/(4*a
^2*c*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])) +
 ((4*c - d)*d^4*Tan[e + f*x])/(a^2*c^2*(c - d)^4*(c + d)*f*Sqrt[a + a*Sec[e
 + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^3(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{a^3c^3x\sqrt{a-ax}} - \frac{1}{a^3(c-d)^3(1+x)^3\sqrt{a-ax}} + \frac{-c+4d}{a^3(c-d)^4(1+x)^2\sqrt{a-ax}} + \frac{-c^2+5cd-10c}{a^3(c-d)^5(1+x)\sqrt{\cdot}}\right.\right.}{f\sqrt{a-a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{ac^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c-4d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{a(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{(1+x)^3\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{a(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(d^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{ac(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{((4c-d)d^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{ac^2(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(d^4(10c^2-5cd+d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{ac^3(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{((c^2-5cd+10d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{a(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$
\begin{aligned}
&= -\frac{\tan(e+fx)}{4a^2(c-d)^3 f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(c-4d)\tan(e+fx)}{2a^2(c-d)^4 f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{d^4\tan(e+fx)}{2a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} \\
&\quad + \frac{(4c-d)d^4\tan(e+fx)}{a^2c^2(c-d)^4(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad + \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{a^2c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{((c-4d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{4a(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{(3\tan(e+fx))\text{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{8a(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(3d^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{4ac(c-d)^3(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{((4c-d)d^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2ac^2(c-d)^4(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{(2d^4(10c^2-5cd+d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{c+d-\frac{dx^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{a^2c^3(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(2(c^2-5cd+10d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{a^2(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(c-4d)\tan(e+fx)}{2a^2(c-d)^4f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{3\tan(e+fx)}{16a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\sqrt{2}(c^2-5cd+10d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2d^{7/2}(10c^2-5cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^3(c-d)^5\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{d^4\tan(e+fx)}{2a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} \\
&\quad + \frac{3d^4\tan(e+fx)}{4a^2c(c-d)^3(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad + \frac{(4c-d)d^4\tan(e+fx)}{a^2c^2(c-d)^4(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\
&\quad - \frac{((c-4d)\tan(e+fx))\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{2a^2(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{(3\tan(e+fx))\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{32a(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{(3d^4\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{8ac(c-d)^3(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{((4c-d)d^4\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2c^2(c-d)^4(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{(c-4d)\tan(e+fx)}{2a^2(c-d)^4f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{3\tan(e+fx)}{16a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{(c-4d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{\sqrt{2}(c^2-5cd+10d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{(4c-d)d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{2d^{7/2}(10c^2-5cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^3(c-d)^5\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{d^4\tan(e+fx)}{2a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}$$
$$+\frac{3d^4\tan(e+fx)}{4a^2c(c-d)^3(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$
$$+\frac{(4c-d)d^4\tan(e+fx)}{a^2c^2(c-d)^4(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$
$$-\frac{(3\tan(e+fx))\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{16a^2(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{(3d^4\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{c+d-\frac{dx^2}{a}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{4a^2c(c-d)^3(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{(c-4d)\tan(e+fx)}{2a^2(c-d)^4f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{3\tan(e+fx)}{16a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{(c-4d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{\sqrt{2}(c^2-5cd+10d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{3d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4a^{3/2}c(c-d)^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{(4c-d)d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{2d^{7/2}(10c^2-5cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^3(c-d)^5\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{d^4\tan(e+fx)}{2a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}$$
$$+\frac{3d^4\tan(e+fx)}{4a^2c(c-d)^3(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$
$$+\frac{(4c-d)d^4\tan(e+fx)}{a^2c^2(c-d)^4(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2904 vs. 2(999) = 1998.

Time = 22.08 (sec) , antiderivative size = 2904, normalized size of antiderivative = 2.91

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] (Cos[(e + f*x)/2]^5*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^6*((-3*(5*c^6 - 3*c
^5*d - 21*c^4*d^2 - 13*c^3*d^3 - 28*c^2*d^4 - 12*c*d^5 + 8*d^6)*Sin[(e + f*
x)/2])/(2*c^3*(-c + d)^4*(c + d)^2) - (4*d^6*Sin[(e + f*x)/2])/(c^3*(-c + d
)^3*(c + d)*(d + c*Cos[e + f*x])^2) + (Sec[(e + f*x)/2]^2*(19*c*Sin[(e + f*
x)/2] - 43*d*Sin[(e + f*x)/2]))/(4*(-c + d)^4) + (2*(-23*c^2*d^5*Sin[(e + f
*x)/2] - 9*c*d^6*Sin[(e + f*x)/2] + 8*d^7*Sin[(e + f*x)/2]))/(c^3*(-c + d)^
4*(c + d)^2*(d + c*Cos[e + f*x])) + (Sec[(e + f*x)/2]^3*Tan[(e + f*x)/2])/(
2*(-c + d)^3)))/(f*(a*(1 + Sec[e + f*x]))^(5/2)*(c + d*Sec[e + f*x])^3) - (
(c^3*(c + d)^2*(43*c^2 - 206*c*d + 355*d^2)*ArcSin[Tan[(e + f*x)/2]] - 32*S
qrt[2]*(c - d)^5*(c + d)^2*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + C
os[e + f*x])]] + (4*Sqrt[2]*d^(7/2)*(99*c^4 + 110*c^3*d - 5*c^2*d^2 - 20*c*
d^3 + 8*d^4)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e +
f*x]/(1 + Cos[e + f*x])])])/Sqrt[-c - d])*Cos[(e + f*x)/2]^5*(d + c*Cos[e +
 f*x])^3*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*((-11*c^4*Sec[(e + f*x)/2])/
(8*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) + (45*c^3*
d*Sec[(e + f*x)/2])/(8*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e
 + f*x]]) - (5*c^2*d^2*Sec[(e + f*x)/2])/(8*(-c + d)^4*(c + d)^2*(d + c*Cos
[e + f*x])*Sqrt[Sec[e + f*x]]) - (317*c*d^3*Sec[(e + f*x)/2])/(8*(-c + d)^4
*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (69*d^4*Sec[(e + f*x)
/2])/(2*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (7*
d^5*Sec[(e + f*x)/2])/(2*c*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[S
ec[e + f*x]]) + (2*d^6*Sec[(e + f*x)/2])/(c^2*(-c + d)^4*(c + d)^2*(d + c*C
os[e + f*x])*Sqrt[Sec[e + f*x]]) + (2*c^4*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x
]])/((-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])) - (43*c^3*d*Sec[(e + f*x)/2
]*Sqrt[Sec[e + f*x]])/(8*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])) - (3*c^
2*d^2*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(8*(-c + d)^4*(c + d)^2*(d + c*C
os[e + f*x])) + (123*c*d^3*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(8*(-c + d)
^4*(c + d)^2*(d + c*Cos[e + f*x])) + (95*d^4*Sec[(e + f*x)/2]*Sqrt[Sec[e +
f*x]])/(8*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])) + (d^5*Sec[(e + f*x)/2
]*Sqrt[Sec[e + f*x]])/(2*c*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])) + (2*
c^4*Cos[2*(e + f*x)]*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/((-c + d)^4*(c +
d)^2*(d + c*Cos[e + f*x])) - (4*c^3*d*Cos[2*(e + f*x)]*Sec[(e + f*x)/2]*Sqr
t[Sec[e + f*x]])/((-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])) - (2*c^2*d^2*C
os[2*(e + f*x)]*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/((-c + d)^4*(c + d)^2*
(d + c*Cos[e + f*x])) + (8*c*d^3*Cos[2*(e + f*x)]*Sec[(e + f*x)/2]*Sqrt[Sec
```

```
[e + f*x]])/((-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])) - (2*d^4*Cos[2*(e +
 f*x)]*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/((-c + d)^4*(c + d)^2*(d + c*Co
s[e + f*x])) - (4*d^5*Cos[2*(e + f*x)]*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])
/(c*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])) + (2*d^6*Cos[2*(e + f*x)]*Se
c[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(c^2*(-c + d)^4*(c + d)^2*(d + c*Cos[e +
 f*x])))*Sec[e + f*x]^(11/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]])/(4*c^3*
(c - d)^5*(c + d)^2*f*(a*(1 + Sec[e + f*x]))^(5/2)*(c + d*Sec[e + f*x])^3*(
-1/8*((c^3*(c + d)^2*(43*c^2 - 206*c*d + 355*d^2)*ArcSin[Tan[(e + f*x)/2]]
- 32*Sqrt[2]*(c - d)^5*(c + d)^2*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/
(1 + Cos[e + f*x])]] + (4*Sqrt[2]*d^(7/2)*(99*c^4 + 110*c^3*d - 5*c^2*d^2 -
 20*c*d^3 + 8*d^4)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])])])/Sqrt[-c - d])*Sqrt[Cos[e + f*x]*Sec[(e +
f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2)*(-(Sec[(e + f*x)/2]^2*Si
n[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]))/(c^3*(c -
d)^5*(c + d)^2) - (Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)
/2]^2*Sec[e + f*x]]*((c^3*(c + d)^2*(43*c^2 - 206*c*d + 355*d^2)*Sec[(e + f
*x)/2]^2)/(2*Sqrt[1 - Tan[(e + f*x)/2]^2]) - (32*Sqrt[2]*(c - d)^5*(c + d)^
2*(Sec[(e + f*x)/2]^2/(2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]) - (((Cos[e
+ f*x]*Sin[e + f*x])/(1 + Cos[e + f*x])^2 - Sin[e + f*x]/(1 + Cos[e + f*x])
)*Tan[(e + f*x)/2])/(2*(Cos[e + f*x]/(1 + Cos[e + f*x]))^(3/2))))/(1 + (1 +
 Cos[e + f*x])*Sec[e + f*x]*Tan[(e + f*x)/2]^2) + (4*Sqrt[2]*d^(7/2)*(99*c^
4 + 110*c^3*d - 5*c^2*d^2 - 20*c*d^3 + 8*d^4)*((Sqrt[d]*Sec[(e + f*x)/2]^2)
/(2*Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]) - (Sqrt[d]*((Cos[e
+ f*x]*Sin[e + f*x])/(1 + Cos[e + f*x])^2 - Sin[e + f*x]/(1 + Cos[e + f*x])
)*Tan[(e + f*x)/2])/(2*Sqrt[-c - d]*(Cos[e + f*x]/(1 + Cos[e + f*x]))^(3/2)
)))/(Sqrt[-c - d]*(1 - (d*(1 + Cos[e + f*x])*Sec[e + f*x]*Tan[(e + f*x)/2]^
2)/(-c - d)))))/(4*c^3*(c - d)^5*(c + d)^2) - ((c^3*(c + d)^2*(43*c^2 - 206
*c*d + 355*d^2)*ArcSin[Tan[(e + f*x)/2]] - 32*Sqrt[2]*(c - d)^5*(c + d)^2*A
rcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] + (4*Sqrt[2]*
d^(7/2)*(99*c^4 + 110*c^3*d - 5*c^2*d^2 - 20*c*d^3 + 8*d^4)*ArcTanh[(Sqrt[d
]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])])/
Sqrt[-c - d])*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*(-(Cos[(e + f*x)/2]*Sec
[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x])
)/(8*c^3*(c - d)^5*(c + d)^2*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]])))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 139612 vs. $2(868) = 1736$.

Time = 23.72 (sec) , antiderivative size = 139613, normalized size of antiderivative = 139.75

method	result	size
default	Expression too large to display	139613

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(c+d\sec\left(e+fx\right)\right)^3}\,dx$$

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima"
)
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Hanged}$$

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3),x)
[Out] \text{Hanged}
```

3.184 $\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$

Optimal result 1257
Rubi [A] (verified) 1257
Mathematica [A] (verified) 1259
Maple [B] (warning: unable to verify) 1259
Fricas [A] (verification not implemented) 1260
Sympy [F] 1262
Maxima [F] 1262
Giac [F] 1262
Mupad [F(-1)] 1262

Optimal result

Integrand size = 29, antiderivative size = 123

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\sqrt{a}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

```
[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2))*a^(1/2)*c^(1/2)/f+2*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(a+a*sec(f*x
+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*a^(1/2)*d^(1/2)/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4017, 4019, 209, 4065, 212}

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\sqrt{a}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e
+ f*x]]*Sqrt[c + d*Sec[e + f*x]])])/f + (2*Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[a]
*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])]
)/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4017

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c +
d*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]
]/Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0]
```

Rule 4019

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 4065

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[-2*(b/f), Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = c\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx + d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= -\frac{(2ac)\text{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} - \frac{(2ad)\text{Subst}\left(\int \frac{1}{1-adx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$= \frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\sqrt{a}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

Mathematica [A] (verified)

Time = 14.89 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.95

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}\left(-2\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+c\cos(e+fx)}}{\sqrt{d}\sqrt{c-c\cos(e+fx)}}\right)\sqrt{c(1+\cos}\right.}{f\sqrt{c(1+\cos(e+fx))}\sqrt{c-c}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (-2*Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]]*(-2*Sq
rt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c -
c*Cos[e + f*x]])]*Sqrt[c*(1 + Cos[e + f*x])]*Sin[(e + f*x)/2]^2 + ArcTan[(S
qrt[c*(1 + Cos[e + f*x])]*Sqrt[d + c*Cos[e + f*x]])/Sqrt[c^2*Sin[e + f*x]^2
]]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[c^2*Sin[e + f*x]^2]))/(f*Sqrt[c*(1 + Cos[e
 + f*x])]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1495 vs. 2(99) = 198.

Time = 5.00 (sec) , antiderivative size = 1496, normalized size of antiderivative = 12.16

method	result	size
default	Expression too large to display	1496

```
[In] int((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(c-d)^(1/2)*2^(1/2)/(-d)^(1/2)/(c^2-2*c*d+d^2)*(a*(sec(f*x+e)+1))^(1/2
)*(c+d*sec(f*x+e))^(1/2)*(2^(1/2)*(-d)^(1/2)*ln(-(c*cot(f*x+e)-d*cot(f*x+e)
-c*csc(f*x+e)+d*csc(f*x+e)-(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c-d)
^(1/2))/(c-d)^(1/2))*c^3-3*2^(1/2)*(-d)^(1/2)*ln(-(c*cot(f*x+e)-d*cot(f*x+e
)-c*csc(f*x+e)+d*csc(f*x+e)-(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c-d
)^(1/2))/(c-d)^(1/2))*c^2*d+3*2^(1/2)*(-d)^(1/2)*ln(-(c*cot(f*x+e)-d*cot(f*
```

```
x+e)-c*csc(f*x+e)+d*csc(f*x+e)-(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(
c-d)^(1/2))/(c-d)^(1/2))*c*d^2-2^(1/2)*(-d)^(1/2)*ln(-(c*cot(f*x+e)-d*cot(f
*x+e)-c*csc(f*x+e)+d*csc(f*x+e)-(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*
(c-d)^(1/2))/(c-d)^(1/2))*d^3-2^(1/2)*(-d)^(1/2)*ln((-2*(d+c*cos(f*x+e))/(c
os(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^3+3*2^
(1/2)*(-d)^(1/2)*ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*
cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^2*d-3*2^(1/2)*(-d)^(1/2)*ln((-2*(d+c*c
os(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x
+e))*c*d^2+2^(1/2)*(-d)^(1/2)*ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*d^3+(c-d)^(1/2)*ln(-2*(2^(1
/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-sin(f*
x+e)*c-sin(f*x+e)*d-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/(cos(f*x+e)-1+sin(f*x+e)
))*c^2*d-2*(c-d)^(1/2)*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)-sin(f*x+e)*c-sin(f*x+e)*d-c*cos(f*x+e)+d*cos(f*
x+e)+c-d)/(cos(f*x+e)-1+sin(f*x+e)))*c*d^2+(c-d)^(1/2)*ln(-2*(2^(1/2)*(-d)^
(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-sin(f*x+e)*c-si
n(f*x+e)*d-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/(cos(f*x+e)-1+sin(f*x+e)))*d^3-(c
-d)^(1/2)*ln(2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)-sin(f*x+e)*c-sin(f*x+e)*d+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(-co
s(f*x+e)+1+sin(f*x+e)))*c^2*d+2*(c-d)^(1/2)*ln(2*(2^(1/2)*(-d)^(1/2)*(-2*(d
+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-sin(f*x+e)*c-sin(f*x+e)*d+c
*cos(f*x+e)-d*cos(f*x+e)-c+d)/(-cos(f*x+e)+1+sin(f*x+e)))*c*d^2-(c-d)^(1/2)
*ln(2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)-sin(f*x+e)*c-sin(f*x+e)*d+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(-cos(f*x+e)+
1+sin(f*x+e)))*d^3+2*(-(c-d)^4*c)^(1/2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*
c)^(1/2)*sin(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/
2))*(c-d)^(1/2)*(-d)^(1/2))*cos(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/
(cos(f*x+e)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 806, normalized size of antiderivative = 6.55

$$\int \sqrt{a + a\sec(e + fx)}\sqrt{c + d\sec(e + fx)}\, dx$$

$$= \left[\frac{\sqrt{ad}\log\left(\frac{2\sqrt{ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac-ad)\cos(fx+e)^2+2\,ad+(ac+ad)\cos(fx+e)}{\cos(fx+e)^2+\cos(fx+e)}\right) + \sqrt{-ac}\,\mathrm{lo}}{f} \right.$$

$$-\frac{2\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{ac\sin(fx+e)}\right) - \sqrt{ad}\log\left(\frac{2\sqrt{ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\,\mathrm{si}}{\cos(fx+}\right.}{f}$$

$$-\frac{2\sqrt{-ad}\arctan\left(\frac{\sqrt{-ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{ad\sin(fx+e)}\right) - \sqrt{-ac}\log\left(\frac{2\,ac\cos(fx+e)^2-2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{}}{}\right.}{f}$$

$$-\frac{2\left(\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{ac\sin(fx+e)}\right) + \sqrt{-ad}\arctan\left(\frac{\sqrt{-ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\,\mathrm{co}}{ad\sin(fx+e)}\right.\right.}{f}$$

```
[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [(sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((
c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c - a*d)*c
os(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)^2 + cos(f*x
 + e))) + sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1
)))/f, -(2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e)))
 - sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c - a*d)*
cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)^2 + cos(f*
x + e))))/f, -(2*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*
x + e))) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) +
1)))/f, -2*(sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e))
) + sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))))/f]
```

Sympy [F]

$$\int \sqrt{a + a\sec(e + fx)}\sqrt{c + d\sec(e + fx)}\,dx = \int \sqrt{a\left(\sec\left(e + fx\right) + 1\right)}\sqrt{c + d\sec\left(e + fx\right)}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a + a\sec(e + fx)}\sqrt{c + d\sec(e + fx)}\,dx = \int \sqrt{a\sec\left(fx + e\right) + a}\sqrt{d\sec\left(fx + e\right) + c}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \sqrt{a + a\sec(e + fx)}\sqrt{c + d\sec(e + fx)}\,dx = \int \sqrt{a\sec\left(fx + e\right) + a}\sqrt{d\sec\left(fx + e\right) + c}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(e + fx)}\sqrt{c + d\sec(e + fx)}\,dx = \int \sqrt{a + \frac{a}{\cos\left(e + f\,x\right)}}\,\sqrt{c + \frac{d}{\cos\left(e + f\,x\right)}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2),x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2), x)
```

3.185 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result . . . 1263
Rubi [A] (verified) . . . 1263
Mathematica [A] (verified) . . . 1264
Maple [B] (verified) . . . 1264
Fricas [A] (verification not implemented) . . . 1265
Sympy [F] . . . 1265
Maxima [F(-2)] . . . 1266
Giac [F] . . . 1266
Mupad [F(-1)] . . . 1266

Optimal result

Integrand size = 29, antiderivative size = 61

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}f}$$

```
[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2))*a^(1/2)/f/c^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4019, 209}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}f}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[c]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4019

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\sqrt{d+c\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{c}f\sqrt{c+d\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]
]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(Sq
rt[c]*f*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(49) = 98.

Time = 2.90 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.72

method	result	size
default	$-\frac{2\sqrt{2}\sqrt{-(c-d)^4 c}\arctan\left(\frac{(c-d)^2 c\sqrt{2}\sin(fx+e)}{\sqrt{-(c-d)^4 c}(\cos(fx+e)+1)\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}}\right)\sqrt{c+d\sec(fx+e)}\sqrt{a(\sec(fx+e)+1)}\cos(fx+e)}{f(c^2-2cd+d^2)c(\cos(fx+e)+1)\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}}$	166

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*2^(1/2)*(-(c-d)^4*c)^(1/2)/(c^2-2*c*d+d^2)/c*arctan((c-d)^2*c*2^(1/2)/
(-(c-d)^4*c)^(1/2)*sin(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+
e)+1))^(1/2))*(c+d*sec(f*x+e))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*cos(f*x+e)/(c
os(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \left[\frac{\sqrt{-\frac{a}{c}}\log\left(-\frac{2\,c\sqrt{-\frac{a}{c}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-2\,ac\cos(fx+e)^2+ac-ad-(ac+ad)\cos(fx+e)}{\cos(fx+e)+1}\right)}{f}, -\frac{2\sqrt{\frac{a}{c}}\arctan\left(\frac{\sqrt{\frac{a}{c}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{a\sin(fx+e)}\right)}{f}\right]$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [sqrt(-a/c)*log(-(2*c*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*a*c*cos
(f*x + e)^2 + a*c - a*d - (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1))/f,
-2*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(c + d*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{\sqrt{c + d\sec(e + fx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{\sqrt{c + d\sec(e + fx)}}\,dx = \int \frac{\sqrt{a\sec\left(fx + e\right) + a}}{\sqrt{d\sec\left(fx + e\right) + c}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{\sqrt{c + d\sec(e + fx)}}\,dx = \int \frac{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}{\sqrt{c + \frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)
```

3.186 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal result 1267
Rubi [A] (verified) 1267
Mathematica [A] (verified) 1269
Maple [B] (verified) 1269
Fricas [B] (verification not implemented) 1270
Sympy [F] 1271
Maxima [F(-2)] 1271
Giac [F] 1271
Mupad [F(-1)] 1272

Optimal result

Integrand size = 29, antiderivative size = 111

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{2ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2))*a^(1/2)/c^(3/2)/f-2*a*d*tan(f*x+e)/c/(c+d)/f/(a+a*sec(f*x+e))^(1/2)
/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4024, 4019, 209, 4072, 37}

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{2ad\tan(e+fx)}{cf(c+d)\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}$$

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(c^(3/2)*f) - (2*a*d*Tan[e + f*x])/(c*(c + d)*f
*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4019

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 4024

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(3/2), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[
c + d*Csc[e + f*x]], x], x] - Dist[d/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e
+ f*x]]/(c + d*Csc[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx}{c}$$

$$= -\frac{(2a)\mathrm{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{cf} + \frac{(a^2d\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-ax}(c+dx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{2ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = -\frac{\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\left(-\sqrt{2}(c+d)^{3/2}\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\sqrt{\frac{d+c\cos(e+fx)}{c+d}} + 2\sqrt{c}d\sin}{c^{3/2}(c+d)f\sqrt{c+d\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(-(Sqrt[2]*(c + d)^(3/2)*Arc
Sin[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f*x
])/(c + d)]) + 2*Sqrt[c]*d*Sin[(e + f*x)/2]))/(c^(3/2)*(c + d)*f*Sqrt[c + d
*Sec[e + f*x]]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. 2(95) = 190.

Time = 2.85 (sec) , antiderivative size = 609, normalized size of antiderivative = 5.49

method	result
default	$\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)\sqrt{\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2-d(1-\cos(fx+e))^2\csc(fx+e)^2-c-d}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\Big($

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/(c+d)/c^2/(c^2-2*c*d+d^2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2
)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*((c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-
cos(f*x+e))^2*csc(f*x+e)^2-c-d)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-
2*c^3*d*(-cot(f*x+e)+csc(f*x+e))+4*c^2*d^2*(-cot(f*x+e)+csc(f*x+e))-2*c*d^3
*(-cot(f*x+e)+csc(f*x+e))+2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((c-d)^2*c*2^(1/
```

```
2)/(-(c-d)^4*c)^(1/2)/(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-c-d)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(c*(1-cos(f*x+e))^2*csc(f*
x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)^(1/2)*c+2^(1/2)*(-(c-d)^4*c)^(1
/2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)/(c*(1-cos(f*x+e))^2*csc(f*x
+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*
(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)^(1/2)
*d)/(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(95) = 190$.

Time = 0.36 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.66

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \left[-\frac{2\,d\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) - \left((c^2+cd)\cos(f\right.}{(c}\right.$$

$$-\frac{2\left(d\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) + \left((c^2+cd)\cos(fx+e)^2 + cd + d^2 + (c^2+2\,cd\right.\right.}{(c^3+c^2d)f\cos(fx+e)^2 + (c^3+2\,c^2d+cd^2)f\cos(fx}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-(2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) - ((c^2 + c*d)*cos(f*x + e)^2 + c*d +
 d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a/c)*log(-(2*c*sqrt(-a/c)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) - 2*a*c*cos(f*x + e)^2 + a*c - a*d - (a*c + a*
d)*cos(f*x + e))/(cos(f*x + e) + 1)))/((c^3 + c^2*d)*f*cos(f*x + e)^2 + (c^
3 + 2*c^2*d + c*d^2)*f*cos(f*x + e) + (c^2*d + c*d^2)*f), -2*(d*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + ((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*
d + d^2)*cos(f*x + e))*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*sin(
f*x + e))))/((c^3 + c^2*d)*f*cos(f*x + e)^2 + (c^3 + 2*c^2*d + c*d^2)*f*cos
(f*x + e) + (c^2*d + c*d^2)*f)]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac"
)
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^{3/2}}\,dx = \int \frac{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}{\left(c + \frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2),x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)
```

3.187 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result 1273
Rubi [A] (verified) 1273
Mathematica [A] (verified) 1275
Maple [B] (warning: unable to verify) 1275
Fricas [A] (verification not implemented) 1276
Sympy [F] 1277
Maxima [F(-2)] 1277
Giac [F] 1277
Mupad [F(-1)] 1278

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2))*c^(1/2)/f/a^(1/2)-arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)
/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*2^(1/2)*(c-d)^(1/2)/f/a^(1/
2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4020, 4019, 209, 4068}

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

```
[In] Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt
[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]])])/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4019

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 4020

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[a/c, Int[Sqrt[c + d*Csc[e + f*x]]/Sqrt[a
+ b*Csc[e + f*x]], x], x] + Dist[(b*c - a*d)/c, Int[Csc[e + f*x]/(Sqrt[a +
b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 4068

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-2*(a/(b*f)), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} + (-c+d)\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx \\
&= -\frac{(2c)\mathrm{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} \\
&\quad + \frac{(2(c-d))\mathrm{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}
\end{aligned}$$

$$= \frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

Mathematica [A] (verified)

Time = 13.07 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{-c+d}\operatorname{arctanh}\left(\frac{\sqrt{-c+d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)+\frac{\sqrt{2}\sqrt{c}\sqrt{c+d}\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}}{\sqrt{d+c\cos(e+fx)}}\right)\sqrt{}}{f\sqrt{d+c\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(Sqrt[-c + d]*ArcTanh[(Sqrt[-c + d]*Sin[(e + f*x)/2])/S
qrt[d + c*Cos[e + f*x]]] + (Sqrt[2]*Sqrt[c]*Sqrt[c + d]*ArcSin[(Sqrt[2]*Sqr
t[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f*x])/(c + d)])/Sqr
t[d + c*Cos[e + f*x]])*Sqrt[c + d*Sec[e + f*x]])/(f*Sqrt[d + c*Cos[e + f*x]
]*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(114) = 228$.

Time = 2.72 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.00

method	result
default	$-\frac{2\sqrt{c+d\sec(fx+e)}\,\sqrt{a(\sec(fx+e)+1)}\left(\sqrt{2}\,\sqrt{-(c-d)^4c}\,\arctan\left(\frac{(c-d)^2c\sqrt{2}\,\sin(fx+e)}{\sqrt{-(c-d)^4c}\,(\cos(fx+e)+1)\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}}\right)\sqrt{c-d}-\ln\left(\sqrt{-2}\right.\right.}{}$

```
[In] int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/a/(c-d)^(1/2)/(c^2-2*c*d+d^2)*(c+d*sec(f*x+e))^(1/2)*(a*(sec(f*x+e)+1)
)^(1/2)*(2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(
1/2)*sin(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2))*
(c-d)^(1/2)-ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f
*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^3+3*ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))
^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^2*d-3*ln((-2*(d+c*c
os(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x
+e))*c*d^2+ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*
```

```
x+e)+(c-d)^(1/2)*csc(f*x+e))*d^3)*cos(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*
x+e))/(cos(f*x+e)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 883, normalized size of antiderivative = 6.26

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[\frac{\sqrt{2}\sqrt{-\frac{c-d}{a}}\log\left(\frac{2\sqrt{2}\sqrt{-\frac{c-d}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(3c-d)\cos(fx+e)^2+2(c+d)\cos(fx+e)-c+3d}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+\cdots}{2f} \right.$$

$$-\frac{\sqrt{2}\sqrt{\frac{c-d}{a}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c-d}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{(c-d)\sin(fx+e)}\right)-\sqrt{-\frac{c}{a}}\log\left(-\frac{2\sqrt{-\frac{c}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e}{\cos(fx+}}\cdots\right.}{f}$$

$$-\frac{\sqrt{2}\sqrt{\frac{c-d}{a}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c-d}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{(c-d)\sin(fx+e)}\right)+2\sqrt{\frac{c}{a}}\arctan\left(\frac{\sqrt{\frac{c}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)\cdots}{\cos(fx+e)}}\cdots}{c\sin(fx+e)}\right.}{f}$$

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-(c - d)/a)*log((2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) - c
+ 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-c/a)*log(-(2*sqrt(-
c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x
+ e) + c - d)/(cos(f*x + e) + 1)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*log((2*
sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x
 + e)^2 + 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e
) + 1)) - 4*sqrt(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(c*sin(f*x + e))))
/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
+ e)/((c - d)*sin(f*x + e))) - sqrt(-c/a)*log(-(2*sqrt(-c/a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x +
```

```
e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x + e) + c - d)/(cos(
f*x + e) + 1)))/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
 + e))*cos(f*x + e)/((c - d)*sin(f*x + e))) + 2*sqrt(c/a)*arctan(sqrt(c/a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x +
 e))*cos(f*x + e)/(c*sin(f*x + e))))/f]
```

Sympy [F]

$$\int \frac{\sqrt{c + d\sec(e + fx)}}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{\sqrt{c + d\sec(e + fx)}}{\sqrt{a(\sec(e + fx) + 1)}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c + d\sec(e + fx)}}{\sqrt{a + a\sec(e + fx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{\sqrt{c + d\sec(e + fx)}}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{\sqrt{d\sec(fx + e) + c}}{\sqrt{a\sec(fx + e) + a}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.188 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result 1279
Rubi [A] (verified) 1279
Mathematica [A] (verified) 1281
Maple [B] (warning: unable to verify) 1281
Fricas [A] (verification not implemented) 1282
Sympy [F] 1283
Maxima [F] 1283
Giac [F] 1283
Mupad [F(-1)] 1284

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

```
[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2))/f/a^(1/2)/c^(1/2)-arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)
/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/
2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4023, 4019, 209, 4068}

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c +
 d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c]*f) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c -
 d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]
])])/(Sqrt[a]*Sqrt[c - d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4019

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 4023

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x
_)]*(d_.) + (c_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqr
t[c + d*Csc[e + f*x]], x], x] - Dist[b/a, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0]
```

Rule 4068

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-2*(a/(b*f)), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - \int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\text{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$= \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \frac{2\left(\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right) - \sqrt{c}\arctan\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{d+c\cos(e+}}{\sqrt{c}\sqrt{c-d}f\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}}$$

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d +
c*Cos[e + f*x]]] - Sqrt[c]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c
*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])/(S
qrt[c]*Sqrt[c - d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(114) = 228.

Time = 2.50 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.60

method	result
default	$-\frac{2\sqrt{a(\sec(fx+e)+1)}\sqrt{c+d\sec(fx+e)}\left(-\ln\left(\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}-\sqrt{c-d}\cot(fx+e)+\sqrt{c-d}\csc(fx+e)\right)c^3+2\ln\left(\sqrt{-\frac{2(d+c\cos(}{\cos(fx+}}\right.\right.}{}$

```
[In] int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/f/a/(c-d)^(1/2)/(c^2-2*c*d+d^2)/c*(a*(sec(f*x+e)+1))^(1/2)*(c+d*sec(f*x+
e))^(1/2)*(-ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f
*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^3+2*ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))
^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e))*c^2*d-ln((-2*(d+c*cos
(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*csc(f*x+e
))*c*d^2+2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(
1/2)*sin(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2))*
(c-d)^(1/2))*cos(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.92 (sec) , antiderivative size = 913, normalized size of antiderivative = 6.48

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= \left[\frac{\sqrt{2}ac\sqrt{-\frac{1}{ac-ad}}\log\left(\frac{2\sqrt{2}(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\sqrt{-\frac{1}{ac-ad}}\cos(fx+e)\sin(fx+e)+(3c-d)\cos(fx+e)^2+2(c+d)\cos(fx+e}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{2} \right.$$

```
[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/
(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c +
d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt(
-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1)))/(a*c*f), 1/
2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/(a*
c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*
cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 4*sqrt(a*c
)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
+ e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e))))/(a*c*f), (sqrt(2)
*a*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
 + e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(
a*c - a*d) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(-a*c)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)
+ 1)))/(a*c*f), (sqrt(2)*a*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a
*d)*sin(f*x + e)))/sqrt(a*c - a*d) - 2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(
f*x + e)/(a*c*sin(f*x + e))))/(a*c*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

[In] integrate(1/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="max
ima")

[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)

Giac [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="gia
c")

[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)), x)
```

3.189 $\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$

Optimal result 1285
Rubi [A] (verified) 1285
Mathematica [A] (verified) 1286
Maple [A] (verified) 1287
Fricas [A] (verification not implemented) 1287
Sympy [F] 1288
Maxima [F(-2)] 1288
Giac [B] (verification not implemented) 1288
Mupad [B] (verification not implemented) 1289

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx=\frac{ax}{c}+\frac{2(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}\,f}$$

```
[Out] a*x/c+2*(-a*d+b*c)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c/f/
(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4004, 3916, 2738, 214}

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx=\frac{2(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}+\frac{ax}{c}$$

```
[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*x)/c + (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]
])/(c*Sqrt[c - d]*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c} \\
&= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{cd} \\
&= \frac{ax}{c} + \frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{cdf} \\
&= \frac{ax}{c} + \frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \frac{a(e+fx) + \frac{2(-bc+ad)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}}{cf}$$

```
[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*(e + f*x) + (2*(-(b*c) + a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c
^2 - d^2]])/Sqrt[c^2 - d^2])/(c*f)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{2(da-bc)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c\sqrt{(c+d)(c-d)}}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}}{f}$
default	$\frac{-\frac{2(da-bc)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c\sqrt{(c+d)(c-d)}}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}}{f}$
risch	$\frac{ax}{c}+\frac{\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)da}{\sqrt{c^2-d^2}\,fc}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)b}{\sqrt{c^2-d^2}\,f}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d}}{\sqrt{c^2-d^2}\,c}\right.}{\sqrt{c^2-d^2}\,fc}$

```
[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(a*d-b*c)/c/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((
c+d)*(c-d))^(1/2))+2*a/c*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.73

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$$
$$=\left[\frac{2\,(ac^2-ad^2)fx-(bc-ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2-2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right.}{2\,(c^3-cd^2)f}\right.$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*c^2 - a*d^2)*f*x - (b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*
x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) +
 c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) +
d^2)))/((c^3 - c*d^2)*f), ((a*c^2 - a*d^2)*f*x + (b*c - a*d)*sqrt(-c^2 + d^
2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))
))/((c^3 - c*d^2)*f)]
```

Sympy [F]

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \int \frac{a+b\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(58) = 116$.

Time = 0.33 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.09

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \frac{\frac{\left(\sqrt{-c^2+d^2}a(c-2d)|-c+d|+\sqrt{-c^2+d^2}bc|-c+d|-\sqrt{-c^2+d^2}a|c||-c+d|+\sqrt{-c^2+d^2}b|c||-c+d|\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-\frac{d+\sqrt{(c+d)(c-d)+d^2}}{c-d}}}\right)\right)}{(c^2-2cd+d^2)c^2+(c^2d-2cd^2+d^3)|c|}}{f}$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] ((sqrt(-c^2 + d^2)*a*(c - 2*d)*abs(-c + d) + sqrt(-c^2 + d^2)*b*c*abs(-c +
d) - sqrt(-c^2 + d^2)*a*abs(c)*abs(-c + d) + sqrt(-c^2 + d^2)*b*abs(c)*abs(
-c + d))*(pi*floor(1/2*(f*x + e)/pi + 1/2) + arctan(tan(1/2*f*x + 1/2*e)/sq
rt(-(d + sqrt((c + d)*(c - d) + d^2))/(c - d))))/((c^2 - 2*c*d + d^2)*c^2 +
 (c^2*d - 2*c*d^2 + d^3)*abs(c)) + (a*c + b*c - 2*a*d + a*abs(c) - b*abs(c)
)*(pi*floor(1/2*(f*x + e)/pi + 1/2) + arctan(tan(1/2*f*x + 1/2*e)/sqrt(-(d
- sqrt((c + d)*(c - d) + d^2))/(c - d))))/(c^2 - d*abs(c)))/f
```

Mupad [B] (verification not implemented)

Time = 15.63 (sec) , antiderivative size = 573, normalized size of antiderivative = 8.55

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \frac{b\,c^2\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}} - \frac{b\,d^2\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}} + \frac{2\,a\,c\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c^2-d^2\right)} - \frac{b\,\ln\left(\frac{c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{\left(c+d\right)\,\left(c-d\right)}}{f\,\left(c^2-d^2\right)} - \frac{a\,c\,d\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}} - \frac{2\,a\,d^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{c\,f\,\left(c^2-d^2\right)} + \frac{a\,d^3\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{c\,f\,{\left(c^2-d^2\right)}^{3/2}} + \frac{a\,d\,\ln\left(\frac{c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{\left(c+d\right)\,\left(c-d\right)}}{c\,f\,\left(c^2-d^2\right)}$$

```
[In] int((a + b/cos(e + f*x))/(c + d/cos(e + f*x)),x)
```

```
[Out] (b*c^2*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2
)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (b*d^2*lo
g((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 -
d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) + (2*a*c*atan(sin(e/
2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)) - (b*log((c*cos(e/2 + (f*
x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/
2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(f*(c^2 - d^2)) - (a*c*d*log((c*sin(
e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2
))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (2*a*d^2*atan(sin(e/2 + (f*
x)/2)/cos(e/2 + (f*x)/2)))/(c*f*(c^2 - d^2)) + (a*d^3*log((c*sin(e/2 + (f*x
)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2
 + (f*x)/2)))/(c*f*(c^2 - d^2)^(3/2)) + (a*d*log((c*cos(e/2 + (f*x)/2) + d*
cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/
2))*((c + d)*(c - d))^(1/2))/(c*f*(c^2 - d^2))
```

3.190 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$

Optimal result 1290
Rubi [A] (verified) 1290
Mathematica [A] (verified) 1292
Maple [A] (verified) 1292
Fricas [B] (verification not implemented) 1293
Sympy [F] 1293
Maxima [F(-2)] 1294
Giac [A] (verification not implemented) 1294
Mupad [B] (verification not implemented) 1294

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \frac{ax}{c^2}+\frac{2(bc^3-2ac^2d+ad^3)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} - \frac{d(bc-ad)\tan(e+fx)}{c\,(c^2-d^2)\,f(c+d\sec(e+fx))}$$

```
[Out] a*x/c^2+2*(-2*a*c^2*d+a*d^3+b*c^3)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(
c+d)^(1/2))/c^2/(c-d)^(3/2)/(c+d)^(3/2)/f-d*(-a*d+b*c)*tan(f*x+e)/c/(c^2-d^
2)/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4008, 4004, 3916, 2738, 214}

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \frac{2(-2ac^2d+ad^3+bc^3)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2f(c-d)^{3/2}(c+d)^{3/2}} - \frac{d(bc-ad)\tan(e+fx)}{cf\,(c^2-d^2)\,(c+d\sec(e+fx))}+\frac{ax}{c^2}$$

```
[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*x)/c^2 + (2*(b*c^3 - 2*a*c^2*d + a*d^3)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*
x)/2])/Sqrt[c + d]])/(c^2*(c - d)^(3/2)*(c + d)^(3/2)*f) - (d*(b*c - a*d)*T
an[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(bc-ad)\tan(e+fx)}{c\,(c^2-d^2)\,f(c+d\sec(e+fx))}-\frac{\int \frac{-a(c^2-d^2)-c(bc-ad)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c\,(c^2-d^2)}\\
&=\frac{ax}{c^2}-\frac{d(bc-ad)\tan(e+fx)}{c\,(c^2-d^2)\,f(c+d\sec(e+fx))}+\frac{(c^2(bc-ad)-ad(c^2-d^2))\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c^2\,(c^2-d^2)}\\
&=\frac{ax}{c^2}-\frac{d(bc-ad)\tan(e+fx)}{c\,(c^2-d^2)\,f(c+d\sec(e+fx))}+\frac{(c^2(bc-ad)-ad(c^2-d^2))\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{c^2d\,(c^2-d^2)}
\end{aligned}$$

$$= \frac{ax}{c^2} - \frac{d(bc-ad)\tan(e+fx)}{c(c^2-d^2)f(c+d\sec(e+fx))} + \frac{(2(c^2(bc-ad)-ad(c^2-d^2)))\operatorname{Subst}\left(\int \frac{1}{1+\frac{c}{d}+(1-\frac{c}{d})x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{c^2d(c^2-d^2)f}$$

$$= \frac{ax}{c^2} + \frac{2(bc^3-2ac^2d+ad^3)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} - \frac{d(bc-ad)\tan(e+fx)}{c(c^2-d^2)f(c+d\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \frac{-\frac{2(bc^3+ad(-2c^2+d^2))\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{ad(c^2-d^2)(e+fx)+ac(c^2-d^2)(e+fx)\cos(e+fx)-cd(bc-ad)\sin(e+fx)}{d+c\cos(e+fx)}}{c^2(c-d)(c+d)f}$$

```
[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]
[Out] ((-2*(b*c^3 + a*d*(-2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2] + (a*d*(c^2 - d^2)*(e + f*x) + a*c*(c^2 - d^2)
*(e + f*x)*Cos[e + f*x] - c*d*(b*c - a*d)*Sin[e + f*x])/(d + c*Cos[e + f*x]
))/(c^2*(c - d)*(c + d)*f)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\frac{-\frac{2d(da-bc)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{2\left(2a\,c^2d-a\,d^3-b\,c^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2}}{f}$
default	$\frac{\frac{-\frac{2d(da-bc)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{2\left(2a\,c^2d-a\,d^3-b\,c^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2}}{f}$
risch	$\frac{ax}{c^2} - \frac{2id(-da+bc)\left(d\,e^{i(fx+e)}+c\right)}{c^2(c^2-d^2)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)} + \frac{2\ln\left(e^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{c^2-d^2}\,d}{c\sqrt{c^2-d^2}}\right)ad}{\sqrt{c^2-d^2}\,(c+d)(c-d)f} - \frac{\ln\left(e^{i(fx+e)}+\frac{-ic^2+id^2}{c\sqrt{c}}\right)}{\sqrt{c^2-d^2}\,(c+d)(c}$

```
[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/c^2*(-d*(a*d-b*c)*c/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)
^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)-(2*a*c^2*d-a*d^3-b*c^3)/(c+d)/(c-d)/((c+d)
*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))+2*a/c^
2*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(114) = 228$.

Time = 0.31 (sec) , antiderivative size = 561, normalized size of antiderivative = 4.56

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$$
$$= \left[\frac{2\,(ac^5-2\,ac^3d^2+acd^4)fx\cos{(fx+e)}+2\,(ac^4d-2\,ac^2d^3+ad^5)fx-(bc^3d-2\,ac^2d^2+ad^4+(bc^4-\cdots}{2\,((c^7\cdots}\right.$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*c^5 - 2*a*c^3*d^2 + a*c*d^4)*f*x*cos(f*x + e) + 2*(a*c^4*d - 2*a
*c^2*d^3 + a*d^5)*f*x - (b*c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3*d
 + a*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(b*c^4
*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3
*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a*c^5 - 2*a*c^3*
d^2 + a*c*d^4)*f*x*cos(f*x + e) + (a*c^4*d - 2*a*c^2*d^3 + a*d^5)*f*x + (b*
c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3*d + a*c*d^3)*cos(f*x + e))*s
qrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*
sin(f*x + e))) - (b*c^4*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*sin(f*x + e))/
((c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)
*f)]
```

Sympy [F]

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \int \frac{a+b\sec{(e+fx)}}{(c+d\sec{(e+fx)})^2}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.63

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{\frac{2\left(bc^3-2ac^2d+ad^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4-c^2d^2)\sqrt{-c^2+d^2}}+\frac{(fx+e)a}{c^2}+\frac{2\left(bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\cdots\right.}{(c^3-cd^2)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\cdots\right.}}{f}$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(b*c^3 - 2*a*c^2*d + a*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c +
 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2
 + d^2)))/((c^4 - c^2*d^2)*sqrt(-c^2 + d^2)) + (f*x + e)*a/c^2 + 2*(b*c*d*t
an(1/2*f*x + 1/2*e) - a*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan(1/2
*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f
```

Mupad [B] (verification not implemented)

Time = 22.21 (sec) , antiderivative size = 3763, normalized size of antiderivative = 30.59

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int((a + b/cos(e + f*x))/(c + d/cos(e + f*x))^2,x)
```

```
[Out] (2*a*atan(((a*((a*((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2
 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c
```

^4*d^2) - (a*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^
4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i
)/c^2 + (32*tan(e/2 + (f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5
 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*
d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))/c^2 - (a*((a*((32*(
a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*c^7*d^2
 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a*tan(e/2 + (
f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^
2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2 - (32*tan(e/2 + (f
x)/2)(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*c
^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^4
*d + c^5 - c^2*d^3 - c^3*d^2)))/c^2)/((64*(a^3*d^5 + a*b^2*c^5 - a^2*b*c^5
 - a^3*c*d^4 + 2*a^3*c^4*d - 3*a^3*c^2*d^3 + 2*a^3*c^3*d^2 + a^2*b*c^2*d^3 +
 a^2*b*c^3*d^2 - 3*a^2*b*c^4*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a*((a
((32(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*
c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (a*tan(
e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2
*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2 + (32*tan(e
/2 + (f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d -
5*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3
))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2))*1i)/c^2 + (a*((a*((32*(a*c^4*d^5 - b*
c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d +
 b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a*tan(e/2 + (f*x)/2)*(2*c^9
*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(
c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2 - (32*tan(e/2 + (f*x)/2)*(a^2*c^
6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a^2
*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c^2
*d^3 - c^3*d^2))*1i)/c^2)))/(c^2*f) + (atan(((((32*tan(e/2 + (f*x)/2)*(a^2*
c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a
^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c
^2*d^3 - c^3*d^2) + (((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*
d^2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3
 - c^4*d^2) - (32*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*
c^3 - 2*a*c^2*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 -
 2*c^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^4
 - 3*c^6*d^2)))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d))/(c^
8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 +
b*c^3 - 2*a*c^2*d)*1i)/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2) + (((32*tan(
e/2 + (f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d -
 5*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^
3))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) - (((32*(a*c^4*d^5 - b*c^9 - a*c^9 -
3*a*c^6*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^
5*d + c^6 - c^3*d^3 - c^4*d^2) + (32*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^
3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c
^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 -

```
c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c
^3 - 2*a*c^2*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*((c + d)^3*(c - d
)^3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d)*1i)/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c
^6*d^2))/((64*(a^3*d^5 + a*b^2*c^5 - a^2*b*c^5 - a^3*c*d^4 + 2*a^3*c^4*d -
3*a^3*c^2*d^3 + 2*a^3*c^3*d^2 + a^2*b*c^2*d^3 + a^2*b*c^3*d^2 - 3*a^2*b*c^4
*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (((32*tan(e/2 + (f*x)/2)*(a^2*c^6
+ 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a^2*c
^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c^2*d
^3 - c^3*d^2) + (((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2
- b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^
4*d^2) - (32*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3
- 2*a*c^2*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c
^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*
c^6*d^2)))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d))/(c^8 -
c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^
3 - 2*a*c^2*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2) - (((32*tan(e/2 + (
f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*
c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^
4*d + c^5 - c^2*d^3 - c^3*d^2) - (((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6
*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c
^6 - c^3*d^3 - c^4*d^2) + (32*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2
)*(a*d^3 + b*c^3 - 2*a*c^2*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4
- 4*c^7*d^3 - 2*c^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6
 + 3*c^4*d^4 - 3*c^6*d^2)))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3 - 2*
a*c^2*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*((c + d)^3*(c - d)^3)^(1
/2)*(a*d^3 + b*c^3 - 2*a*c^2*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*
((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d)*2i)/(f*(c^8 - c^2*d
^6 + 3*c^4*d^4 - 3*c^6*d^2)) - (2*tan(e/2 + (f*x)/2)*(a*d^2 - b*c*d))/(f*(c
 + d)*(c*d - c^2)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
```

3.191 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$

Optimal result . . . 1297
Rubi [A] (verified) . . . 1297
Mathematica [A] (verified) . . . 1300
Maple [A] (verified) . . . 1300
Fricas [B] (verification not implemented) . . . 1301
Sympy [F] . . . 1301
Maxima [F(-2)] . . . 1302
Giac [B] (verification not implemented) . . . 1302
Mupad [B] (verification not implemented) . . . 1303

Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \frac{ax}{c^3}+\frac{(bc^3(2c^2+d^2)-ad(6c^4-5c^2d^2+2d^4))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} - \frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2}-\frac{d(3bc^3-5ac^2d+2ad^3)\tan(e+fx)}{2c^2\left(c^2-d^2\right)^2f(c+d\sec(e+fx))}$$

```
[Out] a*x/c^3+(b*c^3*(2*c^2+d^2)-a*d*(6*c^4-5*c^2*d^2+2*d^4))*arctanh((c-d)^(1/2)
*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d)^(5/2)/f-1/2*d*(-a*d+
b*c)*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))^2-1/2*d*(-5*a*c^2*d+2*a*d^3+
3*b*c^3)*tan(f*x+e)/c^2/(c^2-d^2)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4008, 4145, 4004, 3916, 2738, 214}

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \frac{(bc^3(2c^2+d^2)-ad(6c^4-5c^2d^2+2d^4))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}} - \frac{d(bc-ad)\tan(e+fx)}{2cf\left(c^2-d^2\right)(c+d\sec(e+fx))^2}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{2c^2f\left(c^2-d^2\right)^2(c+d\sec(e+fx))}+\frac{ax}{c^3}$$

```
[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*x)/c^3 + ((b*c^3*(2*c^2 + d^2) - a*d*(6*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan
h[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^3*(c - d)^(5/2)*(c + d)^(
5/2)*f) - (d*(b*c - a*d)*Tan[e + f*x])/(2*c*(c^2 - d^2)*f*(c + d*Sec[e + f*
x])^2) - (d*(3*b*c^3 - 5*a*c^2*d + 2*a*d^3)*Tan[e + f*x])/(2*c^2*(c^2 - d^2
)^2*f*(c + d*Sec[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
```

```
- b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\int \frac{-2a\left(c^2-d^2\right)-2c(bc-ad)\sec(e+fx)+d(bc-ad)\sec^2(e+fx)}{(c+d\sec(e+fx))^2}\,dx}{2c\left(c^2-d^2\right)}\\
&= -\frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{d\left(3bc^3-5ac^2d+2ad^3\right)\tan(e+fx)}{2c^2\left(c^2-d^2\right)^2f(c+d\sec(e+fx))} + \frac{\int \frac{2a\left(c^2-d^2\right)^2-c\left(ad\left(4c^2-d^2\right)-bc\left(2c^2+d^2\right)\right)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{2c^2\left(c^2-d^2\right)^2}\\
&= \frac{ax}{c^3} - \frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{d\left(3bc^3-5ac^2d+2ad^3\right)\tan(e+fx)}{2c^2\left(c^2-d^2\right)^2f(c+d\sec(e+fx))} + \frac{\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{2c^3\left(c^2-d^2\right)^2}\\
&= \frac{ax}{c^3} - \frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{d\left(3bc^3-5ac^2d+2ad^3\right)\tan(e+fx)}{2c^2\left(c^2-d^2\right)^2f(c+d\sec(e+fx))} + \frac{\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{2c^3d\left(c^2-d^2\right)^2}\\
&= \frac{ax}{c^3} - \frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{d\left(3bc^3-5ac^2d+2ad^3\right)\tan(e+fx)}{2c^2\left(c^2-d^2\right)^2f(c+d\sec(e+fx))} + \frac{\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{c^3d\left(c^2-d^2\right)^2f}\\
&= \frac{ax}{c^3} + \frac{\left(2bc^5-6ac^4d+bc^3d^2+5ac^2d^3-2ad^5\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} - \frac{d(bc-ad)\tan(e+fx)}{2c\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{d\left(3bc^3-5ac^2d+2ad^3\right)\tan(e+fx)}{2c^2\left(c^2-d^2\right)^2f(c+d\sec(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.31

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))\left(2a(e+fx)(d+c\cos(e+fx))^2 - \frac{2(bc^3(2c^2+d^2)+ad(-6c^4}{}\right.}{2c^3f(b+a\cos(e}$$

[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]

[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])*(2*a*(e + f*x)*(d + c*Cos[e + f*x])^2 - (2*(b*c^3*(2*c^2 + d^2) + a*d*(-6*c^4 + 5*c^2*d^2 - 2*d^4))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2)/(c^2 - d^2)^(5/2) + (c*d^2*(b*c - a*d)*Sin[e + f*x])/((c - d)*(c + d)) - (c*d*(4*b*c^3 - 6*a*c^2*d - b*c*d^2 + 3*a*d^3)*(d + c*Cos[e + f*x])*Sin[e + f*x])/((c - d)^2*(c + d)^2)))/(2*c^3*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3)

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a\,c^2d+ac\,d^2-2a\,d^3-4b\,c^3-b\,c^2d\right)cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{dc\left(6a\,c^2d-ac\,d^2-2a\,d^3-4b\,c^3+b\,c^2d\right)\tan\left(\frac{fx}{2}\right.}{2(c+d)(c-d)^2}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2 \quad c^3}}{f}$
default	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a\,c^2d+ac\,d^2-2a\,d^3-4b\,c^3-b\,c^2d\right)cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{dc\left(6a\,c^2d-ac\,d^2-2a\,d^3-4b\,c^3+b\,c^2d\right)\tan\left(\frac{fx}{2}\right.}{2(c+d)(c-d)^2}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2 \quad c^3}}{f}$
risch	Expression too large to display

[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(2*a/c^3*arctan(tan(1/2*f*x+1/2*e))+2/c^3*((-1/2*(6*a*c^2*d+a*c*d^2-2*a*d^3-4*b*c^3-b*c^2*d)*c*d/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*c*(6*a*c^2*d-a*c*d^2-2*a*d^3-4*b*c^3+b*c^2*d)/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-1/2*(6*a*c^4*d-5*a*c^2*d^3+2*a*d^5-2*b*c^5-b*c^3*d^2)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(191) = 382$.

Time = 0.34 (sec) , antiderivative size = 1152, normalized size of antiderivative = 5.65

$$\int \frac{a + b\sec(e + fx)}{(c + d\sec(e + fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*c^8 - 3*a*c^6*d^2 + 3*a*c^4*d^4 - a*c^2*d^6)*f*x*cos(f*x + e)^2
+ 8*(a*c^7*d - 3*a*c^5*d^3 + 3*a*c^3*d^5 - a*c*d^7)*f*x*cos(f*x + e) + 4*(a
*c^6*d^2 - 3*a*c^4*d^4 + 3*a*c^2*d^6 - a*d^8)*f*x - (2*b*c^5*d^2 - 6*a*c^4*
d^3 + b*c^3*d^4 + 5*a*c^2*d^5 - 2*a*d^7 + (2*b*c^7 - 6*a*c^6*d + b*c^5*d^2
+ 5*a*c^4*d^3 - 2*a*c^2*d^5)*cos(f*x + e)^2 + 2*(2*b*c^6*d - 6*a*c^5*d^2 +
b*c^4*d^3 + 5*a*c^3*d^4 - 2*a*c*d^6)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c
*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f
*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*
x + e) + d^2)) - 2*(3*b*c^6*d^2 - 5*a*c^5*d^3 - 3*b*c^4*d^4 + 7*a*c^3*d^5 -
 2*a*c*d^7 + (4*b*c^7*d - 6*a*c^6*d^2 - 5*b*c^5*d^3 + 9*a*c^4*d^4 + b*c^3*d
^5 - 3*a*c^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^
4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7
)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*f), 1/2*(2*(
a*c^8 - 3*a*c^6*d^2 + 3*a*c^4*d^4 - a*c^2*d^6)*f*x*cos(f*x + e)^2 + 4*(a*c^
7*d - 3*a*c^5*d^3 + 3*a*c^3*d^5 - a*c*d^7)*f*x*cos(f*x + e) + 2*(a*c^6*d^2
- 3*a*c^4*d^4 + 3*a*c^2*d^6 - a*d^8)*f*x + (2*b*c^5*d^2 - 6*a*c^4*d^3 + b*c
^3*d^4 + 5*a*c^2*d^5 - 2*a*d^7 + (2*b*c^7 - 6*a*c^6*d + b*c^5*d^2 + 5*a*c^4
*d^3 - 2*a*c^2*d^5)*cos(f*x + e)^2 + 2*(2*b*c^6*d - 6*a*c^5*d^2 + b*c^4*d^3
 + 5*a*c^3*d^4 - 2*a*c*d^6)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^
2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (3*b*c^6*d^2 -
5*a*c^5*d^3 - 3*b*c^4*d^4 + 7*a*c^3*d^5 - 2*a*c*d^7 + (4*b*c^7*d - 6*a*c^6*
d^2 - 5*b*c^5*d^3 + 9*a*c^4*d^4 + b*c^3*d^5 - 3*a*c^2*d^6)*cos(f*x + e))*si
n(f*x + e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*
(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^
7*d^4 + 3*c^5*d^6 - c^3*d^8)*f)]
```

Sympy [F]

$$\int \frac{a + b\sec(e + fx)}{(c + d\sec(e + fx))^3}\,dx = \int \frac{a + b\sec{(e + fx)}}{(c + d\sec{(e + fx)})^3}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(191) = 382.

Time = 0.37 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.24

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \frac{\frac{(2\,bc^5-6\,ac^4d+bc^3d^2+5\,ac^2d^3-2\,ad^5)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^7-2\,c^5d^2+c^3d^4)\sqrt{-c^2+d^2}} + \frac{(fx+e)a}{c^3} + \frac{4\,bc^4d\tan(\ldots}{\ldots}}{\ldots}$$

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*b*c^5 - 6*a*c^4*d + b*c^3*d^2 + 5*a*c^2*d^3 - 2*a*d^5)*(pi*floor(1/2*(f
*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan
(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c^5*d^2 + c^3*d^4)*sqrt(-c^
2 + d^2)) + (f*x + e)*a/c^3 + (4*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 6*a*c^3*d
^2*tan(1/2*f*x + 1/2*e)^3 - 3*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 5*a*c^2*d^
3*tan(1/2*f*x + 1/2*e)^3 - b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 3*a*c*d^4*tan
(1/2*f*x + 1/2*e)^3 - 2*a*d^5*tan(1/2*f*x + 1/2*e)^3 - 4*b*c^4*d*tan(1/2*f*
x + 1/2*e) + 6*a*c^3*d^2*tan(1/2*f*x + 1/2*e) - 3*b*c^3*d^2*tan(1/2*f*x + 1
/2*e) + 5*a*c^2*d^3*tan(1/2*f*x + 1/2*e) + b*c^2*d^3*tan(1/2*f*x + 1/2*e) -
 3*a*c*d^4*tan(1/2*f*x + 1/2*e) - 2*a*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 2*c
^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c
- d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 24.05 (sec) , antiderivative size = 6909, normalized size of antiderivative = 33.87

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int((a + b/cos(e + f*x))/(c + d/cos(e + f*x))^3,x)
```

```
[Out] (2*a*atan(((a*((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10
- 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*
d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b
^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 +
 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4
 - 3*c^8*d^3 - 3*c^9*d^2) + (a*((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9 + 2*a
*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a
*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c
^13*d^2 - 12*a*c^14*d - 4*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3
*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (a*tan(e/2 + (f*x)/2)*(8*
c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 4
8*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^1
1 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i
)/c^3))/c^3 + (a*((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^
10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c
^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2
+ b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^
5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*
d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (a*((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9 +
2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 3
4*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*
b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6
+ 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (a*tan(e/2 + (f*x)/2)*
(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6
+ 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d +
c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))
*1i)/c^3))/c^3)/((16*(4*a^3*d^9 + 4*a*b^2*c^9 - 4*a^2*b*c^9 - 2*a^3*c*d^8 +
 12*a^3*c^8*d - 18*a^3*c^2*d^7 + 13*a^3*c^3*d^6 + 36*a^3*c^4*d^5 - 26*a^3*c
^5*d^4 - 34*a^3*c^6*d^3 + 24*a^3*c^7*d^2 + a*b^2*c^5*d^4 + 4*a*b^2*c^7*d^2
- 2*a^2*b*c^2*d^7 - 2*a^2*b*c^3*d^6 + 2*a^2*b*c^4*d^5 + 2*a^2*b*c^6*d^3 + 6
*a^2*b*c^7*d^2 - 20*a^2*b*c^8*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^
8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (a*((8*tan(e/2 + (f*x)/2)*(4
*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^
2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 +
 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9
*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d
```

^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + (a*((8*(4*a *c^15 + 4*b*c^15 - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14*d))/(c ^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c ^11*d^2) - (a*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^ 8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c ^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3)*1i)/c^3 + (a*((8*tan(e/2 + (f*x) /2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32* a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6 *d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a *b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (a*((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9 *d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^ 8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14* d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (a*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^1 3*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3)*1i)/c^3)))/(c^3*f) - ((tan (e/2 + (f*x)/2)^3*(2*a*d^4 - 6*a*c^2*d^2 + b*c^2*d^2 - a*c*d^3 + 4*b*c^3*d))/((c^2*d - c^3)*(c + d)^2) + (tan(e/2 + (f*x)/2)*(2*a*d^4 - 6*a*c^2*d^2 - b*c^2*d^2 + a*c*d^3 + 4*b*c^3*d))/((c + d)*(c^4 - 2*c^3*d + c^2*d^2)))/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) + (atan(((((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8 *a^2*d^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^ 2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d ^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^ 3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + (((8*(4*a*c^15 + 4*b*c^1 5 - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (4*t an(e/2 + (f*x)/2)*((c + d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2* d^3 + b*c^3*d^2 - 6*a*c^4*d)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^ 8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8* c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11 *d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^ 3 - 3*c^9*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d ^3 + b*c^3*d^2 - 6*a*c^4*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 - 6*a*c^4*d)*1i)/(2*(c^13 - c^3*d^10 + 5*c^5*d^8

- 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a^2*c
^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8
+ 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^
2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4
*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c
^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (((8*(4*a*c^15 +
4*b*c^15 - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^
10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^
9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14*d))/(c^12*d +
c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2)
 + (4*tan(e/2 + (f*x)/2)*((c + d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5
*a*c^2*d^3 + b*c^3*d^2 - 6*a*c^4*d)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32
*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d
^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 -
 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3
*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*
a*c^2*d^3 + b*c^3*d^2 - 6*a*c^4*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^
7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2
*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 - 6*a*c^4*d)*1i)/(2*(c^13 - c^3*d^10 + 5*c
^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))/((16*(4*a^3*d^9 + 4*a*b^2*
c^9 - 4*a^2*b*c^9 - 2*a^3*c*d^8 + 12*a^3*c^8*d - 18*a^3*c^2*d^7 + 13*a^3*c^
3*d^6 + 36*a^3*c^4*d^5 - 26*a^3*c^5*d^4 - 34*a^3*c^6*d^3 + 24*a^3*c^7*d^2 +
 a*b^2*c^5*d^4 + 4*a*b^2*c^7*d^2 - 2*a^2*b*c^2*d^7 - 2*a^2*b*c^3*d^6 + 2*a^
2*b*c^4*d^5 + 2*a^2*b*c^6*d^3 + 6*a^2*b*c^7*d^2 - 20*a^2*b*c^8*d))/(c^12*d
+ c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^
2) - (((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*a^2*
c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48
*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d
^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c
^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8
*d^3 - 3*c^9*d^2) + (((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9 + 2*a*c^7*d^8 +
 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*d^3
- 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^2 -
12*a*c^14*d - 4*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 +
 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (4*tan(e/2 + (f*x)/2)*((c + d)^5*(c
 - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 - 6*a*c^4*d)*(8
*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 +
48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 +
5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7
- c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c
- d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 - 6*a*c^4*d))/(2
*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))*((c
 + d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 - 6*a
*c^4*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11
*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*

a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6
- 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c
^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a
*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3
*c^8*d^3 - 3*c^9*d^2) - (((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9 + 2*a*c^7*d
^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*
d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^
2 - 12*a*c^14*d - 4*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d
^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (4*tan(e/2 + (f*x)/2)*((c + d)^
5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 - 6*a*c^4*d
)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^
6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^1
0 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*
d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5
(c - d)^5)^(1/2)(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 - 6*a*c^4*d)
)/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))
((c + d)^5(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 -
 6*a*c^4*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*
c^11*d^2))))*((c + d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 +
 b*c^3*d^2 - 6*a*c^4*d)*1i)/(f*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 +
10*c^9*d^4 - 5*c^11*d^2))

3.192 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$

Optimal result 1307
Rubi [A] (verified) 1307
Mathematica [A] (verified) 1309
Maple [A] (verified) 1309
Fricas [B] (verification not implemented) 1310
Sympy [F] 1311
Maxima [F(-2)] 1311
Giac [A] (verification not implemented) 1311
Mupad [B] (verification not implemented) 1312

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{a^2x}{c^2} + \frac{2(bc-ad)\left(2ac^2-bcd-ad^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} + \frac{(bc-ad)^2\sin(e+fx)}{c\left(c^2-d^2\right)f(d+c\cos(e+fx))}$$

```
[Out] a^2*x/c^2+2*(-a*d+b*c)*(2*a*c^2-a*d^2-b*c*d)*arctanh((c-d)^(1/2)*tan(1/2*f*
x+1/2*e)/(c+d)^(1/2))/c^2/(c-d)^(3/2)/(c+d)^(3/2)/f+(-a*d+b*c)^2*sin(f*x+e)
/c/(c^2-d^2)/f/(d+c*cos(f*x+e))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4026, 2869, 2814, 2738, 214}

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{a^2x}{c^2} + \frac{2(bc-ad)\left(2ac^2-ad^2-bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2f(c-d)^{3/2}(c+d)^{3/2}} + \frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)(c\cos(e+fx)+d)}$$

```
[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*x)/c^2 + (2*(b*c - a*d)*(2*a*c^2 - b*c*d - a*d^2)*ArcTanh[(Sqrt[c - d]
*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^2*(c - d)^(3/2)*(c + d)^(3/2)*f) + ((b*
c - a*d)^2*Sin[e + f*x])/(c*(c^2 - d^2)*f*(d + c*Cos[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(b+a\cos(e+fx))^2}{(d+c\cos(e+fx))^2}\,dx \\
&= \frac{(bc-ad)^2\sin(e+fx)}{c\,(c^2-d^2)\,f(d+c\cos(e+fx))} - \frac{\int \frac{-c\left(2abc-\left(a^2+b^2\right)d\right)-a^2\left(c^2-d^2\right)\cos(e+fx)}{d+c\cos(e+fx)}\,dx}{c\,(c^2-d^2)} \\
&= \frac{a^2x}{c^2} + \frac{(bc-ad)^2\sin(e+fx)}{c\,(c^2-d^2)\,f(d+c\cos(e+fx))} \\
&\quad + \frac{\left(c^2\left(2abc-\left(a^2+b^2\right)d\right)-a^2d\left(c^2-d^2\right)\right)\int \frac{1}{d+c\cos(e+fx)}\,dx}{c^2\,(c^2-d^2)}
\end{aligned}$$

$$= \frac{a^2x}{c^2} + \frac{(bc-ad)^2\sin(e+fx)}{c\,(c^2-d^2)\,f(d+c\cos(e+fx))} + \frac{(2(c^2(2abc-(a^2+b^2)\,d)-a^2d(c^2-d^2)))\,\text{Subst}\left(\int \frac{1}{c+d+(-c+d)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{c^2\,(c^2-d^2)\,f}$$

$$= \frac{a^2x}{c^2} + \frac{2(bc-ad)\,(2ac^2-bcd-ad^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} + \frac{(bc-ad)^2\sin(e+fx)}{c\,(c^2-d^2)\,f(d+c\cos(e+fx))}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{a^2(e+fx) + \frac{2(-2abc^3+b^2c^2d+a^2(2c^2d-d^3))\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}} + \frac{c(bc-ad)^2\sin(e+fx)}{(c-d)(c+d)(d+c\cos(e+fx))}}{c^2f}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(e + f*x) + (2*(-2*a*b*c^3 + b^2*c^2*d + a^2*(2*c^2*d - d^3))*ArcTanh[
((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (c*(b*c -
 a*d)^2*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e + f*x])))/(c^2*f)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2} + \frac{-\frac{2\left(a^2d^2-2dabc+b^2c^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)} - \frac{2\left(2a^2c^2d-a^2d^3-2ab\,c^3+b^2c^2d\right)\text{arctanh}\left(\frac{(c-d)\cdots}{\sqrt{(\cdots}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}}{f}$
default	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2} + \frac{-\frac{2\left(a^2d^2-2dabc+b^2c^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)} - \frac{2\left(2a^2c^2d-a^2d^3-2ab\,c^3+b^2c^2d\right)\text{arctanh}\left(\frac{(c-d)\cdots}{\sqrt{(\cdots}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}}{f}$
risch	$\frac{a^2x}{c^2} + \frac{2i\left(a^2d^2-2dabc+b^2c^2\right)\left(d\,\mathrm{e}^{i(fx+e)}+c\right)}{c^2(c^2-d^2)f\left(\mathrm{e}^{2i(fx+e)}c+2d\,\mathrm{e}^{i(fx+e)}+c\right)} + \frac{2\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{c^2-d^2}\,d}{c\sqrt{c^2-d^2}}\right)a^2d}{\sqrt{c^2-d^2}\,(c+d)(c-d)f} - \frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ic^2+\cdots}{\cdots}\right)\cdots}{\sqrt{c^2-d^2}\,(c-\cdots}$

```
[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*a^2/c^2*arctan(tan(1/2*f*x+1/2*e))+2/c^2*(-(a^2*d^2-2*a*b*c*d+b^2*c^
2)*c/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e
)^2*d-c-d)-(2*a^2*c^2*d-a^2*d^3-2*a*b*c^3+b^2*c^2*d)/(c+d)/(c-d)/((c+d)*(c-
d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(124) = 248$.

Time = 0.32 (sec) , antiderivative size = 671, normalized size of antiderivative = 5.05

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$$
$$= \left[\frac{2\,(a^2c^5-2\,a^2c^3d^2+a^2cd^4)fx\cos{(fx+e)}+2\,(a^2c^4d-2\,a^2c^2d^3+a^2d^5)fx+(2\,abc^3d+a^2d^4-(2\,a^2+b^2\right.$$

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*c^5 - 2*a^2*c^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + 2*(a^2*c^4
*d - 2*a^2*c^2*d^3 + a^2*d^5)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2)*
c^2*d^2 + (2*a*b*c^4 + a^2*c*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sqrt(
c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(
c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^5 - 2*a*b*c^4*d + 2*a*b*c^2*d^
3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*
d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a^2*c^5 - 2*a^2*c
^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + (a^2*c^4*d - 2*a^2*c^2*d^3 + a^2*d^5
)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2)*c^2*d^2 + (2*a*b*c^4 + a^2*c
*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^
2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b^2*c^5 - 2*a*
b*c^4*d + 2*a*b*c^2*d^3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*sin(f*x + e))/((
c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f
)]
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.78

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{\frac{(fx+e)a^2}{c^2} + \frac{2\,(2\,abc^3-2\,a^2c^2d-b^2c^2d+a^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4-c^2d^2)\sqrt{-c^2+d^2}} - \frac{2\,(b^2c^2\tan(\frac{1}{2}}{(c^3-cd}}{f}$$

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^2/c^2 + 2*(2*a*b*c^3 - 2*a^2*c^2*d - b^2*c^2*d + a^2*d^3)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/
2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^4 - c^2*d^2)*sqrt(-c^
2 + d^2)) - 2*(b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b*c*d*tan(1/2*f*x + 1/2*e
) + a^2*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan(1/2*f*x + 1/2*e)^2
- d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f
```

Mupad [B] (verification not implemented)

Time = 22.90 (sec) , antiderivative size = 4934, normalized size of antiderivative = 37.10

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

[In] int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^2,x)

[Out] (2*a^2*atan(((a^2*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) + (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (a^2*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2))/c^2 + (a^2*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) - (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a^2*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2))/c^2)/((64*(a^6*d^5 - 2*a^5*b*c^5 - a^6*c*d^4 + 2*a^6*c^4*d + 4*a^4*b^2*c^5 - 3*a^6*c^2*d^3 + 2*a^6*c^3*d^2 - 4*a^3*b^3*c^4*d - a^4*b^2*c*d^4 + a^4*b^2*c^4*d + 2*a^5*b*c^2*d^3 + 2*a^5*b*c^3*d^2 + a^2*b^4*c^3*d^2 - a^4*b^2*c^2*d^3 + 3*a^4*b^2*c^3*d^2 - 6*a^5*b*c^4*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a^2*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) + (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (a^2*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2)*1i)/c^2 - (a^2*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) - (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4

```
 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*
a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a^2*tan(e/2 + (f*x)/2)*(
2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(
c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2)*1i)/c^2)))/(c^2*f) + (atan
((((a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^4*c^6
 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 +
4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c
^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 -
 c^2*d^3 - c^3*d^2) + (((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^
5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 -
 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c
^3*d^3 - c^4*d^2) - (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^3*(c - d)^3
)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*
d^4 - 4*c^7*d^3 - 2*c^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2
*d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*
d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*(a*d^2 - 2
*a*c^2 + b*c*d)*1i)/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2) + ((a*d - b*c)*
((c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 -
2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 +
 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*
b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*
d^2) - (((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d
^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*
a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^
2) + (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2
 - 2*a*c^2 + b*c*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3
 - 2*c^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^
4 - 3*c^6*d^2)))*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 +
 b*c*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*(a*d^2 - 2*a*c^2 + b*c*d)
*1i)/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))/((64*(a^6*d^5 - 2*a^5*b*c^5 -
 a^6*c*d^4 + 2*a^6*c^4*d + 4*a^4*b^2*c^5 - 3*a^6*c^2*d^3 + 2*a^6*c^3*d^2 -
4*a^3*b^3*c^4*d - a^4*b^2*c*d^4 + a^4*b^2*c^4*d + 2*a^5*b*c^2*d^3 + 2*a^5*b
*c^3*d^2 + a^2*b^4*c^3*d^2 - a^4*b^2*c^2*d^3 + 3*a^4*b^2*c^3*d^2 - 6*a^5*b*
c^4*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + ((a*d - b*c)*((c + d)^3*(c - d)
^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^
4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b
^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*
b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) + (((32*(2*a^
2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 +
 b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*
c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (32*tan(e/2 +
 (f*x)/2)*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d)
*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2))/((c
^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*(
a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^
```

```
2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*(a*d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^2*d^6 +
 3*c^4*d^4 - 3*c^6*d^2) - ((a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*((32*tan
(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^
2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3
*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*
b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) - (((32*(2*a^2*c^8*d - a^2*c^9
+ b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2
*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^
7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(a*d - b
*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d)*(2*c^9*d - 2*c^4*
d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2))/((c^4*d + c^5 - c^2*d
^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*(a*d - b*c)*((c + d
)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^2*d^6 + 3*c^4*d^4
- 3*c^6*d^2))*(a*d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6
*d^2)))*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d)*2
i)/(f*(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2)) - (2*tan(e/2 + (f*x)/2)*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))/(f*(c + d)*(c*d - c^2)*(c + d - tan(e/2 + (f*x
)/2)^2*(c - d)))
```

3.193 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3} dx$

Optimal result 1315
Rubi [A] (verified) 1315
Mathematica [B] (verified) 1318
Maple [A] (verified) 1319
Fricas [B] (verification not implemented) 1319
Sympy [F] 1320
Maxima [F(-2)] 1320
Giac [B] (verification not implemented) 1321
Mupad [B] (verification not implemented) 1321

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3} dx = \frac{a^2x}{c^3} - \frac{(3b^2c^4d-2abc^3(2c^2+d^2)+a^2(6c^4d-5c^2d^3+2d^5))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} - \frac{d(bc-ad)^2\sin(e+fx)}{2c^2(c^2-d^2)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)(3ad(2c^2-d^2)-bc(2c^2+d^2))\sin(e+fx)}{2c^2(c^2-d^2)^2f(d+c\cos(e+fx))}$$

```
[Out] a^2*x/c^3-(3*b^2*c^4*d-2*a*b*c^3*(2*c^2+d^2)+a^2*(6*c^4*d-5*c^2*d^3+2*d^5))
*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d)^
(5/2)/f-1/2*d*(-a*d+b*c)^2*sin(f*x+e)/c^2/(c^2-d^2)/f/(d+c*cos(f*x+e))^2-1/
2*(-a*d+b*c)*(3*a*d*(2*c^2-d^2)-b*c*(2*c^2+d^2))*sin(f*x+e)/c^2/(c^2-d^2)^2
/f/(d+c*cos(f*x+e))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

$= \{4026, 3067, 3100, 2814, 2738, 214\}$

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$
$$= -\frac{\left(a^2(6c^4d-5c^2d^3+2d^5)-2abc^3(2c^2+d^2)+3b^2c^4d\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}$$
$$+\frac{a^2x}{c^3}-\frac{(bc-ad)\left(3ad(2c^2-d^2)-bc(2c^2+d^2)\right)\sin(e+fx)}{2c^2f\left(c^2-d^2\right)^2(c\cos(e+fx)+d)}$$
$$-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

```
[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*x)/c^3 - ((3*b^2*c^4*d - 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^
2*d^3 + 2*d^5))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^3*(
c - d)^(5/2)*(c + d)^(5/2)*f) - (d*(b*c - a*d)^2*Sin[e + f*x])/(2*c^2*(c^2
- d^2)*f*(d + c*Cos[e + f*x])^2) - ((b*c - a*d)*(3*a*d*(2*c^2 - d^2) - b*c*
(2*c^2 + d^2))*Sin[e + f*x])/(2*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
```

```
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \int \frac{\cos(e+fx)(b+a\cos(e+fx))^2}{(d+c\cos(e+fx))^3}\,dx$$

$$= -\frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{\int \frac{-2c(bc-ad)^2+\left(b^2c^2d-2abc\left(2c^2-d^2\right)+a^2\left(2c^2d-d^3\right)\right)\cos(e+fx)-2a^2c\left(c^2-d^2\right)\cos^2(e+fx)}{(d+c\cos(e+fx))^2}\,dx}{2c^2\left(c^2-d^2\right)}$$

$$= -\frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\left(3ad(2c^2-d^2)-bc(2c^2+d^2)\right)\sin(e+fx)}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))} - \frac{\int \frac{-c^2(bc-ad)\left(4ac^2-3bcd-ad^2\right)-2a^2c\left(c^2-d^2\right)^2\cos(e+fx)}{d+c\cos(e+fx)}\,dx}{2c^3\left(c^2-d^2\right)^2}$$

$$= \frac{a^2x}{c^3} - \frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\left(3ad(2c^2-d^2)-bc(2c^2+d^2)\right)\sin(e+fx)}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))} - \frac{\left(3b^2c^4d-2abc^3(2c^2+d^2)+a^2(6c^4d-5c^2d^3+2d^5)\right)\int \frac{1}{d+c\cos(e+fx)}\,dx}{2c^3\left(c^2-d^2\right)^2}$$

$$= \frac{a^2x}{c^3} - \frac{d(bc-ad)^2\sin(e+fx)}{2c^2(c^2-d^2)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)(3ad(2c^2-d^2)-bc(2c^2+d^2))\sin(e+fx)}{2c^2(c^2-d^2)^2f(d+c\cos(e+fx))} - \frac{(3b^2c^4d-2abc^3(2c^2+d^2)+a^2(6c^4d-5c^2d^3+2d^5))\,\text{Subst}\left(\int\frac{1}{c+d+(-c+d)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right.}{c^3(c^2-d^2)^2f}$$

$$= \frac{a^2x}{c^3} + \frac{(4abc^5-6a^2c^4d-3b^2c^4d+2abc^3d^2+5a^2c^2d^3-2a^2d^5)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} - \frac{d(bc-ad)^2\sin(e+fx)}{2c^2(c^2-d^2)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)(3ad(2c^2-d^2)-bc(2c^2+d^2))\sin(e+fx)}{2c^2(c^2-d^2)^2f(d+c\cos(e+fx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 493 vs. 2(237) = 474.

Time = 2.69 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.08

$$\int\frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec(e+fx)(a+b\sec(e+fx))^2\left(\frac{4(3b^2c^4d-2abc^3(2c^2+d^2)+a^2(6c^4d-5c^2d^3+2d^5))\text{arctanh}\left(\frac{(-c+d)\,\text{ta}}{\sqrt{}}\right.}{(c^2-d^2)^{5/2}}\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^2*((4*(3*b^2*c^4*d
- 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^2*d^3 + 2*d^5))*ArcTanh[((-c
 + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2)/(c^2 - d^2
)^(5/2) + (2*a^2*c^6*e - 6*a^2*c^2*d^4*e + 4*a^2*d^6*e + 2*a^2*c^6*f*x - 6*
a^2*c^2*d^4*f*x + 4*a^2*d^6*f*x + 8*a^2*c*d*(c^2 - d^2)^2*(e + f*x)*Cos[e +
 f*x] + 2*a^2*c^2*(c^2 - d^2)^2*(e + f*x)*Cos[2*(e + f*x)] + 2*b^2*c^5*d*Si
n[e + f*x] - 12*a*b*c^4*d^2*Sin[e + f*x] + 10*a^2*c^3*d^3*Sin[e + f*x] + 4*
b^2*c^3*d^3*Sin[e + f*x] - 4*a^2*c*d^5*Sin[e + f*x] + 2*b^2*c^6*Sin[2*(e +
f*x)] - 8*a*b*c^5*d*Sin[2*(e + f*x)] + 6*a^2*c^4*d^2*Sin[2*(e + f*x)] + b^2
*c^4*d^2*Sin[2*(e + f*x)] + 2*a*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^2*c^2*d^4*
Sin[2*(e + f*x)])/(c^2 - d^2)^2))/(4*c^3*f*(b + a*Cos[e + f*x])^2*(c + d*Se
c[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a^2c^2d^2+a^2d^3c-2a^2d^4-8ab\,c^3d-2ab\,c^2d^2+2b^2c^4+b^2c^3d+2b^2c^2d^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{c\left(6a^2c^2\cdots\right.}{\cdots}\cdots\right)}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d\cdots\right.}\cdots$
default	$\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a^2c^2d^2+a^2d^3c-2a^2d^4-8ab\,c^3d-2ab\,c^2d^2+2b^2c^4+b^2c^3d+2b^2c^2d^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{c\left(6a^2c^2\cdots\right.}{\cdots}\cdots\right)}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d\cdots\right.}\cdots$
risch	Expression too large to display

```
[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*a^2/c^3*arctan(tan(1/2*f*x+1/2*e))+2/c^3*((-1/2*(6*a^2*c^2*d^2+a^2*c
*d^3-2*a^2*d^4-8*a*b*c^3*d-2*a*b*c^2*d^2+2*b^2*c^4+b^2*c^3*d+2*b^2*c^2*d^2)
*c/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*c*(6*a^2*c^2*d^2-a^2*c*d^
3-2*a^2*d^4-8*a*b*c^3*d+2*a*b*c^2*d^2+2*b^2*c^4-b^2*c^3*d+2*b^2*c^2*d^2)/(c
+d)/(c-d)^2*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^
2*d-c-d)^2-1/2*(6*a^2*c^4*d-5*a^2*c^2*d^3+2*a^2*d^5-4*a*b*c^5-2*a*b*c^3*d^2
+3*b^2*c^4*d)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2
*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(223) = 446$.

Time = 0.36 (sec) , antiderivative size = 1409, normalized size of antiderivative = 5.95

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*d^4 - a^2*c^2*d^6)*f*x*cos(f*x
 + e)^2 + 8*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a^2*c^3*d^5 - a^2*c*d^7)*f*x*cos
(f*x + e) + 4*(a^2*c^6*d^2 - 3*a^2*c^4*d^4 + 3*a^2*c^2*d^6 - a^2*d^8)*f*x -
 (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^2*d^5 - 2*a^2*d^7 - 3*(2*a^2 + b^
2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^2 + 5*a^2*c^4*d^3 - 2*a^2*c^2*d^5 - 3
*(2*a^2 + b^2)*c^6*d)*cos(f*x + e)^2 + 2*(4*a*b*c^6*d + 2*a*b*c^4*d^3 + 5*a
^2*c^3*d^4 - 2*a^2*c*d^6 - 3*(2*a^2 + b^2)*c^5*d^2)*cos(f*x + e))*sqrt(c^2
- d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2
```

```
- d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2
 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^7*d - 6*a*b*c^6*d^2 + 6*a*b*c^4*d^
4 + 2*a^2*c*d^7 + (5*a^2 + b^2)*c^5*d^3 - (7*a^2 + 2*b^2)*c^3*d^5 + (2*b^2*
c^8 - 8*a*b*c^7*d + 10*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + 3*a^2*c^2*d^6 + (6*a^2
 - b^2)*c^6*d^2 - (9*a^2 + b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11
 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^
3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6
- c^3*d^8)*f), 1/2*(2*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*d^4 - a^2*c^2*d^
6)*f*x*cos(f*x + e)^2 + 4*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a^2*c^3*d^5 - a^2*
c*d^7)*f*x*cos(f*x + e) + 2*(a^2*c^6*d^2 - 3*a^2*c^4*d^4 + 3*a^2*c^2*d^6 -
a^2*d^8)*f*x + (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^2*d^5 - 2*a^2*d^7 -
 3*(2*a^2 + b^2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^2 + 5*a^2*c^4*d^3 - 2*a
^2*c^2*d^5 - 3*(2*a^2 + b^2)*c^6*d)*cos(f*x + e)^2 + 2*(4*a*b*c^6*d + 2*a*b
*c^4*d^3 + 5*a^2*c^3*d^4 - 2*a^2*c*d^6 - 3*(2*a^2 + b^2)*c^5*d^2)*cos(f*x +
 e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
 d^2)*sin(f*x + e))) + (b^2*c^7*d - 6*a*b*c^6*d^2 + 6*a*b*c^4*d^4 + 2*a^2*c
*d^7 + (5*a^2 + b^2)*c^5*d^3 - (7*a^2 + 2*b^2)*c^3*d^5 + (2*b^2*c^8 - 8*a*b
*c^7*d + 10*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + 3*a^2*c^2*d^6 + (6*a^2 - b^2)*c^6
*d^2 - (9*a^2 + b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^
2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d
^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*
f)]
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx=\int \frac{(a+b\sec{(e+fx)})^2}{(c+d\sec{(e+fx)})^3}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(223) = 446$.

Time = 0.40 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.78

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \frac{\frac{(4\,abc^5-6\,a^2c^4d-3\,b^2c^4d+2\,abc^3d^2+5\,a^2c^2d^3-2\,a^2d^5)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^7-2\,c^5d^2+c^3d^4)\sqrt{-c^2+d^2}} + \frac{(fx+e}{c^3}}{}$$

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((4*a*b*c^5 - 6*a^2*c^4*d - 3*b^2*c^4*d + 2*a*b*c^3*d^2 + 5*a^2*c^2*d^3 - 2
*a^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*ta
n(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c
^5*d^2 + c^3*d^4)*sqrt(-c^2 + d^2)) + (f*x + e)*a^2/c^3 - (2*b^2*c^5*tan(1/
2*f*x + 1/2*e)^3 - 8*a*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - b^2*c^4*d*tan(1/2*f
*x + 1/2*e)^3 + 6*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*b*c^3*d^2*tan(1/
2*f*x + 1/2*e)^3 + b^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*a^2*c^2*d^3*tan(1
/2*f*x + 1/2*e)^3 + 2*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b^2*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 - 3*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^2*d^5*tan(1
/2*f*x + 1/2*e)^3 - 2*b^2*c^5*tan(1/2*f*x + 1/2*e) + 8*a*b*c^4*d*tan(1/2*f*
x + 1/2*e) - b^2*c^4*d*tan(1/2*f*x + 1/2*e) - 6*a^2*c^3*d^2*tan(1/2*f*x + 1
/2*e) + 6*a*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - b^2*c^3*d^2*tan(1/2*f*x + 1/2*
e) - 5*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 2*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e
) - 2*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 3*a^2*c*d^4*tan(1/2*f*x + 1/2*e) +
 2*a^2*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*
x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 24.42 (sec) , antiderivative size = 8682, normalized size of antiderivative = 36.63

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^3,x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^3*(2*b^2*c^4 - 2*a^2*d^4 + a^2*c*d^3 + b^2*c^3*d + 6*a
^2*c^2*d^2 + 2*b^2*c^2*d^2 - 8*a*b*c^3*d - 2*a*b*c^2*d^2))/((c^2*d - c^3)*(
c + d)^2) - (tan(e/2 + (f*x)/2)*(2*a^2*d^4 - 2*b^2*c^4 + a^2*c*d^3 + b^2*c^
3*d - 6*a^2*c^2*d^2 - 2*b^2*c^2*d^2 + 8*a*b*c^3*d - 2*a*b*c^2*d^2))/((c + d
)*(c^4 - 2*c^3*d + c^2*d^2)))/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d
```

^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) - (2*a^2*atan(
((a^2*((a^2*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c^6*d^9
+ 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5 + 6*a^2*
c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^2*c^10*d^
5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^15 - 8*a*b
*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a*b*c^13*d^2
))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3
- 3*c^11*d^2) - (a^2*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9
+ 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^
13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^
5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3 + (8*tan(e/2 + (f*x)/2)*(4
*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^2*b^2*c^10 - 32*a
^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5*d^5 - 52*a^4*c^6*
d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 12*a*b^3*c^7*d^3 -
8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2*b^2*c^4*d^6 -
26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d - 48*a^3*b*c^9*d)
)/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 -
3*c^9*d^2)))/c^3 - (a^2*((a^2*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6*b^2*c^14*
d - 4*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2
*c^10*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d
^6 - 6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 +
8*a*b*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6 + 12*a*b*c^12*d^3
- 12*a*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9
*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (a^2*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6
*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 3
2*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 -
c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3 - (8*tan
(e/2 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a
^2*b^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5
*d^5 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 1
2*a*b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12
*a^2*b^2*c^4*d^6 - 26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d
- 48*a^3*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*
d^4 - 3*c^8*d^3 - 3*c^9*d^2)))/c^3)/((a^2*((a^2*((8*(4*a^2*c^15 - 12*a^2*c^
14*d - 6*b^2*c^14*d - 4*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^
2*c^9*d^6 - 36*a^2*c^10*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13
*d^2 + 6*b^2*c^9*d^6 - 6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 +
6*b^2*c^13*d^2 + 8*a*b*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6
+ 12*a*b*c^12*d^3 - 12*a*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 +
3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (a^2*tan(e/2 + (f*x)/2)
*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6
+ 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d +
c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2))
)*1i)/c^3 + (8*tan(e/2 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 -
8*a^4*c^9*d + 16*a^2*b^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^

4*d^6 - 48*a^4*c^5*d^5 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 12*a*b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2*b^2*c^4*d^6 - 26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d - 48*a^3*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2))*1i)/c^3 - (16*(4*a^6*d^9 - 8*a^5*b*c^9 - 2*a^6*c*d^8 + 12*a^6*c^8*d + 16*a^4*b^2*c^9 - 18*a^6*c^2*d^7 + 13*a^6*c^3*d^6 + 36*a^6*c^4*d^5 - 26*a^6*c^5*d^4 - 34*a^6*c^6*d^3 + 24*a^6*c^7*d^2 - 24*a^3*b^3*c^8*d + 6*a^4*b^2*c^8*d - 4*a^5*b*c^2*d^7 - 4*a^5*b*c^3*d^6 + 4*a^5*b*c^4*d^5 + 4*a^5*b*c^6*d^3 + 12*a^5*b*c^7*d^2 + 9*a^2*b^4*c^7*d^2 - 12*a^3*b^3*c^6*d^3 + 6*a^4*b^2*c^3*d^6 + 6*a^4*b^2*c^4*d^5 - 14*a^4*b^2*c^5*d^4 - 12*a^4*b^2*c^6*d^3 + 46*a^4*b^2*c^7*d^2 - 40*a^5*b*c^8*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (a^2*((a^2*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (a^2*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3 - (8*tan(e/2 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^2*b^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5*d^5 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 12*a*b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2*b^2*c^4*d^6 - 26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d - 48*a^3*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2))*1i)/c^3)))/(c^3*f) + (atan(((((8*tan(e/2 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^2*b^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5*d^5 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 12*a*b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2*b^2*c^4*d^6 - 26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d - 48*a^3*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + (((c + d)^5*(c - d)^5)^(1/2)*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (4*tan(e/2 + (f*x)/2)*((c + d)^5*(c - d)^5)^(1/2)*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3

- 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5 *c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c ^8*d^3 - 3*c^9*d^2)))*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*a^2*c^2*d^ 3 - 4*a*b*c^5 - 2*a*b*c^3*d^2))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^ 6 + 10*c^9*d^4 - 5*c^11*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(2*a^2*d^5 + 6*a ^2*c^4*d + 3*b^2*c^4*d - 5*a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2)*1i)/(2* (c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^2*b^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^ 4*c^5*d^5 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^ 2 - 12*a*b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2*b^2*c^4*d^6 - 26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3* c^9*d - 48*a^3*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3 *c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (((c + d)^5*(c - d)^5)^(1/2)*((8*(4*a^2 *c^15 - 12*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a ^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12 *d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^ 7 + 4*a*b*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a*b*c^13*d^2))/(c^12*d + c^13 - c^ 6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (4*tan (e/2 + (f*x)/2)*((c + d)^5*(c - d)^5)^(1/2)*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^ 2*c^4*d - 5*a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^1 2*d^4 - 32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d ^6 + 10*c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^ 5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c ^4*d - 5*a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2))/(2*(c^13 - c^3*d^10 + 5* c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))*((c + d)^5*(c - d)^5)^(1/ 2)*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*a^2*c^2*d^3 - 4*a*b*c^5 - 2*a *b*c^3*d^2)*1i)/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))/((16*(4*a^6*d^9 - 8*a^5*b*c^9 - 2*a^6*c*d^8 + 12*a^6*c^8*d + 16*a^4*b^2*c^9 - 18*a^6*c^2*d^7 + 13*a^6*c^3*d^6 + 36*a^6*c^4*d^5 - 26*a^6 *c^5*d^4 - 34*a^6*c^6*d^3 + 24*a^6*c^7*d^2 - 24*a^3*b^3*c^8*d + 6*a^4*b^2*c ^8*d - 4*a^5*b*c^2*d^7 - 4*a^5*b*c^3*d^6 + 4*a^5*b*c^4*d^5 + 4*a^5*b*c^6*d^ 3 + 12*a^5*b*c^7*d^2 + 9*a^2*b^4*c^7*d^2 - 12*a^3*b^3*c^6*d^3 + 6*a^4*b^2*c ^3*d^6 + 6*a^4*b^2*c^4*d^5 - 14*a^4*b^2*c^5*d^4 - 12*a^4*b^2*c^6*d^3 + 46*a ^4*b^2*c^7*d^2 - 40*a^5*b*c^8*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^ 8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (((8*tan(e/2 + (f*x)/2)*(4*a ^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^2*b^2*c^10 - 32*a^4 *c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5*d^5 - 52*a^4*c^6*d^ 4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 12*a*b^3*c^7*d^3 - 8* a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2*b^2*c^4*d^6 - 2 6*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d - 48*a^3*b*c^9*d))/ (c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3* c^9*d^2) + (((c + d)^5*(c - d)^5)^(1/2)*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6

*b^2*c^14*d - 4*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^
6 - 36*a^2*c^10*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6
*b^2*c^9*d^6 - 6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c
^13*d^2 + 8*a*b*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6 + 12*a*
b*c^12*d^3 - 12*a*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d
^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (4*tan(e/2 + (f*x)/2)*((c + d)^
5*(c - d)^5)^(1/2)*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*a^2*c^2*d^3 -
 4*a*b*c^5 - 2*a*b*c^3*d^2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8
 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c
^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*
d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3
 - 3*c^9*d^2)))*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*a^2*c^2*d^3 - 4*
a*b*c^5 - 2*a*b*c^3*d^2))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10
*c^9*d^4 - 5*c^11*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(2*a^2*d^5 + 6*a^2*c^4
*d + 3*b^2*c^4*d - 5*a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2))/(2*(c^13 - c
^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)) + (((8*tan(e/2
 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^2*b
^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5*d^5
 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 12*a*
b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2
*b^2*c^4*d^6 - 26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d - 4
8*a^3*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4
 - 3*c^8*d^3 - 3*c^9*d^2) - (((c + d)^5*(c - d)^5)^(1/2)*((8*(4*a^2*c^15 - 1
2*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^
7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*
a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^
12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b
*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c
^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (4*tan(e/2 + (f
x)/2)((c + d)^5*(c - d)^5)^(1/2)*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d -
 5*a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*
d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 3
2*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c
^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7
*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*
a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 -
 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(2*a^2
*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^
2))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)
)))*((c + d)^5*(c - d)^5)^(1/2)*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*
a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2)*1i)/(f*(c^13 - c^3*d^10 + 5*c^5*d^
8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2))

3.194 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

Optimal result 1326
Rubi [A] (verified) 1327
Mathematica [A] (verified) 1330
Maple [A] (verified) 1331
Fricas [B] (verification not implemented) 1331
Sympy [F] 1333
Maxima [F(-2)] 1333
Giac [B] (verification not implemented) 1333
Mupad [B] (verification not implemented) 1334

Optimal result

Integrand size = 25, antiderivative size = 377

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \frac{a^2x}{c^4} - \frac{(b^2c^4d(4c^2+d^2)-ab(4c^7+6c^5d^2)+a^2(8c^6d-8c^4d^3+7c^2d^5-2d^7))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^4(c-d)^{7/2}(c+d)^{7/2}f} + \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)(6bc^3-8ac^2d-bcd^2+3ad^3)\sin(e+fx)}{6c^3(c^2-d^2)^2f(d+c\cos(e+fx))^2} - \frac{(2abcd(18c^4-5c^2d^2+2d^4)-a^2d^2(34c^4-28c^2d^2+9d^4)-b^2(6c^6+10c^4d^2-c^2d^4))\sin(e+fx)}{6c^3(c^2-d^2)^3f(d+c\cos(e+fx))}$$

```
[Out] a^2*x/c^4-(b^2*c^4*d*(4*c^2+d^2)-a*b*(4*c^7+6*c^5*d^2)+a^2*(8*c^6*d-8*c^4*d
^3+7*c^2*d^5-2*d^7))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^
4/(c-d)^(7/2)/(c+d)^(7/2)/f+1/3*d^2*(b+a*cos(f*x+e))^2*sin(f*x+e)/c/(c^2-d^
2)/f/(d+c*cos(f*x+e))^3-1/6*d*(-a*d+b*c)*(-8*a*c^2*d+3*a*d^3+6*b*c^3-b*c*d^
2)*sin(f*x+e)/c^3/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^2-1/6*(2*a*b*c*d*(18*c^4-5
*c^2*d^2+2*d^4)-a^2*d^2*(34*c^4-28*c^2*d^2+9*d^4)-b^2*(6*c^6+10*c^4*d^2-c^2
*d^4))*sin(f*x+e)/c^3/(c^2-d^2)^3/f/(d+c*cos(f*x+e))
```

Rubi [A] (verified)

Time = 2.80 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4026, 3127, 3110, 3100, 2814, 2738, 214}

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = -\frac{(a^2(8c^6d-8c^4d^3+7c^2d^5-2d^7)-ab(4c^7+6c^5d^2)+b^2c^4d(4c^2+d^2))\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^4f(c-d)^{7/2}(c+d)^{7/2}} - \frac{(-a^2d^2(34c^4-28c^2d^2+9d^4)+2abcd(18c^4-5c^2d^2+2d^4)-(b^2(6c^6+10c^4d^2-c^2d^4)))\sin(e+fx)}{6c^3f\left(c^2-d^2\right)^3(c\cos(e+fx)+d)} + \frac{a^2x}{c^4} + \frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3} - \frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{6c^3f\left(c^2-d^2\right)^2(c\cos(e+fx)+d)^2}$$

```
[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*x)/c^4 - ((b^2*c^4*d*(4*c^2 + d^2) - a*b*(4*c^7 + 6*c^5*d^2) + a^2*(8*
c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/
2])/Sqrt[c + d]])/(c^4*(c - d)^(7/2)*(c + d)^(7/2)*f) + (d^2*(b + a*Cos[e +
 f*x])^2*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3) - (d*(b*c
 - a*d)*(6*b*c^3 - 8*a*c^2*d - b*c*d^2 + 3*a*d^3)*Sin[e + f*x])/(6*c^3*(c^2
 - d^2)^2*f*(d + c*Cos[e + f*x])^2) - ((2*a*b*c*d*(18*c^4 - 5*c^2*d^2 + 2*d
^4) - a^2*d^2*(34*c^4 - 28*c^2*d^2 + 9*d^4) - b^2*(6*c^6 + 10*c^4*d^2 - c^2
*d^4))*Sin[e + f*x])/(6*c^3*(c^2 - d^2)^3*f*(d + c*Cos[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^2}{(d+c\cos(e+fx))^4}\,dx$$

$$= \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{\int \frac{(b+a\cos(e+fx))\left(-d(3bc-2ad)+\left(3bc^2-3acd-bd^2\right)\cos(e+fx)+3a\left(c^2-d^2\right)\cos^2(e+fx)\right)}{(d+c\cos(e+fx))^3}\,dx}{3c\left(c^2-d^2\right)}$$

$$= \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+3ad^3\right)\sin(e+fx)}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2} - \frac{\int \frac{-2c\left(a^2d^2\left(8c^2-3d^2\right)-2abcd\left(6c^2-d^2\right)+b^2\left(3c^4+2c^2d^2\right)\right)+\left(b^2c^2d\left(6c^2-d^2\right)-2abc\left(6c^4-3c^2d^2+2d^4\right)+a^2\left(12c^4d-10c^2d^3+3d^5\right)\right) c}{(d+c\cos(e+fx))^2}}{6c^3\left(c^2-d^2\right)^2}$$

$$= \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+3ad^3\right)\sin(e+fx)}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2} - \frac{(2abcd(18c^4-5c^2d^2+2d^4)-a^2d^2(34c^4-28c^2d^2+9d^4)-b^2(6c^6+10c^4d^2-c^2d^4))\sin(e+fx}{6c^3\left(c^2-d^2\right)^3f(d+c\cos(e+fx))} - \frac{\int \frac{3c^2\left(b^2c^2d\left(4c^2+d^2\right)-2abc^3\left(2c^2+3d^2\right)+a^2\left(6c^4d-2c^2d^3+d^5\right)\right)-6a^2c\left(c^2-d^2\right)^3\cos(e+fx)}{d+c\cos(e+fx)}\,dx}{6c^4\left(c^2-d^2\right)^3}$$

$$= \frac{a^2x}{c^4} + \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+3ad^3\right)\sin(e+fx)}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2} - \frac{(2abcd(18c^4-5c^2d^2+2d^4)-a^2d^2(34c^4-28c^2d^2+9d^4)-b^2(6c^6+10c^4d^2-c^2d^4))\sin(e+fx}{6c^3\left(c^2-d^2\right)^3f(d+c\cos(e+fx))} - \frac{(b^2c^4d(4c^2+d^2)-2abc^5(2c^2+3d^2)+a^2(8c^6d-8c^4d^3+7c^2d^5-2d^7))\int \frac{1}{d+c\cos(e+fx)}\,dx}{2c^4\left(c^2-d^2\right)^3}$$

$$= \frac{a^2x}{c^4} + \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+3ad^3\right)\sin(e+fx)}{6c^3\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2} - \frac{(2abcd(18c^4-5c^2d^2+2d^4)-a^2d^2(34c^4-28c^2d^2+9d^4)-b^2(6c^6+10c^4d^2-c^2d^4))\sin(e+fx}{6c^3\left(c^2-d^2\right)^3f(d+c\cos(e+fx))} - \frac{(b^2c^4d(4c^2+d^2)-2abc^5(2c^2+3d^2)+a^2(8c^6d-8c^4d^3+7c^2d^5-2d^7))\,\text{Subst}\Bigg(\int \frac{1}{c+d+(-c+d)x^2}\,dx}{c^4\left(c^2-d^2\right)^3f}$$

$$= \frac{a^2 x}{c^4} + \frac{(4abc^7 - 8a^2c^6d - 4b^2c^6d + 6abc^5d^2 + 8a^2c^4d^3 - b^2c^4d^3 - 7a^2c^2d^5 + 2a^2d^7)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan(\frac{1}{2}(e+}{\sqrt{c+d}}\right.}{c^4(c-d)^{7/2}(c+d)^{7/2}f} + \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)(6bc^3-8ac^2d-bcd^2+3ad^3)\sin(e+fx)}{6c^3(c^2-d^2)^2f(d+c\cos(e+fx))^2} - \frac{(2abcd(18c^4-5c^2d^2+2d^4)-a^2d^2(34c^4-28c^2d^2+9d^4)-b^2(6c^6+10c^4d^2-c^2d^4))\sin(e+fx)}{6c^3(c^2-d^2)^3f(d+c\cos(e+fx))}$$

Mathematica [A] (verified)

Time = 4.42 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))^2\left(6a^2(e+fx)(d+c\cos(e+fx))^3+\frac{6(b^2c^4d(4c^2+d^2)-2ab}{}\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^2*(6*a^2*(e + f*x
)*(d + c*Cos[e + f*x])^3 + (6*(b^2*c^4*d*(4*c^2 + d^2) - 2*a*b*c^5*(2*c^2 +
 3*d^2) + a^2*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[((-c + d)*
Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^3)/(c^2 - d^2)^(7/2
) + (2*c*d^2*(b*c - a*d)^2*Sin[e + f*x])/(c^2 - d^2) - (c*d*(a^2*d^2*(12*c^
2 - 7*d^2) + b^2*(6*c^4 - c^2*d^2) + a*b*(-18*c^3*d + 8*c*d^3))*(d + c*Cos[
e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-2*a*b*c*d*(18*c^4 - 5*c^2*d^2
+ 2*d^4) + a^2*d^2*(36*c^4 - 32*c^2*d^2 + 11*d^4) + b^2*(6*c^6 + 10*c^4*d^2
 - c^2*d^4))*(d + c*Cos[e + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3))/(6*c^4*f*
(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^4)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.68

method	result
derivativedivides	$\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^2c^4d^2+4a^2c^3d^3-6a^2d^4c^2-a^2c\,d^5+2a^2d^6-12dab\,c^5-6ab\,c^4d^2-4ab\,d^3c^3+2b^2c^6+2b^2c^5d+6b\cdots\right.}{2(c-d)\left(c^3+3c^2d+3c\,d^2+d^3\right)}\cdots\right.}{\cdots}$
default	$\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^2c^4d^2+4a^2c^3d^3-6a^2d^4c^2-a^2c\,d^5+2a^2d^6-12dab\,c^5-6ab\,c^4d^2-4ab\,d^3c^3+2b^2c^6+2b^2c^5d+6b\cdots\right.}{2(c-d)\left(c^3+3c^2d+3c\,d^2+d^3\right)}\cdots\right.}{\cdots}$
risch	Expression too large to display

```
[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*a^2/c^4*arctan(tan(1/2*f*x+1/2*e))+2/c^4*((-1/2*(12*a^2*c^4*d^2+4*a^
2*c^3*d^3-6*a^2*c^2*d^4-a^2*c*d^5+2*a^2*d^6-12*a*b*c^5*d-6*a*b*c^4*d^2-4*a*
b*c^3*d^3+2*b^2*c^6+2*b^2*c^5*d+6*b^2*c^4*d^2+b^2*c^3*d^3)*c/(c-d)/(c^3+3*c
^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(18*a^2*c^4*d^2-11*a^2*c^2*d^4+3
*a^2*d^6-18*a*b*c^5*d-2*a*b*c^3*d^3+3*b^2*c^6+7*b^2*c^4*d^2)*c/(c^2-2*c*d+d
^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/2*(12*a^2*c^4*d^2-4*a^2*c^3*d^3-
6*a^2*c^2*d^4+a^2*c*d^5+2*a^2*d^6-12*a*b*c^5*d+6*a*b*c^4*d^2-4*a*b*c^3*d^3+
2*b^2*c^6-2*b^2*c^5*d+6*b^2*c^4*d^2-b^2*c^3*d^3)*c/(c+d)/(c^3-3*c^2*d+3*c*d
^2-d^3)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-
c-d)^3-1/2*(8*a^2*c^6*d-8*a^2*c^4*d^3+7*a^2*c^2*d^5-2*a^2*d^7-4*a*b*c^7-6*a
*b*c^5*d^2+4*b^2*c^6*d+b^2*c^4*d^3)/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c
-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1152 vs. 2(362) = 724.

Time = 0.45 (sec) , antiderivative size = 2362, normalized size of antiderivative = 6.27

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^2*c^11 - 4*a^2*c^9*d^2 + 6*a^2*c^7*d^4 - 4*a^2*c^5*d^6 + a^2*c
^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^2*c^10*d - 4*a^2*c^8*d^3 + 6*a^2*c^6*d^5
 - 4*a^2*c^4*d^7 + a^2*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^2*c^9*d^2 - 4*a^
2*c^7*d^4 + 6*a^2*c^5*d^6 - 4*a^2*c^3*d^8 + a^2*c*d^10)*f*x*cos(f*x + e) +
```

```
12*(a^2*c^8*d^3 - 4*a^2*c^6*d^5 + 6*a^2*c^4*d^7 - 4*a^2*c^2*d^9 + a^2*d^11)
*f*x - 3*(4*a*b*c^7*d^3 + 6*a*b*c^5*d^5 - 7*a^2*c^2*d^8 + 2*a^2*d^10 - 4*(2
*a^2 + b^2)*c^6*d^4 + (8*a^2 - b^2)*c^4*d^6 + (4*a*b*c^10 + 6*a*b*c^8*d^2 -
 7*a^2*c^5*d^5 + 2*a^2*c^3*d^7 - 4*(2*a^2 + b^2)*c^9*d + (8*a^2 - b^2)*c^7*
d^3)*cos(f*x + e)^3 + 3*(4*a*b*c^9*d + 6*a*b*c^7*d^3 - 7*a^2*c^4*d^6 + 2*a^
2*c^2*d^8 - 4*(2*a^2 + b^2)*c^8*d^2 + (8*a^2 - b^2)*c^6*d^4)*cos(f*x + e)^2
 + 3*(4*a*b*c^8*d^2 + 6*a*b*c^6*d^4 - 7*a^2*c^3*d^7 + 2*a^2*c*d^9 - 4*(2*a^
2 + b^2)*c^7*d^3 + (8*a^2 - b^2)*c^5*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(2*b^2*c^9*d^2 - 22*a*b*c^8*d^3 + 14*a*b*c^6*d^5 +
8*a*b*c^4*d^7 + 23*a^2*c^3*d^8 - 6*a^2*c*d^10 + (26*a^2 + 11*b^2)*c^7*d^4 -
 (43*a^2 + 13*b^2)*c^5*d^6 + (6*b^2*c^11 - 36*a*b*c^10*d + 46*a*b*c^8*d^3 -
 14*a*b*c^6*d^5 + 4*a*b*c^4*d^7 - 11*a^2*c^3*d^8 + 4*(9*a^2 + b^2)*c^9*d^2
- (68*a^2 + 11*b^2)*c^7*d^4 + (43*a^2 + b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*(2
*b^2*c^10*d - 18*a*b*c^9*d^2 + 16*a*b*c^7*d^4 + 2*a*b*c^5*d^6 - 5*a^2*c^2*d
^9 + (20*a^2 + 7*b^2)*c^8*d^3 - 5*(7*a^2 + 2*b^2)*c^6*d^5 + (20*a^2 + b^2)*
c^4*d^7)*cos(f*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c
^9*d^6 + c^7*d^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 -
4*c^8*d^7 + c^6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^
6 - 4*c^7*d^8 + c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d
^7 - 4*c^6*d^9 + c^4*d^11)*f), 1/6*(6*(a^2*c^11 - 4*a^2*c^9*d^2 + 6*a^2*c^7
*d^4 - 4*a^2*c^5*d^6 + a^2*c^3*d^8)*f*x*cos(f*x + e)^3 + 18*(a^2*c^10*d - 4
*a^2*c^8*d^3 + 6*a^2*c^6*d^5 - 4*a^2*c^4*d^7 + a^2*c^2*d^9)*f*x*cos(f*x + e
)^2 + 18*(a^2*c^9*d^2 - 4*a^2*c^7*d^4 + 6*a^2*c^5*d^6 - 4*a^2*c^3*d^8 + a^2
*c*d^10)*f*x*cos(f*x + e) + 6*(a^2*c^8*d^3 - 4*a^2*c^6*d^5 + 6*a^2*c^4*d^7
- 4*a^2*c^2*d^9 + a^2*d^11)*f*x + 3*(4*a*b*c^7*d^3 + 6*a*b*c^5*d^5 - 7*a^2*
c^2*d^8 + 2*a^2*d^10 - 4*(2*a^2 + b^2)*c^6*d^4 + (8*a^2 - b^2)*c^4*d^6 + (4
*a*b*c^10 + 6*a*b*c^8*d^2 - 7*a^2*c^5*d^5 + 2*a^2*c^3*d^7 - 4*(2*a^2 + b^2)
*c^9*d + (8*a^2 - b^2)*c^7*d^3)*cos(f*x + e)^3 + 3*(4*a*b*c^9*d + 6*a*b*c^7
*d^3 - 7*a^2*c^4*d^6 + 2*a^2*c^2*d^8 - 4*(2*a^2 + b^2)*c^8*d^2 + (8*a^2 - b
^2)*c^6*d^4)*cos(f*x + e)^2 + 3*(4*a*b*c^8*d^2 + 6*a*b*c^6*d^4 - 7*a^2*c^3*
d^7 + 2*a^2*c*d^9 - 4*(2*a^2 + b^2)*c^7*d^3 + (8*a^2 - b^2)*c^5*d^5)*cos(f*
x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^
2 - d^2)*sin(f*x + e))) + (2*b^2*c^9*d^2 - 22*a*b*c^8*d^3 + 14*a*b*c^6*d^5
+ 8*a*b*c^4*d^7 + 23*a^2*c^3*d^8 - 6*a^2*c*d^10 + (26*a^2 + 11*b^2)*c^7*d^4
 - (43*a^2 + 13*b^2)*c^5*d^6 + (6*b^2*c^11 - 36*a*b*c^10*d + 46*a*b*c^8*d^3
 - 14*a*b*c^6*d^5 + 4*a*b*c^4*d^7 - 11*a^2*c^3*d^8 + 4*(9*a^2 + b^2)*c^9*d^
2 - (68*a^2 + 11*b^2)*c^7*d^4 + (43*a^2 + b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*
(2*b^2*c^10*d - 18*a*b*c^9*d^2 + 16*a*b*c^7*d^4 + 2*a*b*c^5*d^6 - 5*a^2*c^2
*d^9 + (20*a^2 + 7*b^2)*c^8*d^3 - 5*(7*a^2 + 2*b^2)*c^6*d^5 + (20*a^2 + b^2
)*c^4*d^7)*cos(f*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4
*c^9*d^6 + c^7*d^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5
- 4*c^8*d^7 + c^6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*
d^6 - 4*c^7*d^8 + c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8
```

```
*d^7 - 4*c^6*d^9 + c^4*d^11)*f)]
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(362) = 724$.

Time = 0.40 (sec) , antiderivative size = 1201, normalized size of antiderivative = 3.19

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(4*a*b*c^7 - 8*a^2*c^6*d - 4*b^2*c^6*d + 6*a*b*c^5*d^2 + 8*a^2*c^4*d
^3 - b^2*c^4*d^3 - 7*a^2*c^2*d^5 + 2*a^2*d^7)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/
2*e))/sqrt(-c^2 + d^2)))/((c^10 - 3*c^8*d^2 + 3*c^6*d^4 - c^4*d^6)*sqrt(-c^
2 + d^2)) + 3*(f*x + e)*a^2/c^4 - (6*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 36*a*
b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 6*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 36*a^
2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 54*a*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 +
12*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^2*c^5*d^3*tan(1/2*f*x + 1/2*e)
```

```
^5 - 12*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 27*b^2*c^5*d^3*tan(1/2*f*x + 1
/2*e)^5 - 6*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*c^4*d^4*tan(1/2*f*x
+ 1/2*e)^5 + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 45*a^2*c^3*d^5*tan(1/2
*f*x + 1/2*e)^5 - 12*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 + 3*b^2*c^3*d^5*tan
(1/2*f*x + 1/2*e)^5 - 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 15*a^2*c*d^7*t
an(1/2*f*x + 1/2*e)^5 + 6*a^2*d^8*tan(1/2*f*x + 1/2*e)^5 - 12*b^2*c^8*tan(1
/2*f*x + 1/2*e)^3 + 72*a*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 72*a^2*c^6*d^2*ta
n(1/2*f*x + 1/2*e)^3 - 16*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 64*a*b*c^5*d
^3*tan(1/2*f*x + 1/2*e)^3 + 116*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 28*b^2
*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 56
*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 12*a^2*d^8*tan(1/2*f*x + 1/2*e)^3 + 6
*b^2*c^8*tan(1/2*f*x + 1/2*e) - 36*a*b*c^7*d*tan(1/2*f*x + 1/2*e) + 6*b^2*c
^7*d*tan(1/2*f*x + 1/2*e) + 36*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e) - 54*a*b*c^
6*d^2*tan(1/2*f*x + 1/2*e) + 12*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e) + 60*a^2*c
^5*d^3*tan(1/2*f*x + 1/2*e) - 12*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e) + 27*b^2*
c^5*d^3*tan(1/2*f*x + 1/2*e) - 6*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e) - 6*a*b*c
^4*d^4*tan(1/2*f*x + 1/2*e) + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) - 45*a^2*
c^3*d^5*tan(1/2*f*x + 1/2*e) - 12*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e) - 3*b^2*
c^3*d^5*tan(1/2*f*x + 1/2*e) - 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e) + 15*a^2*
c*d^7*tan(1/2*f*x + 1/2*e) + 6*a^2*d^8*tan(1/2*f*x + 1/2*e))/((c^9 - 3*c^7*
d^2 + 3*c^5*d^4 - c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*
e)^2 - c - d)^3))/f
```

Mupad [B] (verification not implemented)

Time = 27.00 (sec) , antiderivative size = 12818, normalized size of antiderivative = 34.00

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

```
[In] int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^4,x)
```

```
[Out] (2*a^2*atan(((a^2*((8*tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c
*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11
+ 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 +
156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*
a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^
9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*
c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a
^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13
*d - 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5
*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d
^3 - 5*c^15*d^2) + (a^2*((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*
a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^
2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2
```

*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^1
1*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6
 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8
*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*
d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b*c^18*d^3
 - 12*a*b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5
*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*
d^3 - 5*c^18*d^2) - (a^2*tan(e/2 + (f*x)/2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*
d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^1
4*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*
d^3 - 8*c^20*d^2)*8i)/(c^4*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9
 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^
14*d^3 - 5*c^15*d^2)))*1i)/c^4))/c^4 + (a^2*((8*tan(e/2 + (f*x)/2)*(4*a^4*c
^14 + 8*a^4*d^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c
^2*d^12 + 48*a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^
6*d^8 + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d
^4 + 48*a^4*c^11*d^3 + 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*
b^4*c^12*d^2 - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68
*a^3*b*c^7*d^7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10
 - 2*a^2*b^2*c^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^
2*c^12*d^2 - 32*a*b^3*c^13*d - 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11
 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^
5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) - (a^2*((8*(4*a^2*c^21 - 16*a^2*
c^20*d - 8*b^2*c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11
 - 14*a^2*c^11*d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 -
 30*a^2*c^15*d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 1
2*a^2*c^19*d^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*
c^14*d^7 + 18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^1
8*d^3 + 8*b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a
*b*c^13*d^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*
c^17*d^4 + 12*a*b*c^18*d^3 - 12*a*b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 -
c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d
^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (a^2*tan(e/2 + (f*x)/2)*(8*c^
21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^1
0 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^
5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)*8i)/(c^4*(c^16*d + c^17 - c^6*d
^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^1
2*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*1i)/c^4))/c^4)/((16*(4*a^6
*d^13 - 8*a^5*b*c^13 - 2*a^6*c*d^12 + 16*a^6*c^12*d + 16*a^4*b^2*c^13 - 26*
a^6*c^2*d^11 + 11*a^6*c^3*d^10 + 70*a^6*c^4*d^9 - 34*a^6*c^5*d^8 - 110*a^6*
c^6*d^7 + 66*a^6*c^7*d^6 + 110*a^6*c^8*d^5 - 64*a^6*c^9*d^4 - 64*a^6*c^10*d
^3 + 48*a^6*c^11*d^2 - 32*a^3*b^3*c^12*d + 8*a^4*b^2*c^12*d + 12*a^5*b*c^4*
d^9 + 12*a^5*b*c^5*d^8 - 40*a^5*b*c^6*d^7 - 28*a^5*b*c^7*d^6 + 28*a^5*b*c^8
*d^5 + 12*a^5*b*c^9*d^4 - 44*a^5*b*c^10*d^3 + 12*a^5*b*c^11*d^2 + a^2*b^4*c
^7*d^6 + 8*a^2*b^4*c^9*d^4 + 16*a^2*b^4*c^11*d^2 - 12*a^3*b^3*c^8*d^5 - 56*

a^3*b^3*c^10*d^3 - 2*a^4*b^2*c^3*d^10 - 2*a^4*b^2*c^4*d^9 - 2*a^4*b^2*c^6*d
^7 + 22*a^4*b^2*c^7*d^6 + 18*a^4*b^2*c^8*d^5 + 10*a^4*b^2*c^9*d^4 - 22*a^4*
b^2*c^10*d^3 + 104*a^4*b^2*c^11*d^2 - 56*a^5*b*c^12*d))/(c^19*d + c^20 - c^
9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 +
10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (a^2*((8*tan(e/2 + (
f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2
*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*
d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5
 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^
4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^
3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a
^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*
d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13*d - 64*a^3*b*c^13*d))/(c^16*d +
c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*
d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) + (a^2*((8*(4*a^
2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 2
6*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 11
0*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*
a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c
^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*
d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b
*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^
16*d^5 - 12*a*b*c^17*d^4 + 12*a*b*c^18*d^3 - 12*a*b*c^19*d^2))/(c^19*d + c^
20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14
*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (a^2*tan(e/2
+ (f*x)/2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^1
1 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d
^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)*8i)/(c^4*(c^16*
d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c
^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*1i)/c^4)*1
i)/c^4 + (a^2*((8*tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c*d^1
3 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11 + 11
7*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 + 156*
a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*a^4*
c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^9*d^
5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*c^9*
d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a^2*b
^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13*d -
 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9
*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 -
 5*c^15*d^2) - (a^2*((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*a^2*
c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^2*c^
12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2*c^1
6*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^11*d^
10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6 - 1

8*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8*a*b
*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*d^7
+ 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b*c^18*d^3 - 1
2*a*b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^1
2*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3
- 5*c^18*d^2) + (a^2*tan(e/2 + (f*x)/2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13
 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^
8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3
- 8*c^20*d^2)*8i)/(c^4*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5
*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d
^3 - 5*c^15*d^2)))*1i)/c^4)*1i)/c^4)))/(c^4*f) - ((tan(e/2 + (f*x)/2)^5*(2*
a^2*d^6 + 2*b^2*c^6 - a^2*c*d^5 + 2*b^2*c^5*d - 6*a^2*c^2*d^4 + 4*a^2*c^3*d
^3 + 12*a^2*c^4*d^2 + b^2*c^3*d^3 + 6*b^2*c^4*d^2 - 12*a*b*c^5*d - 4*a*b*c^
3*d^3 - 6*a*b*c^4*d^2))/((c^3*d - c^4)*(c + d)^3) + (4*tan(e/2 + (f*x)/2)^3
*(3*a^2*d^6 + 3*b^2*c^6 - 11*a^2*c^2*d^4 + 18*a^2*c^4*d^2 + 7*b^2*c^4*d^2 -
 18*a*b*c^5*d - 2*a*b*c^3*d^3))/(3*(c + d)^2*(c^5 - 2*c^4*d + c^3*d^2)) + (
tan(e/2 + (f*x)/2)*(2*a^2*d^6 + 2*b^2*c^6 + a^2*c*d^5 - 2*b^2*c^5*d - 6*a^2
*c^2*d^4 - 4*a^2*c^3*d^3 + 12*a^2*c^4*d^2 - b^2*c^3*d^3 + 6*b^2*c^4*d^2 - 1
2*a*b*c^5*d - 4*a*b*c^3*d^3 + 6*a*b*c^4*d^2))/((c + d)*(3*c^5*d - c^6 + c^3
*d^3 - 3*c^4*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3
*d^3) - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2
+ 3*c^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3
))) + (atan(((((8*tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c*d^1
3 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11 + 11
7*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 + 156*
a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*a^4*
c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^9*d^
5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*c^9*
d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a^2*b
^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13*d -
 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9
*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 -
 5*c^15*d^2) + (((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*a^2*c^8*
d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^2*c^12*d
^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2*c^16*d^
5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^11*d^10 +
 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6 - 18*b^
2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8*a*b*c^2
1 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*d^7 + 28
*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b*c^18*d^3 - 12*a*
b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^
8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*
c^18*d^2) - (4*tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 -
8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4
*a*b*c^7 + 6*a*b*c^5*d^2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^1

2 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d
^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2))
/((c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 +
 21*c^14*d^4 - 7*c^16*d^2)*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9
 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^
14*d^3 - 5*c^15*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d
 - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 +
6*a*b*c^5*d^2))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^
8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(
2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2
*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2)*1i)/(2*(c^18 - c^4*d^14 + 7*c^6*d^12
- 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)) + ((
(8*tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c*d^13 - 8*a^4*c^13*
d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11 + 117*a^4*c^4*d^10
- 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 1
20*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*a^4*c^12*d^2 + b^4*
c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^
11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*
c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12
*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13*d - 64*a^3*b*c^13*
d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*
d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) -
(((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^
9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^1
3*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17
*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9
 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22
*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*
d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 +
 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b*c^18*d^3 - 12*a*b*c^19*d^2))/(c
^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7
 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (4*
tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4
*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*
b*c^5*d^2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^1
1 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d
^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2))/((c^18 - c^4*d
^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 -
7*c^16*d^2)*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 -
10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15
d^2)))((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d
- 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2))
/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6
 + 21*c^14*d^4 - 7*c^16*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a
^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*

b*c^7 + 6*a*b*c^5*d^2)*1i)/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)))/((16*(4*a^6*d^13 - 8*a^5*b*c^13 - 2*a^6*c*d^12 + 16*a^6*c^12*d + 16*a^4*b^2*c^13 - 26*a^6*c^2*d^11 + 11*a^6*c^3*d^10 + 70*a^6*c^4*d^9 - 34*a^6*c^5*d^8 - 110*a^6*c^6*d^7 + 66*a^6*c^7*d^6 + 110*a^6*c^8*d^5 - 64*a^6*c^9*d^4 - 64*a^6*c^10*d^3 + 48*a^6*c^11*d^2 - 32*a^3*b^3*c^12*d + 8*a^4*b^2*c^12*d + 12*a^5*b*c^4*d^9 + 12*a^5*b*c^5*d^8 - 40*a^5*b*c^6*d^7 - 28*a^5*b*c^7*d^6 + 28*a^5*b*c^8*d^5 + 12*a^5*b*c^9*d^4 - 44*a^5*b*c^10*d^3 + 12*a^5*b*c^11*d^2 + a^2*b^4*c^7*d^6 + 8*a^2*b^4*c^9*d^4 + 16*a^2*b^4*c^11*d^2 - 12*a^3*b^3*c^8*d^5 - 56*a^3*b^3*c^10*d^3 - 2*a^4*b^2*c^3*d^10 - 2*a^4*b^2*c^4*d^9 - 2*a^4*b^2*c^6*d^7 + 22*a^4*b^2*c^7*d^6 + 18*a^4*b^2*c^8*d^5 + 10*a^4*b^2*c^9*d^4 - 22*a^4*b^2*c^10*d^3 + 104*a^4*b^2*c^11*d^2 - 56*a^5*b*c^12*d))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (((8*tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13*d - 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) + (((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b*c^18*d^3 - 12*a*b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (4*tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2))/((c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d

^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d ^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48* a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a ^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c ^11*d^3 + 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^ 7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c ^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13*d - 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d ^4 - 5*c^14*d^3 - 5*c^15*d^2) - (((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^ 20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^1 0 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2 *c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^1 9*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28* a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b *c^18*d^3 - 12*a*b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^1 1*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (4*tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1 /2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d ^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14 *d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d ^3 - 8*c^20*d^2))/((c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^ 8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)*(c^16*d + c^17 - c^6*d^11 - c^7 *d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 1 0*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2* d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d ^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8* d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8* a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c ^16*d^2))))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^ 6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d ^2)*1i)/(f*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c ^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2))

3.195 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

Optimal result 1341
Rubi [A] (verified) 1341
Mathematica [B] (verified) 1344
Maple [A] (verified) 1345
Fricas [B] (verification not implemented) 1345
Sympy [F] 1346
Maxima [F(-2)] 1347
Giac [B] (verification not implemented) 1347
Mupad [B] (verification not implemented) 1348

Optimal result

Integrand size = 25, antiderivative size = 254

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \frac{a^3x}{c^3} - \frac{(bc-ad)\left(2abcd(4c^2-d^2)-b^2c^2(c^2+2d^2)-a^2(6c^4-5c^2d^2+2d^4)\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} + \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} + \frac{(bc-ad)^2\left(5ac^2-3bcd-2ad^2\right)\sin(e+fx)}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))}$$

```
[Out] a^3*x/c^3-(-a*d+b*c)*(2*a*b*c*d*(4*c^2-d^2)-b^2*c^2*(c^2+2*d^2)-a^2*(6*c^4-
5*c^2*d^2+2*d^4))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3/(
c-d)^(5/2)/(c+d)^(5/2)/f+1/2*(-a*d+b*c)^2*(b+a*cos(f*x+e))*sin(f*x+e)/c/(c^
2-d^2)/f/(d+c*cos(f*x+e))^2+1/2*(-a*d+b*c)^2*(5*a*c^2-2*a*d^2-3*b*c*d)*sin(
f*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4026, 2871, 3100, 2814, 2738, 214}

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \frac{a^3x}{c^3} - \frac{(bc-ad)\left(-(a^2(6c^4-5c^2d^2+2d^4))+2abcd(4c^2-d^2)-b^2c^2(c^2+2d^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}} + \frac{(bc-ad)^2\left(5ac^2-2ad^2-3bcd\right)\sin(e+fx)}{2c^2f\left(c^2-d^2\right)^2(c\cos(e+fx)+d)} + \frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

```
[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*x)/c^3 - ((b*c - a*d)*(2*a*b*c*d*(4*c^2 - d^2) - b^2*c^2*(c^2 + 2*d^2)
 - a^2*(6*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/
Sqrt[c + d]])/(c^3*(c - d)^(5/2)*(c + d)^(5/2)*f) + ((b*c - a*d)^2*(b + a*C
os[e + f*x])*Sin[e + f*x])/(2*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) + ((b
*c - a*d)^2*(5*a*c^2 - 3*b*c*d - 2*a*d^2)*Sin[e + f*x])/(2*c^2*(c^2 - d^2)^
2*f*(d + c*Cos[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \int \frac{(b + a\cos(e + fx))^3}{(d + c\cos(e + fx))^3}\,dx$$

$$= \frac{(bc - ad)^2(b + a\cos(e + fx))\sin(e + fx)}{2c\,(c^2 - d^2)\,f(d + c\cos(e + fx))^2} + \frac{\int \frac{5ab^2c^2 - 4a^2bcd - 2b^3cd + a^3d^2 + (b^3c^2 - 2a^3cd - 4ab^2cd + a^2b(6c^2 - d^2))\cos(e + fx) + 2a^3(c^2 - d^2)\cos^2(e + fx)}{(d + c\cos(e + fx))^2}\,dx}{2c\,(c^2 - d^2)}$$

$$= \frac{(bc - ad)^2(b + a\cos(e + fx))\sin(e + fx)}{2c\,(c^2 - d^2)\,f(d + c\cos(e + fx))^2} + \frac{(bc - ad)^2\,(5ac^2 - 3bcd - 2ad^2)\sin(e + fx)}{2c^2\,(c^2 - d^2)^2\,f(d + c\cos(e + fx))} + \frac{\int \frac{-c(9ab^2c^2d - 3a^2bc(2c^2 + d^2) - b^3c(c^2 + 2d^2) + a^3(4c^2d - d^3)) + 2a^3(c^2 - d^2)^2\cos(e + fx)}{d + c\cos(e + fx)}\,dx}{2c^2\,(c^2 - d^2)^2}$$

$$= \frac{a^3x}{c^3} + \frac{(bc - ad)^2(b + a\cos(e + fx))\sin(e + fx)}{2c\,(c^2 - d^2)\,f(d + c\cos(e + fx))^2} + \frac{(bc - ad)^2\,(5ac^2 - 3bcd - 2ad^2)\sin(e + fx)}{2c^2\,(c^2 - d^2)^2\,f(d + c\cos(e + fx))} - \frac{(9ab^2c^4d - 3a^2bc^3(2c^2 + d^2) - b^3c^3(c^2 + 2d^2) + a^3(6c^4d - 5c^2d^3 + 2d^5))\int \frac{1}{d + c\cos(e + fx)}\,dx}{2c^3\,(c^2 - d^2)^2}$$

$$= \frac{a^3x}{c^3} + \frac{(bc - ad)^2(b + a\cos(e + fx))\sin(e + fx)}{2c\,(c^2 - d^2)\,f(d + c\cos(e + fx))^2} + \frac{(bc - ad)^2\,(5ac^2 - 3bcd - 2ad^2)\sin(e + fx)}{2c^2\,(c^2 - d^2)^2\,f(d + c\cos(e + fx))} - \frac{(9ab^2c^4d - 3a^2bc^3(2c^2 + d^2) - b^3c^3(c^2 + 2d^2) + a^3(6c^4d - 5c^2d^3 + 2d^5))\,\text{Subst}\left(\int \frac{1}{c + d + (-c + d)x^2}\,d\right.}{c^3\,(c^2 - d^2)^2\,f}$$

$$= \frac{a^3x}{c^3} + \frac{(bc-ad)\left(6a^2c^4+b^2c^4-8abc^3d-5a^2c^2d^2+2b^2c^2d^2+2abcd^3+2a^2d^4\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} + \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} + \frac{(bc-ad)^2\left(5ac^2-3bcd-2ad^2\right)\sin(e+fx)}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 517 vs. $2(254) = 508$.

Time = 2.25 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \frac{-\frac{4\left(-9ab^2c^4d+3a^2bc^3\left(2c^2+d^2\right)+b^3c^3\left(c^2+2d^2\right)+a^3\left(-6c^4d+5c^2d^3-2d^5\right)\right)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}} + \frac{2a^3c^6e-6a^3c^2d^4e+4a^3d^6e+2a\ldots}{\ldots}}{\ldots}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((-4*(-9*a*b^2*c^4*d + 3*a^2*b*c^3*(2*c^2 + d^2) + b^3*c^3*(c^2 + 2*d^2) +
a^3*(-6*c^4*d + 5*c^2*d^3 - 2*d^5))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqr
t[c^2 - d^2]])/(c^2 - d^2)^(5/2) + (2*a^3*c^6*e - 6*a^3*c^2*d^4*e + 4*a^3*d
^6*e + 2*a^3*c^6*f*x - 6*a^3*c^2*d^4*f*x + 4*a^3*d^6*f*x + 8*a^3*c*d*(c^2 -
 d^2)^2*(e + f*x)*Cos[e + f*x] + 2*a^3*(c^3 - c*d^2)^2*(e + f*x)*Cos[2*(e +
 f*x)] + 2*b^3*c^6*Sin[e + f*x] + 6*a*b^2*c^5*d*Sin[e + f*x] - 18*a^2*b*c^4
*d^2*Sin[e + f*x] - 8*b^3*c^4*d^2*Sin[e + f*x] + 10*a^3*c^3*d^3*Sin[e + f*x
] + 12*a*b^2*c^3*d^3*Sin[e + f*x] - 4*a^3*c*d^5*Sin[e + f*x] + 6*a*b^2*c^6*
Sin[2*(e + f*x)] - 12*a^2*b*c^5*d*Sin[2*(e + f*x)] - 3*b^3*c^5*d*Sin[2*(e +
 f*x)] + 6*a^3*c^4*d^2*Sin[2*(e + f*x)] + 3*a*b^2*c^4*d^2*Sin[2*(e + f*x)]
+ 3*a^2*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^3*c^2*d^4*Sin[2*(e + f*x)])/((c^2
- d^2)^2*(d + c*Cos[e + f*x])^2))/(4*c^3*f)
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{2a^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a^3c^2d^2+a^3d^3c-2a^3d^4-12a^2b\,c^3d-3a^2b\,c^2d^2+6a\,b^2c^4+3a\,b^2c^3d+6a\,b^2c^2d^2-b^3c^4-4b^3c^3d\right)c\,t}{2(c-d)\left(c^2+2cd+d^2\right)}\right.}{\left(\tan\left(\frac{f}{2}\right.\right.}$
default	$\frac{2a^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a^3c^2d^2+a^3d^3c-2a^3d^4-12a^2b\,c^3d-3a^2b\,c^2d^2+6a\,b^2c^4+3a\,b^2c^3d+6a\,b^2c^2d^2-b^3c^4-4b^3c^3d\right)c\,t}{2(c-d)\left(c^2+2cd+d^2\right)}\right.}{\left(\tan\left(\frac{fx}{2}\right.\right.}$
risch	Expression too large to display

```
[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*a^3/c^3*arctan(tan(1/2*f*x+1/2*e))+2/c^3*((-1/2*(6*a^3*c^2*d^2+a^3*c
*d^3-2*a^3*d^4-12*a^2*b*c^3*d-3*a^2*b*c^2*d^2+6*a*b^2*c^4+3*a*b^2*c^3*d+6*a
*b^2*c^2*d^2-b^3*c^4-4*b^3*c^3*d)*c/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e
)^3+1/2*c*(6*a^3*c^2*d^2-a^3*c*d^3-2*a^3*d^4-12*a^2*b*c^3*d+3*a^2*b*c^2*d^2
+6*a*b^2*c^4-3*a*b^2*c^3*d+6*a*b^2*c^2*d^2+b^3*c^4-4*b^3*c^3*d)/(c+d)/(c-d)
^2*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^
2-1/2*(6*a^3*c^4*d-5*a^3*c^2*d^3+2*a^3*d^5-6*a^2*b*c^5-3*a^2*b*c^3*d^2+9*a*
b^2*c^4*d-b^3*c^5-2*b^3*c^3*d^2)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*ar
ctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. 2(238) = 476.

Time = 0.40 (sec) , antiderivative size = 1629, normalized size of antiderivative = 6.41

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d^4 - a^3*c^2*d^6)*f*x*cos(f*x
 + e)^2 + 8*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^3*c^3*d^5 - a^3*c*d^7)*f*x*cos
(f*x + e) + 4*(a^3*c^6*d^2 - 3*a^3*c^4*d^4 + 3*a^3*c^2*d^6 - a^3*d^8)*f*x -
 (5*a^3*c^2*d^5 - 2*a^3*d^7 + (6*a^2*b + b^3)*c^5*d^2 - 3*(2*a^3 + 3*a*b^2)
*c^4*d^3 + (3*a^2*b + 2*b^3)*c^3*d^4 + (5*a^3*c^4*d^3 - 2*a^3*c^2*d^5 + (6*
a^2*b + b^3)*c^7 - 3*(2*a^3 + 3*a*b^2)*c^6*d + (3*a^2*b + 2*b^3)*c^5*d^2)*c
os(f*x + e)^2 + 2*(5*a^3*c^3*d^4 - 2*a^3*c*d^6 + (6*a^2*b + b^3)*c^6*d - 3*
```

```
(2*a^3 + 3*a*b^2)*c^5*d^2 + (3*a^2*b + 2*b^3)*c^4*d^3)*cos(f*x + e))*sqrt(c
^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c
^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e
)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^3*c^8 + 3*a*b^2*c^7*d + 2*a^3*c*d^7
 - (9*a^2*b + 5*b^3)*c^6*d^2 + (5*a^3 + 3*a*b^2)*c^5*d^3 + (9*a^2*b + 4*b^3
)*c^4*d^4 - (7*a^3 + 6*a*b^2)*c^3*d^5 + 3*(2*a*b^2*c^8 - a^2*b*c^3*d^5 + a^
3*c^2*d^6 - (4*a^2*b + b^3)*c^7*d + (2*a^3 - a*b^2)*c^6*d^2 + (5*a^2*b + b^
3)*c^5*d^3 - (3*a^3 + a*b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 -
3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 +
 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c
^3*d^8)*f), 1/2*(2*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d^4 - a^3*c^2*d^6)*
f*x*cos(f*x + e)^2 + 4*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^3*c^3*d^5 - a^3*c*d
^7)*f*x*cos(f*x + e) + 2*(a^3*c^6*d^2 - 3*a^3*c^4*d^4 + 3*a^3*c^2*d^6 - a^3
*d^8)*f*x + (5*a^3*c^2*d^5 - 2*a^3*d^7 + (6*a^2*b + b^3)*c^5*d^2 - 3*(2*a^3
 + 3*a*b^2)*c^4*d^3 + (3*a^2*b + 2*b^3)*c^3*d^4 + (5*a^3*c^4*d^3 - 2*a^3*c^
2*d^5 + (6*a^2*b + b^3)*c^7 - 3*(2*a^3 + 3*a*b^2)*c^6*d + (3*a^2*b + 2*b^3)
*c^5*d^2)*cos(f*x + e)^2 + 2*(5*a^3*c^3*d^4 - 2*a^3*c*d^6 + (6*a^2*b + b^3)
*c^6*d - 3*(2*a^3 + 3*a*b^2)*c^5*d^2 + (3*a^2*b + 2*b^3)*c^4*d^3)*cos(f*x +
 e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
 d^2)*sin(f*x + e))) + (b^3*c^8 + 3*a*b^2*c^7*d + 2*a^3*c*d^7 - (9*a^2*b +
5*b^3)*c^6*d^2 + (5*a^3 + 3*a*b^2)*c^5*d^3 + (9*a^2*b + 4*b^3)*c^4*d^4 - (7
*a^3 + 6*a*b^2)*c^3*d^5 + 3*(2*a*b^2*c^8 - a^2*b*c^3*d^5 + a^3*c^2*d^6 - (4
*a^2*b + b^3)*c^7*d + (2*a^3 - a*b^2)*c^6*d^2 + (5*a^2*b + b^3)*c^5*d^3 - (
3*a^3 + a*b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2 + 3*
c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c
^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*f)]
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(238) = 476$.

Time = 0.41 (sec) , antiderivative size = 818, normalized size of antiderivative = 3.22

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^3/c^3 + (6*a^2*b*c^5 + b^3*c^5 - 6*a^3*c^4*d - 9*a*b^2*c^4*d +
 3*a^2*b*c^3*d^2 + 2*b^3*c^3*d^2 + 5*a^3*c^2*d^3 - 2*a^3*d^5)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*
tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c^5*d^2 + c^3*d^4)*sqrt(
-c^2 + d^2)) - (6*a*b^2*c^5*tan(1/2*f*x + 1/2*e)^3 - b^3*c^5*tan(1/2*f*x +
1/2*e)^3 - 12*a^2*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^4*d*tan(1/2*f*
x + 1/2*e)^3 - 3*b^3*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^3*c^3*d^2*tan(1/2*f
*x + 1/2*e)^3 + 9*a^2*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*a*b^2*c^3*d^2*ta
n(1/2*f*x + 1/2*e)^3 + 4*b^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*a^3*c^2*d^3
*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*
c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^3
*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*c^5*tan(1/2*f*x + 1/2*e) - b^3*c^5*ta
n(1/2*f*x + 1/2*e) + 12*a^2*b*c^4*d*tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^4*d*ta
n(1/2*f*x + 1/2*e) + 3*b^3*c^4*d*tan(1/2*f*x + 1/2*e) - 6*a^3*c^3*d^2*tan(1
/2*f*x + 1/2*e) + 9*a^2*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^3*d^2*ta
n(1/2*f*x + 1/2*e) + 4*b^3*c^3*d^2*tan(1/2*f*x + 1/2*e) - 5*a^3*c^2*d^3*tan
(1/2*f*x + 1/2*e) - 3*a^2*b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 6*a*b^2*c^2*d^3*
tan(1/2*f*x + 1/2*e) + 3*a^3*c*d^4*tan(1/2*f*x + 1/2*e) + 2*a^3*d^5*tan(1/2
*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*t
an(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 27.04 (sec) , antiderivative size = 10759, normalized size of antiderivative = 42.36

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^3,x)
```

```
[Out] (atan(((((8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^10 + b^6*c^10 - 8*a^6*
c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^10 - 32*a^6*c^2*d^8 +
32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 + 32*a^6*
c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c^7*d^3
 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*c^7*d^
3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16*a^3*b
^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^6*d^4 +
 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^10*d + c^11 - c
^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + ((a*d -
 b*c)*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^14*d - 2*b^3*
c^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*c^9*d^6 - 3
6*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*d^2 - 4*b^3*
c^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18*a*b^2*c^9*d^
6 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3 + 18*a*b^2*c^
13*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d^3 - 18*a^2*b*c
^13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^
7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (4*tan(e/2 + (f*
x)/2)*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*
c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*(8*c^15*d
- 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*
d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^
8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^
6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c - d)^5)^
(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*
a*b*c*d^3 - 8*a*b*c^3*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10
*c^9*d^4 - 5*c^11*d^2)))*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4
 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*
b*c^3*d)*1i)/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*
c^11*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^10 + b^6*c^10 -
8*a^6*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^10 - 32*a^6*c^2*
d^8 + 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 + 3
2*a^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c
^7*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*
c^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16
*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^6
*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^10*d + c^
```

11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (
(a*d - b*c)*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^14*d -
2*b^3*c^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*c^9*d
^6 - 36*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*d^2 -
4*b^3*c^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18*a*b^2*
c^9*d^6 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3 + 18*a*
b^2*c^13*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d^3 - 18*a
^2*b*c^13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^13 - c^6*d^
7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (4*tan(e/2
 + (f*x)/2)*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4
+ b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*(8*c
^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48
*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*
c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 -
c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c -
d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^
2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^
6 + 10*c^9*d^4 - 5*c^11*d^2)))*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a
^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3
 - 8*a*b*c^3*d)*1i)/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^
4 - 5*c^11*d^2)))/((16*(4*a^9*d^9 - 12*a^8*b*c^9 - 2*a^9*c*d^8 + 12*a^9*c^8
*d + a^3*b^6*c^9 + 12*a^5*b^4*c^9 - 2*a^6*b^3*c^9 + 36*a^7*b^2*c^9 - 18*a^9
*c^2*d^7 + 13*a^9*c^3*d^6 + 36*a^9*c^4*d^5 - 26*a^9*c^5*d^4 - 34*a^9*c^6*d^
3 + 24*a^9*c^7*d^2 - 18*a^4*b^5*c^8*d - 118*a^6*b^3*c^8*d + 18*a^7*b^2*c^8*
d - 6*a^8*b*c^2*d^7 - 6*a^8*b*c^3*d^6 + 6*a^8*b*c^4*d^5 + 6*a^8*b*c^6*d^3 +
 18*a^8*b*c^7*d^2 + 4*a^3*b^6*c^5*d^4 + 4*a^3*b^6*c^7*d^2 - 36*a^4*b^5*c^6*
d^3 + 12*a^5*b^4*c^5*d^4 + 111*a^5*b^4*c^7*d^2 - 4*a^6*b^3*c^2*d^7 - 4*a^6*
b^3*c^3*d^6 + 10*a^6*b^3*c^4*d^5 + 6*a^6*b^3*c^5*d^4 - 68*a^6*b^3*c^6*d^3 +
 18*a^7*b^2*c^3*d^6 + 18*a^7*b^2*c^4*d^5 - 45*a^7*b^2*c^5*d^4 - 36*a^7*b^2*
c^6*d^3 + 126*a^7*b^2*c^7*d^2 - 60*a^8*b*c^8*d))/(c^12*d + c^13 - c^6*d^7 -
 c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (((8*tan(e/2
+ (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^10 + b^6*c^10 - 8*a^6*c*d^9 - 8*a^6*c^9*d
+ 12*a^2*b^4*c^10 + 36*a^4*b^2*c^10 - 32*a^6*c^2*d^8 + 32*a^6*c^3*d^7 + 57*
a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 + 32*a^6*c^7*d^3 + 24*a^6*c^8
*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c^7*d^3 - 120*a^3*b^3*c^9*d
 - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*c^7*d^3 + 12*a^2*b^4*c^6*d
^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16*a^3*b^3*c^5*d^5 - 68*a^3*
b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^6*d^4 + 144*a^4*b^2*c^8*d^2
 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3
*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + ((a*d - b*c)*((8*(4*a^3*c^1
5 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^14*d - 2*b^3*c^14*d - 4*a^3*c^6*d
^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*c^9*d^6 - 36*a^3*c^10*d^5 + 6*a
^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*d^2 - 4*b^3*c^8*d^7 + 4*b^3*c^9*
d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18*a*b^2*c^9*d^6 - 18*a*b^2*c^10*d^
5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3 + 18*a*b^2*c^13*d^2 - 6*a^2*b*c^8

*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d^3 - 18*a^2*b*c^13*d^2 - 18*a*b^2*c
^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 +
3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (4*tan(e/2 + (f*x)/2)*(a*d - b*c)*((
c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2
+ 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*(8*c^15*d - 8*c^6*d^10 + 8*c^7
*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 -
32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*
c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^
7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2
*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^
3*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^
2)))*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c
^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(c^13 -
c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)) + (((8*tan(e
/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^10 + b^6*c^10 - 8*a^6*c*d^9 - 8*a^6*c^9
*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^10 - 32*a^6*c^2*d^8 + 32*a^6*c^3*d^7 +
57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 + 32*a^6*c^7*d^3 + 24*a^6*
c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c^7*d^3 - 120*a^3*b^3*c^
9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*c^7*d^3 + 12*a^2*b^4*c^
6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16*a^3*b^3*c^5*d^5 - 68*a
^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^6*d^4 + 144*a^4*b^2*c^8*
d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6
+ 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - ((a*d - b*c)*((8*(4*a^3*
c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^14*d - 2*b^3*c^14*d - 4*a^3*c^
6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*c^9*d^6 - 36*a^3*c^10*d^5 +
6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*d^2 - 4*b^3*c^8*d^7 + 4*b^3*c
^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18*a*b^2*c^9*d^6 - 18*a*b^2*c^10
*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3 + 18*a*b^2*c^13*d^2 - 6*a^2*b*
c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d^3 - 18*a^2*b*c^13*d^2 - 18*a*b^
2*c^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5
+ 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (4*tan(e/2 + (f*x)/2)*(a*d - b*c)
((c + d)^5(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d
^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*(8*c^15*d - 8*c^6*d^10 + 8*
c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4
- 32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 +
10*c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3
*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4
+ 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b
*c^3*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11
d^2)))(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^
2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(c^1
3 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2))))*(a*d -
b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c
^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*1i)/(f*(c^13 - c^3*d^10
+ 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)) - (2*a^3*atan(((a^3*(

(a^3*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^14*d - 2*b^3*c
^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*c^9*d^6 - 36
*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*d^2 - 4*b^3*c
^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18*a*b^2*c^9*d^6
 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3 + 18*a*b^2*c^1
3*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d^3 - 18*a^2*b*c^
13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7
*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (a^3*tan(e/2 + (f
x)/2)(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^
10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^
10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9
*d^2)))*1i)/c^3 + (8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^10 + b^6*c^10
 - 8*a^6*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^10 - 32*a^6*c
^2*d^8 + 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4
+ 32*a^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^
5*c^7*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5
*b*c^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 +
 16*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*
c^6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^10*d +
 c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2))
)/c^3 - (a^3*((a^3*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^
14*d - 2*b^3*c^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^
3*c^9*d^6 - 36*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13
*d^2 - 4*b^3*c^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18
*a*b^2*c^9*d^6 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3
+ 18*a*b^2*c^13*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d^3
 - 18*a^2*b*c^13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^13 -
 c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (a^
3*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c
^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2
)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c
^8*d^3 - 3*c^9*d^2)))*1i)/c^3 - (8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d
^10 + b^6*c^10 - 8*a^6*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c
^10 - 32*a^6*c^2*d^8 + 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 5
2*a^6*c^6*d^4 + 32*a^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8
*d^2 - 36*a*b^5*c^7*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^
5*d^5 + 24*a^5*b*c^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3
*b^3*c^3*d^7 + 16*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6
 - 81*a^4*b^2*c^6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9
*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3
 - 3*c^9*d^2)))/c^3)/((a^3*((a^3*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^
15 - 12*a^3*c^14*d - 2*b^3*c^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*
c^8*d^7 - 4*a^3*c^9*d^6 - 36*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^
3 - 8*a^3*c^13*d^2 - 4*b^3*c^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3
*c^11*d^4 + 18*a*b^2*c^9*d^6 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a

*b^2*c^12*d^3 + 18*a*b^2*c^13*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*
a^2*b*c^12*d^3 - 18*a^2*b*c^13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d))/(c
^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c
^11*d^2) - (a^3*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*
c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^
3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3
*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3 + (8*tan(e/2 + (f*x)/2)*(4*a^6*
c^10 + 8*a^6*d^10 + b^6*c^10 - 8*a^6*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10
+ 36*a^4*b^2*c^10 - 32*a^6*c^2*d^8 + 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a
^6*c^5*d^5 - 52*a^6*c^6*d^4 + 32*a^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d
^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c^7*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^
7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*c^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c
^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a
^4*b^2*c^4*d^6 - 81*a^4*b^2*c^6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d
- 72*a^5*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d
^4 - 3*c^8*d^3 - 3*c^9*d^2))*1i)/c^3 - (16*(4*a^9*d^9 - 12*a^8*b*c^9 - 2*a^
9*c*d^8 + 12*a^9*c^8*d + a^3*b^6*c^9 + 12*a^5*b^4*c^9 - 2*a^6*b^3*c^9 + 36*
a^7*b^2*c^9 - 18*a^9*c^2*d^7 + 13*a^9*c^3*d^6 + 36*a^9*c^4*d^5 - 26*a^9*c^5
*d^4 - 34*a^9*c^6*d^3 + 24*a^9*c^7*d^2 - 18*a^4*b^5*c^8*d - 118*a^6*b^3*c^8
*d + 18*a^7*b^2*c^8*d - 6*a^8*b*c^2*d^7 - 6*a^8*b*c^3*d^6 + 6*a^8*b*c^4*d^5
+ 6*a^8*b*c^6*d^3 + 18*a^8*b*c^7*d^2 + 4*a^3*b^6*c^5*d^4 + 4*a^3*b^6*c^7*d
^2 - 36*a^4*b^5*c^6*d^3 + 12*a^5*b^4*c^5*d^4 + 111*a^5*b^4*c^7*d^2 - 4*a^6*
b^3*c^2*d^7 - 4*a^6*b^3*c^3*d^6 + 10*a^6*b^3*c^4*d^5 + 6*a^6*b^3*c^5*d^4 -
68*a^6*b^3*c^6*d^3 + 18*a^7*b^2*c^3*d^6 + 18*a^7*b^2*c^4*d^5 - 45*a^7*b^2*c
^5*d^4 - 36*a^7*b^2*c^6*d^3 + 126*a^7*b^2*c^7*d^2 - 60*a^8*b*c^8*d))/(c^12*
d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*
d^2) + (a^3*((a^3*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^1
4*d - 2*b^3*c^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3
*c^9*d^6 - 36*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*
d^2 - 4*b^3*c^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18*
a*b^2*c^9*d^6 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3 +
18*a*b^2*c^13*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d^3
- 18*a^2*b*c^13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^13 -
c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (a^3
*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^
9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)
8i)/(c^3(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^
8*d^3 - 3*c^9*d^2)))*1i)/c^3 - (8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^
10 + b^6*c^10 - 8*a^6*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^
10 - 32*a^6*c^2*d^8 + 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52
*a^6*c^6*d^4 + 32*a^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*
d^2 - 36*a*b^5*c^7*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5
*d^5 + 24*a^5*b*c^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*
b^3*c^3*d^7 + 16*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6
- 81*a^4*b^2*c^6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*

```
d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3
- 3*c^9*d^2))*1i)/c^3)))/(c^3*f) - ((tan(e/2 + (f*x)/2)^3*(2*a^3*d^4 + b^3*
c^4 - 6*a*b^2*c^4 - a^3*c*d^3 + 4*b^3*c^3*d - 6*a^3*c^2*d^2 - 6*a*b^2*c^2*d
^2 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d + 12*a^2*b*c^3*d))/((c^2*d - c^3)*(c +
 d)^2) + (tan(e/2 + (f*x)/2)*(2*a^3*d^4 - b^3*c^4 - 6*a*b^2*c^4 + a^3*c*d^3
 + 4*b^3*c^3*d - 6*a^3*c^2*d^2 - 6*a*b^2*c^2*d^2 - 3*a^2*b*c^2*d^2 + 3*a*b^
2*c^3*d + 12*a^2*b*c^3*d))/((c + d)*(c^4 - 2*c^3*d + c^2*d^2)))/(f*(2*c*d -
 tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d +
 d^2) + c^2 + d^2))
```

3.196 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

Optimal result 1354
Rubi [A] (verified) 1355
Mathematica [A] (verified) 1358
Maple [A] (verified) 1359
Fricas [B] (verification not implemented) 1360
Sympy [F] 1361
Maxima [F(-2)] 1361
Giac [B] (verification not implemented) 1362
Mupad [B] (verification not implemented) 1363

Optimal result

Integrand size = 25, antiderivative size = 412

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \frac{a^3x}{c^4} - \frac{(3ab^2c^4d(4c^2+d^2)-b^3c^5(c^2+4d^2)-a^2b(6c^7+9c^5d^2)+a^3(8c^6d-8c^4d^3+7c^2d^5-2d^7))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^4\sqrt{c-d}\sqrt{c+d}\,(c^2-d^2)^3 f} - \frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\,(3bc^3-8ac^2d+2bcd^2+3ad^3)\sin(e+fx)}{6c^3\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\,(b^2c^2d(13c^2+2d^2)-abc(18c^4+17c^2d^2-5d^4)+a^2(34c^4d-28c^2d^3+9d^5))\sin(e+fx)}{6c^3\,(c^2-d^2)^3\,f(d+c\cos(e+fx))}$$

```
[Out] a^3*x/c^4-1/3*d*(-a*d+b*c)*(b+a*cos(f*x+e))^2*sin(f*x+e)/c/(c^2-d^2)/f/(d+c
*cos(f*x+e))^3+1/6*(-a*d+b*c)^2*(-8*a*c^2*d+3*a*d^3+3*b*c^3+2*b*c*d^2)*sin(
f*x+e)/c^3/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^2-1/6*(-a*d+b*c)*(b^2*c^2*d*(13*c
^2+2*d^2)-a*b*c*(18*c^4+17*c^2*d^2-5*d^4)+a^2*(34*c^4*d-28*c^2*d^3+9*d^5))*
sin(f*x+e)/c^3/(c^2-d^2)^3/f/(d+c*cos(f*x+e))-(3*a*b^2*c^4*d*(4*c^2+d^2)-b^
3*c^5*(c^2+4*d^2)-a^2*b*(6*c^7+9*c^5*d^2)+a^3*(8*c^6*d-8*c^4*d^3+7*c^2*d^5-
2*d^7))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^4/(c^2-d^2)^3
/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4026, 3068, 3110, 3100, 2814, 2738, 214}

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \frac{a^3x}{c^4} - \frac{(bc-ad)\left(a^2(34c^4d-28c^2d^3+9d^5)-abc(18c^4+17c^2d^2-5d^4)+b^2c^2d(13c^2+2d^2)\right)\sin(e+fx)}{6c^3f\left(c^2-d^2\right)^3(c\cos(e+fx)+d)} - \frac{\left(a^3(8c^6d-8c^4d^3+7c^2d^5-2d^7)-a^2b(6c^7+9c^5d^2)+3ab^2c^4d(4c^2+d^2)-b^3c^5(c^2+4d^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{c-}}{}\right.}{c^4f\sqrt{c-d}\sqrt{c+d}\left(c^2-d^2\right)^3} - \frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3} + \frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{6c^3f\left(c^2-d^2\right)^2(c\cos(e+fx)+d)^2}$$

```
[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*x)/c^4 - ((3*a*b^2*c^4*d*(4*c^2 + d^2) - b^3*c^5*(c^2 + 4*d^2) - a^2*b
*(6*c^7 + 9*c^5*d^2) + a^3*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTa
nh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^4*Sqrt[c - d]*Sqrt[c + d
]*(c^2 - d^2)^3*f) - (d*(b*c - a*d)*(b + a*Cos[e + f*x])^2*Sin[e + f*x])/(3
*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3) + ((b*c - a*d)^2*(3*b*c^3 - 8*a*c^
2*d + 2*b*c*d^2 + 3*a*d^3)*Sin[e + f*x])/(6*c^3*(c^2 - d^2)^2*f*(d + c*Cos[
e + f*x])^2) - ((b*c - a*d)*(b^2*c^2*d*(13*c^2 + 2*d^2) - a*b*c*(18*c^4 + 1
7*c^2*d^2 - 5*d^4) + a^2*(34*c^4*d - 28*c^2*d^3 + 9*d^5))*Sin[e + f*x])/(6*
c^3*(c^2 - d^2)^3*f*(d + c*Cos[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \int \frac{\cos(e+fx)(b+a\cos(e+fx))^3}{(d+c\cos(e+fx))^4}\,dx$$

$$= -\frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} + \frac{\int \frac{(b+a\cos(e+fx))\left((3bc-2ad)(bc-ad)-\left(3a^2cd+2b^2cd-ab\left(6c^2-d^2\right)\right)\cos(e+fx)+3a^2\left(c^2-d^2\right)\cos^2(e+fx)\right)}{(d+c\cos(e+fx))^3}\,dx}{3c\,(c^2-d^2)}$$

$$= -\frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\,(3bc^3-8ac^2d+2bcd^2+3ad^3)\sin(e+fx)}{6c^3\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^2} - \frac{\int \frac{-2c(bc-ad)\left(9abc^3-8a^2c^2d-5b^2c^2d+abcd^2+3a^2d^3\right)+\left(3ab^2c^2d\left(6c^2-d^2\right)-b^3c^3\left(3c^2+2d^2\right)-a^2bc\left(18c^4-7c^2d^2+4d^4\right)+a^3\left(12c^4d\right.\right.}{(d+c\cos(e+fx))^2}}{6c^3\,(c^2-d^2)^2}$$

$$= -\frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\,(3bc^3-8ac^2d+2bcd^2+3ad^3)\sin(e+fx)}{6c^3\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\,(b^2c^2d(13c^2+2d^2)-abc(18c^4+17c^2d^2-5d^4)+a^2(34c^4d-28c^2d^3+9d^5))\sin(e+f}{6c^3\,(c^2-d^2)^3\,f(d+c\cos(e+fx))} - \frac{\int \frac{-3c^2(bc-ad)\left(6a^2c^4+b^2c^4-11abc^3d-2a^2c^2d^2+4b^2c^2d^2+abcd^3+a^2d^4\right)-6a^3c\left(c^2-d^2\right)^3\cos(e+fx)}{d+c\cos(e+fx)}\,dx}{6c^4\,(c^2-d^2)^3}$$

$$= \frac{a^3x}{c^4} - \frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\,(3bc^3-8ac^2d+2bcd^2+3ad^3)\sin(e+fx)}{6c^3\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\,(b^2c^2d(13c^2+2d^2)-abc(18c^4+17c^2d^2-5d^4)+a^2(34c^4d-28c^2d^3+9d^5))\sin(e+f}{6c^3\,(c^2-d^2)^3\,f(d+c\cos(e+fx))} - \frac{(3ab^2c^4d(4c^2+d^2)-b^3c^5(c^2+4d^2)-a^2b(6c^7+9c^5d^2)+a^3(8c^6d-8c^4d^3+7c^2d^5-2d^7))\int \frac{}{d+}}{2c^4\,(c^2-d^2)^3}$$

$$= \frac{a^3x}{c^4} - \frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))^3}$$
$$+ \frac{(bc-ad)^2(3bc^3-8ac^2d+2bcd^2+3ad^3)\sin(e+fx)}{6c^3(c^2-d^2)^2f(d+c\cos(e+fx))^2}$$
$$- \frac{(bc-ad)(b^2c^2d(13c^2+2d^2)-abc(18c^4+17c^2d^2-5d^4)+a^2(34c^4d-28c^2d^3+9d^5))\sin(e+fx}{6c^3(c^2-d^2)^3f(d+c\cos(e+fx))}$$
$$- \frac{(3ab^2c^4d(4c^2+d^2)-b^3c^5(c^2+4d^2)-a^2b(6c^7+9c^5d^2)+a^3(8c^6d-8c^4d^3+7c^2d^5-2d^7))\,\mathrm{Subst}}{c^4(c^2-d^2)^3f}$$

$$= \frac{a^3x}{c^4}$$
$$- \frac{(3ab^2c^4d(4c^2+d^2)-b^3c^5(c^2+4d^2)-a^2b(6c^7+9c^5d^2)+a^3(8c^6d-8c^4d^3+7c^2d^5-2d^7))\arctan}{c^4\sqrt{c-d}\sqrt{c+d}(c^2-d^2)^3f}$$
$$- \frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))^3}$$
$$+ \frac{(bc-ad)^2(3bc^3-8ac^2d+2bcd^2+3ad^3)\sin(e+fx)}{6c^3(c^2-d^2)^2f(d+c\cos(e+fx))^2}$$
$$- \frac{(bc-ad)(b^2c^2d(13c^2+2d^2)-abc(18c^4+17c^2d^2-5d^4)+a^2(34c^4d-28c^2d^3+9d^5))\sin(e+fx}{6c^3(c^2-d^2)^3f(d+c\cos(e+fx))}$$

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$
$$= \frac{(d+c\cos(e+fx))\sec(e+fx)(a+b\sec(e+fx))^3\left(6a^3(e+fx)(d+c\cos(e+fx))^3-\frac{6(-3ab^2c^4d(4c^2+d^2)+b}{}\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^3*(6*a^3*(e + f*x)*
(d + c*Cos[e + f*x])^3 - (6*(-3*a*b^2*c^4*d*(4*c^2 + d^2) + b^3*c^5*(c^2 +
4*d^2) + a^2*b*(6*c^7 + 9*c^5*d^2) + a^3*(-8*c^6*d + 8*c^4*d^3 - 7*c^2*d^5
+ 2*d^7))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e
 + f*x])^3)/(c^2 - d^2)^(7/2) - (2*c*d*(b*c - a*d)^3*Sin[e + f*x])/(c^2 - d
^2) + (c*(b*c - a*d)^2*(3*b*c^3 - 12*a*c^2*d + 2*b*c*d^2 + 7*a*d^3)*(d + c*
Cos[e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-(b^3*c^3*d*(13*c^2 + 2*d^2
)) + 3*a*b^2*c^2*(6*c^4 + 10*c^2*d^2 - d^4) - 3*a^2*b*c*d*(18*c^4 - 5*c^2*d
^2 + 2*d^4) + a^3*(36*c^4*d^2 - 32*c^2*d^4 + 11*d^6))*(d + c*Cos[e + f*x])^
```

2*Sin[e + f*x])/(c^2 - d^2)^3))/(6*c^4*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[
e + f*x])^4)

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.91

method	result
derivativedivides	$\frac{2a^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^3c^4d^2+4a^3c^3d^3-6a^3c^2d^4-a^3c\,d^5+2a^3d^6-18a^2b\,c^5d-9a^2b\,c^4d^2-6a^2b\,c^3d^3+6a\,b^2c^6+6a\,b^2\right.}{2(c-d)\left(c^3+3c^2d+3c\,d^2\right.}}{}$
default	$\frac{2a^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^3c^4d^2+4a^3c^3d^3-6a^3c^2d^4-a^3c\,d^5+2a^3d^6-18a^2b\,c^5d-9a^2b\,c^4d^2-6a^2b\,c^3d^3+6a\,b^2c^6+6a\,b^2\right.}{2(c-d)\left(c^3+3c^2d+3c\,d^2\right.}}{}$
risch	Expression too large to display

[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)

[Out] 1/f*(2*a^3/c^4*arctan(tan(1/2*f*x+1/2*e))+2/c^4*((-1/2*(12*a^3*c^4*d^2+4*a^
3*c^3*d^3-6*a^3*c^2*d^4-a^3*c*d^5+2*a^3*d^6-18*a^2*b*c^5*d-9*a^2*b*c^4*d^2-
6*a^2*b*c^3*d^3+6*a*b^2*c^6+6*a*b^2*c^5*d+18*a*b^2*c^4*d^2+3*a*b^2*c^3*d^3-
b^3*c^6-6*b^3*c^5*d-2*b^3*c^4*d^2-2*b^3*c^3*d^3)*c/(c-d)/(c^3+3*c^2*d+3*c*d
^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(18*a^3*c^4*d^2-11*a^3*c^2*d^4+3*a^3*d^6-2
7*a^2*b*c^5*d-3*a^2*b*c^3*d^3+9*a*b^2*c^6+21*a*b^2*c^4*d^2-7*b^3*c^5*d-3*b^
3*c^3*d^3)*c/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/2*(12*a
^3*c^4*d^2-4*a^3*c^3*d^3-6*a^3*c^2*d^4+a^3*c*d^5+2*a^3*d^6-18*a^2*b*c^5*d+9
*a^2*b*c^4*d^2-6*a^2*b*c^3*d^3+6*a*b^2*c^6-6*a*b^2*c^5*d+18*a*b^2*c^4*d^2-3
*a*b^2*c^3*d^3+b^3*c^6-6*b^3*c^5*d+2*b^3*c^4*d^2-2*b^3*c^3*d^3)*c/(c+d)/(c^
3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*
f*x+1/2*e)^2*d-c-d)^3-1/2*(8*a^3*c^6*d-8*a^3*c^4*d^3+7*a^3*c^2*d^5-2*a^3*d^
7-6*a^2*b*c^7-9*a^2*b*c^5*d^2+12*a*b^2*c^6*d+3*a*b^2*c^4*d^3-b^3*c^7-4*b^3*
c^5*d^2)/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*ta
n(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(396) = 792$.

Time = 0.48 (sec) , antiderivative size = 2776, normalized size of antiderivative = 6.74

$$\int \frac{(a + b\sec(e + fx))^3}{(c + d\sec(e + fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^3*c^11 - 4*a^3*c^9*d^2 + 6*a^3*c^7*d^4 - 4*a^3*c^5*d^6 + a^3*c
^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^3*c^10*d - 4*a^3*c^8*d^3 + 6*a^3*c^6*d^5
 - 4*a^3*c^4*d^7 + a^3*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^3*c^9*d^2 - 4*a^
3*c^7*d^4 + 6*a^3*c^5*d^6 - 4*a^3*c^3*d^8 + a^3*c*d^10)*f*x*cos(f*x + e) +
12*(a^3*c^8*d^3 - 4*a^3*c^6*d^5 + 6*a^3*c^4*d^7 - 4*a^3*c^2*d^9 + a^3*d^11)
*f*x + 3*(7*a^3*c^2*d^8 - 2*a^3*d^10 - (6*a^2*b + b^3)*c^7*d^3 + 4*(2*a^3 +
 3*a*b^2)*c^6*d^4 - (9*a^2*b + 4*b^3)*c^5*d^5 - (8*a^3 - 3*a*b^2)*c^4*d^6 +
 (7*a^3*c^5*d^5 - 2*a^3*c^3*d^7 - (6*a^2*b + b^3)*c^10 + 4*(2*a^3 + 3*a*b^2
)*c^9*d - (9*a^2*b + 4*b^3)*c^8*d^2 - (8*a^3 - 3*a*b^2)*c^7*d^3)*cos(f*x +
e)^3 + 3*(7*a^3*c^4*d^6 - 2*a^3*c^2*d^8 - (6*a^2*b + b^3)*c^9*d + 4*(2*a^3
+ 3*a*b^2)*c^8*d^2 - (9*a^2*b + 4*b^3)*c^7*d^3 - (8*a^3 - 3*a*b^2)*c^6*d^4)
*cos(f*x + e)^2 + 3*(7*a^3*c^3*d^7 - 2*a^3*c*d^9 - (6*a^2*b + b^3)*c^8*d^2
+ 4*(2*a^3 + 3*a*b^2)*c^7*d^3 - (9*a^2*b + 4*b^3)*c^6*d^4 - (8*a^3 - 3*a*b^
2)*c^5*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^3*c
^10*d + 6*a*b^2*c^9*d^2 + 23*a^3*c^3*d^8 - 6*a^3*c*d^10 - 11*(3*a^2*b + b^3
)*c^8*d^3 + (26*a^3 + 33*a*b^2)*c^7*d^4 + (21*a^2*b + 4*b^3)*c^6*d^5 - (43*
a^3 + 39*a*b^2)*c^5*d^6 + 6*(2*a^2*b + b^3)*c^4*d^7 + (18*a*b^2*c^11 + 6*a^
2*b*c^4*d^7 - 11*a^3*c^3*d^8 - (54*a^2*b + 13*b^3)*c^10*d + 12*(3*a^3 + a*b
^2)*c^9*d^2 + (69*a^2*b + 11*b^3)*c^8*d^3 - (68*a^3 + 33*a*b^2)*c^7*d^4 - (
21*a^2*b - 2*b^3)*c^6*d^5 + (43*a^3 + 3*a*b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*
(b^3*c^11 + 6*a*b^2*c^10*d - 5*a^3*c^2*d^9 - (27*a^2*b + 10*b^3)*c^9*d^2 +
(20*a^3 + 21*a*b^2)*c^8*d^3 + (24*a^2*b + 7*b^3)*c^7*d^4 - 5*(7*a^3 + 6*a*b
^2)*c^6*d^5 + (3*a^2*b + 2*b^3)*c^5*d^6 + (20*a^3 + 3*a*b^2)*c^4*d^7)*cos(f
*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c^9*d^6 + c^7*d
^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 - 4*c^8*d^7 + c^
6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^6 - 4*c^7*d^8
+ c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d^7 - 4*c^6*d^9
 + c^4*d^11)*f), 1/6*(6*(a^3*c^11 - 4*a^3*c^9*d^2 + 6*a^3*c^7*d^4 - 4*a^3*c
^5*d^6 + a^3*c^3*d^8)*f*x*cos(f*x + e)^3 + 18*(a^3*c^10*d - 4*a^3*c^8*d^3 +
 6*a^3*c^6*d^5 - 4*a^3*c^4*d^7 + a^3*c^2*d^9)*f*x*cos(f*x + e)^2 + 18*(a^3*
c^9*d^2 - 4*a^3*c^7*d^4 + 6*a^3*c^5*d^6 - 4*a^3*c^3*d^8 + a^3*c*d^10)*f*x*c
os(f*x + e) + 6*(a^3*c^8*d^3 - 4*a^3*c^6*d^5 + 6*a^3*c^4*d^7 - 4*a^3*c^2*d^
9 + a^3*d^11)*f*x - 3*(7*a^3*c^2*d^8 - 2*a^3*d^10 - (6*a^2*b + b^3)*c^7*d^3
```

```
 + 4*(2*a^3 + 3*a*b^2)*c^6*d^4 - (9*a^2*b + 4*b^3)*c^5*d^5 - (8*a^3 - 3*a*b
^2)*c^4*d^6 + (7*a^3*c^5*d^5 - 2*a^3*c^3*d^7 - (6*a^2*b + b^3)*c^10 + 4*(2*
a^3 + 3*a*b^2)*c^9*d - (9*a^2*b + 4*b^3)*c^8*d^2 - (8*a^3 - 3*a*b^2)*c^7*d^
3)*cos(f*x + e)^3 + 3*(7*a^3*c^4*d^6 - 2*a^3*c^2*d^8 - (6*a^2*b + b^3)*c^9*
d + 4*(2*a^3 + 3*a*b^2)*c^8*d^2 - (9*a^2*b + 4*b^3)*c^7*d^3 - (8*a^3 - 3*a*
b^2)*c^6*d^4)*cos(f*x + e)^2 + 3*(7*a^3*c^3*d^7 - 2*a^3*c*d^9 - (6*a^2*b +
b^3)*c^8*d^2 + 4*(2*a^3 + 3*a*b^2)*c^7*d^3 - (9*a^2*b + 4*b^3)*c^6*d^4 - (8
*a^3 - 3*a*b^2)*c^5*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 +
 d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b^3*c^10*d + 6*a*
b^2*c^9*d^2 + 23*a^3*c^3*d^8 - 6*a^3*c*d^10 - 11*(3*a^2*b + b^3)*c^8*d^3 +
(26*a^3 + 33*a*b^2)*c^7*d^4 + (21*a^2*b + 4*b^3)*c^6*d^5 - (43*a^3 + 39*a*b
^2)*c^5*d^6 + 6*(2*a^2*b + b^3)*c^4*d^7 + (18*a*b^2*c^11 + 6*a^2*b*c^4*d^7
- 11*a^3*c^3*d^8 - (54*a^2*b + 13*b^3)*c^10*d + 12*(3*a^3 + a*b^2)*c^9*d^2
+ (69*a^2*b + 11*b^3)*c^8*d^3 - (68*a^3 + 33*a*b^2)*c^7*d^4 - (21*a^2*b - 2
*b^3)*c^6*d^5 + (43*a^3 + 3*a*b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*(b^3*c^11 +
6*a*b^2*c^10*d - 5*a^3*c^2*d^9 - (27*a^2*b + 10*b^3)*c^9*d^2 + (20*a^3 + 21
*a*b^2)*c^8*d^3 + (24*a^2*b + 7*b^3)*c^7*d^4 - 5*(7*a^3 + 6*a*b^2)*c^6*d^5
+ (3*a^2*b + 2*b^3)*c^5*d^6 + (20*a^3 + 3*a*b^2)*c^4*d^7)*cos(f*x + e))*sin
(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c^9*d^6 + c^7*d^8)*f*cos(f*
x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 - 4*c^8*d^7 + c^6*d^9)*f*cos
(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^6 - 4*c^7*d^8 + c^5*d^10)*
f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d^7 - 4*c^6*d^9 + c^4*d^11)
*f)]
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx=\int \frac{(a+b\sec{(e+fx)})^3}{(c+d\sec{(e+fx)})^4}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**4,x)
[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. 2(396) = 792.

Time = 0.43 (sec) , antiderivative size = 1572, normalized size of antiderivative = 3.82

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(6*a^2*b*c^7 + b^3*c^7 - 8*a^3*c^6*d - 12*a*b^2*c^6*d + 9*a^2*b*c^5*
d^2 + 4*b^3*c^5*d^2 + 8*a^3*c^4*d^3 - 3*a*b^2*c^4*d^3 - 7*a^3*c^2*d^5 + 2*a
^3*d^7)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(
1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^10 - 3*c^
8*d^2 + 3*c^6*d^4 - c^4*d^6)*sqrt(-c^2 + d^2)) + 3*(f*x + e)*a^3/c^4 - (18*
a*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 3*b^3*c^8*tan(1/2*f*x + 1/2*e)^5 - 54*a^
2*b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 18*a*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^5 -
12*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 36*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5
 + 81*a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 36*a*b^2*c^6*d^2*tan(1/2*f*x +
 1/2*e)^5 + 27*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^3*c^5*d^3*tan(1/2*
f*x + 1/2*e)^5 - 18*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 81*a*b^2*c^5*d^3
*tan(1/2*f*x + 1/2*e)^5 - 12*b^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 6*a^3*c^4
*d^4*tan(1/2*f*x + 1/2*e)^5 + 9*a^2*b*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 36*a
*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*b^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5
 + 45*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^3*d^5*tan(1/2*f*x + 1/
2*e)^5 + 9*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 6*b^3*c^3*d^5*tan(1/2*f*x
 + 1/2*e)^5 - 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 15*a^3*c*d^7*tan(1/2*f
*x + 1/2*e)^5 + 6*a^3*d^8*tan(1/2*f*x + 1/2*e)^5 - 36*a*b^2*c^8*tan(1/2*f*x
 + 1/2*e)^3 + 108*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 28*b^3*c^7*d*tan(1/2
*f*x + 1/2*e)^3 - 72*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 48*a*b^2*c^6*d^2*
tan(1/2*f*x + 1/2*e)^3 - 96*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 16*b^3*c
^5*d^3*tan(1/2*f*x + 1/2*e)^3 + 116*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 84
*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 12*a^2*b*c^3*d^5*tan(1/2*f*x + 1/2*
e)^3 - 12*b^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 56*a^3*c^2*d^6*tan(1/2*f*x +
 1/2*e)^3 + 12*a^3*d^8*tan(1/2*f*x + 1/2*e)^3 + 18*a*b^2*c^8*tan(1/2*f*x +
1/2*e) + 3*b^3*c^8*tan(1/2*f*x + 1/2*e) - 54*a^2*b*c^7*d*tan(1/2*f*x + 1/2*
e) + 18*a*b^2*c^7*d*tan(1/2*f*x + 1/2*e) - 12*b^3*c^7*d*tan(1/2*f*x + 1/2*e
) + 36*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e) - 81*a^2*b*c^6*d^2*tan(1/2*f*x + 1/
2*e) + 36*a*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e) - 27*b^3*c^6*d^2*tan(1/2*f*x +
 1/2*e) + 60*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e) - 18*a^2*b*c^5*d^3*tan(1/2*f*
x + 1/2*e) + 81*a*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e) - 12*b^3*c^5*d^3*tan(1/2
*f*x + 1/2*e) - 6*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e) - 9*a^2*b*c^4*d^4*tan(1/
2*f*x + 1/2*e) + 36*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) - 6*b^3*c^4*d^4*tan(
1/2*f*x + 1/2*e) - 45*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e) - 18*a^2*b*c^3*d^5*t
an(1/2*f*x + 1/2*e) - 9*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e) - 6*b^3*c^3*d^5*
```

```
tan(1/2*f*x + 1/2*e) - 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e) + 15*a^3*c*d^7*ta
n(1/2*f*x + 1/2*e) + 6*a^3*d^8*tan(1/2*f*x + 1/2*e))/((c^9 - 3*c^7*d^2 + 3*
c^5*d^4 - c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c
 - d)^3))/f
```

Mupad [B] (verification not implemented)

Time = 28.48 (sec) , antiderivative size = 15647, normalized size of antiderivative = 37.98

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

```
[In] int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^4,x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^5*(b^3*c^6 - 2*a^3*d^6 - 6*a*b^2*c^6 + a^3*c*d^5 + 6*b
^3*c^5*d + 6*a^3*c^2*d^4 - 4*a^3*c^3*d^3 - 12*a^3*c^4*d^2 + 2*b^3*c^3*d^3 +
 2*b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 - 18*a*b^2*c^4*d^2 + 6*a^2*b*c^3*d^3 + 9*a
^2*b*c^4*d^2 - 6*a*b^2*c^5*d + 18*a^2*b*c^5*d))/((c^3*d - c^4)*(c + d)^3) +
 (4*tan(e/2 + (f*x)/2)^3*(7*b^3*c^5*d - 9*a*b^2*c^6 - 3*a^3*d^6 + 11*a^3*c^
2*d^4 - 18*a^3*c^4*d^2 + 3*b^3*c^3*d^3 - 21*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3
 + 27*a^2*b*c^5*d))/(3*(c + d)^2*(c^5 - 2*c^4*d + c^3*d^2)) - (tan(e/2 + (f
*x)/2)*(2*a^3*d^6 + b^3*c^6 + 6*a*b^2*c^6 + a^3*c*d^5 - 6*b^3*c^5*d - 6*a^3
*c^2*d^4 - 4*a^3*c^3*d^3 + 12*a^3*c^4*d^2 - 2*b^3*c^3*d^3 + 2*b^3*c^4*d^2 -
 3*a*b^2*c^3*d^3 + 18*a*b^2*c^4*d^2 - 6*a^2*b*c^3*d^3 + 9*a^2*b*c^4*d^2 - 6
*a*b^2*c^5*d - 18*a^2*b*c^5*d))/((c + d)*(3*c^5*d - c^6 + c^3*d^3 - 3*c^4*d
^2)))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/
2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^
3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) - (2*a^3*a
tan(((a^3*((a^3*((8*(4*a^3*c^21 + 2*b^3*c^21 + 12*a^2*b*c^21 - 16*a^3*c^20*
d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^12 + 26*a^3*c^10*d^11 - 14*
a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^8 + 110*a^3*c^14*d^7 - 30*a
^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4 + 64*a^3*c^18*d^3 - 12*a^3
*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22*b^3*c^14*d^7 + 22*b^3*c^15
*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3*c^18*d^3 + 2*b^3*c^19*d^2
- 6*a*b^2*c^11*d^10 + 6*a*b^2*c^12*d^9 - 6*a*b^2*c^13*d^8 + 6*a*b^2*c^14*d^
7 + 54*a*b^2*c^15*d^6 - 54*a*b^2*c^16*d^5 - 66*a*b^2*c^17*d^4 + 66*a*b^2*c^
18*d^3 + 24*a*b^2*c^19*d^2 + 18*a^2*b*c^12*d^9 - 18*a^2*b*c^13*d^8 - 42*a^2
*b*c^14*d^7 + 42*a^2*b*c^15*d^6 + 18*a^2*b*c^16*d^5 - 18*a^2*b*c^17*d^4 + 1
8*a^2*b*c^18*d^3 - 18*a^2*b*c^19*d^2 - 24*a*b^2*c^20*d - 12*a^2*b*c^20*d))/
(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d
^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (
a^3*tan(e/2 + (f*x)/2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 -
 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7
- 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)*8i)
/(c^4*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^1
```

0*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2))
)*1i)/c^4 + (8*tan(e/2 + (f*x)/2)*(4*a^6*c^14 + 8*a^6*d^14 + b^6*c^14 - 8*a
^6*c*d^13 - 8*a^6*c^13*d + 12*a^2*b^4*c^14 + 36*a^4*b^2*c^14 - 48*a^6*c^2*d
^12 + 48*a^6*c^3*d^11 + 117*a^6*c^4*d^10 - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^
8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^10*d^4 +
48*a^6*c^11*d^3 + 44*a^6*c^12*d^2 + 16*b^6*c^10*d^4 + 8*b^6*c^12*d^2 - 24*
a*b^5*c^9*d^5 - 102*a*b^5*c^11*d^3 - 160*a^3*b^3*c^13*d + 36*a^5*b*c^5*d^9
- 102*a^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5 - 48*a^5*b*c^11*d^3 + 9*a^2*b^4*c^8*
d^6 + 144*a^2*b^4*c^10*d^4 + 210*a^2*b^4*c^12*d^2 + 16*a^3*b^3*c^5*d^9 - 52
*a^3*b^3*c^7*d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^11*d^3 - 12*a^4*b^2*c^
4*d^10 - 6*a^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^10*d^4 + 30
0*a^4*b^2*c^12*d^2 - 24*a*b^5*c^13*d - 96*a^5*b*c^13*d))/(c^16*d + c^17 - c
^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10
*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))/c^4 - (a^3*((a^3*((8*(
4*a^3*c^21 + 2*b^3*c^21 + 12*a^2*b*c^21 - 16*a^3*c^20*d - 2*b^3*c^20*d - 4*
a^3*c^8*d^13 + 2*a^3*c^9*d^12 + 26*a^3*c^10*d^11 - 14*a^3*c^11*d^10 - 70*a^
3*c^12*d^9 + 30*a^3*c^13*d^8 + 110*a^3*c^14*d^7 - 30*a^3*c^15*d^6 - 110*a^3
*c^16*d^5 + 20*a^3*c^17*d^4 + 64*a^3*c^18*d^3 - 12*a^3*c^19*d^2 + 8*b^3*c^1
2*d^9 - 8*b^3*c^13*d^8 - 22*b^3*c^14*d^7 + 22*b^3*c^15*d^6 + 18*b^3*c^16*d^
5 - 18*b^3*c^17*d^4 - 2*b^3*c^18*d^3 + 2*b^3*c^19*d^2 - 6*a*b^2*c^11*d^10 +
6*a*b^2*c^12*d^9 - 6*a*b^2*c^13*d^8 + 6*a*b^2*c^14*d^7 + 54*a*b^2*c^15*d^6
- 54*a*b^2*c^16*d^5 - 66*a*b^2*c^17*d^4 + 66*a*b^2*c^18*d^3 + 24*a*b^2*c^1
9*d^2 + 18*a^2*b*c^12*d^9 - 18*a^2*b*c^13*d^8 - 42*a^2*b*c^14*d^7 + 42*a^2*
b*c^15*d^6 + 18*a^2*b*c^16*d^5 - 18*a^2*b*c^17*d^4 + 18*a^2*b*c^18*d^3 - 18
*a^2*b*c^19*d^2 - 24*a*b^2*c^20*d - 12*a^2*b*c^20*d))/(c^19*d + c^20 - c^9*
d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10
*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (a^3*tan(e/2 + (f*x)/2
)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c
^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*
c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)*8i)/(c^4*(c^16*d + c^17
- c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 +
10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*1i)/c^4 - (8*tan(e/
2 + (f*x)/2)*(4*a^6*c^14 + 8*a^6*d^14 + b^6*c^14 - 8*a^6*c*d^13 - 8*a^6*c^1
3*d + 12*a^2*b^4*c^14 + 36*a^4*b^2*c^14 - 48*a^6*c^2*d^12 + 48*a^6*c^3*d^11
+ 117*a^6*c^4*d^10 - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 +
156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^10*d^4 + 48*a^6*c^11*d^3 + 44
*a^6*c^12*d^2 + 16*b^6*c^10*d^4 + 8*b^6*c^12*d^2 - 24*a*b^5*c^9*d^5 - 102*a
*b^5*c^11*d^3 - 160*a^3*b^3*c^13*d + 36*a^5*b*c^5*d^9 - 102*a^5*b*c^7*d^7 +
60*a^5*b*c^9*d^5 - 48*a^5*b*c^11*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^2*b^4*c^1
0*d^4 + 210*a^2*b^4*c^12*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7*d^7 - 4*
a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^11*d^3 - 12*a^4*b^2*c^4*d^10 - 6*a^4*b^2*c^
6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^10*d^4 + 300*a^4*b^2*c^12*d^2 -
24*a*b^5*c^13*d - 96*a^5*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 +
5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d
^4 - 5*c^14*d^3 - 5*c^15*d^2)))/c^4)/((a^3*((a^3*((8*(4*a^3*c^21 + 2*b^3*c^

21 + 12*a^2*b*c^21 - 16*a^3*c^20*d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*
c^9*d^12 + 26*a^3*c^10*d^11 - 14*a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c
^13*d^8 + 110*a^3*c^14*d^7 - 30*a^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^
17*d^4 + 64*a^3*c^18*d^3 - 12*a^3*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^
8 - 22*b^3*c^14*d^7 + 22*b^3*c^15*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 -
2*b^3*c^18*d^3 + 2*b^3*c^19*d^2 - 6*a*b^2*c^11*d^10 + 6*a*b^2*c^12*d^9 - 6
*a*b^2*c^13*d^8 + 6*a*b^2*c^14*d^7 + 54*a*b^2*c^15*d^6 - 54*a*b^2*c^16*d^5
- 66*a*b^2*c^17*d^4 + 66*a*b^2*c^18*d^3 + 24*a*b^2*c^19*d^2 + 18*a^2*b*c^12
*d^9 - 18*a^2*b*c^13*d^8 - 42*a^2*b*c^14*d^7 + 42*a^2*b*c^15*d^6 + 18*a^2*b
*c^16*d^5 - 18*a^2*b*c^17*d^4 + 18*a^2*b*c^18*d^3 - 18*a^2*b*c^19*d^2 - 24*
a*b^2*c^20*d - 12*a^2*b*c^20*d))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*
c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d
^4 - 5*c^17*d^3 - 5*c^18*d^2) - (a^3*tan(e/2 + (f*x)/2)*(8*c^21*d - 8*c^8*d
^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d
^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^
4 - 48*c^19*d^3 - 8*c^20*d^2)*8i)/(c^4*(c^16*d + c^17 - c^6*d^11 - c^7*d^10
+ 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^1
3*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*1i)/c^4 + (8*tan(e/2 + (f*x)/2)*(4*a^6*c
^14 + 8*a^6*d^14 + b^6*c^14 - 8*a^6*c*d^13 - 8*a^6*c^13*d + 12*a^2*b^4*c^14
+ 36*a^4*b^2*c^14 - 48*a^6*c^2*d^12 + 48*a^6*c^3*d^11 + 117*a^6*c^4*d^10 -
120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6 - 12
0*a^6*c^9*d^5 - 92*a^6*c^10*d^4 + 48*a^6*c^11*d^3 + 44*a^6*c^12*d^2 + 16*b^
6*c^10*d^4 + 8*b^6*c^12*d^2 - 24*a*b^5*c^9*d^5 - 102*a*b^5*c^11*d^3 - 160*a
^3*b^3*c^13*d + 36*a^5*b*c^5*d^9 - 102*a^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5 - 4
8*a^5*b*c^11*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^2*b^4*c^10*d^4 + 210*a^2*b^4*c
^12*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7*d^7 - 4*a^3*b^3*c^9*d^5 - 300
*a^3*b^3*c^11*d^3 - 12*a^4*b^2*c^4*d^10 - 6*a^4*b^2*c^6*d^8 + 120*a^4*b^2*c
^8*d^6 - 63*a^4*b^2*c^10*d^4 + 300*a^4*b^2*c^12*d^2 - 24*a*b^5*c^13*d - 96*
a^5*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8
- 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c
^15*d^2))*1i)/c^4 - (16*(4*a^9*d^13 - 12*a^8*b*c^13 - 2*a^9*c*d^12 + 16*a^9
*c^12*d + a^3*b^6*c^13 + 12*a^5*b^4*c^13 - 2*a^6*b^3*c^13 + 36*a^7*b^2*c^13
- 26*a^9*c^2*d^11 + 11*a^9*c^3*d^10 + 70*a^9*c^4*d^9 - 34*a^9*c^5*d^8 - 11
0*a^9*c^6*d^7 + 66*a^9*c^7*d^6 + 110*a^9*c^8*d^5 - 64*a^9*c^9*d^4 - 64*a^9*
c^10*d^3 + 48*a^9*c^11*d^2 - 24*a^4*b^5*c^12*d - 158*a^6*b^3*c^12*d + 24*a^
7*b^2*c^12*d + 18*a^8*b*c^4*d^9 + 18*a^8*b*c^5*d^8 - 60*a^8*b*c^6*d^7 - 42*
a^8*b*c^7*d^6 + 42*a^8*b*c^8*d^5 + 18*a^8*b*c^9*d^4 - 66*a^8*b*c^10*d^3 + 1
8*a^8*b*c^11*d^2 + 16*a^3*b^6*c^9*d^4 + 8*a^3*b^6*c^11*d^2 - 24*a^4*b^5*c^8
*d^5 - 102*a^4*b^5*c^10*d^3 + 9*a^5*b^4*c^7*d^6 + 144*a^5*b^4*c^9*d^4 + 210
*a^5*b^4*c^11*d^2 + 8*a^6*b^3*c^4*d^9 + 8*a^6*b^3*c^5*d^8 - 30*a^6*b^3*c^6*
d^7 - 22*a^6*b^3*c^7*d^6 - 22*a^6*b^3*c^8*d^5 + 18*a^6*b^3*c^9*d^4 - 298*a^
6*b^3*c^10*d^3 - 2*a^6*b^3*c^11*d^2 - 6*a^7*b^2*c^3*d^10 - 6*a^7*b^2*c^4*d^
9 - 6*a^7*b^2*c^6*d^7 + 66*a^7*b^2*c^7*d^6 + 54*a^7*b^2*c^8*d^5 + 3*a^7*b^2
*c^9*d^4 - 66*a^7*b^2*c^10*d^3 + 276*a^7*b^2*c^11*d^2 - 84*a^8*b*c^12*d))/(
c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^

7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (a
^3*((a^3*((8*(4*a^3*c^21 + 2*b^3*c^21 + 12*a^2*b*c^21 - 16*a^3*c^20*d - 2*b
^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^12 + 26*a^3*c^10*d^11 - 14*a^3*c^1
1*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^8 + 110*a^3*c^14*d^7 - 30*a^3*c^15
*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4 + 64*a^3*c^18*d^3 - 12*a^3*c^19*d
^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22*b^3*c^14*d^7 + 22*b^3*c^15*d^6 +
18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3*c^18*d^3 + 2*b^3*c^19*d^2 - 6*a*b
^2*c^11*d^10 + 6*a*b^2*c^12*d^9 - 6*a*b^2*c^13*d^8 + 6*a*b^2*c^14*d^7 + 54*
a*b^2*c^15*d^6 - 54*a*b^2*c^16*d^5 - 66*a*b^2*c^17*d^4 + 66*a*b^2*c^18*d^3
+ 24*a*b^2*c^19*d^2 + 18*a^2*b*c^12*d^9 - 18*a^2*b*c^13*d^8 - 42*a^2*b*c^14
*d^7 + 42*a^2*b*c^15*d^6 + 18*a^2*b*c^16*d^5 - 18*a^2*b*c^17*d^4 + 18*a^2*b
*c^18*d^3 - 18*a^2*b*c^19*d^2 - 24*a*b^2*c^20*d - 12*a^2*b*c^20*d))/(c^19*d
 + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10
*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (a^3*tan
(e/2 + (f*x)/2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^1
1*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c
^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)*8i)/(c^4*(
c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 -
 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*1i)/c
^4 - (8*tan(e/2 + (f*x)/2)*(4*a^6*c^14 + 8*a^6*d^14 + b^6*c^14 - 8*a^6*c*d^
13 - 8*a^6*c^13*d + 12*a^2*b^4*c^14 + 36*a^4*b^2*c^14 - 48*a^6*c^2*d^12 + 4
8*a^6*c^3*d^11 + 117*a^6*c^4*d^10 - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160
*a^6*c^7*d^7 + 156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^10*d^4 + 48*a^6
*c^11*d^3 + 44*a^6*c^12*d^2 + 16*b^6*c^10*d^4 + 8*b^6*c^12*d^2 - 24*a*b^5*c
^9*d^5 - 102*a*b^5*c^11*d^3 - 160*a^3*b^3*c^13*d + 36*a^5*b*c^5*d^9 - 102*a
^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5 - 48*a^5*b*c^11*d^3 + 9*a^2*b^4*c^8*d^6 + 1
44*a^2*b^4*c^10*d^4 + 210*a^2*b^4*c^12*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^
3*c^7*d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^11*d^3 - 12*a^4*b^2*c^4*d^10
- 6*a^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^10*d^4 + 300*a^4*b
^2*c^12*d^2 - 24*a*b^5*c^13*d - 96*a^5*b*c^13*d))/(c^16*d + c^17 - c^6*d^11
 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d
^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2))*1i)/c^4)))/(c^4*f) + (atan((((
(8*tan(e/2 + (f*x)/2)*(4*a^6*c^14 + 8*a^6*d^14 + b^6*c^14 - 8*a^6*c*d^13 -
8*a^6*c^13*d + 12*a^2*b^4*c^14 + 36*a^4*b^2*c^14 - 48*a^6*c^2*d^12 + 48*a^6
*c^3*d^11 + 117*a^6*c^4*d^10 - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*
c^7*d^7 + 156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^10*d^4 + 48*a^6*c^11
*d^3 + 44*a^6*c^12*d^2 + 16*b^6*c^10*d^4 + 8*b^6*c^12*d^2 - 24*a*b^5*c^9*d^
5 - 102*a*b^5*c^11*d^3 - 160*a^3*b^3*c^13*d + 36*a^5*b*c^5*d^9 - 102*a^5*b*
c^7*d^7 + 60*a^5*b*c^9*d^5 - 48*a^5*b*c^11*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^
2*b^4*c^10*d^4 + 210*a^2*b^4*c^12*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7
*d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^11*d^3 - 12*a^4*b^2*c^4*d^10 - 6*a
^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^10*d^4 + 300*a^4*b^2*c^
12*d^2 - 24*a*b^5*c^13*d - 96*a^5*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^
7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 +
10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) + (((8*(4*a^3*c^21 + 2*b^3*c^21 + 12

*a^2*b*c^21 - 16*a^3*c^20*d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^1
2 + 26*a^3*c^10*d^11 - 14*a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^8
 + 110*a^3*c^14*d^7 - 30*a^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4
+ 64*a^3*c^18*d^3 - 12*a^3*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22*
b^3*c^14*d^7 + 22*b^3*c^15*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3*
c^18*d^3 + 2*b^3*c^19*d^2 - 6*a*b^2*c^11*d^10 + 6*a*b^2*c^12*d^9 - 6*a*b^2*
c^13*d^8 + 6*a*b^2*c^14*d^7 + 54*a*b^2*c^15*d^6 - 54*a*b^2*c^16*d^5 - 66*a*
b^2*c^17*d^4 + 66*a*b^2*c^18*d^3 + 24*a*b^2*c^19*d^2 + 18*a^2*b*c^12*d^9 -
18*a^2*b*c^13*d^8 - 42*a^2*b*c^14*d^7 + 42*a^2*b*c^15*d^6 + 18*a^2*b*c^16*d
^5 - 18*a^2*b*c^17*d^4 + 18*a^2*b*c^18*d^3 - 18*a^2*b*c^19*d^2 - 24*a*b^2*c
^20*d - 12*a^2*b*c^20*d))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^
9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*
c^17*d^3 - 5*c^18*d^2) - (4*tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1/2)*
(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^
4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d)
*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^
12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c
^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2))/((c^18 - c^4*d^14 + 7*c^
6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2
)*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^
7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*((
c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d
- 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b
*c^5*d^2 - 12*a*b^2*c^6*d))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10
+ 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)))*((c + d)^7*(c - d
)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5
 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a
*b^2*c^6*d)*1i)/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^
8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a
^6*c^14 + 8*a^6*d^14 + b^6*c^14 - 8*a^6*c*d^13 - 8*a^6*c^13*d + 12*a^2*b^4*
c^14 + 36*a^4*b^2*c^14 - 48*a^6*c^2*d^12 + 48*a^6*c^3*d^11 + 117*a^6*c^4*d^
10 - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6
- 120*a^6*c^9*d^5 - 92*a^6*c^10*d^4 + 48*a^6*c^11*d^3 + 44*a^6*c^12*d^2 + 1
6*b^6*c^10*d^4 + 8*b^6*c^12*d^2 - 24*a*b^5*c^9*d^5 - 102*a*b^5*c^11*d^3 - 1
60*a^3*b^3*c^13*d + 36*a^5*b*c^5*d^9 - 102*a^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5
 - 48*a^5*b*c^11*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^2*b^4*c^10*d^4 + 210*a^2*b
^4*c^12*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7*d^7 - 4*a^3*b^3*c^9*d^5 -
 300*a^3*b^3*c^11*d^3 - 12*a^4*b^2*c^4*d^10 - 6*a^4*b^2*c^6*d^8 + 120*a^4*b
^2*c^8*d^6 - 63*a^4*b^2*c^10*d^4 + 300*a^4*b^2*c^12*d^2 - 24*a*b^5*c^13*d -
 96*a^5*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9
*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 -
 5*c^15*d^2) - (((8*(4*a^3*c^21 + 2*b^3*c^21 + 12*a^2*b*c^21 - 16*a^3*c^20*
d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^12 + 26*a^3*c^10*d^11 - 14*
a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^8 + 110*a^3*c^14*d^7 - 30*a
^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4 + 64*a^3*c^18*d^3 - 12*a^3

*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22*b^3*c^14*d^7 + 22*b^3*c^15
*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3*c^18*d^3 + 2*b^3*c^19*d^2
- 6*a*b^2*c^11*d^10 + 6*a*b^2*c^12*d^9 - 6*a*b^2*c^13*d^8 + 6*a*b^2*c^14*d^
7 + 54*a*b^2*c^15*d^6 - 54*a*b^2*c^16*d^5 - 66*a*b^2*c^17*d^4 + 66*a*b^2*c^
18*d^3 + 24*a*b^2*c^19*d^2 + 18*a^2*b*c^12*d^9 - 18*a^2*b*c^13*d^8 - 42*a^2
*b*c^14*d^7 + 42*a^2*b*c^15*d^6 + 18*a^2*b*c^16*d^5 - 18*a^2*b*c^17*d^4 + 1
8*a^2*b*c^18*d^3 - 18*a^2*b*c^19*d^2 - 24*a*b^2*c^20*d - 12*a^2*b*c^20*d))/
(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d
^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (
4*tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a
^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*
a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d)*(8*c^21*d - 8*c^8*d^14 +
8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 1
60*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48
*c^19*d^3 - 8*c^20*d^2))/((c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*
c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)*(c^16*d + c^17 - c^6*d^1
1 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*
d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*
(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^
4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d)
)/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^
6 + 21*c^14*d^4 - 7*c^16*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^
3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c
^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d)*1i)/(2*(c^18 -
c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*
d^4 - 7*c^16*d^2)))/((16*(4*a^9*d^13 - 12*a^8*b*c^13 - 2*a^9*c*d^12 + 16*a^
9*c^12*d + a^3*b^6*c^13 + 12*a^5*b^4*c^13 - 2*a^6*b^3*c^13 + 36*a^7*b^2*c^1
3 - 26*a^9*c^2*d^11 + 11*a^9*c^3*d^10 + 70*a^9*c^4*d^9 - 34*a^9*c^5*d^8 - 1
10*a^9*c^6*d^7 + 66*a^9*c^7*d^6 + 110*a^9*c^8*d^5 - 64*a^9*c^9*d^4 - 64*a^9
*c^10*d^3 + 48*a^9*c^11*d^2 - 24*a^4*b^5*c^12*d - 158*a^6*b^3*c^12*d + 24*a
^7*b^2*c^12*d + 18*a^8*b*c^4*d^9 + 18*a^8*b*c^5*d^8 - 60*a^8*b*c^6*d^7 - 42
*a^8*b*c^7*d^6 + 42*a^8*b*c^8*d^5 + 18*a^8*b*c^9*d^4 - 66*a^8*b*c^10*d^3 +
18*a^8*b*c^11*d^2 + 16*a^3*b^6*c^9*d^4 + 8*a^3*b^6*c^11*d^2 - 24*a^4*b^5*c^
8*d^5 - 102*a^4*b^5*c^10*d^3 + 9*a^5*b^4*c^7*d^6 + 144*a^5*b^4*c^9*d^4 + 21
0*a^5*b^4*c^11*d^2 + 8*a^6*b^3*c^4*d^9 + 8*a^6*b^3*c^5*d^8 - 30*a^6*b^3*c^6
*d^7 - 22*a^6*b^3*c^7*d^6 - 22*a^6*b^3*c^8*d^5 + 18*a^6*b^3*c^9*d^4 - 298*a
^6*b^3*c^10*d^3 - 2*a^6*b^3*c^11*d^2 - 6*a^7*b^2*c^3*d^10 - 6*a^7*b^2*c^4*d
^9 - 6*a^7*b^2*c^6*d^7 + 66*a^7*b^2*c^7*d^6 + 54*a^7*b^2*c^8*d^5 + 3*a^7*b^
2*c^9*d^4 - 66*a^7*b^2*c^10*d^3 + 276*a^7*b^2*c^11*d^2 - 84*a^8*b*c^12*d))/
(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d
^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (
((8*tan(e/2 + (f*x)/2)*(4*a^6*c^14 + 8*a^6*d^14 + b^6*c^14 - 8*a^6*c*d^13 -
8*a^6*c^13*d + 12*a^2*b^4*c^14 + 36*a^4*b^2*c^14 - 48*a^6*c^2*d^12 + 48*a^
6*c^3*d^11 + 117*a^6*c^4*d^10 - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6
*c^7*d^7 + 156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^10*d^4 + 48*a^6*c^1

1*d^3 + 44*a^6*c^12*d^2 + 16*b^6*c^10*d^4 + 8*b^6*c^12*d^2 - 24*a*b^5*c^9*d
^5 - 102*a*b^5*c^11*d^3 - 160*a^3*b^3*c^13*d + 36*a^5*b*c^5*d^9 - 102*a^5*b
*c^7*d^7 + 60*a^5*b*c^9*d^5 - 48*a^5*b*c^11*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a
^2*b^4*c^10*d^4 + 210*a^2*b^4*c^12*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^
7*d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^11*d^3 - 12*a^4*b^2*c^4*d^10 - 6*
a^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^10*d^4 + 300*a^4*b^2*c
^12*d^2 - 24*a*b^5*c^13*d - 96*a^5*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c
^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 +
10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) + (((8*(4*a^3*c^21 + 2*b^3*c^21 + 1
2*a^2*b*c^21 - 16*a^3*c^20*d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^
12 + 26*a^3*c^10*d^11 - 14*a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^
8 + 110*a^3*c^14*d^7 - 30*a^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4
+ 64*a^3*c^18*d^3 - 12*a^3*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22
*b^3*c^14*d^7 + 22*b^3*c^15*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3
*c^18*d^3 + 2*b^3*c^19*d^2 - 6*a*b^2*c^11*d^10 + 6*a*b^2*c^12*d^9 - 6*a*b^2
*c^13*d^8 + 6*a*b^2*c^14*d^7 + 54*a*b^2*c^15*d^6 - 54*a*b^2*c^16*d^5 - 66*a
*b^2*c^17*d^4 + 66*a*b^2*c^18*d^3 + 24*a*b^2*c^19*d^2 + 18*a^2*b*c^12*d^9 -
18*a^2*b*c^13*d^8 - 42*a^2*b*c^14*d^7 + 42*a^2*b*c^15*d^6 + 18*a^2*b*c^16*
d^5 - 18*a^2*b*c^17*d^4 + 18*a^2*b*c^18*d^3 - 18*a^2*b*c^19*d^2 - 24*a*b^2*
c^20*d - 12*a^2*b*c^20*d))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d
^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5
*c^17*d^3 - 5*c^18*d^2) - (4*tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1/2)
*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c
^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d
)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c
^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*
c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2))/((c^18 - c^4*d^14 + 7*c
^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^
2)*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d
^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*(
(c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d
- 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*
b*c^5*d^2 - 12*a*b^2*c^6*d))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10
+ 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)))*((c + d)^7*(c -
d)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^
5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*
a*b^2*c^6*d))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8
- 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a^6
*c^14 + 8*a^6*d^14 + b^6*c^14 - 8*a^6*c*d^13 - 8*a^6*c^13*d + 12*a^2*b^4*c^
14 + 36*a^4*b^2*c^14 - 48*a^6*c^2*d^12 + 48*a^6*c^3*d^11 + 117*a^6*c^4*d^10
- 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6 -
120*a^6*c^9*d^5 - 92*a^6*c^10*d^4 + 48*a^6*c^11*d^3 + 44*a^6*c^12*d^2 + 16*
b^6*c^10*d^4 + 8*b^6*c^12*d^2 - 24*a*b^5*c^9*d^5 - 102*a*b^5*c^11*d^3 - 160
*a^3*b^3*c^13*d + 36*a^5*b*c^5*d^9 - 102*a^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5 -
48*a^5*b*c^11*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^2*b^4*c^10*d^4 + 210*a^2*b^4

*c^12*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7*d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^11*d^3 - 12*a^4*b^2*c^4*d^10 - 6*a^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^10*d^4 + 300*a^4*b^2*c^12*d^2 - 24*a*b^5*c^13*d - 96*a^5*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) - (((8*(4*a^3*c^21 + 2*b^3*c^21 + 12*a^2*b*c^21 - 16*a^3*c^20*d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^12 + 26*a^3*c^10*d^11 - 14*a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^8 + 110*a^3*c^14*d^7 - 30*a^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4 + 64*a^3*c^18*d^3 - 12*a^3*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22*b^3*c^14*d^7 + 22*b^3*c^15*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3*c^18*d^3 + 2*b^3*c^19*d^2 - 6*a*b^2*c^11*d^10 + 6*a*b^2*c^12*d^9 - 6*a*b^2*c^13*d^8 + 6*a*b^2*c^14*d^7 + 54*a*b^2*c^15*d^6 - 54*a*b^2*c^16*d^5 - 66*a*b^2*c^17*d^4 + 66*a*b^2*c^18*d^3 + 24*a*b^2*c^19*d^2 + 18*a^2*b*c^12*d^9 - 18*a^2*b*c^13*d^8 - 42*a^2*b*c^14*d^7 + 42*a^2*b*c^15*d^6 + 18*a^2*b*c^16*d^5 - 18*a^2*b*c^17*d^4 + 18*a^2*b*c^18*d^3 - 18*a^2*b*c^19*d^2 - 24*a*b^2*c^20*d - 12*a^2*b*c^20*d))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (4*tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2))/((c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2))))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d)*1i)/(f*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2))

3.197 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

Optimal result 1371
Rubi [A] (verified) 1372
Mathematica [A] (verified) 1376
Maple [B] (verified) 1377
Fricas [B] (verification not implemented) 1378
Sympy [F] 1380
Maxima [F(-2)] 1380
Giac [B] (verification not implemented) 1381
Mupad [B] (verification not implemented) 1383

Optimal result

Integrand size = 25, antiderivative size = 622

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \frac{a^3x}{c^5}$$
$$-\frac{(15ab^2c^6d(4c^2+3d^2)-3a^2bc^5(8c^4+24c^2d^2+3d^4)-b^3c^5(4c^4+27c^2d^2+4d^4)+a^3(40c^8d-40c^6d^3+(}{4c^5\sqrt{c-d}\sqrt{c+d}\,(c^2-d^2)^4 f}$$
$$+\frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\,(c^2-d^2)\,f(d+c\cos(e+fx))^4}$$
$$-\frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\,(b+a\cos(e+fx))^2\sin(e+fx)}{12c^2\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^3}$$
$$-\frac{(bc-ad)\,(2abcd(32c^4+c^2d^2+2d^4)-a^2d^2(58c^4-35c^2d^2+12d^4)-b^2(12c^6+25c^4d^2-2c^2d^4))\sin(e+}{24c^4\,(c^2-d^2)^3\,f(d+c\cos(e+fx))^2}$$
$$-\frac{(b^3c^3d(68c^4+39c^2d^2-2d^4)+a^2bcd(272c^6+10c^4d^2+49c^2d^4-16d^6)-3ab^2c^2(24c^6+84c^4d^2-5c^2d^4}{24c^4\,(c^2-d^2)^4\,f(d+c\cos(e+fx))}$$

```
[Out] a^3*x/c^5+1/4*d^2*(b+a*cos(f*x+e))^3*sin(f*x+e)/c/(c^2-d^2)/f/(d+c*cos(f*x+
e))^4-1/12*d*(-11*a*c^2*d+4*a*d^3+8*b*c^3-b*c*d^2)*(b+a*cos(f*x+e))^2*sin(f
*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^3-1/24*(-a*d+b*c)*(2*a*b*c*d*(32*c
^4+c^2*d^2+2*d^4)-a^2*d^2*(58*c^4-35*c^2*d^2+12*d^4)-b^2*(12*c^6+25*c^4*d^2
-2*c^2*d^4))*sin(f*x+e)/c^4/(c^2-d^2)^3/f/(d+c*cos(f*x+e))^2-1/24*(b^3*c^3*
d*(68*c^4+39*c^2*d^2-2*d^4)+a^2*b*c*d*(272*c^6+10*c^4*d^2+49*c^2*d^4-16*d^6
)-3*a*b^2*c^2*(24*c^6+84*c^4*d^2-5*c^2*d^4+2*d^6)-a^3*(212*c^6*d^2-210*c^4*
d^4+139*c^2*d^6-36*d^8))*sin(f*x+e)/c^4/(c^2-d^2)^4/f/(d+c*cos(f*x+e))-1/4*
(15*a*b^2*c^6*d*(4*c^2+3*d^2)-3*a^2*b*c^5*(8*c^4+24*c^2*d^2+3*d^4)-b^3*c^5*
(4*c^4+27*c^2*d^2+4*d^4)+a^3*(40*c^8*d-40*c^6*d^3+63*c^4*d^5-36*c^2*d^7+8*d
^9))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^5/(c^2-d^2)^4/f/
(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 2.70 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4026, 3127, 3126, 3110, 3100, 2814, 2738, 214}

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \frac{a^3x}{c^5}$$
$$-\frac{(bc-ad)\left(-a^2d^2(58c^4-35c^2d^2+12d^4)+2abcd(32c^4+c^2d^2+2d^4)-(b^2(12c^6+25c^4d^2-2c^2d^4))\right)\sin(\ldots}{24c^4f\left(c^2-d^2\right)^3(c\cos(e+fx)+d)^2}$$
$$-\frac{(a^3(40c^8d-40c^6d^3+63c^4d^5-36c^2d^7+8d^9)-3a^2bc^5(8c^4+24c^2d^2+3d^4)+15ab^2c^6d(4c^2+3d^2)-b^3c\ldots}{4c^5f\sqrt{c-d}\sqrt{c+d}\left(c^2-d^2\right)^4}$$
$$-\frac{(-(a^3(212c^6d^2-210c^4d^4+139c^2d^6-36d^8))+a^2bcd(272c^6+10c^4d^2+49c^2d^4-16d^6)-3ab^2c^2(24c^6+\ldots}{24c^4f\left(c^2-d^2\right)^4(c\cos(e+fx)+d)}$$
$$+\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}$$
$$-\frac{d(-11ac^2d+4ad^3+8bc^3-bcd^2)\sin(e+fx)(a\cos(e+fx)+b)^2}{12c^2f\left(c^2-d^2\right)^2(c\cos(e+fx)+d)^3}$$

```
[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*x)/c^5 - ((15*a*b^2*c^6*d*(4*c^2 + 3*d^2) - 3*a^2*b*c^5*(8*c^4 + 24*c^
2*d^2 + 3*d^4) - b^3*c^5*(4*c^4 + 27*c^2*d^2 + 4*d^4) + a^3*(40*c^8*d - 40*
c^6*d^3 + 63*c^4*d^5 - 36*c^2*d^7 + 8*d^9))*ArcTanh[(Sqrt[c - d]*Tan[(e + f
*x)/2])/Sqrt[c + d]])/(4*c^5*Sqrt[c - d]*Sqrt[c + d]*(c^2 - d^2)^4*f) + (d^
2*(b + a*Cos[e + f*x])^3*Sin[e + f*x])/(4*c*(c^2 - d^2)*f*(d + c*Cos[e + f*
x])^4) - (d*(8*b*c^3 - 11*a*c^2*d - b*c*d^2 + 4*a*d^3)*(b + a*Cos[e + f*x])
^2*Sin[e + f*x])/(12*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x])^3) - ((b*c -
a*d)*(2*a*b*c*d*(32*c^4 + c^2*d^2 + 2*d^4) - a^2*d^2*(58*c^4 - 35*c^2*d^2 +
 12*d^4) - b^2*(12*c^6 + 25*c^4*d^2 - 2*c^2*d^4))*Sin[e + f*x])/(24*c^4*(c^
2 - d^2)^3*f*(d + c*Cos[e + f*x])^2) - ((b^3*c^3*d*(68*c^4 + 39*c^2*d^2 - 2
*d^4) + a^2*b*c*d*(272*c^6 + 10*c^4*d^2 + 49*c^2*d^4 - 16*d^6) - 3*a*b^2*c^
2*(24*c^6 + 84*c^4*d^2 - 5*c^2*d^4 + 2*d^6) - a^3*(212*c^6*d^2 - 210*c^4*d^
4 + 139*c^2*d^6 - 36*d^8))*Sin[e + f*x])/(24*c^4*(c^2 - d^2)^4*f*(d + c*Cos
[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^3}{(d+c\cos(e+fx))^5}\,dx$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} + \frac{\int \frac{(b+a\cos(e+fx))^2\left(-d(4bc-3ad)+\left(4bc^2-4acd-bd^2\right)\cos(e+fx)+4a\left(c^2-d^2\right)\cos^2(e+fx)\right)}{(d+c\cos(e+fx))^4}\,dx}{4c\left(c^2-d^2\right)}$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d\left(8bc^3-11ac^2d-bcd^2+4ad^3\right)(b+a\cos(e+fx))^2\sin(e+fx)}{12c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^3} + \frac{\int \frac{(b+a\cos(e+fx))\big(2a^2d^2\left(11c^2-4d^2\right)-5abcd\left(8c^2-d^2\right)+3b^2\left(4c^4+3c^2d^2\right)-\big(2b^2cd\left(8c^2-d^2\right)+3a^2\left(8c^3d-cd^3\right)-ab\big(24c^4+7c^2d^2+4d^4}{(d+c\cos(e+fx))^3}}{12c^2\left(c^2-d^2\right)^2}$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\,(c^2-d^2)\,f(d+c\cos(e+fx))^4} - \frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\,(b+a\cos(e+fx))^2\sin(e+fx)}{12c^2\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^3} - \frac{(bc-ad)\,(2abcd(32c^4+c^2d^2+2d^4)-a^2d^2(58c^4-35c^2d^2+12d^4)-b^2(12c^6+25c^4d^2-2c^2d^4)}{24c^4\,(c^2-d^2)^3\,f(d+c\cos(e+fx))^2} - \frac{\int \frac{2c(7b^3c^3d(4c^2+d^2)-3ab^2c^2(12c^4+24c^2d^2-d^4)+a^2bcd(100c^4-3c^2d^2+8d^4)-a^3(58c^4d^2-35c^2d^4+12d^6))-(b^3c^3(12c^4+25c^2d^2}{}}{}$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\,(c^2-d^2)\,f(d+c\cos(e+fx))^4} - \frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\,(b+a\cos(e+fx))^2\sin(e+fx)}{12c^2\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^3} - \frac{(bc-ad)\,(2abcd(32c^4+c^2d^2+2d^4)-a^2d^2(58c^4-35c^2d^2+12d^4)-b^2(12c^6+25c^4d^2-2c^2d^4)}{24c^4\,(c^2-d^2)^3\,f(d+c\cos(e+fx))^2} - \frac{(b^3c^3d(68c^4+39c^2d^2-2d^4)+a^2bcd(272c^6+10c^4d^2+49c^2d^4-16d^6)-3ab^2c^2(24c^6+84c^4d^2}{24c^4\,(c^2-d^2)^4\,f(d+c\cos(e+}} - \frac{\int \frac{3c(15ab^2c^5d(4c^2+3d^2)-3a^2bc^4(8c^4+24c^2d^2+3d^4)-b^3c^4(4c^4+27c^2d^2+4d^4)+a^3(32c^7d-8c^5d^3+15c^3d^5-4cd^7))-24a^3c(c^2-d^2}{d+c\cos(e+fx)}}{24c^5\,(c^2-d^2)^4}$$

$$= \frac{a^3x}{c^5} + \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\,(c^2-d^2)\,f(d+c\cos(e+fx))^4} - \frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\,(b+a\cos(e+fx))^2\sin(e+fx)}{12c^2\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^3} - \frac{(bc-ad)\,(2abcd(32c^4+c^2d^2+2d^4)-a^2d^2(58c^4-35c^2d^2+12d^4)-b^2(12c^6+25c^4d^2-2c^2d^4)}{24c^4\,(c^2-d^2)^3\,f(d+c\cos(e+fx))^2} - \frac{(b^3c^3d(68c^4+39c^2d^2-2d^4)+a^2bcd(272c^6+10c^4d^2+49c^2d^4-16d^6)-3ab^2c^2(24c^6+84c^4d^2}{24c^4\,(c^2-d^2)^4\,f(d+c\cos(e+}} - \frac{(15ab^2c^6d(4c^2+3d^2)-3a^2bc^5(8c^4+24c^2d^2+3d^4)-b^3c^5(4c^4+27c^2d^2+4d^4)+a^3(40c^8d-40}{8c^5\,(c^2-d^2)^4}$$

$$= \frac{a^3x}{c^5} + \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\,(c^2-d^2)\,f(d+c\cos(e+fx))^4}$$
$$- \frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\,(b+a\cos(e+fx))^2\sin(e+fx)}{12c^2\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^3}$$
$$- \frac{(bc-ad)\,(2abcd(32c^4+c^2d^2+2d^4)-a^2d^2(58c^4-35c^2d^2+12d^4)-b^2(12c^6+25c^4d^2-2c^2d^4))}{24c^4\,(c^2-d^2)^3\,f(d+c\cos(e+fx))^2}$$
$$- \frac{(b^3c^3d(68c^4+39c^2d^2-2d^4)+a^2bcd(272c^6+10c^4d^2+49c^2d^4-16d^6)-3ab^2c^2(24c^6+84c^4d^2-}{24c^4\,(c^2-d^2)^4\,f(d+c\cos(e+}$$
$$- \frac{(15ab^2c^6d(4c^2+3d^2)-3a^2bc^5(8c^4+24c^2d^2+3d^4)-b^3c^5(4c^4+27c^2d^2+4d^4)+a^3(40c^8d-40c}{4c^5\,(c^2-d^2)^4\,f}$$

$$= \frac{a^3x}{c^5}$$
$$- \frac{(15ab^2c^6d(4c^2+3d^2)-3a^2bc^5(8c^4+24c^2d^2+3d^4)-b^3c^5(4c^4+27c^2d^2+4d^4)+a^3(40c^8d-40c}{4c^5\sqrt{c-d}\sqrt{c+d}\,(c^2-d^2)^4\,f}$$
$$+ \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\,(c^2-d^2)\,f(d+c\cos(e+fx))^4}$$
$$- \frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\,(b+a\cos(e+fx))^2\sin(e+fx)}{12c^2\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^3}$$
$$- \frac{(bc-ad)\,(2abcd(32c^4+c^2d^2+2d^4)-a^2d^2(58c^4-35c^2d^2+12d^4)-b^2(12c^6+25c^4d^2-2c^2d^4))}{24c^4\,(c^2-d^2)^3\,f(d+c\cos(e+fx))^2}$$
$$- \frac{(b^3c^3d(68c^4+39c^2d^2-2d^4)+a^2bcd(272c^6+10c^4d^2+49c^2d^4-16d^6)-3ab^2c^2(24c^6+84c^4d^2-}{24c^4\,(c^2-d^2)^4\,f(d+c\cos(e+}$$

Mathematica [A] (verified)

Time = 9.72 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$$
$$= \frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))^3\left(24a^3(e+fx)(d+c\cos(e+fx))^4 - \frac{6(-15ab^2c^6d(4c^2+3c}{}\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^3*(24*a^3*(e + f*
x)*(d + c*Cos[e + f*x])^4 - (6*(-15*a*b^2*c^6*d*(4*c^2 + 3*d^2) + 3*a^2*b*c
^5*(8*c^4 + 24*c^2*d^2 + 3*d^4) + b^3*c^5*(4*c^4 + 27*c^2*d^2 + 4*d^4) + a^
3*(-40*c^8*d + 40*c^6*d^3 - 63*c^4*d^5 + 36*c^2*d^7 - 8*d^9))*ArcTanh[((-c
```

```
+ d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^4)/(c^2 - d^2)
^(9/2) + (6*c*d^2*(b*c - a*d)^3*Sin[e + f*x])/(c^2 - d^2) - (2*c*d*(b*c - a
*d)^2*(8*b*c^3 - 20*a*c^2*d - b*c*d^2 + 13*a*d^3)*(d + c*Cos[e + f*x])*Sin[
e + f*x])/(c^2 - d^2)^2 + (c*(a^3*d^3*(-120*c^4 + 131*c^2*d^2 - 46*d^4) - 3
*a*b^2*c^2*d*(36*c^4 - 3*c^2*d^2 + 2*d^4) + 3*a^2*b*c*d^2*(72*c^4 - 55*c^2*
d^2 + 18*d^4) + b^3*(12*c^7 + 25*c^5*d^2 - 2*c^3*d^4))*(d + c*Cos[e + f*x])
^2*Sin[e + f*x])/(c^2 - d^2)^3 + (c*(b^3*c^3*d*(-68*c^4 - 39*c^2*d^2 + 2*d^
4) - 3*a^2*b*c*d*(96*c^6 - 8*c^4*d^2 + 23*c^2*d^4 - 6*d^6) + 3*a*b^2*c^2*(2
4*c^6 + 84*c^4*d^2 - 5*c^2*d^4 + 2*d^6) + 5*a^3*(48*c^6*d^2 - 56*c^4*d^4 +
39*c^2*d^6 - 10*d^8))*(d + c*Cos[e + f*x])^3*Sin[e + f*x])/(c^2 - d^2)^4))/
(24*c^5*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e + f*x])^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(603) = 1206$.

Time = 2.38 (sec) , antiderivative size = 1345, normalized size of antiderivative = 2.16

method	result	size
derivativedivides	Expression too large to display	1345
default	Expression too large to display	1345
risch	Expression too large to display	5129

```
[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*a^3/c^5*arctan(tan(1/2*f*x+1/2*e))+2/c^5*((-1/8*(80*a^3*c^6*d^2+40*a
^3*c^5*d^3-40*a^3*c^4*d^4-15*a^3*c^3*d^5+32*a^3*c^2*d^6+4*a^3*c*d^7-8*a^3*d
^8-96*a^2*b*c^7*d-72*a^2*b*c^6*d^2-96*a^2*b*c^5*d^3-15*a^2*b*c^4*d^4+24*a*b
^2*c^8+36*a*b^2*c^7*d+144*a*b^2*c^6*d^2+51*a*b^2*c^5*d^3+24*a*b^2*c^4*d^4-4
*b^3*c^8-32*b^3*c^7*d-21*b^3*c^6*d^2-32*b^3*c^5*d^3-4*b^3*c^4*d^4)*c/(c-d)/
(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+d^4)*tan(1/2*f*x+1/2*e)^7+1/24*c*(720*a^3*c^
6*d^2+120*a^3*c^5*d^3-520*a^3*c^4*d^4-69*a^3*c^3*d^5+320*a^3*c^2*d^6+12*a^3
*c*d^7-72*a^3*d^8-864*a^2*b*c^7*d-216*a^2*b*c^6*d^2-480*a^2*b*c^5*d^3+27*a^
2*b*c^4*d^4+216*a*b^2*c^8+108*a*b^2*c^7*d+1008*a*b^2*c^6*d^2+81*a*b^2*c^5*d
^3+120*a*b^2*c^4*d^4-12*b^3*c^8-224*b^3*c^7*d-39*b^3*c^6*d^2-224*b^3*c^5*d^
3-12*b^3*c^4*d^4)/(c^3+3*c^2*d+3*c*d^2+d^3)/(c-d)^2*tan(1/2*f*x+1/2*e)^5-1/
24*c*(720*a^3*c^6*d^2-120*a^3*c^5*d^3-520*a^3*c^4*d^4+69*a^3*c^3*d^5+320*a^
3*c^2*d^6-12*a^3*c*d^7-72*a^3*d^8-864*a^2*b*c^7*d+216*a^2*b*c^6*d^2-480*a^2
*b*c^5*d^3-27*a^2*b*c^4*d^4+216*a*b^2*c^8-108*a*b^2*c^7*d+1008*a*b^2*c^6*d^
2-81*a*b^2*c^5*d^3+120*a*b^2*c^4*d^4+12*b^3*c^8-224*b^3*c^7*d+39*b^3*c^6*d^
2-224*b^3*c^5*d^3+12*b^3*c^4*d^4)/(c-d)^3/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e
)^3+1/8*(80*a^3*c^6*d^2-40*a^3*c^5*d^3-40*a^3*c^4*d^4+15*a^3*c^3*d^5+32*a^3
*c^2*d^6-4*a^3*c*d^7-8*a^3*d^8-96*a^2*b*c^7*d+72*a^2*b*c^6*d^2-96*a^2*b*c^5
*d^3+15*a^2*b*c^4*d^4+24*a*b^2*c^8-36*a*b^2*c^7*d+144*a*b^2*c^6*d^2-51*a*b^
2*c^5*d^3+24*a*b^2*c^4*d^4+4*b^3*c^8-32*b^3*c^7*d+21*b^3*c^6*d^2-32*b^3*c^5
```

```
*d^3+4*b^3*c^4*d^4)*c/(c+d)/(c^4-4*c^3*d+6*c^2*d^2-4*c*d^3+d^4)*tan(1/2*f*x
+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^4-1/8*(40*a^3*
c^8*d-40*a^3*c^6*d^3+63*a^3*c^4*d^5-36*a^3*c^2*d^7+8*a^3*d^9-24*a^2*b*c^9-7
2*a^2*b*c^7*d^2-9*a^2*b*c^5*d^4+60*a*b^2*c^8*d+45*a*b^2*c^6*d^3-4*b^3*c^9-2
7*b^3*c^7*d^2-4*b^3*c^5*d^4)/(c^8-4*c^6*d^2+6*c^4*d^4-4*c^2*d^6+d^8)/((c+d)
*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2144 vs. $2(603) = 1206$.

Time = 0.69 (sec) , antiderivative size = 4346, normalized size of antiderivative = 6.99

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/48*(48*(a^3*c^14 - 5*a^3*c^12*d^2 + 10*a^3*c^10*d^4 - 10*a^3*c^8*d^6 + 5
*a^3*c^6*d^8 - a^3*c^4*d^10)*f*x*cos(f*x + e)^4 + 192*(a^3*c^13*d - 5*a^3*c
^11*d^3 + 10*a^3*c^9*d^5 - 10*a^3*c^7*d^7 + 5*a^3*c^5*d^9 - a^3*c^3*d^11)*f
*x*cos(f*x + e)^3 + 288*(a^3*c^12*d^2 - 5*a^3*c^10*d^4 + 10*a^3*c^8*d^6 - 1
0*a^3*c^6*d^8 + 5*a^3*c^4*d^10 - a^3*c^2*d^12)*f*x*cos(f*x + e)^2 + 192*(a^
3*c^11*d^3 - 5*a^3*c^9*d^5 + 10*a^3*c^7*d^7 - 10*a^3*c^5*d^9 + 5*a^3*c^3*d^
11 - a^3*c*d^13)*f*x*cos(f*x + e) + 48*(a^3*c^10*d^4 - 5*a^3*c^8*d^6 + 10*a
^3*c^6*d^8 - 10*a^3*c^4*d^10 + 5*a^3*c^2*d^12 - a^3*d^14)*f*x + 3*(63*a^3*c
^4*d^9 - 36*a^3*c^2*d^11 + 8*a^3*d^13 - 4*(6*a^2*b + b^3)*c^9*d^4 + 20*(2*a
^3 + 3*a*b^2)*c^8*d^5 - 9*(8*a^2*b + 3*b^3)*c^7*d^6 - 5*(8*a^3 - 9*a*b^2)*c
^6*d^7 - (9*a^2*b + 4*b^3)*c^5*d^8 + (63*a^3*c^8*d^5 - 36*a^3*c^6*d^7 + 8*a
^3*c^4*d^9 - 4*(6*a^2*b + b^3)*c^13 + 20*(2*a^3 + 3*a*b^2)*c^12*d - 9*(8*a^
2*b + 3*b^3)*c^11*d^2 - 5*(8*a^3 - 9*a*b^2)*c^10*d^3 - (9*a^2*b + 4*b^3)*c^
9*d^4)*cos(f*x + e)^4 + 4*(63*a^3*c^7*d^6 - 36*a^3*c^5*d^8 + 8*a^3*c^3*d^10
 - 4*(6*a^2*b + b^3)*c^12*d + 20*(2*a^3 + 3*a*b^2)*c^11*d^2 - 9*(8*a^2*b +
3*b^3)*c^10*d^3 - 5*(8*a^3 - 9*a*b^2)*c^9*d^4 - (9*a^2*b + 4*b^3)*c^8*d^5)*
cos(f*x + e)^3 + 6*(63*a^3*c^6*d^7 - 36*a^3*c^4*d^9 + 8*a^3*c^2*d^11 - 4*(6
*a^2*b + b^3)*c^11*d^2 + 20*(2*a^3 + 3*a*b^2)*c^10*d^3 - 9*(8*a^2*b + 3*b^3
)*c^9*d^4 - 5*(8*a^3 - 9*a*b^2)*c^8*d^5 - (9*a^2*b + 4*b^3)*c^7*d^6)*cos(f*
x + e)^2 + 4*(63*a^3*c^5*d^8 - 36*a^3*c^3*d^10 + 8*a^3*c*d^12 - 4*(6*a^2*b
+ b^3)*c^10*d^3 + 20*(2*a^3 + 3*a*b^2)*c^9*d^4 - 9*(8*a^2*b + 3*b^3)*c^8*d^
5 - 5*(8*a^3 - 9*a*b^2)*c^7*d^6 - (9*a^2*b + 4*b^3)*c^6*d^7)*cos(f*x + e))*
sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*
sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f
*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*b^3*c^12*d^2 + 18*a*b^2*c^11*
d^3 - 116*a^3*c^3*d^11 + 24*a^3*c*d^13 - (150*a^2*b + 41*b^3)*c^10*d^4 + 77
*(2*a^3 + 3*a*b^2)*c^9*d^5 - (15*a^2*b + 29*b^3)*c^8*d^6 - (271*a^3 + 201*a
*b^2)*c^7*d^7 + (165*a^2*b + 68*b^3)*c^6*d^8 + (209*a^3 - 48*a*b^2)*c^5*d^9
```

+ (72*a*b^2*c^14 - 18*a^2*b*c^5*d^9 + 50*a^3*c^4*d^10 - 4*(72*a^2*b + 17*b^3)*c^13*d + 60*(4*a^3 + 3*a*b^2)*c^12*d^2 + (312*a^2*b + 29*b^3)*c^11*d^3 - (520*a^3 + 267*a*b^2)*c^10*d^4 - (93*a^2*b - 41*b^3)*c^9*d^5 + (475*a^3 + 21*a*b^2)*c^8*d^6 + (87*a^2*b - 2*b^3)*c^7*d^7 - (245*a^3 + 6*a*b^2)*c^6*d^8)*cos(f*x + e)^3 + (12*b^3*c^14 + 108*a*b^2*c^13*d + 104*a^3*c^3*d^11 - (648*a^2*b + 203*b^3)*c^12*d^2 + 15*(40*a^3 + 51*a*b^2)*c^11*d^3 + (339*a^2*b + 47*b^3)*c^10*d^4 - (1189*a^3 + 933*a*b^2)*c^9*d^5 + (321*a^2*b + 152*b^3)*c^8*d^6 + (997*a^3 + 84*a*b^2)*c^7*d^7 - 4*(3*a^2*b + 2*b^3)*c^6*d^8 - 8*(64*a^3 + 3*a*b^2)*c^5*d^9)*cos(f*x + e)^2 + (8*b^3*c^13*d + 72*a*b^2*c^12*d^2 - 407*a^3*c^4*d^10 + 84*a^3*c^2*d^12 - 8*(66*a^2*b + 19*b^3)*c^11*d^3 + 8*(65*a^3 + 93*a*b^2)*c^10*d^4 + (84*a^2*b - 47*b^3)*c^9*d^5 - (964*a^3 + 759*a*b^2)*c^8*d^6 + (471*a^2*b + 203*b^3)*c^7*d^7 + (767*a^3 - 57*a*b^2)*c^6*d^8 - 3*(9*a^2*b + 4*b^3)*c^5*d^9)*cos(f*x + e))*sin(f*x + e))/((c^19 - 5*c^17*d^2 + 10*c^15*d^4 - 10*c^13*d^6 + 5*c^11*d^8 - c^9*d^10)*f*cos(f*x + e)^4 + 4*(c^18*d - 5*c^16*d^3 + 10*c^14*d^5 - 10*c^12*d^7 + 5*c^10*d^9 - c^8*d^11)*f*cos(f*x + e)^3 + 6*(c^17*d^2 - 5*c^15*d^4 + 10*c^13*d^6 - 10*c^11*d^8 + 5*c^9*d^10 - c^7*d^12)*f*cos(f*x + e)^2 + 4*(c^16*d^3 - 5*c^14*d^5 + 10*c^12*d^7 - 10*c^10*d^9 + 5*c^8*d^11 - c^6*d^13)*f*cos(f*x + e) + (c^15*d^4 - 5*c^13*d^6 + 10*c^11*d^8 - 10*c^9*d^10 + 5*c^7*d^12 - c^5*d^14)*f), 1/24*(24*(a^3*c^14 - 5*a^3*c^12*d^2 + 10*a^3*c^10*d^4 - 10*a^3*c^8*d^6 + 5*a^3*c^6*d^8 - a^3*c^4*d^10)*f*x*cos(f*x + e)^4 + 96*(a^3*c^13*d - 5*a^3*c^11*d^3 + 10*a^3*c^9*d^5 - 10*a^3*c^7*d^7 + 5*a^3*c^5*d^9 - a^3*c^3*d^11)*f*x*cos(f*x + e)^3 + 144*(a^3*c^12*d^2 - 5*a^3*c^10*d^4 + 10*a^3*c^8*d^6 - 10*a^3*c^6*d^8 + 5*a^3*c^4*d^10 - a^3*c^2*d^12)*f*x*cos(f*x + e)^2 + 96*(a^3*c^11*d^3 - 5*a^3*c^9*d^5 + 10*a^3*c^7*d^7 - 10*a^3*c^5*d^9 + 5*a^3*c^3*d^11 - a^3*c*d^13)*f*x*cos(f*x + e) + 24*(a^3*c^10*d^4 - 5*a^3*c^8*d^6 + 10*a^3*c^6*d^8 - 10*a^3*c^4*d^10 + 5*a^3*c^2*d^12 - a^3*d^14)*f*x - 3*(63*a^3*c^4*d^9 - 36*a^3*c^2*d^11 + 8*a^3*d^13 - 4*(6*a^2*b + b^3)*c^9*d^4 + 20*(2*a^3 + 3*a*b^2)*c^8*d^5 - 9*(8*a^2*b + 3*b^3)*c^7*d^6 - 5*(8*a^3 - 9*a*b^2)*c^6*d^7 - (9*a^2*b + 4*b^3)*c^5*d^8 + (63*a^3*c^8*d^5 - 36*a^3*c^6*d^7 + 8*a^3*c^4*d^9 - 4*(6*a^2*b + b^3)*c^13 + 20*(2*a^3 + 3*a*b^2)*c^12*d - 9*(8*a^2*b + 3*b^3)*c^11*d^2 - 5*(8*a^3 - 9*a*b^2)*c^10*d^3 - (9*a^2*b + 4*b^3)*c^9*d^4)*cos(f*x + e)^4 + 4*(63*a^3*c^7*d^6 - 36*a^3*c^5*d^8 + 8*a^3*c^3*d^10 - 4*(6*a^2*b + b^3)*c^12*d + 20*(2*a^3 + 3*a*b^2)*c^11*d^2 - 9*(8*a^2*b + 3*b^3)*c^10*d^3 - 5*(8*a^3 - 9*a*b^2)*c^9*d^4 - (9*a^2*b + 4*b^3)*c^8*d^5)*cos(f*x + e)^3 + 6*(63*a^3*c^6*d^7 - 36*a^3*c^4*d^9 + 8*a^3*c^2*d^11 - 4*(6*a^2*b + b^3)*c^11*d^2 + 20*(2*a^3 + 3*a*b^2)*c^10*d^3 - 9*(8*a^2*b + 3*b^3)*c^9*d^4 - 5*(8*a^3 - 9*a*b^2)*c^8*d^5 - (9*a^2*b + 4*b^3)*c^7*d^6)*cos(f*x + e)^2 + 4*(63*a^3*c^5*d^8 - 36*a^3*c^3*d^10 + 8*a^3*c*d^12 - 4*(6*a^2*b + b^3)*c^10*d^3 + 20*(2*a^3 + 3*a*b^2)*c^9*d^4 - 9*(8*a^2*b + 3*b^3)*c^8*d^5 - 5*(8*a^3 - 9*a*b^2)*c^7*d^6 - (9*a^2*b + 4*b^3)*c^6*d^7)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (2*b^3*c^12*d^2 + 18*a*b^2*c^11*d^3 - 116*a^3*c^3*d^11 + 24*a^3*c*d^13 - (150*a^2*b + 41*b^3)*c^10*d^4 + 77*(2*a^3 + 3*a*b^2)*c^9*d^5 - (15*a^2*b + 29*b^3)*c^8*d^6 - (271*a^3 + 201*a*b^2)*c^7*d^7 + (165*a^2*b

```
+ 68*b^3)*c^6*d^8 + (209*a^3 - 48*a*b^2)*c^5*d^9 + (72*a*b^2*c^14 - 18*a^2*
b*c^5*d^9 + 50*a^3*c^4*d^10 - 4*(72*a^2*b + 17*b^3)*c^13*d + 60*(4*a^3 + 3*
a*b^2)*c^12*d^2 + (312*a^2*b + 29*b^3)*c^11*d^3 - (520*a^3 + 267*a*b^2)*c^1
0*d^4 - (93*a^2*b - 41*b^3)*c^9*d^5 + (475*a^3 + 21*a*b^2)*c^8*d^6 + (87*a^
2*b - 2*b^3)*c^7*d^7 - (245*a^3 + 6*a*b^2)*c^6*d^8)*cos(f*x + e)^3 + (12*b^
3*c^14 + 108*a*b^2*c^13*d + 104*a^3*c^3*d^11 - (648*a^2*b + 203*b^3)*c^12*d
^2 + 15*(40*a^3 + 51*a*b^2)*c^11*d^3 + (339*a^2*b + 47*b^3)*c^10*d^4 - (118
9*a^3 + 933*a*b^2)*c^9*d^5 + (321*a^2*b + 152*b^3)*c^8*d^6 + (997*a^3 + 84*
a*b^2)*c^7*d^7 - 4*(3*a^2*b + 2*b^3)*c^6*d^8 - 8*(64*a^3 + 3*a*b^2)*c^5*d^9
)*cos(f*x + e)^2 + (8*b^3*c^13*d + 72*a*b^2*c^12*d^2 - 407*a^3*c^4*d^10 + 8
4*a^3*c^2*d^12 - 8*(66*a^2*b + 19*b^3)*c^11*d^3 + 8*(65*a^3 + 93*a*b^2)*c^1
0*d^4 + (84*a^2*b - 47*b^3)*c^9*d^5 - (964*a^3 + 759*a*b^2)*c^8*d^6 + (471*
a^2*b + 203*b^3)*c^7*d^7 + (767*a^3 - 57*a*b^2)*c^6*d^8 - 3*(9*a^2*b + 4*b^
3)*c^5*d^9)*cos(f*x + e))*sin(f*x + e))/((c^19 - 5*c^17*d^2 + 10*c^15*d^4 -
 10*c^13*d^6 + 5*c^11*d^8 - c^9*d^10)*f*cos(f*x + e)^4 + 4*(c^18*d - 5*c^16
*d^3 + 10*c^14*d^5 - 10*c^12*d^7 + 5*c^10*d^9 - c^8*d^11)*f*cos(f*x + e)^3
+ 6*(c^17*d^2 - 5*c^15*d^4 + 10*c^13*d^6 - 10*c^11*d^8 + 5*c^9*d^10 - c^7*d
^12)*f*cos(f*x + e)^2 + 4*(c^16*d^3 - 5*c^14*d^5 + 10*c^12*d^7 - 10*c^10*d^
9 + 5*c^8*d^11 - c^6*d^13)*f*cos(f*x + e) + (c^15*d^4 - 5*c^13*d^6 + 10*c^1
1*d^8 - 10*c^9*d^10 + 5*c^7*d^12 - c^5*d^14)*f)]
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx=\int \frac{(a+b\sec{(e+fx)})^3}{(c+d\sec{(e+fx)})^5}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3173 vs. 2(603) = 1206.

Time = 0.55 (sec) , antiderivative size = 3173, normalized size of antiderivative = 5.10

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*(24*a^2*b*c^9 + 4*b^3*c^9 - 40*a^3*c^8*d - 60*a*b^2*c^8*d + 72*a^2*
b*c^7*d^2 + 27*b^3*c^7*d^2 + 40*a^3*c^6*d^3 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^
5*d^4 + 4*b^3*c^5*d^4 - 63*a^3*c^4*d^5 + 36*a^3*c^2*d^7 - 8*a^3*d^9)*(pi*fl
oor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*
e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^13 - 4*c^11*d^2 + 6*c^9
*d^4 - 4*c^7*d^6 + c^5*d^8)*sqrt(-c^2 + d^2)) + 12*(f*x + e)*a^3/c^5 - (72*
a*b^2*c^11*tan(1/2*f*x + 1/2*e)^7 - 12*b^3*c^11*tan(1/2*f*x + 1/2*e)^7 - 28
8*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^7 - 108*a*b^2*c^10*d*tan(1/2*f*x + 1/2*
e)^7 - 60*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^7 + 240*a^3*c^9*d^2*tan(1/2*f*x +
 1/2*e)^7 + 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e)^7 + 324*a*b^2*c^9*d^2*ta
n(1/2*f*x + 1/2*e)^7 + 189*b^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^7 - 600*a^3*c^8
*d^3*tan(1/2*f*x + 1/2*e)^7 - 504*a^2*b*c^8*d^3*tan(1/2*f*x + 1/2*e)^7 - 89
1*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^7 - 183*b^3*c^8*d^3*tan(1/2*f*x + 1/2*
e)^7 + 240*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 459*a^2*b*c^7*d^4*tan(1/2*f
*x + 1/2*e)^7 + 801*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 183*b^3*c^7*d^4*
tan(1/2*f*x + 1/2*e)^7 + 435*a^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 513*a^2*b
*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 189*a*b^2*c^6*d^5*tan(1/2*f*x + 1/2*e)^7
- 189*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 249*a^3*c^5*d^6*tan(1/2*f*x + 1/
2*e)^7 + 153*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 + 63*a*b^2*c^5*d^6*tan(1/
2*f*x + 1/2*e)^7 + 60*b^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 - 291*a^3*c^4*d^7*
tan(1/2*f*x + 1/2*e)^7 + 45*a^2*b*c^4*d^7*tan(1/2*f*x + 1/2*e)^7 - 72*a*b^2
*c^4*d^7*tan(1/2*f*x + 1/2*e)^7 + 12*b^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^7 + 2
73*a^3*c^3*d^8*tan(1/2*f*x + 1/2*e)^7 + 12*a^3*c^2*d^9*tan(1/2*f*x + 1/2*e)
^7 - 84*a^3*c*d^10*tan(1/2*f*x + 1/2*e)^7 + 24*a^3*d^11*tan(1/2*f*x + 1/2*e
)^7 - 216*a*b^2*c^11*tan(1/2*f*x + 1/2*e)^5 + 12*b^3*c^11*tan(1/2*f*x + 1/2
*e)^5 + 864*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^5 + 108*a*b^2*c^10*d*tan(1/2*
f*x + 1/2*e)^5 + 212*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^5 - 720*a^3*c^9*d^2*ta
n(1/2*f*x + 1/2*e)^5 - 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e)^5 - 684*a*b^2
*c^9*d^2*tan(1/2*f*x + 1/2*e)^5 - 197*b^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^5 +
600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e)^5 - 600*a^2*b*c^8*d^3*tan(1/2*f*x + 1/
2*e)^5 + 819*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^5 - 27*b^3*c^8*d^3*tan(1/2*
f*x + 1/2*e)^5 + 1360*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^5 + 141*a^2*b*c^7*d^
4*tan(1/2*f*x + 1/2*e)^5 + 861*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e)^5 - 27*b^
3*c^7*d^4*tan(1/2*f*x + 1/2*e)^5 - 1051*a^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^5
- 237*a^2*b*c^6*d^5*tan(1/2*f*x + 1/2*e)^5 - 807*a*b^2*c^6*d^5*tan(1/2*f*x
```

+ 1/2*e)^5 - 197*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^5 - 1029*a^3*c^5*d^6*tan(
1/2*f*x + 1/2*e)^5 + 507*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^5 + 39*a*b^2*c^
5*d^6*tan(1/2*f*x + 1/2*e)^5 + 212*b^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^5 + 759
*a^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^5 - 27*a^2*b*c^4*d^7*tan(1/2*f*x + 1/2*e)
^5 - 120*a*b^2*c^4*d^7*tan(1/2*f*x + 1/2*e)^5 + 12*b^3*c^4*d^7*tan(1/2*f*x
+ 1/2*e)^5 + 473*a^3*c^3*d^8*tan(1/2*f*x + 1/2*e)^5 - 380*a^3*c^2*d^9*tan(1
/2*f*x + 1/2*e)^5 - 84*a^3*c*d^10*tan(1/2*f*x + 1/2*e)^5 + 72*a^3*d^11*tan(
1/2*f*x + 1/2*e)^5 + 216*a*b^2*c^11*tan(1/2*f*x + 1/2*e)^3 + 12*b^3*c^11*ta
n(1/2*f*x + 1/2*e)^3 - 864*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^3 + 108*a*b^2*
c^10*d*tan(1/2*f*x + 1/2*e)^3 - 212*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^3 + 720
*a^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^3 - 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e
)^3 + 684*a*b^2*c^9*d^2*tan(1/2*f*x + 1/2*e)^3 - 197*b^3*c^9*d^2*tan(1/2*f*
x + 1/2*e)^3 + 600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e)^3 + 600*a^2*b*c^8*d^3*t
an(1/2*f*x + 1/2*e)^3 + 819*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^3 + 27*b^3*c
^8*d^3*tan(1/2*f*x + 1/2*e)^3 - 1360*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^3 + 1
41*a^2*b*c^7*d^4*tan(1/2*f*x + 1/2*e)^3 - 861*a*b^2*c^7*d^4*tan(1/2*f*x + 1
/2*e)^3 - 27*b^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^3 - 1051*a^3*c^6*d^5*tan(1/2*
f*x + 1/2*e)^3 + 237*a^2*b*c^6*d^5*tan(1/2*f*x + 1/2*e)^3 - 807*a*b^2*c^6*d
^5*tan(1/2*f*x + 1/2*e)^3 + 197*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^3 + 1029*a
^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^3 + 507*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^
3 - 39*a*b^2*c^5*d^6*tan(1/2*f*x + 1/2*e)^3 + 212*b^3*c^5*d^6*tan(1/2*f*x +
1/2*e)^3 + 759*a^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^3 + 27*a^2*b*c^4*d^7*tan(1
/2*f*x + 1/2*e)^3 - 120*a*b^2*c^4*d^7*tan(1/2*f*x + 1/2*e)^3 - 12*b^3*c^4*d
^7*tan(1/2*f*x + 1/2*e)^3 - 473*a^3*c^3*d^8*tan(1/2*f*x + 1/2*e)^3 - 380*a^
3*c^2*d^9*tan(1/2*f*x + 1/2*e)^3 + 84*a^3*c*d^10*tan(1/2*f*x + 1/2*e)^3 + 7
2*a^3*d^11*tan(1/2*f*x + 1/2*e)^3 - 72*a*b^2*c^11*tan(1/2*f*x + 1/2*e) - 12
*b^3*c^11*tan(1/2*f*x + 1/2*e) + 288*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e) - 10
8*a*b^2*c^10*d*tan(1/2*f*x + 1/2*e) + 60*b^3*c^10*d*tan(1/2*f*x + 1/2*e) -
240*a^3*c^9*d^2*tan(1/2*f*x + 1/2*e) + 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*
e) - 324*a*b^2*c^9*d^2*tan(1/2*f*x + 1/2*e) + 189*b^3*c^9*d^2*tan(1/2*f*x +
1/2*e) - 600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e) + 504*a^2*b*c^8*d^3*tan(1/2*
f*x + 1/2*e) - 891*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e) + 183*b^3*c^8*d^3*tan
(1/2*f*x + 1/2*e) - 240*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e) + 459*a^2*b*c^7*d^
4*tan(1/2*f*x + 1/2*e) - 801*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e) + 183*b^3*c
^7*d^4*tan(1/2*f*x + 1/2*e) + 435*a^3*c^6*d^5*tan(1/2*f*x + 1/2*e) + 513*a^
2*b*c^6*d^5*tan(1/2*f*x + 1/2*e) - 189*a*b^2*c^6*d^5*tan(1/2*f*x + 1/2*e) +
189*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e) + 249*a^3*c^5*d^6*tan(1/2*f*x + 1/2*e
) + 153*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e) - 63*a*b^2*c^5*d^6*tan(1/2*f*x +
1/2*e) + 60*b^3*c^5*d^6*tan(1/2*f*x + 1/2*e) - 291*a^3*c^4*d^7*tan(1/2*f*x
+ 1/2*e) - 45*a^2*b*c^4*d^7*tan(1/2*f*x + 1/2*e) - 72*a*b^2*c^4*d^7*tan(1/
2*f*x + 1/2*e) - 12*b^3*c^4*d^7*tan(1/2*f*x + 1/2*e) - 273*a^3*c^3*d^8*tan(
1/2*f*x + 1/2*e) + 12*a^3*c^2*d^9*tan(1/2*f*x + 1/2*e) + 84*a^3*c*d^10*tan(
1/2*f*x + 1/2*e) + 24*a^3*d^11*tan(1/2*f*x + 1/2*e))/((c^12 - 4*c^10*d^2 +
6*c^8*d^4 - 4*c^6*d^6 + c^4*d^8)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x
+ 1/2*e)^2 - c - d)^4))/f

Mupad [B] (verification not implemented)

Time = 28.96 (sec) , antiderivative size = 21021, normalized size of antiderivative = 33.80

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

```
[In] int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^5,x)
```

```
[Out] (atan(((((c + d)^9*(c - d)^9)^(1/2)*((tan(e/2 + (f*x)/2)*(64*a^6*c^18 + 128
*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*b^4*c^1
8 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*a^6*c^4
*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + 8385*a^
6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^7 + 484
8*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^15*d^3
+ 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c^14*d^4
 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 3600*a*b^5
*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c^7*d^11 +
 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 - 3840*a^5*
b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*a^2*b^4*c^1
4*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b^3*c^7*d^11
+ 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*c^13*d^5 - 12
640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^8*d^10 + 1431*
a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14*d^4 + 8256*a^4
*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d))/(2*(c^22*d + c^23 -
c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^
10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 +
21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) + (((32*a^3*c^27 + 16*b^3*c^27 + 96
*a^2*b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 32*a^3*c^10*d^17 + 16*a^3*c^
11*d^16 + 272*a^3*c^12*d^15 - 132*a^3*c^13*d^14 - 1020*a^3*c^14*d^13 + 528*
a^3*c^15*d^12 + 2160*a^3*c^16*d^11 - 1112*a^3*c^17*d^10 - 2920*a^3*c^18*d^9
 + 1280*a^3*c^19*d^8 + 2752*a^3*c^20*d^7 - 836*a^3*c^21*d^6 - 1852*a^3*c^22
*d^5 + 352*a^3*c^23*d^4 + 800*a^3*c^24*d^3 - 128*a^3*c^25*d^2 - 16*b^3*c^14
*d^13 + 16*b^3*c^15*d^12 - 44*b^3*c^16*d^11 + 44*b^3*c^17*d^10 + 320*b^3*c^
18*d^9 - 320*b^3*c^19*d^8 - 520*b^3*c^20*d^7 + 520*b^3*c^21*d^6 + 320*b^3*c
^22*d^5 - 320*b^3*c^23*d^4 - 44*b^3*c^24*d^3 + 44*b^3*c^25*d^2 + 180*a*b^2*
c^15*d^12 - 180*a*b^2*c^16*d^11 - 480*a*b^2*c^17*d^10 + 480*a*b^2*c^18*d^9
+ 120*a*b^2*c^19*d^8 - 120*a*b^2*c^20*d^7 + 720*a*b^2*c^21*d^6 - 720*a*b^2*
c^22*d^5 - 780*a*b^2*c^23*d^4 + 780*a*b^2*c^24*d^3 + 240*a*b^2*c^25*d^2 - 3
6*a^2*b*c^14*d^13 + 36*a^2*b*c^15*d^12 - 144*a^2*b*c^16*d^11 + 144*a^2*b*c^
17*d^10 + 840*a^2*b*c^18*d^9 - 840*a^2*b*c^19*d^8 - 1200*a^2*b*c^20*d^7 + 1
200*a^2*b*c^21*d^6 + 540*a^2*b*c^22*d^5 - 540*a^2*b*c^23*d^4 + 96*a^2*b*c^2
4*d^3 - 96*a^2*b*c^25*d^2 - 240*a*b^2*c^26*d - 96*a^2*b*c^26*d)/(c^26*d + c
^27 - c^12*d^15 - c^13*d^14 + 7*c^14*d^13 + 7*c^15*d^12 - 21*c^16*d^11 - 21
*c^17*d^10 + 35*c^18*d^9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*c^21*d^6 + 21*c^2
```

2*d^5 + 21*c^23*d^4 - 7*c^24*d^3 - 7*c^25*d^2) - (tan(e/2 + (f*x)/2)*((c + d)^9*(c - d)^9)^(1/2)*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d)*(128*c^27*d - 128*c^10*d^18 + 128*c^11*d^17 + 1024*c^12*d^16 - 1024*c^13*d^15 - 3584*c^14*d^14 + 3584*c^15*d^13 + 7168*c^16*d^12 - 7168*c^17*d^11 - 8960*c^18*d^10 + 8960*c^19*d^9 + 7168*c^20*d^8 - 7168*c^21*d^7 - 3584*c^22*d^6 + 3584*c^23*d^5 + 1024*c^24*d^4 - 1024*c^25*d^3 - 128*c^26*d^2))/(16*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)))*((c + d)^9*(c - d)^9)^(1/2)*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d))/(8*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)))*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d)*1i)/(8*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)) + (((c + d)^9*(c - d)^9)^(1/2)*((tan(e/2 + (f*x)/2)*(64*a^6*c^18 + 128*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*b^4*c^18 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*a^6*c^4*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + 8385*a^6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^7 + 4848*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^15*d^3 + 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c^14*d^4 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 3600*a*b^5*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c^7*d^11 + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 - 3840*a^5*b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*a^2*b^4*c^14*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b^3*c^7*d^11 + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*c^13*d^5 - 12640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^8*d^10 + 1431*a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14*d^4 + 8256*a^4*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d))/(2*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) - (((32*a^3*c^27 + 16*b^3*c^27 + 96*a^2*b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 32*a^3*c^10*d^17 + 16*a^3*c^11*d^16 + 272*a^3*c^12*d^15 - 132*a^3*c^13*d^14 - 1020*a^3*c^14*d^13 + 528*a^3*c^15*d^12 + 2160*a^3*c^16*d^11 - 1112*a^3*c^17*d^10 - 2920*a^3*c^18*d^9 + 1280*a^3*c^19*d^8 + 2752*a^3*c^20*d^7 - 836*a^3*c^21*d^

6 - 1852*a^3*c^22*d^5 + 352*a^3*c^23*d^4 + 800*a^3*c^24*d^3 - 128*a^3*c^25*
d^2 - 16*b^3*c^14*d^13 + 16*b^3*c^15*d^12 - 44*b^3*c^16*d^11 + 44*b^3*c^17*
d^10 + 320*b^3*c^18*d^9 - 320*b^3*c^19*d^8 - 520*b^3*c^20*d^7 + 520*b^3*c^2
1*d^6 + 320*b^3*c^22*d^5 - 320*b^3*c^23*d^4 - 44*b^3*c^24*d^3 + 44*b^3*c^25
*d^2 + 180*a*b^2*c^15*d^12 - 180*a*b^2*c^16*d^11 - 480*a*b^2*c^17*d^10 + 48
0*a*b^2*c^18*d^9 + 120*a*b^2*c^19*d^8 - 120*a*b^2*c^20*d^7 + 720*a*b^2*c^21
*d^6 - 720*a*b^2*c^22*d^5 - 780*a*b^2*c^23*d^4 + 780*a*b^2*c^24*d^3 + 240*a
*b^2*c^25*d^2 - 36*a^2*b*c^14*d^13 + 36*a^2*b*c^15*d^12 - 144*a^2*b*c^16*d^
11 + 144*a^2*b*c^17*d^10 + 840*a^2*b*c^18*d^9 - 840*a^2*b*c^19*d^8 - 1200*a
^2*b*c^20*d^7 + 1200*a^2*b*c^21*d^6 + 540*a^2*b*c^22*d^5 - 540*a^2*b*c^23*d
^4 + 96*a^2*b*c^24*d^3 - 96*a^2*b*c^25*d^2 - 240*a*b^2*c^26*d - 96*a^2*b*c^
26*d)/(c^26*d + c^27 - c^12*d^15 - c^13*d^14 + 7*c^14*d^13 + 7*c^15*d^12 -
21*c^16*d^11 - 21*c^17*d^10 + 35*c^18*d^9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*
c^21*d^6 + 21*c^22*d^5 + 21*c^23*d^4 - 7*c^24*d^3 - 7*c^25*d^2) + (tan(e/2
+ (f*x)/2)*((c + d)^9*(c - d)^9)^(1/2)*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^
9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3
*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c
^7*d^2 - 60*a*b^2*c^8*d)*(128*c^27*d - 128*c^10*d^18 + 128*c^11*d^17 + 1024
*c^12*d^16 - 1024*c^13*d^15 - 3584*c^14*d^14 + 3584*c^15*d^13 + 7168*c^16*d
^12 - 7168*c^17*d^11 - 8960*c^18*d^10 + 8960*c^19*d^9 + 7168*c^20*d^8 - 716
8*c^21*d^7 - 3584*c^22*d^6 + 3584*c^23*d^5 + 1024*c^24*d^4 - 1024*c^25*d^3
- 128*c^26*d^2))/(16*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*
d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^
2)*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^
12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*
d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)))*((c + d)^9*(c
- d)^9)^(1/2)*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3
*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2
- 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d))
/(8*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d
^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)))*(4*b^3*c^9 -
8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5
+ 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^
2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d)*1i)/(8*(c^23 - c^5*d^18 +
9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84
*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)))/((64*a^9*d^17 - 192*a^8*b*c^17 - 32
*a^9*c*d^16 + 320*a^9*c^16*d + 16*a^3*b^6*c^17 + 192*a^5*b^4*c^17 - 32*a^6*
b^3*c^17 + 576*a^7*b^2*c^17 - 544*a^9*c^2*d^15 + 264*a^9*c^3*d^14 + 2040*a^
9*c^4*d^13 - 856*a^9*c^5*d^12 - 4320*a^9*c^6*d^11 + 1649*a^9*c^7*d^10 + 584
0*a^9*c^8*d^9 - 2416*a^9*c^9*d^8 - 5504*a^9*c^10*d^7 + 2936*a^9*c^11*d^6 +
3704*a^9*c^12*d^5 - 1600*a^9*c^13*d^4 - 1600*a^9*c^14*d^3 + 1280*a^9*c^15*d
^2 - 480*a^4*b^5*c^16*d - 3168*a^6*b^3*c^16*d + 480*a^7*b^2*c^16*d - 72*a^8
*b*c^4*d^13 - 72*a^8*b*c^5*d^12 - 216*a^8*b*c^6*d^11 - 288*a^8*b*c^7*d^10 +
1986*a^8*b*c^8*d^9 + 1680*a^8*b*c^9*d^8 - 4224*a^8*b*c^10*d^7 - 2400*a^8*b
*c^11*d^6 + 936*a^8*b*c^12*d^5 + 1080*a^8*b*c^13*d^4 - 4032*a^8*b*c^14*d^3

+ 192*a^8*b*c^15*d^2 + 16*a^3*b^6*c^9*d^8 + 216*a^3*b^6*c^11*d^6 + 761*a^3*
b^6*c^13*d^4 + 216*a^3*b^6*c^15*d^2 - 360*a^4*b^5*c^10*d^7 - 2910*a^4*b^5*c
^12*d^5 - 3600*a^4*b^5*c^14*d^3 + 72*a^5*b^4*c^9*d^8 + 3087*a^5*b^4*c^11*d^
6 + 9552*a^5*b^4*c^13*d^4 + 5472*a^5*b^4*c^15*d^2 - 32*a^6*b^3*c^4*d^13 - 3
2*a^6*b^3*c^5*d^12 - 56*a^6*b^3*c^6*d^11 - 88*a^6*b^3*c^7*d^10 + 736*a^6*b^
3*c^8*d^9 + 640*a^6*b^3*c^9*d^8 - 2564*a^6*b^3*c^10*d^7 - 1040*a^6*b^3*c^11
*d^6 - 6864*a^6*b^3*c^12*d^5 + 640*a^6*b^3*c^13*d^4 - 12552*a^6*b^3*c^14*d^
3 - 88*a^6*b^3*c^15*d^2 + 360*a^7*b^2*c^5*d^12 + 360*a^7*b^2*c^6*d^11 - 132
0*a^7*b^2*c^7*d^10 - 960*a^7*b^2*c^8*d^9 + 1191*a^7*b^2*c^9*d^8 + 240*a^7*b
^2*c^10*d^7 + 3816*a^7*b^2*c^11*d^6 + 1440*a^7*b^2*c^12*d^5 + 5976*a^7*b^2*
c^13*d^4 - 1560*a^7*b^2*c^14*d^3 + 7776*a^7*b^2*c^15*d^2 - 1728*a^8*b*c^16*
d)/(c^26*d + c^27 - c^12*d^15 - c^13*d^14 + 7*c^14*d^13 + 7*c^15*d^12 - 21*
c^16*d^11 - 21*c^17*d^10 + 35*c^18*d^9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*c^2
1*d^6 + 21*c^22*d^5 + 21*c^23*d^4 - 7*c^24*d^3 - 7*c^25*d^2) - (((c + d)^9*
(c - d)^9)^(1/2)*((tan(e/2 + (f*x)/2)*(64*a^6*c^18 + 128*a^6*d^18 + 16*b^6*
c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*b^4*c^18 + 576*a^4*b^2*c^1
8 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*a^6*c^4*d^14 - 3584*a^6*c^
5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + 8385*a^6*c^8*d^10 - 8960*a
^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^7 + 4848*a^6*c^12*d^6 - 35
84*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^15*d^3 + 1152*a^6*c^16*d^2
 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c^14*d^4 + 216*b^6*c^16*d^2
 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 3600*a*b^5*c^15*d^3 - 3200*a^
3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c^7*d^11 + 3666*a^5*b*c^9*d^9
 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 - 3840*a^5*b*c^15*d^3 + 72*a^2
*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*a^2*b^4*c^14*d^4 + 5472*a^2*b^
4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b^3*c^7*d^11 + 1376*a^3*b^3*c^9*
d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*c^13*d^5 - 12640*a^3*b^3*c^15*d^
3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^8*d^10 + 1431*a^4*b^2*c^10*d^8 +
5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14*d^4 + 8256*a^4*b^2*c^16*d^2 - 480
*a*b^5*c^17*d - 1920*a^5*b*c^17*d))/(2*(c^22*d + c^23 - c^8*d^15 - c^9*d^14
 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 +
35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^2
0*d^3 - 7*c^21*d^2)) + (((32*a^3*c^27 + 16*b^3*c^27 + 96*a^2*b*c^27 - 160*a
^3*c^26*d - 16*b^3*c^26*d - 32*a^3*c^10*d^17 + 16*a^3*c^11*d^16 + 272*a^3*c
^12*d^15 - 132*a^3*c^13*d^14 - 1020*a^3*c^14*d^13 + 528*a^3*c^15*d^12 + 216
0*a^3*c^16*d^11 - 1112*a^3*c^17*d^10 - 2920*a^3*c^18*d^9 + 1280*a^3*c^19*d^
8 + 2752*a^3*c^20*d^7 - 836*a^3*c^21*d^6 - 1852*a^3*c^22*d^5 + 352*a^3*c^23
*d^4 + 800*a^3*c^24*d^3 - 128*a^3*c^25*d^2 - 16*b^3*c^14*d^13 + 16*b^3*c^15
*d^12 - 44*b^3*c^16*d^11 + 44*b^3*c^17*d^10 + 320*b^3*c^18*d^9 - 320*b^3*c^
19*d^8 - 520*b^3*c^20*d^7 + 520*b^3*c^21*d^6 + 320*b^3*c^22*d^5 - 320*b^3*c
^23*d^4 - 44*b^3*c^24*d^3 + 44*b^3*c^25*d^2 + 180*a*b^2*c^15*d^12 - 180*a*b
^2*c^16*d^11 - 480*a*b^2*c^17*d^10 + 480*a*b^2*c^18*d^9 + 120*a*b^2*c^19*d^
8 - 120*a*b^2*c^20*d^7 + 720*a*b^2*c^21*d^6 - 720*a*b^2*c^22*d^5 - 780*a*b^
2*c^23*d^4 + 780*a*b^2*c^24*d^3 + 240*a*b^2*c^25*d^2 - 36*a^2*b*c^14*d^13 +
 36*a^2*b*c^15*d^12 - 144*a^2*b*c^16*d^11 + 144*a^2*b*c^17*d^10 + 840*a^2*b

*c^18*d^9 - 840*a^2*b*c^19*d^8 - 1200*a^2*b*c^20*d^7 + 1200*a^2*b*c^21*d^6 + 540*a^2*b*c^22*d^5 - 540*a^2*b*c^23*d^4 + 96*a^2*b*c^24*d^3 - 96*a^2*b*c^25*d^2 - 240*a*b^2*c^26*d - 96*a^2*b*c^26*d)/(c^26*d + c^27 - c^12*d^15 - c^13*d^14 + 7*c^14*d^13 + 7*c^15*d^12 - 21*c^16*d^11 - 21*c^17*d^10 + 35*c^18*d^9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*c^21*d^6 + 21*c^22*d^5 + 21*c^23*d^4 - 7*c^24*d^3 - 7*c^25*d^2) - (tan(e/2 + (f*x)/2)*((c + d)^9*(c - d)^9)^(1/2)*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d)*(128*c^27*d - 128*c^10*d^18 + 128*c^11*d^17 + 1024*c^12*d^16 - 1024*c^13*d^15 - 3584*c^14*d^14 + 3584*c^15*d^13 + 7168*c^16*d^12 - 7168*c^17*d^11 - 8960*c^18*d^10 + 8960*c^19*d^9 + 7168*c^20*d^8 - 7168*c^21*d^7 - 3584*c^22*d^6 + 3584*c^23*d^5 + 1024*c^24*d^4 - 1024*c^25*d^3 - 128*c^26*d^2))/(16*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)))*((c + d)^9*(c - d)^9)^(1/2)*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d))/(8*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)))*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d))/(8*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)) + (((c + d)^9*(c - d)^9)^(1/2)*((tan(e/2 + (f*x)/2)*(64*a^6*c^18 + 128*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*b^4*c^18 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*a^6*c^4*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + 8385*a^6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^7 + 4848*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^15*d^3 + 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c^14*d^4 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 3600*a*b^5*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c^7*d^11 + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 - 3840*a^5*b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*a^2*b^4*c^14*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b^3*c^7*d^11 + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*c^13*d^5 - 12640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^8*d^10 + 1431*a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14*d^4 + 8256*a^4*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d))/(2*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^

19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) - (((32*a^3*c^27 + 16*b^3*c^27 + 96*a^2*
b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 32*a^3*c^10*d^17 + 16*a^3*c^11*d^
16 + 272*a^3*c^12*d^15 - 132*a^3*c^13*d^14 - 1020*a^3*c^14*d^13 + 528*a^3*c
^15*d^12 + 2160*a^3*c^16*d^11 - 1112*a^3*c^17*d^10 - 2920*a^3*c^18*d^9 + 12
80*a^3*c^19*d^8 + 2752*a^3*c^20*d^7 - 836*a^3*c^21*d^6 - 1852*a^3*c^22*d^5
+ 352*a^3*c^23*d^4 + 800*a^3*c^24*d^3 - 128*a^3*c^25*d^2 - 16*b^3*c^14*d^13
 + 16*b^3*c^15*d^12 - 44*b^3*c^16*d^11 + 44*b^3*c^17*d^10 + 320*b^3*c^18*d^
9 - 320*b^3*c^19*d^8 - 520*b^3*c^20*d^7 + 520*b^3*c^21*d^6 + 320*b^3*c^22*d
^5 - 320*b^3*c^23*d^4 - 44*b^3*c^24*d^3 + 44*b^3*c^25*d^2 + 180*a*b^2*c^15*
d^12 - 180*a*b^2*c^16*d^11 - 480*a*b^2*c^17*d^10 + 480*a*b^2*c^18*d^9 + 120
*a*b^2*c^19*d^8 - 120*a*b^2*c^20*d^7 + 720*a*b^2*c^21*d^6 - 720*a*b^2*c^22*
d^5 - 780*a*b^2*c^23*d^4 + 780*a*b^2*c^24*d^3 + 240*a*b^2*c^25*d^2 - 36*a^2
*b*c^14*d^13 + 36*a^2*b*c^15*d^12 - 144*a^2*b*c^16*d^11 + 144*a^2*b*c^17*d^
10 + 840*a^2*b*c^18*d^9 - 840*a^2*b*c^19*d^8 - 1200*a^2*b*c^20*d^7 + 1200*a
^2*b*c^21*d^6 + 540*a^2*b*c^22*d^5 - 540*a^2*b*c^23*d^4 + 96*a^2*b*c^24*d^3
 - 96*a^2*b*c^25*d^2 - 240*a*b^2*c^26*d - 96*a^2*b*c^26*d)/(c^26*d + c^27 -
 c^12*d^15 - c^13*d^14 + 7*c^14*d^13 + 7*c^15*d^12 - 21*c^16*d^11 - 21*c^17
*d^10 + 35*c^18*d^9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*c^21*d^6 + 21*c^22*d^5
 + 21*c^23*d^4 - 7*c^24*d^3 - 7*c^25*d^2) + (tan(e/2 + (f*x)/2)*((c + d)^9*
(c - d)^9)^(1/2)*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*
a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*
d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*
d)*(128*c^27*d - 128*c^10*d^18 + 128*c^11*d^17 + 1024*c^12*d^16 - 1024*c^13
*d^15 - 3584*c^14*d^14 + 3584*c^15*d^13 + 7168*c^16*d^12 - 7168*c^17*d^11 -
 8960*c^18*d^10 + 8960*c^19*d^9 + 7168*c^20*d^8 - 7168*c^21*d^7 - 3584*c^22
*d^6 + 3584*c^23*d^5 + 1024*c^24*d^4 - 1024*c^25*d^3 - 128*c^26*d^2))/(16*(
c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 +
 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)*(c^22*d + c^23 - c^
8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10
 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21
*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)))*((c + d)^9*(c - d)^9)^(1/2)*(4*b^3*c
^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*
d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 +
9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d))/(8*(c^23 - c^5*d^18 +
 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 8
4*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)))*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*
c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b
^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b
*c^7*d^2 - 60*a*b^2*c^8*d))/(8*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14
+ 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 -
 9*c^21*d^2))))*((c + d)^9*(c - d)^9)^(1/2)*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2
*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 +
4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^
2*b*c^7*d^2 - 60*a*b^2*c^8*d)*1i)/(4*f*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c
^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^

19*d^4 - 9*c^21*d^2)) - (2*a^3*atan(-((a^3*((tan(e/2 + (f*x)/2)*(64*a^6*c^1
8 + 128*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*
b^4*c^18 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*
a^6*c^4*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 +
8385*a^6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^
7 + 4848*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^
15*d^3 + 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c
^14*d^4 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 360
0*a*b^5*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c^7
*d^11 + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 - 38
40*a^5*b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*a^2*
b^4*c^14*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b^3*c^
7*d^11 + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*c^13*d
^5 - 12640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^8*d^10
+ 1431*a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14*d^4 + 8
256*a^4*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d))/(2*(c^22*d +
c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*
c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18
*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) + (a^3*((32*a^3*c^27 + 16*b^
3*c^27 + 96*a^2*b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 32*a^3*c^10*d^17
+ 16*a^3*c^11*d^16 + 272*a^3*c^12*d^15 - 132*a^3*c^13*d^14 - 1020*a^3*c^14*
d^13 + 528*a^3*c^15*d^12 + 2160*a^3*c^16*d^11 - 1112*a^3*c^17*d^10 - 2920*a
^3*c^18*d^9 + 1280*a^3*c^19*d^8 + 2752*a^3*c^20*d^7 - 836*a^3*c^21*d^6 - 18
52*a^3*c^22*d^5 + 352*a^3*c^23*d^4 + 800*a^3*c^24*d^3 - 128*a^3*c^25*d^2 -
16*b^3*c^14*d^13 + 16*b^3*c^15*d^12 - 44*b^3*c^16*d^11 + 44*b^3*c^17*d^10 +
320*b^3*c^18*d^9 - 320*b^3*c^19*d^8 - 520*b^3*c^20*d^7 + 520*b^3*c^21*d^6
+ 320*b^3*c^22*d^5 - 320*b^3*c^23*d^4 - 44*b^3*c^24*d^3 + 44*b^3*c^25*d^2 +
180*a*b^2*c^15*d^12 - 180*a*b^2*c^16*d^11 - 480*a*b^2*c^17*d^10 + 480*a*b^
2*c^18*d^9 + 120*a*b^2*c^19*d^8 - 120*a*b^2*c^20*d^7 + 720*a*b^2*c^21*d^6 -
720*a*b^2*c^22*d^5 - 780*a*b^2*c^23*d^4 + 780*a*b^2*c^24*d^3 + 240*a*b^2*c
^25*d^2 - 36*a^2*b*c^14*d^13 + 36*a^2*b*c^15*d^12 - 144*a^2*b*c^16*d^11 + 1
44*a^2*b*c^17*d^10 + 840*a^2*b*c^18*d^9 - 840*a^2*b*c^19*d^8 - 1200*a^2*b*c
^20*d^7 + 1200*a^2*b*c^21*d^6 + 540*a^2*b*c^22*d^5 - 540*a^2*b*c^23*d^4 + 9
6*a^2*b*c^24*d^3 - 96*a^2*b*c^25*d^2 - 240*a*b^2*c^26*d - 96*a^2*b*c^26*d)/
(c^26*d + c^27 - c^12*d^15 - c^13*d^14 + 7*c^14*d^13 + 7*c^15*d^12 - 21*c^1
6*d^11 - 21*c^17*d^10 + 35*c^18*d^9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*c^21*d
^6 + 21*c^22*d^5 + 21*c^23*d^4 - 7*c^24*d^3 - 7*c^25*d^2) - (a^3*tan(e/2 +
(f*x)/2)*(128*c^27*d - 128*c^10*d^18 + 128*c^11*d^17 + 1024*c^12*d^16 - 102
4*c^13*d^15 - 3584*c^14*d^14 + 3584*c^15*d^13 + 7168*c^16*d^12 - 7168*c^17*
d^11 - 8960*c^18*d^10 + 8960*c^19*d^9 + 7168*c^20*d^8 - 7168*c^21*d^7 - 358
4*c^22*d^6 + 3584*c^23*d^5 + 1024*c^24*d^4 - 1024*c^25*d^3 - 128*c^26*d^2)*
1i)/(2*c^5*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12
- 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 -
35*c^17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)))*1i)/c^
5))/c^5 + (a^3*((tan(e/2 + (f*x)/2)*(64*a^6*c^18 + 128*a^6*d^18 + 16*b^6*c^

18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*b^4*c^18 + 576*a^4*b^2*c^18
- 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*a^6*c^4*d^14 - 3584*a^6*c^5*
d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + 8385*a^6*c^8*d^10 - 8960*a^6
*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^7 + 4848*a^6*c^12*d^6 - 3584
*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^15*d^3 + 1152*a^6*c^16*d^2 +
16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c^14*d^4 + 216*b^6*c^16*d^2 -
360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 3600*a*b^5*c^15*d^3 - 3200*a^3*
b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c^7*d^11 + 3666*a^5*b*c^9*d^9 -
6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 - 3840*a^5*b*c^15*d^3 + 72*a^2*b
^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*a^2*b^4*c^14*d^4 + 5472*a^2*b^4*
c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b^3*c^7*d^11 + 1376*a^3*b^3*c^9*d^
9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*c^13*d^5 - 12640*a^3*b^3*c^15*d^3
+ 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^8*d^10 + 1431*a^4*b^2*c^10*d^8 + 52
56*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14*d^4 + 8256*a^4*b^2*c^16*d^2 - 480*a
*b^5*c^17*d - 1920*a^5*b*c^17*d))/(2*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 +
7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35
*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*
d^3 - 7*c^21*d^2)) - (a^3*((32*a^3*c^27 + 16*b^3*c^27 + 96*a^2*b*c^27 - 160
*a^3*c^26*d - 16*b^3*c^26*d - 32*a^3*c^10*d^17 + 16*a^3*c^11*d^16 + 272*a^3
*c^12*d^15 - 132*a^3*c^13*d^14 - 1020*a^3*c^14*d^13 + 528*a^3*c^15*d^12 + 2
160*a^3*c^16*d^11 - 1112*a^3*c^17*d^10 - 2920*a^3*c^18*d^9 + 1280*a^3*c^19*
d^8 + 2752*a^3*c^20*d^7 - 836*a^3*c^21*d^6 - 1852*a^3*c^22*d^5 + 352*a^3*c^
23*d^4 + 800*a^3*c^24*d^3 - 128*a^3*c^25*d^2 - 16*b^3*c^14*d^13 + 16*b^3*c^
15*d^12 - 44*b^3*c^16*d^11 + 44*b^3*c^17*d^10 + 320*b^3*c^18*d^9 - 320*b^3*
c^19*d^8 - 520*b^3*c^20*d^7 + 520*b^3*c^21*d^6 + 320*b^3*c^22*d^5 - 320*b^3
*c^23*d^4 - 44*b^3*c^24*d^3 + 44*b^3*c^25*d^2 + 180*a*b^2*c^15*d^12 - 180*a
*b^2*c^16*d^11 - 480*a*b^2*c^17*d^10 + 480*a*b^2*c^18*d^9 + 120*a*b^2*c^19*
d^8 - 120*a*b^2*c^20*d^7 + 720*a*b^2*c^21*d^6 - 720*a*b^2*c^22*d^5 - 780*a*
b^2*c^23*d^4 + 780*a*b^2*c^24*d^3 + 240*a*b^2*c^25*d^2 - 36*a^2*b*c^14*d^13
+ 36*a^2*b*c^15*d^12 - 144*a^2*b*c^16*d^11 + 144*a^2*b*c^17*d^10 + 840*a^2
*b*c^18*d^9 - 840*a^2*b*c^19*d^8 - 1200*a^2*b*c^20*d^7 + 1200*a^2*b*c^21*d^
6 + 540*a^2*b*c^22*d^5 - 540*a^2*b*c^23*d^4 + 96*a^2*b*c^24*d^3 - 96*a^2*b*
c^25*d^2 - 240*a*b^2*c^26*d - 96*a^2*b*c^26*d)/(c^26*d + c^27 - c^12*d^15 -
c^13*d^14 + 7*c^14*d^13 + 7*c^15*d^12 - 21*c^16*d^11 - 21*c^17*d^10 + 35*c
^18*d^9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*c^21*d^6 + 21*c^22*d^5 + 21*c^23*d
^4 - 7*c^24*d^3 - 7*c^25*d^2) + (a^3*tan(e/2 + (f*x)/2)*(128*c^27*d - 128*c
^10*d^18 + 128*c^11*d^17 + 1024*c^12*d^16 - 1024*c^13*d^15 - 3584*c^14*d^14
+ 3584*c^15*d^13 + 7168*c^16*d^12 - 7168*c^17*d^11 - 8960*c^18*d^10 + 8960
*c^19*d^9 + 7168*c^20*d^8 - 7168*c^21*d^7 - 3584*c^22*d^6 + 3584*c^23*d^5 +
1024*c^24*d^4 - 1024*c^25*d^3 - 128*c^26*d^2)*1i)/(2*c^5*(c^22*d + c^23 -
c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^
10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 +
21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)))*1i)/c^5))/c^5)/((64*a^9*d^17 - 192
*a^8*b*c^17 - 32*a^9*c*d^16 + 320*a^9*c^16*d + 16*a^3*b^6*c^17 + 192*a^5*b^
4*c^17 - 32*a^6*b^3*c^17 + 576*a^7*b^2*c^17 - 544*a^9*c^2*d^15 + 264*a^9*c^

3*d^14 + 2040*a^9*c^4*d^13 - 856*a^9*c^5*d^12 - 4320*a^9*c^6*d^11 + 1649*a^9*c^7*d^10 + 5840*a^9*c^8*d^9 - 2416*a^9*c^9*d^8 - 5504*a^9*c^10*d^7 + 2936*a^9*c^11*d^6 + 3704*a^9*c^12*d^5 - 1600*a^9*c^13*d^4 - 1600*a^9*c^14*d^3 + 1280*a^9*c^15*d^2 - 480*a^4*b^5*c^16*d - 3168*a^6*b^3*c^16*d + 480*a^7*b^2*c^16*d - 72*a^8*b*c^4*d^13 - 72*a^8*b*c^5*d^12 - 216*a^8*b*c^6*d^11 - 288*a^8*b*c^7*d^10 + 1986*a^8*b*c^8*d^9 + 1680*a^8*b*c^9*d^8 - 4224*a^8*b*c^10*d^7 - 2400*a^8*b*c^11*d^6 + 936*a^8*b*c^12*d^5 + 1080*a^8*b*c^13*d^4 - 4032*a^8*b*c^14*d^3 + 192*a^8*b*c^15*d^2 + 16*a^3*b^6*c^9*d^8 + 216*a^3*b^6*c^11*d^6 + 761*a^3*b^6*c^13*d^4 + 216*a^3*b^6*c^15*d^2 - 360*a^4*b^5*c^10*d^7 - 2910*a^4*b^5*c^12*d^5 - 3600*a^4*b^5*c^14*d^3 + 72*a^5*b^4*c^9*d^8 + 3087*a^5*b^4*c^11*d^6 + 9552*a^5*b^4*c^13*d^4 + 5472*a^5*b^4*c^15*d^2 - 32*a^6*b^3*c^4*d^13 - 32*a^6*b^3*c^5*d^12 - 56*a^6*b^3*c^6*d^11 - 88*a^6*b^3*c^7*d^10 + 736*a^6*b^3*c^8*d^9 + 640*a^6*b^3*c^9*d^8 - 2564*a^6*b^3*c^10*d^7 - 1040*a^6*b^3*c^11*d^6 - 6864*a^6*b^3*c^12*d^5 + 640*a^6*b^3*c^13*d^4 - 12552*a^6*b^3*c^14*d^3 - 88*a^6*b^3*c^15*d^2 + 360*a^7*b^2*c^5*d^12 + 360*a^7*b^2*c^6*d^11 - 1320*a^7*b^2*c^7*d^10 - 960*a^7*b^2*c^8*d^9 + 1191*a^7*b^2*c^9*d^8 + 240*a^7*b^2*c^10*d^7 + 3816*a^7*b^2*c^11*d^6 + 1440*a^7*b^2*c^12*d^5 + 5976*a^7*b^2*c^13*d^4 - 1560*a^7*b^2*c^14*d^3 + 7776*a^7*b^2*c^15*d^2 - 1728*a^8*b*c^16*d)/(c^26*d + c^27 - c^12*d^15 - c^13*d^14 + 7*c^14*d^13 + 7*c^15*d^12 - 21*c^16*d^11 - 21*c^17*d^10 + 35*c^18*d^9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*c^21*d^6 + 21*c^22*d^5 + 21*c^23*d^4 - 7*c^24*d^3 - 7*c^25*d^2) - (a^3*((tan(e/2 + (f*x)/2)*(64*a^6*c^18 + 128*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*b^4*c^18 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*a^6*c^4*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + 8385*a^6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^7 + 4848*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^15*d^3 + 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c^14*d^4 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 3600*a*b^5*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c^7*d^11 + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 - 3840*a^5*b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*a^2*b^4*c^14*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b^3*c^7*d^11 + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*c^13*d^5 - 12640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^8*d^10 + 1431*a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14*d^4 + 8256*a^4*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d))/(2*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) + (a^3*((32*a^3*c^27 + 16*b^3*c^27 + 96*a^2*b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 32*a^3*c^10*d^17 + 16*a^3*c^11*d^16 + 272*a^3*c^12*d^15 - 132*a^3*c^13*d^14 - 1020*a^3*c^14*d^13 + 528*a^3*c^15*d^12 + 2160*a^3*c^16*d^11 - 1112*a^3*c^17*d^10 - 2920*a^3*c^18*d^9 + 1280*a^3*c^19*d^8 + 2752*a^3*c^20*d^7 - 836*a^3*c^21*d^6 - 1852*a^3*c^22*d^5 + 352*a^3*c^23*d^4 + 800*a^3*c^24*d^3 - 128*a^3*c^25*d^2 - 16*b^3*c^14*d^13 + 16*b^3*c^15*d^1

2 - 44*b^3*c^16*d^11 + 44*b^3*c^17*d^10 + 320*b^3*c^18*d^9 - 320*b^3*c^19*d
^8 - 520*b^3*c^20*d^7 + 520*b^3*c^21*d^6 + 320*b^3*c^22*d^5 - 320*b^3*c^23*
d^4 - 44*b^3*c^24*d^3 + 44*b^3*c^25*d^2 + 180*a*b^2*c^15*d^12 - 180*a*b^2*c
^16*d^11 - 480*a*b^2*c^17*d^10 + 480*a*b^2*c^18*d^9 + 120*a*b^2*c^19*d^8 -
120*a*b^2*c^20*d^7 + 720*a*b^2*c^21*d^6 - 720*a*b^2*c^22*d^5 - 780*a*b^2*c^
23*d^4 + 780*a*b^2*c^24*d^3 + 240*a*b^2*c^25*d^2 - 36*a^2*b*c^14*d^13 + 36*
a^2*b*c^15*d^12 - 144*a^2*b*c^16*d^11 + 144*a^2*b*c^17*d^10 + 840*a^2*b*c^1
8*d^9 - 840*a^2*b*c^19*d^8 - 1200*a^2*b*c^20*d^7 + 1200*a^2*b*c^21*d^6 + 54
0*a^2*b*c^22*d^5 - 540*a^2*b*c^23*d^4 + 96*a^2*b*c^24*d^3 - 96*a^2*b*c^25*d
^2 - 240*a*b^2*c^26*d - 96*a^2*b*c^26*d)/(c^26*d + c^27 - c^12*d^15 - c^13*
d^14 + 7*c^14*d^13 + 7*c^15*d^12 - 21*c^16*d^11 - 21*c^17*d^10 + 35*c^18*d^
9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*c^21*d^6 + 21*c^22*d^5 + 21*c^23*d^4 - 7
*c^24*d^3 - 7*c^25*d^2) - (a^3*tan(e/2 + (f*x)/2)*(128*c^27*d - 128*c^10*d^
18 + 128*c^11*d^17 + 1024*c^12*d^16 - 1024*c^13*d^15 - 3584*c^14*d^14 + 358
4*c^15*d^13 + 7168*c^16*d^12 - 7168*c^17*d^11 - 8960*c^18*d^10 + 8960*c^19*
d^9 + 7168*c^20*d^8 - 7168*c^21*d^7 - 3584*c^22*d^6 + 3584*c^23*d^5 + 1024*
c^24*d^4 - 1024*c^25*d^3 - 128*c^26*d^2)*1i)/(2*c^5*(c^22*d + c^23 - c^8*d^
15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 3
5*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^1
9*d^4 - 7*c^20*d^3 - 7*c^21*d^2)))*1i)/c^5)*1i)/c^5 + (a^3*((tan(e/2 + (f*x
)/2)*(64*a^6*c^18 + 128*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c
^17*d + 192*a^2*b^4*c^18 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*
c^3*d^15 + 3584*a^6*c^4*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168
*a^6*c^7*d^11 + 8385*a^6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 +
7168*a^6*c^11*d^7 + 4848*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d
^4 + 1024*a^6*c^15*d^3 + 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12
*d^6 + 761*b^6*c^14*d^4 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^
5*c^13*d^5 - 3600*a*b^5*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13
 - 504*a^5*b*c^7*d^11 + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5
*b*c^13*d^5 - 3840*a^5*b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12
*d^6 + 9552*a^2*b^4*c^14*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13
 - 144*a^3*b^3*c^7*d^11 + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 622
4*a^3*b^3*c^13*d^5 - 12640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a
^4*b^2*c^8*d^10 + 1431*a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*
b^2*c^14*d^4 + 8256*a^4*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d
))/(2*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21
*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^
17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) - (a^3*((32*
a^3*c^27 + 16*b^3*c^27 + 96*a^2*b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 3
2*a^3*c^10*d^17 + 16*a^3*c^11*d^16 + 272*a^3*c^12*d^15 - 132*a^3*c^13*d^14
 - 1020*a^3*c^14*d^13 + 528*a^3*c^15*d^12 + 2160*a^3*c^16*d^11 - 1112*a^3*c^
17*d^10 - 2920*a^3*c^18*d^9 + 1280*a^3*c^19*d^8 + 2752*a^3*c^20*d^7 - 836*a
^3*c^21*d^6 - 1852*a^3*c^22*d^5 + 352*a^3*c^23*d^4 + 800*a^3*c^24*d^3 - 128
*a^3*c^25*d^2 - 16*b^3*c^14*d^13 + 16*b^3*c^15*d^12 - 44*b^3*c^16*d^11 + 44
*b^3*c^17*d^10 + 320*b^3*c^18*d^9 - 320*b^3*c^19*d^8 - 520*b^3*c^20*d^7 + 5

20*b^3*c^21*d^6 + 320*b^3*c^22*d^5 - 320*b^3*c^23*d^4 - 44*b^3*c^24*d^3 + 4
4*b^3*c^25*d^2 + 180*a*b^2*c^15*d^12 - 180*a*b^2*c^16*d^11 - 480*a*b^2*c^17
*d^10 + 480*a*b^2*c^18*d^9 + 120*a*b^2*c^19*d^8 - 120*a*b^2*c^20*d^7 + 720*
a*b^2*c^21*d^6 - 720*a*b^2*c^22*d^5 - 780*a*b^2*c^23*d^4 + 780*a*b^2*c^24*d
^3 + 240*a*b^2*c^25*d^2 - 36*a^2*b*c^14*d^13 + 36*a^2*b*c^15*d^12 - 144*a^2
*b*c^16*d^11 + 144*a^2*b*c^17*d^10 + 840*a^2*b*c^18*d^9 - 840*a^2*b*c^19*d^
8 - 1200*a^2*b*c^20*d^7 + 1200*a^2*b*c^21*d^6 + 540*a^2*b*c^22*d^5 - 540*a^
2*b*c^23*d^4 + 96*a^2*b*c^24*d^3 - 96*a^2*b*c^25*d^2 - 240*a*b^2*c^26*d - 9
6*a^2*b*c^26*d)/(c^26*d + c^27 - c^12*d^15 - c^13*d^14 + 7*c^14*d^13 + 7*c^
15*d^12 - 21*c^16*d^11 - 21*c^17*d^10 + 35*c^18*d^9 + 35*c^19*d^8 - 35*c^20
*d^7 - 35*c^21*d^6 + 21*c^22*d^5 + 21*c^23*d^4 - 7*c^24*d^3 - 7*c^25*d^2) +
(a^3*tan(e/2 + (f*x)/2)*(128*c^27*d - 128*c^10*d^18 + 128*c^11*d^17 + 1024
*c^12*d^16 - 1024*c^13*d^15 - 3584*c^14*d^14 + 3584*c^15*d^13 + 7168*c^16*d
^12 - 7168*c^17*d^11 - 8960*c^18*d^10 + 8960*c^19*d^9 + 7168*c^20*d^8 - 716
8*c^21*d^7 - 3584*c^22*d^6 + 3584*c^23*d^5 + 1024*c^24*d^4 - 1024*c^25*d^3
- 128*c^26*d^2)*1i)/(2*c^5*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^
13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8
- 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^
21*d^2)))*1i)/c^5)*1i)/c^5)))/(c^5*f) - ((tan(e/2 + (f*x)/2)^7*(8*a^3*d^8 +
4*b^3*c^8 - 24*a*b^2*c^8 - 4*a^3*c*d^7 + 32*b^3*c^7*d - 32*a^3*c^2*d^6 + 1
5*a^3*c^3*d^5 + 40*a^3*c^4*d^4 - 40*a^3*c^5*d^3 - 80*a^3*c^6*d^2 + 4*b^3*c^
4*d^4 + 32*b^3*c^5*d^3 + 21*b^3*c^6*d^2 - 24*a*b^2*c^4*d^4 - 51*a*b^2*c^5*d
^3 - 144*a*b^2*c^6*d^2 + 15*a^2*b*c^4*d^4 + 96*a^2*b*c^5*d^3 + 72*a^2*b*c^6
*d^2 - 36*a*b^2*c^7*d + 96*a^2*b*c^7*d))/(4*(c^4*d - c^5)*(c + d)^4) - (tan
(e/2 + (f*x)/2)*(4*b^3*c^8 - 8*a^3*d^8 + 24*a*b^2*c^8 - 4*a^3*c*d^7 - 32*b^
3*c^7*d + 32*a^3*c^2*d^6 + 15*a^3*c^3*d^5 - 40*a^3*c^4*d^4 - 40*a^3*c^5*d^3
+ 80*a^3*c^6*d^2 + 4*b^3*c^4*d^4 - 32*b^3*c^5*d^3 + 21*b^3*c^6*d^2 + 24*a*
b^2*c^4*d^4 - 51*a*b^2*c^5*d^3 + 144*a*b^2*c^6*d^2 + 15*a^2*b*c^4*d^4 - 96*
a^2*b*c^5*d^3 + 72*a^2*b*c^6*d^2 - 36*a*b^2*c^7*d - 96*a^2*b*c^7*d))/(4*(c
+ d)*(c^8 - 4*c^7*d + c^4*d^4 - 4*c^5*d^3 + 6*c^6*d^2)) + (tan(e/2 + (f*x)/
2)^5*(72*a^3*d^8 + 12*b^3*c^8 - 216*a*b^2*c^8 - 12*a^3*c*d^7 + 224*b^3*c^7*
d - 320*a^3*c^2*d^6 + 69*a^3*c^3*d^5 + 520*a^3*c^4*d^4 - 120*a^3*c^5*d^3 -
720*a^3*c^6*d^2 + 12*b^3*c^4*d^4 + 224*b^3*c^5*d^3 + 39*b^3*c^6*d^2 - 120*a
*b^2*c^4*d^4 - 81*a*b^2*c^5*d^3 - 1008*a*b^2*c^6*d^2 - 27*a^2*b*c^4*d^4 + 4
80*a^2*b*c^5*d^3 + 216*a^2*b*c^6*d^2 - 108*a*b^2*c^7*d + 864*a^2*b*c^7*d))/
(12*(c + d)^3*(c^6 - 2*c^5*d + c^4*d^2)) + (tan(e/2 + (f*x)/2)^3*(72*a^3*d^
8 - 12*b^3*c^8 - 216*a*b^2*c^8 + 12*a^3*c*d^7 + 224*b^3*c^7*d - 320*a^3*c^2
*d^6 - 69*a^3*c^3*d^5 + 520*a^3*c^4*d^4 + 120*a^3*c^5*d^3 - 720*a^3*c^6*d^2
- 12*b^3*c^4*d^4 + 224*b^3*c^5*d^3 - 39*b^3*c^6*d^2 - 120*a*b^2*c^4*d^4 +
81*a*b^2*c^5*d^3 - 1008*a*b^2*c^6*d^2 + 27*a^2*b*c^4*d^4 + 480*a^2*b*c^5*d^
3 - 216*a^2*b*c^6*d^2 + 108*a*b^2*c^7*d + 864*a^2*b*c^7*d))/(12*(c + d)^2*(
3*c^6*d - c^7 + c^4*d^3 - 3*c^5*d^2)))/(f*(tan(e/2 + (f*x)/2)^4*(6*c^4 + 6*
d^4 - 12*c^2*d^2) + tan(e/2 + (f*x)/2)^2*(8*c*d^3 - 8*c^3*d - 4*c^4 + 4*d^4
) - tan(e/2 + (f*x)/2)^6*(8*c*d^3 - 8*c^3*d + 4*c^4 - 4*d^4) + tan(e/2 + (f
x)/2)^8(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2) + 4*c*d^3 + 4*c^3*d +

```
c^4 + d^4 + 6*c^2*d^2))
```

3.198 $\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$

Optimal result 1395
Rubi [A] (verified) 1396
Mathematica [A] (verified) 1398
Maple [B] (verified) 1398
Fricas [F] 1399
Sympy [F] 1399
Maxima [F] 1400
Giac [F] 1400
Mupad [F(-1)] 1400

Optimal result

Integrand size = 25, antiderivative size = 320

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = -\frac{2(a-b)\sqrt{a+b}d\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf}$$
$$+\frac{2\sqrt{a+b}(b(c-d)+ad)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf}$$
$$-\frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

```
[Out] -2*(a-b)*d*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/
(a-b))^(1/2)/b/f+2*(b*(c-d)+a*d)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/
2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f-2*c*cot(f*x+e)*EllipticPi((a+b*sec(f
*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec
(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4001, 3869, 4090, 3917, 4089}

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$= \frac{2\sqrt{a+b}(ad+b(c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}$$
$$-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$
$$-\frac{2d(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bf}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*d*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) + (2*Sqrt[a + b]*(b*(c - d) +
 a*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e
+ f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4001

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a*c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
 Int[Csc[e + f*x]*((b*c + a*d + b*d*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]])
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\text{integral} = (ac)\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx + \int \frac{\sec(e+fx)(bc+ad+bd\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= -\frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+\cdots}{a-b}}}{f}$$

$$+ (bd)\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$+ (b(c-d)+ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}d\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf}$$

$$+\frac{2\sqrt{a+b}(b(c-d)+ad)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\cdots}}{bf}$$

$$-\frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+\cdots}{a-b}}}{f}$$

Mathematica [A] (verified)

Time = 10.20 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.18

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$= \frac{\cos(e+fx)\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\left(2d\sin(e+fx)-\frac{\cos^2\left(\frac{1}{2}(e+fx)\right)\left(4(a+b)d\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\cdots\right.}{\cdots}\right)}{\cdots}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])*(2*d*Sin[e + f*
x] - (Cos[(e + f*x)/2]^2*(4*(a + b)*d*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]
*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*EllipticE[ArcSin[T
an[(e + f*x)/2]], (a - b)/(a + b)] + 4*(a*(c - d) - b*(c + d))*Sqrt[Cos[e +
 f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f
*x]))]*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 8*a*c*Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[
e + f*x]))]*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + d*(
b + a*Cos[e + f*x])*Sec[(e + f*x)/2]^3*(-Sin[(e + f*x)/2] + Sin[(3*(e + f*x
))/2])))/(b + a*Cos[e + f*x])))/(f*(d + c*Cos[e + f*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. 2(293) = 586.

Time = 20.48 (sec) , antiderivative size = 1249, normalized size of antiderivative = 3.90

method	result	size
parts	Expression too large to display	1249
default	Expression too large to display	1840

```
[In] int((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c/f*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*
a-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi(cot
(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(
b+a*cos(f*x+e))-2*d/f*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))
*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*a*cos(f*x+e)^2+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))
*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*b*cos(f*x+e)^2-(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))
```

```
^(1/2))*a*cos(f*x+e)^2-(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))
^(1/2))*b*cos(f*x+e)^2+2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2
))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*a*cos(f*x+e)+2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2
))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+
1))^(1/2)*b*cos(f*x+e)-2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b
))^(1/2))*a*cos(f*x+e)-2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b
))^(1/2))*b*cos(f*x+e)+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))
*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*a+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*b-(1/
(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a-(1/(a+b)*(b+a*co
s(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE
(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b-cos(f*x+e)*sin(f*x+e)*a-sin(f
*x+e)*b)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))/(cos(f*x+e)+1)
```

Fricas [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)\,dx$$

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

Sympy [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))\,\,dx$$

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{b\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)\,dx$$

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{b\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)\,dx$$

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)\,dx$$

```
[In] int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)),x)
[Out] int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)), x)
```

3.199 $\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result 1401
Rubi [A] (verified) 1401
Mathematica [A] (verified) 1403
Maple [A] (verified) 1403
Fricas [F(-1)] 1404
Sympy [F] 1404
Maxima [F] 1404
Giac [F] 1405
Mupad [F(-1)] 1405

Optimal result

Integrand size = 27, antiderivative size = 220

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{cf} + \frac{2(bc-ad)\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{c(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))
/(a-b))^(1/2)/c/f+2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),
2*d/(c+d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e
)/c/(c+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used

$= \{4011, 3869, 4058\}$

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$
$$= \frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$
$$-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{cf}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
 + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(c*f) + (2*(b*c - a*d)*EllipticPi
[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt
[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*Sec[e
 + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 4011

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a/c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
 Dist[(b*c - a*d)/c, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{a\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx}{c} + \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{c}$$

$$= -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+f}{a-b}}}{cf}$$

$$+\frac{2(bc-ad)\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{c(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 8.25 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$
$$= \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(-((a-b)c(c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(-((a - b)*c*(c + d)*EllipticF[Arc
Sin[Tan[(e + f*x)/2]], (a - b)/(a + b)]) + 2*a*(c^2 - d^2)*EllipticPi[-1, A
rcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*d*(-(b*c) + a*d)*EllipticPi[(
c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[
e + f*x]])/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.82

method	result
default	$\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a\,c^2+\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)acd-\text{EllipticF}\left(\cot(fx+\right.\right.}{}$

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/c/(c-d)/(c+d)*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a
+b))^(1/2))*a*c^2+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*c*
d-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b*c^2-EllipticF(cot(
```

```
f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b*c*d-2*EllipticPi(cot(f*x+e)-csc(f*
x+e),-1,((a-b)/(a+b))^(1/2))*a*c^2+2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((
a-b)/(a+b))^(1/2))*a*d^2-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a
-b)/(a+b))^(1/2))*a*d^2+2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-
b)/(a+b))^(1/2))*b*c*d)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
[Out] Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}}{d\sec(fx+e)+c}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{d\sec\left(fx+e\right)+c}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)),x)
[Out] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)), x)
```

3.200 $\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

Optimal result 1406
Rubi [A] (verified) 1407
Mathematica [B] (warning: unable to verify) 1409
Maple [B] (verified) 1410
Fricas [F] 1412
Sympy [F] 1412
Maxima [F] 1412
Giac [F] 1412
Mupad [F(-1)] 1413

Optimal result

Integrand size = 25, antiderivative size = 380

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = -\frac{2(a-b)\sqrt{a+b}(3bc+4ad)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{3bf}$$
$$+\frac{2\sqrt{a+b}(ab(6c-4d)-b^2(3c-d)+3a^2d)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+f}{a+b}}}{3bf}$$
$$-\frac{2a\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$
$$+\frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f}$$

```
[Out] -2/3*(a-b)*(4*a*d+3*b*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(
1+sec(f*x+e))/(a-b))^(1/2)/b/f+2/3*(a*b*(6*c-4*d)-b^2*(3*c-d)+3*a^2*d)*cot(
f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a
+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/
f-2*a*c*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((
a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*
x+e))/(a-b))^(1/2)/f+2/3*b*d*(a+b*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4003, 4143, 4006, 3869, 3917, 4089}

$$\int (a + b\sec(e + fx))^{3/2}(c + d\sec(e+fx))\,dx = \frac{2\sqrt{a+b}(3a^2d + ab(6c-4d) - b^2(3c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}}{3bf}$$

$$-\frac{2(a-b)\sqrt{a+b}(4ad+3bc)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bf}$$

$$-\frac{2ac\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$$+\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*c + 4*a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*b*f) + (2*Sqrt[a +
 b]*(a*b*(6*c - 4*d) - b^2*(3*c - d) + 3*a^2*d)*Cot[e + f*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*b*f) - (2*
a*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (2*b*d*Sqrt[a + b*Sec[e + f*x
]]*Tan[e + f*x])/(3*f)
```

<u>Rule 3869</u>

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

<u>Rule 3917</u>

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f} \\ &\quad +\frac{2}{3}\int\frac{\frac{3a^2c}{2}+\frac{1}{2}(6abc+3a^2d+b^2d)\sec(e+fx)+\frac{1}{2}b(3bc+4ad)\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx\end{aligned}$$

$$= \frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f} + \frac{2}{3}\int \frac{\frac{3a^2c}{2} + \left(-\frac{1}{2}b(3bc+4ad) + \frac{1}{2}(6abc+3a^2d+b^2d)\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx + \frac{1}{3}(b(3bc+4ad))\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}dx$$

$$= -\frac{2(a-b)\sqrt{a+b}(3bc+4ad)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{3bf} + \frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f} + (a^2c)\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx + \frac{1}{3}(ab(6c-4d) - b^2(3c-d) + 3a^2d)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx$$

$$= -\frac{2(a-b)\sqrt{a+b}(3bc+4ad)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{3bf} + \frac{2\sqrt{a+b}(ab(6c-4d) - b^2(3c-d) + 3a^2d)\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right), \frac{a+b}{a-b}\right)}{3bf} - \frac{2a\sqrt{a+b}c\cot(e+fx)\operatorname{EllipticPi}\left(\frac{a+b}{a}, \arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right), \frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e}{a-b}}}{f} + \frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6063 vs. $2(380) = 760$.

Time = 23.57 (sec) , antiderivative size = 6063, normalized size of antiderivative = 15.96

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2983 vs. 2(345) = 690.

Time = 24.79 (sec) , antiderivative size = 2984, normalized size of antiderivative = 7.85

method	result	size
parts	Expression too large to display	2984
default	Expression too large to display	2988

```
[In] int((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c/f*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*c
os(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2*cos(
f*x+e)^2-2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b
+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b*
cos(f*x+e)^2-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*
(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^
2*cos(f*x+e)^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*
a*b*cos(f*x+e)^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2)
)*b^2*cos(f*x+e)^2-2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b
))^(1/2))*a^2*cos(f*x+e)^2+2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*a^2*cos(f*x+e)-4*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b)
)^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*a*b*cos(f*x+e)-2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+
b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*b^2*cos(f*x+e)+2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),(
(a-b)/(a+b))^(1/2))*a*b*cos(f*x+e)+2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e)
,((a-b)/(a+b))^(1/2))*b^2*cos(f*x+e)-4*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x
+e),-1,((a-b)/(a+b))^(1/2))*a^2*cos(f*x+e)+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f
*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*a^2-2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))
^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*a*b-(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*Elli
pticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*b^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f
*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b
+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f
*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2-2*(1/(a+b)
```

```
*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1
,((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2+sin(f*x+e)*cos(
f*x+e)*a*b+sin(f*x+e)*b^2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))/(cos(f*x
+e)+1)-2/3*d/f*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))/(cos(f*x+e)+1)*(3*El
lipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2*cos(f*x+e)^2+
4*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*
x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b*cos(f*x+e
)^2+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(
f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2*cos(f*x
+e)^2-4*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*cos
(f*x+e)^2-4*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b
*cos(f*x+e)^2+6*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+
b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*a^2*cos(f*x+e)+8*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(
a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*a*b*cos(f*x+e)+2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1
/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*b^2*cos(f*x+e)-8*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))
^(1/2)*a^2*cos(f*x+e)-8*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b)
)^(1/2))*a*b*cos(f*x+e)+3*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*E
llipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*a^2+4*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(
cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*a*b+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-
csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2-4*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1
/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^2-4*(1/(a+b)*(b+
a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/
(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b-4*a^2*cos(f*x+e)*sin(f*
x+e)-sin(f*x+e)*cos(f*x+e)*a*b-5*a*b*sin(f*x+e)-sin(f*x+e)*b^2-b^2*tan(f*x+
e))
```

Fricas [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((b*d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e))*sqrt(b*sec(f
*x + e) + a), x)
```

Sympy [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (a+b\sec(e+fx))^{\frac{3}{2}}\,(c+d\sec(e+fx))\,\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
[Out] Integral((a + b*sec(e + f*x))**(3/2)*(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

```
[In] int((a + b/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)),x)
[Out] int((a + b/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)), x)
```

3.201 $\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

Optimal result 1414
Rubi [A] (verified) 1414
Mathematica [A] (verified) 1417
Maple [A] (verified) 1417
Fricas [F(-1)] 1418
Sympy [F] 1418
Maxima [F] 1418
Giac [F] 1418
Mupad [F(-1)] 1419

Optimal result

Integrand size = 27, antiderivative size = 326

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{2b\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{df}$$

$$-\frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{cf}$$

$$-\frac{2(bc-ad)^2\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{cd(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[Out] 2*b*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^
(1/2)/d/f-2*a*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b
)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+
sec(f*x+e))/(a-b))^(1/2)/c/f-2*(-a*d+b*c)^2*EllipticPi(1/2*(1-sec(f*x+e))^(
1/2)*2^(1/2),2*d/(c+d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1
/2)*tan(f*x+e)/c/d/(c+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

$= \{4013, 4006, 3869, 3917, 4058\}$

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = -\frac{2(bc-ad)^2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{cdf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$
$$-\frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{cf}$$
$$+\frac{2b\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{df}$$

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*b*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*a*Sqrt[a + b]*Cot[e + f*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
 b))])/(c*f) - (2*(b*c - a*d)^2*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - S
ec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Ta
n[e + f*x])/(c*d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Dist[1/(c*d), Int[(a^2*d + b^2*c*Csc[e + f*x])/S
qrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)^2/(c*d), Int[Csc[e + f*x
]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{a^2d+b^2c\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{cd} - \frac{(bc-ad)^2\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{cd} \\
&= -\frac{2(bc-ad)^2\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{cd(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}} \\
&\quad + \frac{a^2\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx}{c} + \frac{b^2\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} \\
&= \frac{2b\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{df} \\
&\quad - \frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+f}{a-b}}}{cf} \\
&\quad - \frac{2(bc-ad)^2\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{cd(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.68 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx =$$

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a-b)^2c(c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-}{a-}\right.\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)^2*c*(c + d)*EllipticF[Ar
cSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*(a^2*(c^2 - d^2)*EllipticPi[-1
, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + (b*c - a*d)^2*EllipticPi[(c
- d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))*Sqrt[a + b*Sec[e
 + f*x]])/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [A] (verified)

Time = 8.07 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.63

method	result
default	$\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a^2c^2+\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a^2cd-2\,\text{EllipticF}\left(\cot(f\right.\right.}{}$

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/c/(c+d)/(c-d)*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a
+b))^(1/2))*a^2*c^2+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^
2*c*d-2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b*c^2-2*Elli
pticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b*c*d+EllipticF(cot(f*x+
e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^2*c^2+EllipticF(cot(f*x+e)-csc(f*x+e),
((a-b)/(a+b))^(1/2))*b^2*c*d-2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(
a+b))^(1/2))*a^2*c^2+2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1
/2))*a^2*d^2-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(
1/2))*a^2*d^2+4*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^
(1/2))*a*b*c*d-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b))
^(1/2))*b^2*c^2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(a+b\sec{(e+fx)})^{\frac{3}{2}}}{c+d\sec{(e+fx)}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
[Out] Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{3}{2}}}{d\sec{(fx+e)}+c}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{3}{2}}}{d\sec{(fx+e)}+c}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)),x)
[Out] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)), x)
```

3.202 $\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

Optimal result 1420
Rubi [A] (verified) 1421
Mathematica [B] (warning: unable to verify) 1424
Maple [B] (verified) 1424
Fricas [F] 1426
Sympy [F] 1427
Maxima [F] 1427
Giac [F] 1427
Mupad [F(-1)] 1427

Optimal result

Integrand size = 25, antiderivative size = 442

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(35abc+23a^2d+9b^2d)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{15bf}$$

$$+\frac{2\sqrt{a+b}(a^2b(45c-23d)-ab^2(35c-17d)+b^3(5c-9d)+15a^3d)\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{15bf}$$

$$-\frac{2a^2\sqrt{a+b}c\cot(e+fx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

$$+\frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{15f}+\frac{2bd(a+b\sec(e+fx))^{3/2}\tan(e+fx)}{5f}$$

```
[Out] -2/15*(a-b)*(23*a^2*d+35*a*b*c+9*b^2*d)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+
b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f+2/15*(a^2*b*(45*c-23*d)-a*b^2
*(35*c-17*d)+b^3*(5*c-9*d)+15*a^3*d)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^
(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))
^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f-2*a^2*c*cot(f*x+e)*EllipticPi((a
+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(
b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f+2/5*b*d*(a+
b*sec(f*x+e))^(3/2)*tan(f*x+e)/f+2/15*b*(8*a*d+5*b*c)*(a+b*sec(f*x+e))^(1/2
)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4003, 4141, 4143, 4006, 3869, 3917, 4089}

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(23a^2d+35abc+9b^2d)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx}}{\sqrt{a+b}}\right.\right.}{15bf}$$

$$-\frac{2a^2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$$+\frac{2\sqrt{a+b}(15a^3d+a^2b(45c-23d)-ab^2(35c-17d)+b^3(5c-9d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx}{a-b}}}{15bf}$$

$$+\frac{2b(8ad+5bc)\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{15f}+\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*b*c + 23*a^2*d + 9*b^2*d)*Cot[e + f*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(15*b
*f) + (2*Sqrt[a + b]*(a^2*b*(45*c - 23*d) - a*b^2*(35*c - 17*d) + b^3*(5*c
- 9*d) + 15*a^3*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[e + f*x]))/(a - b))])/(15*b*f) - (2*a^2*Sqrt[a + b]*c*Cot[e + f*
x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*
x]))/(a - b))])/f + (2*b*(5*b*c + 8*a*d)*Sqrt[a + b*Sec[e + f*x]]*Tan[e + f
*x])/(15*f) + (2*b*d*(a + b*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
```

```
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2bd(a+b\sec(e+fx))^{3/2}\tan(e+fx)}{5f} + \frac{2}{5}\int \sqrt{a+b\sec(e+fx)}\bigg(\frac{5a^2c}{2} + \frac{1}{2}(10abc+5a^2d+3b^2d)\sec(e+fx) + \frac{1}{2}b(5bc+8ad)\sec^2(e+fx)\bigg)\,dx$$

$$= \frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2bd(a+b\sec(e+fx))^{3/2}\tan(e+fx)}{5f} + \frac{4}{15}\int \frac{\frac{15a^3c}{4} + \frac{1}{4}(45a^2bc+5b^3c+15a^3d+17ab^2d)\sec(e+fx) + \frac{1}{4}b(35abc+23a^2d+9b^2d)\sec^2(}{\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2bd(a+b\sec(e+fx))^{3/2}\tan(e+fx)}{5f} + \frac{4}{15}\int \frac{\frac{15a^3c}{4} + \left(-\frac{1}{4}b(35abc+23a^2d+9b^2d) + \frac{1}{4}(45a^2bc+5b^3c+15a^3d+17ab^2d)\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,d$$
$$+ \frac{1}{15}\left(b\left(35abc+23a^2d+9b^2d\right)\right)\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}(35abc+23a^2d+9b^2d)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+}{a+b}}}{15bf}$$
$$+ \frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2bd(a+b\sec(e+fx))^{3/2}\tan(e+fx)}{5f} + (a^3c)\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx$$
$$+ \frac{1}{15}\left(a^2b(45c-23d)-ab^2(35c-17d)+b^3(5c-9d)+15a^3d\right)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}(35abc+23a^2d+9b^2d)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+}{a+b}}}{15bf}$$
$$+ \frac{2\sqrt{a+b}(a^2b(45c-23d)-ab^2(35c-17d)+b^3(5c-9d)+15a^3d)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\right.}{15bf}$$
$$- \frac{2a^2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(}{a-}}}{f}$$
$$+ \frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2bd(a+b\sec(e+fx))^{3/2}\tan(e+fx)}{5f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7138 vs. 2(442) = 884.

Time = 26.04 (sec) , antiderivative size = 7138, normalized size of antiderivative = 16.15

$$\int (a + b\sec(e + fx))^{5/2}(c + d\sec(e + fx))\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4160 vs. 2(403) = 806.

Time = 32.02 (sec) , antiderivative size = 4161, normalized size of antiderivative = 9.41

method	result	size
parts	Expression too large to display	4161
default	Expression too large to display	4180

```
[In] int((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 2/3*c/f*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))/(cos(f*x+e)+1)*(-14*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1
/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2*cos(f*x+e)+14*El
lipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b*cos(f*x+e)+
14*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2*cos(f*
x+e)-18*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),(
(a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b*c
os(f*x+e)-2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+
e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*b^3
*cos(f*x+e)+7*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*
x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a
^2*b+7*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((
a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2-9*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a
+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b-7*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1
/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2-(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(
a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*b^3-6*(cos(f*x+e)/(cos(f*x+e)+1
```

))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticPi(cot(f*x
+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^3*cos(f*x+e)^2+3*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(
cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^3*cos(f*x+e)^2-(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*Elliptic
F(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^3*cos(f*x+e)^2-12*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*Ell
ipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^3*cos(f*x+e)+6*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/
2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^3*cos(f*x+e)+8*a*
b^2*sin(f*x+e)+7*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2)
)*a^2*b*cos(f*x+e)^2+7*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(
f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))
^(1/2))*a*b^2*cos(f*x+e)^2-9*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+
a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/
(a+b))^(1/2))*a^2*b*cos(f*x+e)^2-7*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+
b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),(
(a-b)/(a+b))^(1/2))*a*b^2*cos(f*x+e)^2+7*a^2*b*cos(f*x+e)*sin(f*x+e)-6*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/
2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^3+3*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*El
lipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^3+a*b^2*cos(f*x+e)*sin
(f*x+e)+b^3*sin(f*x+e)+b^3*tan(f*x+e))+2/15*d/f*(a+b*sec(f*x+e))^(1/2)/(b+a
*cos(f*x+e))/(cos(f*x+e)+1)*(-34*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos
(f*x+e)+1))^(1/2)*a*b^2*cos(f*x+e)+46*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b
)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))
/(cos(f*x+e)+1))^(1/2)*a^2*b*cos(f*x+e)+18*EllipticE(cot(f*x+e)-csc(f*x+e),
((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*
x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2*cos(f*x+e)-46*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*c
os(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b*cos(f*x+e)-18*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(
b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*b^3*cos(f*x+e)+23*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b
)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b+9*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*
cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2-23*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*
x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b-17*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/
(cos(f*x+e)+1))^(1/2)*a*b^2-9*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(c
ot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*
x+e)+1))^(1/2)*b^3-15*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f

```
*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^
(1/2))*a^3*cos(f*x+e)^2-9*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*c
os(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+
b))^(1/2))*b^3*cos(f*x+e)^2-30*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(
b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b
)/(a+b))^(1/2))*a^3*cos(f*x+e)+14*a*b^2*sin(f*x+e)+23*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(
f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*cos(f*x+e)^2+9*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*Elliptic
E(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*cos(f*x+e)^2-23*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*E
llipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*cos(f*x+e)^2-17*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^
(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*cos(f*x+e)
^2+14*a*b^2*tan(f*x+e)+23*a^3*cos(f*x+e)*sin(f*x+e)+3*b^3*sec(f*x+e)*tan(f*
x+e)+34*a^2*b*sin(f*x+e)+11*a^2*b*cos(f*x+e)*sin(f*x+e)-15*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF
(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^3+9*a*b^2*cos(f*x+e)*sin(f*x+
e)+23*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+
e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^3+9*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1
/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^3+9*b^3*sin(f*x+
e)+3*b^3*tan(f*x+e)+23*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(
f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))
^(1/2))*a^3*cos(f*x+e)^2+9*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*
cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a
+b))^(1/2))*b^3*cos(f*x+e)^2+46*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*
(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-
b)/(a+b))^(1/2))*a^3*cos(f*x+e)+18*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+
b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),(
(a-b)/(a+b))^(1/2))*b^3*cos(f*x+e))
```

Fricas [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx=\int (b\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*sec(f*x + e)^3 + a^2*c + (b^2*c + 2*a*b*d)*sec(f*x + e)^2 +
 (2*a*b*c + a^2*d)*sec(f*x + e))*sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (a+b\sec{(e+fx)})^{\frac{5}{2}}\,(c+d\sec{(e+fx)})\,\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)
[Out] Integral((a + b*sec(e + f*x))**(5/2)*(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)^{5/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

```
[In] int((a + b/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)),x)
[Out] int((a + b/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)), x)
```

3.203 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal result 1428
Rubi [A] (verified) 1428
Mathematica [A] (verified) 1430
Maple [A] (verified) 1430
Fricas [F(-1)] 1431
Sympy [F] 1431
Maxima [F] 1431
Giac [F] 1431
Mupad [F(-1)] 1432

Optimal result

Integrand size = 25, antiderivative size = 208

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \frac{2\sqrt{a+b}d\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf} - \frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{af}$$

```
[Out] 2*d*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^
(1/2)/b/f-2*c*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b
)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+
sec(f*x+e))/(a-b))^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{4006, 3869, 3917\}$

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$
$$= \frac{2d\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}$$
$$-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}$$

```
[In] Int[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
 b))])/(a*f)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = c\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx + d\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= \frac{2\sqrt{a+b}d\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf}$$

$$-\frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+f}{a-b}}}{af}$$

Mathematica [A] (verified)

Time = 3.78 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.70

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((-c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)+2c\right.}{f\sqrt{a+b\sec(e+fx)}}$$

```
[In] Integrate[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((-c + d)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] + 2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]],
 (a - b)/(a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 17.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.88

method	result
default	$\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)c-\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)d-2\,\text{EllipticPi}\left(\cot(fx+e)-c\right.\right.}{f(b+a\cos(fx+e))}$
parts	$\frac{2c(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)-2\,\text{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),-1,\sqrt{\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}}{f(b+a\cos(fx+e))}$

```
[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*c-
EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*d-2*EllipticPi(cot(f*x
+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*c)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*
```

```
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+
a*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\text{Timed out}$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{c+d\sec{(e+fx)}}{\sqrt{a+b\sec{(e+fx)}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral((c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{d\sec{(fx+e)}+c}{\sqrt{b\sec{(fx+e)}+a}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{d\sec{(fx+e)}+c}{\sqrt{b\sec{(fx+e)}+a}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{c+\frac{d}{\cos(e+f\,x)}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(1/2),x)
[Out] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(1/2), x)
```

3.204 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result 1433
Rubi [A] (verified) 1433
Mathematica [A] (verified) 1435
Maple [A] (verified) 1435
Fricas [F(-1)] 1436
Sympy [F] 1436
Maxima [F] 1436
Giac [F] 1437
Mupad [F(-1)] 1437

Optimal result

Integrand size = 27, antiderivative size = 216

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{acf} - \frac{2d\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{c(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))
/(a-b))^(1/2)/a/c/f-2*d*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+
d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/c/(c+
d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used

$= \{4015, 3869, 4058\}$

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$$
$$= -\frac{2d\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$
$$-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{acf}$$

```
[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*c*f) - (2*d*EllipticPi[(2*d)/(
c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*
Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*
Sqrt[-Tan[e + f*x]^2])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 4015

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Dist[1/c, Int[1/Sqrt[a + b*Csc[e + f*x]], x],
x] - Dist[d/c, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{c}$$
$$= -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+f}{a-b}}}{acf}$$
$$-\frac{2d\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{c(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 21.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{2\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}(d+c\cos(e+fx))\left(c(c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)-2\left((c^2-d^2\right.\right.}{c(c-}$$

```
[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(d + c*Cos[e +
f*x])*(c*(c + d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*(
(c^2 - d^2)*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + d^2
*EllipticPi[(c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))*S
qrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e + f*
x]])/(c*(c - d)*(c + d)*f*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e + f*x]]
*(c + d*Sec[e + f*x]))
```

Maple [A] (verified)

Time = 7.08 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.30

method	result
default	$\frac{2(\cos(fx+e)+1)\left(c^2\,\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)+d\,\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)c-2c^2\,\text{EllipticPi}\left(\cot(f\right.\right.}{}$

```
[In] int(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/c/(c+d)/(c-d)*(cos(f*x+e)+1)*(c^2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b
)/(a+b))^(1/2))+d*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*c-2*
```

```
c^2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))+2*EllipticPi(c
ot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*d^2-2*EllipticPi(cot(f*x+e)-cs
c(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*d^2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2
)/(b+a*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a+b\sec{(e+fx)}}\,(c+d\sec{(e+fx)})}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)}+c)}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
[Out] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.205 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal result 1438
Rubi [A] (verified) 1439
Mathematica [B] (verified) 1441
Maple [B] (warning: unable to verify) 1442
Fricas [F] 1444
Sympy [F] 1444
Maxima [F] 1444
Giac [F] 1445
Mupad [F(-1)] 1445

Optimal result

Integrand size = 25, antiderivative size = 376

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{s}\ldots}{}}}{ab\sqrt{a+b}f}$$

$$-\frac{2(bc-ad)\cot(e+fx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{ab\sqrt{a+b}f}$$

$$-\frac{2\sqrt{a+b}c\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a^2f}$$

$$+\frac{2b(bc-ad)\tan(e+fx)}{a(a^2-b^2)f\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2*(-a*d+b*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/
2)/a/b/f/(a+b)^(1/2)-2*(-a*d+b*c)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/
2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+s
ec(f*x+e))/(a-b))^(1/2)/a/b/f/(a+b)^(1/2)-2*c*cot(f*x+e)*EllipticPi((a+b*se
c(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-
sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a^2/f+2*b*(-a*d+b*
c)*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4008, 4143, 4006, 3869, 3917, 4089}

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = -\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^2f}+\frac{2b(bc-ad)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}-\frac{2(bc-ad)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abf\sqrt{a+b}}+\frac{2(bc-ad)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abf\sqrt{a+b}}$$

```
[In] Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*c - a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(a*b*Sqrt[a + b]*f) - (2*(b*c - a*d)*Cot[e + f
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b
))])/(a*b*Sqrt[a + b]*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a^2*
f) + (2*b*(b*c - a*d)*Tan[e + f*x])/(a*(a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x
]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2b(bc-ad)\tan(e+fx)}{a\,(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}} - \frac{2\int \frac{-\frac{1}{2}(a^2-b^2)c+\frac{1}{2}a(bc-ad)\sec(e+fx)+\frac{1}{2}b(bc-ad)\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\,(a^2-b^2)}$$

$$= \frac{2b(bc-ad)\tan(e+fx)}{a(a^2-b^2)f\sqrt{a+b\sec(e+fx)}} - \frac{2\int \frac{-\frac{1}{2}(a^2-b^2)c+\left(\frac{1}{2}a(bc-ad)-\frac{1}{2}b(bc-ad)\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{a(a^2-b^2)}$$
$$- \frac{(b(bc-ad))\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}dx}{a(a^2-b^2)}$$

$$= \frac{2(bc-ad)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{ab\sqrt{a+b}f}$$
$$+ \frac{2b(bc-ad)\tan(e+fx)}{a(a^2-b^2)f\sqrt{a+b\sec(e+fx)}} + \frac{c\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx}{a} - \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{a(a+b)}$$

$$= \frac{2(bc-ad)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{ab\sqrt{a+b}f}$$
$$- \frac{2(bc-ad)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{ab\sqrt{a+b}f}$$
$$- \frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+}{a-b}}}{a^2f}$$
$$+ \frac{2b(bc-ad)\tan(e+fx)}{a(a^2-b^2)f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1138 vs. $2(376) = 752$.

Time = 13.36 (sec) , antiderivative size = 1138, normalized size of antiderivative = 3.03

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}dx = \frac{(b+a\cos(e+fx))^2\sec(e+fx)(c+d\sec(e+fx))\left(\frac{2(-bc+ad)\sin(e+fx)}{a(a^2-b^2)} - \frac{2(-b^2}{a}\right.}{f(d+c\cos(e+fx))(a+b\sec(e+fx))^{3/2}}$$
$$- \frac{2(b+a\cos(e+fx))^{3/2}\sqrt{\sec(e+fx)}(c+d\sec(e+fx))\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+b\tan^2\left(\frac{1}{2}(e+fx)\right)}{1+\tan^2\left(\frac{1}{2}(e+fx)\right)}}\left(-abc\tan\left(\frac{1}{2}(e\right.\right.}{}$$

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[e + f*x])^2*Sec[e + f*x]*(c + d*Sec[e + f*x])*((2*(-(b*c) + a*d
)*Sin[e + f*x])/(a*(a^2 - b^2)) - (2*(-(b^2*c*Sin[e + f*x]) + a*b*d*Sin[e +
 f*x]))/(a*(a^2 - b^2)*(b + a*Cos[e + f*x]))))/(f*(d + c*Cos[e + f*x])*(a +
 b*Sec[e + f*x])^(3/2)) - (2*(b + a*Cos[e + f*x])^(3/2)*Sqrt[Sec[e + f*x]]*
(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2
```

```
]^2)/(1 + Tan[(e + f*x)/2]^2)]*(-(a*b*c*Tan[(e + f*x)/2]) - b^2*c*Tan[(e +
f*x)/2] + a^2*d*Tan[(e + f*x)/2] + a*b*d*Tan[(e + f*x)/2] + 2*a*b*c*Tan[(e
+ f*x)/2]^3 - 2*a^2*d*Tan[(e + f*x)/2]^3 - a*b*c*Tan[(e + f*x)/2]^5 + b^2*c
*Tan[(e + f*x)/2]^5 + a^2*d*Tan[(e + f*x)/2]^5 - a*b*d*Tan[(e + f*x)/2]^5 -
 2*a^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
 Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2
]^2)/(a + b)] + 2*b^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a
 + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*
Tan[(e + f*x)/2]^2)/(a + b)] - 2*a^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/
2]], (a - b)/(a + b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[
(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + 2*b^2*c*El
lipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Tan[(e + f*x)/2]^2*
Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e
+ f*x)/2]^2)/(a + b)] + (a + b)*(-(b*c) + a*d)*EllipticE[ArcSin[Tan[(e + f*
x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]
^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + a
*(a + b)*(c - d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[
1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f
*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(a*(a^2 - b^2)*f*(d + c*Cos[e +
 f*x])*(a + b*Sec[e + f*x])^(3/2)*(-1 + Tan[(e + f*x)/2]^2)*Sqrt[(1 + Tan[(
e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(a*(-1 + Tan[(e + f*x)/2]^2) - b*(
1 + Tan[(e + f*x)/2]^2)))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2882 vs. $2(347) = 694$.

Time = 14.50 (sec) , antiderivative size = 2883, normalized size of antiderivative = 7.67

method	result	size
parts	Expression too large to display	2883
default	Expression too large to display	2987

```
[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c/f/a/(a+b)/(a-b)*(-((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*
csc(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))
^2*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e)
)^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(
a+b))^(1/2))*a^2-((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f
*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*cs
c(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))
^(1/2))*a*b+((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)
^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*csc(f*x
```

```
+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*
x+e)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2
))*a*b+((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-
b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*csc(f*x+e)^2
+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^
2-a-b)/(a+b))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^
2+2*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)*
((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*csc(f*x+e)^2+1)
^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a
-b)/(a+b))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*a
^2-2*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)
*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*csc(f*x+e)^2+1
)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-
a-b)/(a+b))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*
b^2-((1-cos(f*x+e))^4*a*csc(f*x+e)^4-(1-cos(f*x+e))^4*b*csc(f*x+e)^4-2*a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a*b*(1-cos(f*x+e))^3*csc(f*x+e)^3+((
1-cos(f*x+e))^4*a*csc(f*x+e)^4-(1-cos(f*x+e))^4*b*csc(f*x+e)^4-2*a*(1-cos(f
*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+((1-cos(
f*x+e))^4*a*csc(f*x+e)^4-(1-cos(f*x+e))^4*b*csc(f*x+e)^4-2*a*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)^(1/2)*a*b*(-cot(f*x+e)+csc(f*x+e))-((1-cos(f*x+e))^4*a
*csc(f*x+e)^4-(1-cos(f*x+e))^4*b*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+
e)^2+a+b)^(1/2)*b^2*(-cot(f*x+e)+csc(f*x+e)))*((a*(1-cos(f*x+e))^2*csc(f*x+
e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
)^(1/2)/((1-cos(f*x+e))^4*a*csc(f*x+e)^4-(1-cos(f*x+e))^4*b*csc(f*x+e)^4-2*
a*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)/(a*(1-cos(f*x+e))^2*csc(f*x+e)^2
-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)-2*d/f/(a-b)/(a+b)*(EllipticF(cot(f*x+
e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*cos(f*x+e)^2+EllipticF(cot(f*x+
e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b*cos(f*x+e)^2-(1/(a+b)*(b+a*cos(
f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(c
ot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*cos(f*x+e)^2-(1/(a+b)*(b+a*cos(
f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(c
ot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b*cos(f*x+e)^2+2*EllipticF(cot(f*
x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*cos(f*x+e)+2*EllipticF(cot(f*
x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(
a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*b*cos(f*x+e)-2*(1/(a+b)*(b+a*co
s(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE
(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*cos(f*x+e)-2*(1/(a+b)*(b+a*co
s(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE
(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b*cos(f*x+e)+EllipticF(cot(f*x+
e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a+EllipticF(cot(f*x+e)-csc(f*x+e)
,((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f
```

```
*x+e))/(cos(f*x+e)+1))^(1/2)*b-(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b
)/(a+b))^(1/2))*a-(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2
))*b-cos(f*x+e)*sin(f*x+e)*a+cos(f*x+e)*sin(f*x+e)*b)*(a+b*sec(f*x+e))^(1/2
)/(b+a*cos(f*x+e))/(cos(f*x+e)+1)
```

Fricas [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)/(b^2*sec(f*x + e)^2
+ 2*a*b*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{c+d\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(3/2),x)
[Out] Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{c+\frac{d}{\cos(e+f\,x)}}{{\left(a+\frac{b}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

```
[In] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(3/2),x)
[Out] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(3/2), x)
```

3.206 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$

Optimal result 1446
Rubi [A] (verified) 1447
Mathematica [B] (verified) 1450
Maple [B] (verified) 1451
Fricas [F] 1451
Sympy [F] 1451
Maxima [F] 1452
Giac [F] 1452
Mupad [F(-1)] 1452

Optimal result

Integrand size = 25, antiderivative size = 495

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx = \frac{2(7a^2bc-3b^3c-4a^3d)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+\cdots}{a+b}}\cdots}{3a^2(a-b)b(a+b)^{3/2}f}$$
$$-\frac{2(6a^2bc-ab^2c-3b^3c-3a^3d+a^2bd)\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{\cdots}}{3a^2(a-b)b(a+b)^{3/2}f}$$
$$-\frac{2\sqrt{a+b}c\cot(e+fx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a^3f}$$
$$+\frac{2b(bc-ad)\tan(e+fx)}{3a(a^2-b^2)f(a+b\sec(e+fx))^{3/2}}+\frac{2b(7a^2bc-3b^3c-4a^3d)\tan(e+fx)}{3a^2(a^2-b^2)^2f\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2/3*(-4*a^3*d+7*a^2*b*c-3*b^3*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+se
c(f*x+e))/(a-b))^(1/2)/a^2/(a-b)/b/(a+b)^(3/2)/f-2/3*(-3*a^3*d+6*a^2*b*c+a^
2*b*d-a*b^2*c-3*b^3*c)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/
(a-b))^(1/2)/a^2/(a-b)/b/(a+b)^(3/2)/f-2*c*cot(f*x+e)*EllipticPi((a+b*sec(f
*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec
(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a^3/f+2/3*b*(-a*d+b*c
)*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(f*x+e))^(3/2)+2/3*b*(-4*a^3*d+7*a^2*b*c
-3*b^3*c)*tan(f*x+e)/a^2/(a^2-b^2)^2/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4008, 4145, 4143, 4006, 3869, 3917, 4089}

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^3f}$$

$$+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$+\frac{2(-4a^3d+7a^2bc-3b^3c)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^2bf(a-b)(a+b)^{3/2}}$$

$$-\frac{2(-3a^3d+6a^2bc+a^2bd-ab^2c-3b^3c)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{a\right.\right.}{3a^2bf(a-b)(a+b)^{3/2}}$$

$$+\frac{2b(-4a^3d+7a^2bc-3b^3c)\tan(e+fx)}{3a^2f\left(a^2-b^2\right)^2\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(
3/2)*f) - (2*(6*a^2*b*c - a*b^2*c - 3*b^3*c - 3*a^3*d + a^2*b*d)*Cot[e + f*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/(3*a^2*(a - b)*b*(a + b)^(3/2)*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*Ellip
ticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a
- b))])/(a^3*f) + (2*b*(b*c - a*d)*Tan[e + f*x])/(3*a*(a^2 - b^2)*f*(a + b*
Sec[e + f*x])^(3/2)) + (2*b*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*Tan[e + f*x])/(
3*a^2*(a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
```

```
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2b(bc-ad)\tan(e+fx)}{3a(a^2-b^2)f(a+b\sec(e+fx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(a^2-b^2)c+\frac{3}{2}a(bc-ad)\sec(e+fx)-\frac{1}{2}b(bc-ad)\sec^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx}{3a(a^2-b^2)}$$

$$= \frac{2b(bc-ad)\tan(e+fx)}{3a(a^2-b^2)f(a+b\sec(e+fx))^{3/2}} + \frac{2b(7a^2bc-3b^3c-4a^3d)\tan(e+fx)}{3a^2(a^2-b^2)^2f\sqrt{a+b\sec(e+fx)}} + \frac{4\int \frac{\frac{3}{4}(a^2-b^2)^2c-\frac{1}{4}a(6a^2bc-2b^3c-3a^3d-ab^2d)\sec(e+fx)-\frac{1}{4}b(7a^2bc-3b^3c-4a^3d)\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{3a^2(a^2-b^2)^2}$$

$$= \frac{2b(bc-ad)\tan(e+fx)}{3a(a^2-b^2)f(a+b\sec(e+fx))^{3/2}} + \frac{2b(7a^2bc-3b^3c-4a^3d)\tan(e+fx)}{3a^2(a^2-b^2)^2f\sqrt{a+b\sec(e+fx)}} + \frac{4\int \frac{\frac{3}{4}(a^2-b^2)^2c+\left(\frac{1}{4}b(7a^2bc-3b^3c-4a^3d)-\frac{1}{4}a(6a^2bc-2b^3c-3a^3d-ab^2d)\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{3a^2(a^2-b^2)^2} - \frac{(b(7a^2bc-3b^3c-4a^3d))\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx}{3a^2(a^2-b^2)^2}$$

$$= \frac{2(7a^2bc-3b^3c-4a^3d)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx}{a-b}}}{3a^2(a-b)b(a+b)^{3/2}f} + \frac{2b(bc-ad)\tan(e+fx)}{3a(a^2-b^2)f(a+b\sec(e+fx))^{3/2}} + \frac{2b(7a^2bc-3b^3c-4a^3d)\tan(e+fx)}{3a^2(a^2-b^2)^2f\sqrt{a+b\sec(e+fx)}} + \frac{c\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx}{a^2} + \frac{(ab^2c+3b^3c+3a^3d-a^2b(6c+d))\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{3a^2(a-b)(a+b)^2}$$

$$= \frac{2(7a^2bc-3b^3c-4a^3d)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx}{a-b}}}{3a^2(a-b)b(a+b)^{3/2}f} + \frac{2(ab^2c+3b^3c+3a^3d-a^2b(6c+d))\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{3a^2(a-b)b(a+b)^{3/2}f} - \frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+}{a-b}}}{a^3f} + \frac{2b(bc-ad)\tan(e+fx)}{3a(a^2-b^2)f(a+b\sec(e+fx))^{3/2}} + \frac{2b(7a^2bc-3b^3c-4a^3d)\tan(e+fx)}{3a^2(a^2-b^2)^2f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1589 vs. 2(495) = 990.

Time = 14.93 (sec) , antiderivative size = 1589, normalized size of antiderivative = 3.21

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx = \frac{(b+a\cos(e+fx))^3\sec^2(e+fx)(c+d\sec(e+fx))\left(\frac{2(-7a^2bc+3b^3c+4a^3d)\sin(e+fx}{3a^2(a^2-b^2)^2}\right.}{f(d+c\cos(e+}$$
$$-\frac{2(b+a\cos(e+fx))^{5/2}\sec^{\frac{3}{2}}(e+fx)(c+d\sec(e+fx))\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+b\tan^2\left(\frac{1}{2}(e+fx)\right)}{1+\tan^2\left(\frac{1}{2}(e+fx)\right)}}\left(-7a^3bc\tan\left(\frac{1}{2}(e\right.\right.}{}$$

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[e + f*x])^3*Sec[e + f*x]^2*(c + d*Sec[e + f*x])*((2*(-7*a^2*b*c
 + 3*b^3*c + 4*a^3*d)*Sin[e + f*x])/(3*a^2*(a^2 - b^2)^2) - (2*(b^3*c*Sin[e
 + f*x] - a*b^2*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[e + f*x])^2)
 - (2*(-8*a^2*b^2*c*Sin[e + f*x] + 4*b^4*c*Sin[e + f*x] + 5*a^3*b*d*Sin[e +
 f*x] - a*b^3*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[e + f*x]))))
/(f*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(5/2)) - (2*(b + a*Cos[e + f*
x])^(5/2)*Sec[e + f*x]^(3/2)*(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Tan[(e +
f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)]*(-7*a^3*b*c*Tan
[(e + f*x)/2] - 7*a^2*b^2*c*Tan[(e + f*x)/2] + 3*a*b^3*c*Tan[(e + f*x)/2] +
 3*b^4*c*Tan[(e + f*x)/2] + 4*a^4*d*Tan[(e + f*x)/2] + 4*a^3*b*d*Tan[(e + f
*x)/2] + 14*a^3*b*c*Tan[(e + f*x)/2]^3 - 6*a*b^3*c*Tan[(e + f*x)/2]^3 - 8*a
^4*d*Tan[(e + f*x)/2]^3 - 7*a^3*b*c*Tan[(e + f*x)/2]^5 + 7*a^2*b^2*c*Tan[(e
 + f*x)/2]^5 + 3*a*b^3*c*Tan[(e + f*x)/2]^5 - 3*b^4*c*Tan[(e + f*x)/2]^5 +
4*a^4*d*Tan[(e + f*x)/2]^5 - 4*a^3*b*d*Tan[(e + f*x)/2]^5 - 6*a^4*c*Ellipti
cPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2
]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] +
12*a^2*b^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt
[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*
x)/2]^2)/(a + b)] - 6*b^4*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2
+ b*Tan[(e + f*x)/2]^2)/(a + b)] - 6*a^4*c*EllipticPi[-1, ArcSin[Tan[(e + f
*x)/2]], (a - b)/(a + b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*S
qrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + 12*a^2
*b^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Tan[(e + f
*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 +
b*Tan[(e + f*x)/2]^2)/(a + b)] - 6*b^4*c*EllipticPi[-1, ArcSin[Tan[(e + f*x
)/2]], (a - b)/(a + b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqr
t[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (a + b)*
(-7*a^2*b*c + 3*b^3*c + 4*a^3*d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a +
```

```
b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + a*(a + b)*(-2*b
^2*c + 3*a^2*(c - d) + a*b*(3*c - d))*EllipticF[ArcSin[Tan[(e + f*x)/2]], (
a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[
(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(3*a^2*(a^
2 - b^2)^2*f*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(5/2)*(-1 + Tan[(e +
 f*x)/2]^2)*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(a*(-1
+ Tan[(e + f*x)/2]^2) - b*(1 + Tan[(e + f*x)/2]^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7172 vs. $2(456) = 912$.

Time = 17.34 (sec) , antiderivative size = 7173, normalized size of antiderivative = 14.49

method	result	size
parts	Expression too large to display	7173
default	Expression too large to display	7241

```
[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)/(b^3*sec(f*x + e)^3
+ 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e) + a^3), x)
```

Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{c+d\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(5/2),x)
[Out] Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{c+\frac{d}{\cos(e+f\,x)}}{{\left(a+\frac{b}{\cos(e+f\,x)}\right)}^{5/2}}\,dx$$

```
[In] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(5/2),x)
[Out] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(5/2), x)
```

3.207 $\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$

Optimal result 1453
Rubi [A] (verified) 1454
Mathematica [C] (warning: unable to verify) 1455
Maple [A] (verified) 1456
Fricas [F(-1)] 1456
Sympy [F] 1456
Maxima [F] 1457
Giac [F] 1457
Mupad [F(-1)] 1457

Optimal result

Integrand size = 29, antiderivative size = 389

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{}}{\sqrt{a+b}f} + \frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad}{(c-d)(}}}{\sqrt{\frac{a+b}{c+d}}f}$$

```
[Out] 2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec
(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*
x+e))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)
*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/f/((a+b)/(c+d))^(1/2)-2*cot(f
*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sec(f*
x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*x+e
))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((
-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4017, 4021, 4067}

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$

$$= \frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\operatorname{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}}{\sqrt{a}}\right.\right.}{f\sqrt{\frac{a+b}{c+d}}}$$

$$-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\operatorname{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\operatorname{arcs}\right.}{f\sqrt{a+b}}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*f) + (2*
Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c +
d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a + b*
Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*Sec[
e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[(a + b)/(c + d)]*f)
```

Rule 4017

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c +
d*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]
]/Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0]
```

Rule 4021

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4067

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*((a + b*Csc[
e + f*x])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]))*Sqrt[(-(b*c - a*d))*((1
 - Csc[e + f*x])/((c + d)*(a + b*Csc[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + Cs
c[e + f*x])/((c - d)*(a + b*Csc[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a +
b))), ArcSin[Sqrt[(a + b)/(c + d)]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc
[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = c\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx + d\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= -\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e}{(c+d)(a+b\sec(e}}}{\sqrt{a+b}f} + \frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}}{\sqrt{\frac{a+b}{c+d}}f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.77 (sec) , antiderivative size = 40517, normalized size of antiderivative = 104.16

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 11.87 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\left(2\,\mathrm{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\cot(fx+e)-\csc(fx+e)),-\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)ac-\mathrm{Ellip}$

[In] int((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*c-EllipticF(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c+EllipticF(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*d+EllipticF(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c-EllipticF(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*d+2*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*d)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$

[In] integrate((a+b*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x)), x)

Maxima [F]

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \int \sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2),x)
[Out] int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2), x)
```

3.208 $\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result 1458
Rubi [A] (verified) 1458
Mathematica [A] (verified) 1459
Maple [A] (verified) 1460
Fricas [F] 1460
Sympy [F] 1460
Maxima [F] 1461
Giac [F] 1461
Mupad [F(-1)] 1461

Optimal result

Integrand size = 29, antiderivative size = 198

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(b}{(c}}}{\sqrt{a+b}cf}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a+
b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*s
ec(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/c/f/(a+b)^(1
/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {4021}

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\text{arcs}\right.}{cf\sqrt{a+b}}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c*f)
```

Rule 4021

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

integral =

$$-\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\ldots}}{\sqrt{a+b}cf}$$

Mathematica [A] (verified)

Time = 5.31 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.70

$$\int\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$$

$$=\frac{4\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}\csc(e+fx)\left((a+b)c\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(a+b)(d+c\cos\ldots}{\ldots}}\right.\right.\right.}{(a+b)\ldots}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*((a + b)*c*EllipticF
[ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*
d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))] - a*(c + d)*EllipticPi[(b*
c - a*d)/(a*c + b*c), ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*
x)/2]^2)/(-(b*c) + a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))])*Sqrt
[a + b*Sec[e + f*x]]*Sin[(e + f*x)/2]^2)/((a + b)*c*f*Sqrt[((c + d)*(b + a*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 10.63 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.63

method	result
default	$-\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\frac{(\cdots}{(\cdots}}\right.\right.}{f\cdots}$

[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b-2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))

Fricas [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a+b\sec\left(e+fx\right)}}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a + b*sec(e + f*x))/sqrt(c + d*sec(e + f*x)), x)

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxim
a")

[Out] integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)

Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac"
)

[Out] integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2),x)

[Out] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)

3.209 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal result 1462
Rubi [A] (verified) 1463
Mathematica [B] (warning: unable to verify) 1465
Maple [B] (warning: unable to verify) 1467
Fricas [F(-1)] 1468
Sympy [F] 1468
Maxima [F] 1468
Giac [F] 1469
Mupad [F(-1)] 1469

Optimal result

Integrand size = 29, antiderivative size = 598

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-}{(c-d}}}{\sqrt{a+b}c^2f}$$

$$-\frac{2\sqrt{a+b}d\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)(1+\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}}{c(c-d)\sqrt{c+d}f\sqrt{-\frac{(bc-ad)(1+\sec(e+fx))}{(a-b)(c+d\sec(e+fx))}}}$$

$$-\frac{2(a-b)\sqrt{a+b}d\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-}{(a-}}}{c(c-d)\sqrt{c+d}(bc-ad)f}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a+
b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*s
ec(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/c^2/f/(a+b)^
(1/2)-2*d*cot(f*x+e)*EllipticE((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/
2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1+sec(f*x+e))*(
a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)/c/(c-d)
/f/(c+d)^(1/2)/(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)-2*
(a-b)*d*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)
/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(
a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d
+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/c/(c-d)/(-a*d+b*c)/f/(c+
d)^(1/2)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4024, 4021, 4071, 4069, 4079}

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\text{arcsi}\right.}{c^2f\sqrt{a+b}}$$
$$-\frac{2d(a-b)\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\text{EllipticF}\left(\text{arcsi}\right.}{cf(c-d)\sqrt{c+d}(bc-ad)}$$
$$-\frac{2d\sqrt{a+b}\cot(e+fx)(\sec(e+fx)+1)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{cf(c-d)\sqrt{c+d}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c^2*f) -
 (2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*(1 + Sec[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a
 + b)*(c + d*Sec[e + f*x]))])/(c*(c - d)*Sqrt[c + d]*f*Sqrt[-(((b*c - a*d)*
(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]) - (2*(a - b)*Sqrt[a +
 b]*d*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])/
(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d)
)]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x]))]*Sq
rt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]*(c +
 d*Sec[e + f*x]))/(c*(c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 4021

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4024

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(3/2), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[
c + d*Csc[e + f*x]], x], x] - Dist[d/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e
+ f*x]]/(c + d*Csc[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4071

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(3/2), x_Symbol] :> Dist[(a - b)/(c -
d), Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x
], x] + Dist[(b*c - a*d)/(c - d), Int[Csc[e + f*x]*((1 + Csc[e + f*x])/(Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(3/2))), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
 0]
```

Rule 4079

```
Int[(sec[(e_.) + (f_.)*(x_)]*((A_) + (B_.)*sec[(e_.) + (f_.)*(x_)]))/(Sqrt[
(a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sec[(e_.) + (f_.)*(x_)]
)^(3/2)), x_Symbol] :> Simp[2*A*(1 + Sec[e + f*x])*(Sqrt[(b*c - a*d)*((1 -
Sec[e + f*x])/((a + b)*(c + d*Sec[e + f*x])))]/(f*(b*c - a*d)*Rt[(c + d)/(a
 + b), 2]*Tan[e + f*x]*Sqrt[(-(b*c - a*d))*((1 + Sec[e + f*x])/((a - b)*(c
+ d*Sec[e + f*x])))]))*EllipticE[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*
Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]])], (a + b)*((c - d)/((a - b)*(c + d)
))], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx}{c}$$

$$= -\frac{2\sqrt{c+d}\cot(e+fx)\operatorname{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(\cdots}{(c+d)(a+b\sec(\cdots}}}{\sqrt{a+b}c^2f} - \frac{((a-b)d)\int\frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}dx}{c(c-d)} - \frac{(d(bc-ad))\int\frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}dx}{c(c-d)}$$

$$= -\frac{2\sqrt{c+d}\cot(e+fx)\operatorname{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(\cdots}{(c+d)(a+b\sec(\cdots}}}{\sqrt{a+b}c^2f} - \frac{2\sqrt{a+b}d\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)(1+\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec\cdots}{(a+b)(c+d\sec\cdots}}}{c(c-d)\sqrt{c+d}f\sqrt{-\frac{(bc-ad)(1+\sec(e+fx))}{(a-b)(c+d\sec(e+fx))}}} - \frac{2(a-b)\sqrt{a+b}d\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e\cdots}{(a+b)(c+d\sec(e\cdots}}}{c(c-d)\sqrt{c+d}(bc-ad)f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1708 vs. $2(598) = 1196$.

Time = 14.55 (sec) , antiderivative size = 1708, normalized size of antiderivative = 2.86

$$\int\frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}dx = \frac{(d+c\cos(e+fx))^{3/2}\sec(e+fx)\sqrt{a+b\sec(e+fx)}\left(\frac{4bc(bc-ad)\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e\cdots\right.}{c-d}}\cdots}{\cdots}\right.}{\cdots} + \frac{2d(d+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{(-c^2+d^2)f(c+d\sec(e+fx))^{3/2}}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*((4*b*c*(
b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e
```

```
+ f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt
[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]],
(2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqr
t[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(a*c + b*d)
*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*
Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a -
b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c
- a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*
Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2
)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Cs
c[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c
*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/(
(a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*S
qrt[d + c*Cos[e + f*x]])) + 2*a*d*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e +
 f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*
Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a
 + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x]
)]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b
)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]]) - (2*(b*c - a*d)*(
((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)
*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d +
c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[Arc
Sin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sq
rt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c
+ d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt
[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*C
sc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e
+ f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c),
ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*
c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*
Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))/((c - d)*(c + d
)*f*Sqrt[b + a*Cos[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)) + (2*d*(d + c*Cos[
e + f*x])*Sqrt[a + b*Sec[e + f*x]]*Tan[e + f*x])/((-c^2 + d^2)*f*(c + d*Sec
[e + f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2443 vs. 2(553) = 1106.

Time = 14.38 (sec) , antiderivative size = 2444, normalized size of antiderivative = 4.09

method	result	size
default	Expression too large to display	2444

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/((a-b)/(a+b))^(1/2)/(c-d)/(c+d)/c*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*
(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*((c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-c
os(f*x+e))^2*csc(f*x+e)^2-c-d)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(((
a-b)/(a+b))^(1/2)*a*c*d*(1-cos(f*x+e))^3*csc(f*x+e)^3-((a-b)/(a+b))^(1/2)*a
*d^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-((a-b)/(a+b))^(1/2)*b*c*d*(1-cos(f*x+e))
^3*csc(f*x+e)^3+((a-b)/(a+b))^(1/2)*b*d^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*(
-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b
))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2
-c-d)/(c+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),
-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*c^2-2*(-(a*(1-cos(f
*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(
c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))
^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),-(a+b)/(a-b)
,((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*d^2-(-(a*(1-cos(f*x+e))^2*csc(f
*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e
))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*Ellipti
cF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(
1/2))*a*c^2-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e
)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2
*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+
csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c*d+(-(a*(1-cos(f*x+e))^2*cs
c(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*
x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*Elli
pticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d)
)^(1/2))*b*c^2+(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*
x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e)
)^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+
e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c*d+(-(a*(1-cos(f*x+e))^2
*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos
(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*E
llipticE(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))*a*c*d+(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc
(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x
+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(f
```

```
*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*d^2-(-(a*(1-cos(f*x+e)
)^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-
cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2
)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)
/(c+d))^(1/2))*b*c*d-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*
csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(
f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-co
t(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*d^2-((a-b)/(a+b))^(
1/2)*a*c*d*(-cot(f*x+e)+csc(f*x+e))+((a-b)/(a+b))^(1/2)*a*d^2*(-cot(f*x+e)+
csc(f*x+e))-((a-b)/(a+b))^(1/2)*b*c*d*(-cot(f*x+e)+csc(f*x+e))+((a-b)/(a+b)
)^(1/2)*b*d^2*(-cot(f*x+e)+csc(f*x+e)))/(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*
(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-co
s(f*x+e))^2*csc(f*x+e)^2-c-d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a+b\sec{(e+fx)}}}{(c+d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}+a}}{(d\sec{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{{\left(c+\frac{d}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)
```

3.210 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal result 1470
Rubi [A] (verified) 1471
Mathematica [B] (warning: unable to verify) 1474
Maple [B] (warning: unable to verify) 1476
Fricas [F] 1476
Sympy [F] 1477
Maxima [F] 1477
Giac [F] 1477
Mupad [F(-1)] 1477

Optimal result

Integrand size = 29, antiderivative size = 899

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{2(a-b)\sqrt{a+b}d(6bc^3-7ac^2d-2bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-a}{(a-b)}}}{3c^2(c-d)^2(c+d)^{3/2}}$$

$$+\frac{2\sqrt{a+b}(bc^2(3c^2+3cd-2d^2)-ad(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(}{3c^3(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+}}$$

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\text{ar}\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2d^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2/3*d^2*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))/(c
+d*sec(f*x+e))^(1/2)+2/3*(a-b)*d*(-7*a*c^2*d+3*a*d^3+6*b*c^3-2*b*c*d^2)*(d+
c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)
/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^
(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c
)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^2/(
c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(
1/2)+2/3*(b*c^2*(3*c^2+3*c*d-2*d^2)-a*d*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*(d+c
*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/
(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(
1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)
*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/(c
-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2
)-2*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e
```

```
))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a
-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*
x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b
*sec(f*x+e))^(1/2)/c^3/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2)
```

Rubi [A] (verified)

Time = 2.68 (sec) , antiderivative size = 899, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4027, 3127, 3132, 2890, 3077, 2897, 3075}

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{2\sqrt{a+b\sec(e+fx)}\sin(e+fx)d^2}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2(a-b)\sqrt{a+b}\left(6bc^3-7adc^2-2bd^2c+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+f}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)^2f\sqrt{b+a\cos(e+}}$$

$$+\frac{2\sqrt{a+b}\left(bc^2\left(3c^2+3dc-2d^2\right)-ad\left(9c^3-2dc^2-6d^2c+3d^3\right)\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}}{3c^3(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b-}}$$

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\operatorname{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx}}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*d*(6*b*c^3 - 7*a*c^2*d - 2*b*c*d^2 + 3*a*d^3)*Sqrt[-
(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-((
(b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos
[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[
e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a -
 b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*(b*c
 - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a
+ b]*(b*c^2*(3*c^2 + 3*c*d - 2*d^2) - a*d*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^
3))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))
]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*
(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3*(c - d)^2*(c + d)^
(3/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2
*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e
+ f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e +
f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*
(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d
 + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e
```

```
 + f*x]])/(c^3*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*
x]]) + (2*d^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d
 + c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\cos^2(e+fx)\sqrt{b+a\cos(e+fx)}}{(d+c\cos(e+fx))^{5/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{-\frac{1}{2}d(3bc-ad)+\frac{1}{2}(3bc^2-3acd-2bd^2)\cos(e+fx)+\frac{3}{2}a(c^2-d^2)\cos^2(e}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}}{3c\,(c^2-d^2)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(a\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^3\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{-\frac{1}{2}c^2d(3bc-ad)-\frac{3}{2}ad^2\left(c^2-d^2\right)+c\left(-3ad\left(c^2-d^2\right)+\frac{1}{2}c(3bc^2-3acd-2}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}}{3c^3\left(c^2-d^2\right)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\,\mathrm{s}}} + \frac{2d^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} - \frac{\left(d(6bc^3-7ac^2d-2bcd^2+3ad^3)\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{1+\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(}}{3c^2(c-d)\left(c^2-d^2\right)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} + \frac{\left((bc^2(3c^2+3cd-2d^2)-ad(9c^3-2c^2d-6cd^2+3d^3))\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)}{3c^3(c-d)\left(c^2-d^2\right)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2(a-b)\sqrt{a+b}d(6bc^3-7ac^2d-2bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\,\mathrm{c}}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)^2f\sqrt{b+a\,\mathrm{c}}} + \frac{2\sqrt{a+b}(bc^2(3c^2+3cd-2d^2)-ad(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1}{(a-b)(d+}}}{3c^3(c-d)^2(c+d)^{3/2}(bc-} - \frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\,\mathrm{s}}} + \frac{2d^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1990 vs. 2(899) = 1798.

Time = 7.33 (sec) , antiderivative size = 1990, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{(d+c\cos(e+fx))^3\sec^2(e+fx)\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2\sin(e+fx)}{3c(c^2-d^2)(d+c\cos(e+fx))^2} -\right.}{f(c+d\sec(e+fx))^5}$$

$$+\frac{(d+c\cos(e+fx))^{5/2}\sec^2(e+fx)\sqrt{a+b\sec(e+fx)}\left(\frac{4(bc-ad)(3b^2c^4-3abc^3d-a^2c^2d^2+b^2c^2d^2-abcd^3+a^2d^4)\sqrt{\frac{(c+d)\cot}{}}}{}\right.}{}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]]*((2*d^2*Sin
[e + f*x])/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x])^2) - (2*(6*b*c^3*d*Sin[e +
 f*x] - 7*a*c^2*d^2*Sin[e + f*x] - 2*b*c*d^3*Sin[e + f*x] + 3*a*d^4*Sin[e +
 f*x]))/(3*c*(b*c - a*d)*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f*(c + d*Se
c[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*Sqrt[a + b*
Sec[e + f*x]]*((4*(b*c - a*d)*(3*b^2*c^4 - 3*a*b*c^3*d - a^2*c^2*d^2 + b^2*
c^2*d^2 - a*b*c*d^3 + a^2*d^4)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*S
qrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-
a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*E
llipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/
((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*
c - a*d)*(3*a*b*c^4 - 3*a^2*c^3*d + 6*b^2*c^3*d - 7*a*b*c^2*d^2 - a^2*c*d^3
 - 2*b^2*c*d^3 + 4*a*b*d^4)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sq
rt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a
 - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*El
lipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
 - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/(
(a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[
((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Cs
c[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e +
 f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), A
rcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/
Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c
*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(6*a*b*c^3*d - 7*a
^2*c^2*d^2 - 2*a*b*c*d^3 + 3*a^2*d^4)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[
```

```
(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a +
b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/
((-a + b)*(c + d))]])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e +
f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a
 + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]]) - (2*(b*c - a*
d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c
+ d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(
d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF
[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)
]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)
*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*
Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x
])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc
[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*
c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a
*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a +
 b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d
+ c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))/(3*c*(c - d
)^2*(c + d)^2*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*(c + d*Sec[e + f*x])^(
5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 18679 vs. 2(820) = 1640.

Time = 17.71 (sec) , antiderivative size = 18680, normalized size of antiderivative = 20.78

method	result	size
default	Expression too large to display	18680

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] result too large to display
```

Fricas [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="frica
s")

[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)/(d^3*sec(f*x + e
)^3 + 3*c*d^2*sec(f*x + e)^2 + 3*c^2*d*sec(f*x + e) + c^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{a+b\sec{(e+fx)}}}{(c+d\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(5/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{b\sec{(fx+e)}+a}}{(d\sec{(fx+e)}+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{b\sec{(fx+e)}+a}}{(d\sec{(fx+e)}+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac"
)
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\text{Hanged}$$

```
[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(5/2),x)
[Out] \text{Hanged}
```

3.211 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal result 1478
Rubi [A] (verified) 1479
Mathematica [B] (warning: unable to verify) 1482
Maple [B] (warning: unable to verify) 1483
Fricas [F] 1485
Sympy [F] 1485
Maxima [F] 1485
Giac [F] 1486
Mupad [F(-1)] 1486

Optimal result

Integrand size = 29, antiderivative size = 744

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = -\frac{2(a-b)\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)E\left(\arcsin\left(\frac{\sqrt{c}}{\sqrt{a}}\right.\right.}{c(c-d)\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} - \frac{2\sqrt{a+b}(bc-a(2c-d))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{Ellip}}{c^2(c-d)\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(}} - \frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},a\right.}{c^2\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

```
[Out] -2*(a-b)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f
*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(
1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)
*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))
^(1/2)/c/(c-d)/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-
2*(b*c-a*(2*c-d))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(
b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)
/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e
)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*se
c(f*x+e))^(1/2)/c^2/(c-d)/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x
+e))^(1/2)-2*a*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+
a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+
b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/
(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))
```

```
^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^2/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d
*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4027, 2877, 2890, 3077, 2897, 3075}

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{a+b}(bc-a(2c-d))\csc(e+fx)\sqrt{a+b\sec(e+fx)}(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\cdots}}{c^2f(c-d)\sqrt{c+d}\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec\cdots}}$$
$$-\frac{2a\sqrt{a+b}\csc(e+fx)\sqrt{a+b\sec(e+fx)}(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+\cdots}{(a-b)(c\cos(e+fx)+\cdots}}}{c^2f\sqrt{c+d}\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+f\cdots}}$$
$$-\frac{2(a-b)\sqrt{a+b}\csc(e+fx)\sqrt{a+b\sec(e+fx)}(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e\cdots}{(a-b)(c\cos(e\cdots}}}{cf(c-d)\sqrt{c+d}\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]

[Out] (-2*(a - b)*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d
 + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d +
 c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSi
n[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*
x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c*(c
 - d)*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2
*Sqrt[a + b]*(b*c - a*(2*c - d))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((
a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a
- b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*Ellipt
icF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*C
os[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x
]])/(c^2*(c - d)*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e +
f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*
(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d
 + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((
a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[
a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqr
t[a + b*Sec[e + f*x]])/(c^2*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c +
 d*Sec[e + f*x]])
```

Rule 2877

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
```

```
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{3/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}\,dx}{c^2\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} + \frac{\left((bc-ad)\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{bc+ad+2ac\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}\,dx}{c^2\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{Elliptic}}{c^2\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d}} - \frac{\left((bc-a(2c-d))(bc-ad)\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b+a\cos(e+fx)}\sqrt{d+c\cos(e+fx}}}{c^2(c-d)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} + \frac{\left((bc-ad)^2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{1+\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}\,dx}{c(c-d)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2(a-b)\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)E}{c(c-d)\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\,\text{s}}} - \frac{2\sqrt{a+b}(bc-a(2c-d))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(}{c^2(c-d)\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}} - \frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{Elliptic}}{c^2\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1750 vs. 2(744) = 1488.

Time = 19.22 (sec) , antiderivative size = 1750, normalized size of antiderivative = 2.35

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \frac{2(d+c\cos(e+fx))(a+b\sec(e+fx))^{3/2}(-bc\sin(e+fx)+ad\sin(e+fx))}{(-c^2+d^2)\,f(b+a\cos(e+fx))(c+d\sec(e+fx))^{3/2}}$$

$$+\frac{(d+c\cos(e+fx))^{3/2}(a+b\sec(e+fx))^{3/2}\left(\frac{4(bc-ad)\left(abc-b^2d\right)\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{bc-ad}}\sqrt{(\cdots}}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(3/2)*(-(b*c*Sin[e + f*x]) + a
*d*Sin[e + f*x]))/((-c^2 + d^2)*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])
^(3/2)) + ((d + c*Cos[e + f*x])^(3/2)*(a + b*Sec[e + f*x])^(3/2)*((4*(b*c -
 a*d)*(a*b*c - b^2*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c +
 d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d
 + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[
ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*
(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(a^2*c - b^2
*c)*(b*c - a*d)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*
(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c
*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcS
in[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqr
t[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c +
 d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot
[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/
2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/
(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[(
(-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2
*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*
Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-(a*b*c) + a^2*d)*((Sqrt[(-a
+ b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[A
rcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(
```

```
a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e +
f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e
 + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e
 + f*x]))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x
)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(
e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Si
n[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[
e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[(
(c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b
)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*Ellipt
icPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Cs
c[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]
*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e +
 f*x]])))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos
[e + f*x]])))))/((c - d)*(c + d)*f*(b + a*Cos[e + f*x])^(3/2)*(c + d*Sec[e +
 f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3568 vs. $2(675) = 1350$.

Time = 13.97 (sec) , antiderivative size = 3569, normalized size of antiderivative = 4.80

method	result	size
default	Expression too large to display	3569

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/((a-b)/(a+b))^(1/2)/(c-d)/(c+d)/c*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*
(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*((c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-c
os(f*x+e))^2*csc(f*x+e)^2-c-d)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-2
*((a-b)/(a+b))^(1/2)*a*b*c*d*(-cot(f*x+e)+csc(f*x+e))+(-(a*(1-cos(f*x+e))^2
*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos
(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*E
llipticE(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))*a^2*c*d-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f
*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot
(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*b*c^2-(-(a*(1-cos(f*
x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c
*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^
(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(
a-b)/(c+d))^(1/2))*a*b*d^2+(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+
```

e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(
1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2
)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b^2*c*d-(-(a*(1
-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/
2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/
(c+d))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*
(c-d)/(a-b)/(c+d))^(1/2))*a^2*c*d+2*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1
-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x
+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticF(((a-b)/(a
+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*b*c^
2-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(
a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e
)^2-c-d)/(c+d))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)
),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b^2*c*d+((a-b)/(a+b))^(1/2)*a^2*c*d*(1-c
os(f*x+e))^3*csc(f*x+e)^3-((a-b)/(a+b))^(1/2)*a*b*c^2*(1-cos(f*x+e))^3*csc(
f*x+e)^3+((a-b)/(a+b))^(1/2)*a*b*d^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-((a-b)/(
a+b))^(1/2)*b^2*c*d*(1-cos(f*x+e))^3*csc(f*x+e)^3+(-(a*(1-cos(f*x+e))^2*csc
(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x
+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*Ellip
ticE(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*b^2*c^2-((a-b)/(a+b))^(1/2)*a^2*c*d*(-cot(f*x+e)+csc(f*x+e))+((a-b)
/(a+b))^(1/2)*a*b*c^2*(-cot(f*x+e)+csc(f*x+e))+((a-b)/(a+b))^(1/2)*a*b*d^2*
(-cot(f*x+e)+csc(f*x+e))-((a-b)/(a+b))^(1/2)*b^2*c*d*(-cot(f*x+e)+csc(f*x+e
))-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/
(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+
e)^2-c-d)/(c+d))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e
)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a^2*c^2-(-(a*(1-cos(f*x+e))^2*csc(f*x+e
)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2
*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticF((
(a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2)
)*b^2*c^2+2*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e
)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2
*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)
+csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a^2*c^2-
2*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(
a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e
)^2-c-d)/(c+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e
)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a^2*d^2+(-(a*(1-co
s(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*
(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+
d))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-
d)/(a-b)/(c+d))^(1/2))*a^2*d^2-((a-b)/(a+b))^(1/2)*a^2*d^2*(1-cos(f*x+e))^3
*csc(f*x+e)^3+((a-b)/(a+b))^(1/2)*b^2*c^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*(
-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b
))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2

```
-c-d)/(c+d))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),(
(a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*b*c*d-2*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^
2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticE(((a
-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*
a*b*c*d+((a-b)/(a+b))^(1/2)*a^2*d^2*(-cot(f*x+e)+csc(f*x+e))+((a-b)/(a+b))^
(1/2)*b^2*c^2*(-cot(f*x+e)+csc(f*x+e)))/(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*
(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-co
s(f*x+e))^2*csc(f*x+e)^2-c-d)
```

Fricas [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="frica
s")
[Out] integral((b*sec(f*x + e) + a)^(3/2)*sqrt(d*sec(f*x + e) + c)/(d^2*sec(f*x +
 e)^2 + 2*c*d*sec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\sec(e+fx))^{\frac{3}{2}}}{(c+d\sec(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(3/2),x)
[Out] Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(3/2), x)
```

3.212 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal result 1487
Rubi [A] (verified) 1488
Mathematica [B] (warning: unable to verify) 1492
Maple [B] (warning: unable to verify) 1493
Fricas [F(-1)] 1493
Sympy [F] 1494
Maxima [F] 1494
Giac [F] 1494
Mupad [F(-1)] 1494

Optimal result

Integrand size = 29, antiderivative size = 919

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = -\frac{2(a-b)\sqrt{a+b}(3bc^3-7ac^2d+bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+f}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+a\cos(e+f}}$$
$$-\frac{2\sqrt{a+b}(b^2c^3(3c+d)-2abc^2(3c^2+2cd-d^2)+a^2d(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-}}{3c^3(c-d)^2(c+d)^{3/2}(bc-}$$
$$-\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)}\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$
$$-\frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

```
[Out] -2/3*d*(-a*d+b*c)*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d+c*cos(
f*x+e))/(c+d*sec(f*x+e))^(1/2)-2/3*(a-b)*(-7*a*c^2*d+3*a*d^3+3*b*c^3+b*c*d^
2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))
^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*
(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a
*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)
/c^2/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e
))^(1/2)-2/3*(b^2*c^3*(3*c+d)-2*a*b*c^2*(3*c^2+2*c*d-d^2)+a^2*d*(9*c^3-2*c^
2*d-6*c*d^2+3*d^3))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)
*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-
b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x
+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*
```

```
sec(f*x+e))^(1/2)/c^3/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/
2)/(c+d*sec(f*x+e))^(1/2)-2*a*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi(
(c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)
*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos
(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d
+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/f/(c+d)^(1/2)/(b+a*cos(f*x
+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 3.15 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4027, 3068, 3132, 2890, 3077, 2897, 3075}

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{2(a-b)\sqrt{a+b}(3bc^3-7adc^2+bd^2c+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}\csc(e+fx)E\Big(\mathrm{arc}}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+a\cos(e+fx}}$$
$$-\frac{2\sqrt{a+b}(b^2(3c+d)c^3-2ab(3c^2+2dc-d^2)\,c^2+a^2d(9c^3-2dc^2-6d^2c+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-}}{3c^3(c-d)^2(c+d)^{3/2}(bc-a}$$
$$-\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}\csc(e+fx)\,\mathrm{EllipticPi}\Big(\frac{(a+b)c}{a(c+d)},\arcsin\Big(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+}}{\sqrt{a+b}\sqrt{d+c\cos(e+}}}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx}}$$
$$-\frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)\,f\sqrt{c+d\sec(e+fx)}(d+c\cos(e+fx))}$$

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*c^3 - 7*a*c^2*d + b*c*d^2 + 3*a*d^3)*Sqrt[-(((
b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*
c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e
+ f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e +
 f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)
*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*(b*c -
a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[a + b]*
(b^2*c^3*(3*c + d) - 2*a*b*c^2*(3*c^2 + 2*c*d - d^2) + a^2*d*(9*c^3 - 2*c^2
*d - 6*c*d^2 + 3*d^3))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d
+ c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d +
c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin
[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x
]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3
*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*
```

```
Sec[e + f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/(
(a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a
 - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*Ellip
ticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]]
)/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sqrt[a + b*Sec[e + f*x]])/(c^3*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]]) - (2*d*(b*c - a*d)*Sqrt[a + b*Sec[e + f*x]]*Sin[e
 + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

<u>Rule 2890</u>

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

<u>Rule 2897</u>

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

<u>Rule 3068</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

<u>Rule 3075</u>

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\cos(e+fx)(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{5/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\frac{1}{2}(bc-ad)(3bc-ad)-\frac{1}{2}\left(3a^2cd+b^2cd-2ab\left(3c^2-d^2\right)\right)\cos(e+fx)+\frac{3}{2}a}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}}{3c\left(c^2-d^2\right)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(a^2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^3\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\frac{1}{2}c^2(bc-ad)(3bc-ad)-\frac{3}{2}a^2d^2\left(c^2-d^2\right)+c\left(-3a^2d\left(c^2-d^2\right)+\frac{1}{2}c\left(-3a\right.\right.}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))}}{3c^3\left(c^2-d^2\right)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{Elliptic}}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+}} - \frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left((bc-ad)\left(3bc^3-7ac^2d+bcd^2+3ad^3\right)\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{1+}{\sqrt{b+a\cos(e+f}}}{3c^2(c-d)\left(c^2-d^2\right)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} - \frac{\left(\left(b^2c^3(3c+d)-2abc^2(3c^2+2cd-d^2)+a^2d(9c^3-2c^2d-6cd^2+3d^3)\right)\sqrt{d+c\cos(e+fx)}\sqrt{}\right.}{3c^3(c-d)\left(c^2-d^2\right)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(}}$$

$$= -\frac{2(a-b)\sqrt{a+b}(3bc^3-7ac^2d+bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+}} - \frac{2\sqrt{a+b}(b^2c^3(3c+d)-2abc^2(3c^2+2cd-d^2)+a^2d(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos}{(a+b)(d+c\cos}}}{3c^3(c-d)^2(c+}} - \frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{Elliptic}}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+}} - \frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1960 vs. $2(919) = 1838$.

Time = 7.17 (sec) , antiderivative size = 1960, normalized size of antiderivative = 2.13

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{(d+c\cos(e+fx))^3\sec(e+fx)(a+b\sec(e+fx))^{3/2}\left(\frac{2(-bcd\sin(e+fx)+ad^2\sin(e+\cdots}{3c(c^2-d^2)(d+c\cos(e+fx))\cdots}\right.}{f(b+a\cos(e+fx))(c+d\sec\cdots}$$

$$+\frac{(d+c\cos(e+fx))^{5/2}\sec(e+fx)(a+b\sec(e+fx))^{3/2}\left(\frac{4(bc-ad)(3abc^3+a^2c^2d-4b^2c^2d+abcd^2-a^2d^3)\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+f\cdots\right.}{c-d}}\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((2*(-(b*c*
d*Sin[e + f*x]) + a*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x]
)^2) + (2*(3*b*c^3*Sin[e + f*x] - 7*a*c^2*d*Sin[e + f*x] + b*c*d^2*Sin[e +
f*x] + 3*a*d^3*Sin[e + f*x]))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f
*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(
5/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((4*(b*c - a*d)*(3*a*b*c^3 + a
^2*c^2*d - 4*b^2*c^2*d + a*b*c*d^2 - a^2*d^3)*Sqrt[((c + d)*Cot[(e + f*x)/2
]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
 a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e +
 f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(
e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e +
 f*x]]) + 4*(b*c - a*d)*(3*a^2*c^3 - 3*b^2*c^3 + 4*a*b*c^2*d + a^2*c*d^2 -
b^2*c*d^2 - 4*a*b*d^3)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((
c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)
*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*Ellipti
cF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a +
b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c +
 d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e
+ f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)
/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin
[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[
```

```
2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt
[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-3*a*b*c^3 + 7*a^2*c^2
*d - a*b*c*d^2 - 3*a^2*d^3)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/
2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e
 + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a + b)*
(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqr
t[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d +
 c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) - (2*(b*c - a*d)*(((b*c
+ (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b +
a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[
e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sq
rt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]]
, (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*S
qrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c +
 d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e
+ f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)
/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin
[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[
2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt
[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*Cos[e
+ f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))/(3*c*(c - d)^2*(c + d
)^2*f*(b + a*Cos[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 15313 vs. $2(840) = 1680$.

Time = 16.55 (sec) , antiderivative size = 15314, normalized size of antiderivative = 16.66

method	result	size
default	Expression too large to display	15314

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a + b\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\sec{(e+fx)})^{\frac{3}{2}}}{(c+d\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(5/2),x)
[Out] Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{3}{2}}}{(d\sec{(fx+e)}+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{3}{2}}}{(d\sec{(fx+e)}+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Hanged}$$

```
[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(5/2),x)
[Out] \text{Hanged}
```

3.213 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

Optimal result 1495
Rubi [A] (verified) 1496
Mathematica [B] (warning: unable to verify) 1500
Maple [B] (warning: unable to verify) 1502
Fricas [F(-1)] 1502
Sympy [F(-1)] 1502
Maxima [F] 1503
Giac [F] 1503
Mupad [F(-1)] 1503

Optimal result

Integrand size = 29, antiderivative size = 1122

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \frac{2(a-b)\sqrt{a+b}(2abcd(35c^4-8c^2d^2+5d^4)-a^2d^2(58c^4-41c^2d^2+15d^4)-b}{}$$

$$-\frac{2\sqrt{a+b}(b^2c^3(15c^3+10c^2d+9cd^2-2d^3)-2abc^2(15c^4+20c^3d-4c^2d^2-4cd^3+5d^4)+a^2d(60c^5-2c^4d}{}$$

$$-\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\operatorname{EllipticPi}\left(\frac{(a+b)c}{a(c+d)}\right.}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$

$$+\frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$

$$-\frac{2d(10bc^3-13ac^2d-2bcd^2+5ad^3)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2(c^2-d^2)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2/5*d^2*(b+a*cos(f*x+e))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d
+c*cos(f*x+e))^2/(c+d*sec(f*x+e))^(1/2)-2/15*d*(-13*a*c^2*d+5*a*d^3+10*b*c^
3-2*b*c*d^2)*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^2/(c^2-d^2)^2/f/(d+c*cos(f
*x+e))/(c+d*sec(f*x+e))^(1/2)+2/15*(a-b)*(2*a*b*c*d*(35*c^4-8*c^2*d^2+5*d^4
)-a^2*d^2*(58*c^4-41*c^2*d^2+15*d^4)-b^2*(15*c^6+19*c^4*d^2-2*c^2*d^4))*(d+
c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)
/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^
(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c
)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/(
c-d)^3/(c+d)^(5/2)/(-a*d+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(
1/2)-2/15*(b^2*c^3*(15*c^3+10*c^2*d+9*c*d^2-2*d^3)-2*a*b*c^2*(15*c^4+20*c^3
*d-4*c^2*d^2-4*c*d^3+5*d^4)+a^2*d*(60*c^5-2*c^4*d-66*c^3*d^2+25*c^2*d^3+30*
```

```
c*d^4-15*d^5))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a
*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))
^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f
*x+e))^(1/2)/c^4/(c-d)^3/(c+d)^(5/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c
+d*sec(f*x+e))^(1/2)-2*a*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)
^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/
(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+
e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*co
s(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^4/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^
(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 3.66 (sec) , antiderivative size = 1122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {4027, 3127, 3126, 3132, 2890, 3077, 2897, 3075}

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \frac{2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)d^2}{5c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$

$$-\frac{2(10bc^3-13adc^2-2bd^2c+5ad^3)\sqrt{a+b\sec(e+fx)}\sin(e+fx)d}{15c^2\,(c^2-d^2)^2\,f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2(a-b)\sqrt{a+b}(-((15c^6+19d^2c^4-2d^4c^2)\,b^2)+2acd(35c^4-8d^2c^2+5d^4)\,b-a^2d^2(58c^4-41d^2c^2+15d^4))}{15c^3(c-d)^3(c+d)^5}$$

$$-\frac{2\sqrt{a+b}(b^2(15c^3+10dc^2+9d^2c-2d^3)\,c^3-2ab(15c^4+20dc^3-4d^2c^2-4d^3c+5d^4)\,c^2+a^2d(60c^5-2dc^4}{}$$

$$-\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},a\right.}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*a*b*c*d*(35*c^4 - 8*c^2*d^2 + 5*d^4) - a^2*d^2*(5
8*c^4 - 41*c^2*d^2 + 15*d^4) - b^2*(15*c^6 + 19*c^4*d^2 - 2*c^2*d^4))*Sqrt[
-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(
((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos
[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(15*c^3*(c - d)^3*(c + d)^(5/2)*(b
*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[
a + b]*(b^2*c^3*(15*c^3 + 10*c^2*d + 9*c*d^2 - 2*d^3) - 2*a*b*c^2*(15*c^4 +
 20*c^3*d - 4*c^2*d^2 - 4*c*d^3 + 5*d^4) + a^2*d*(60*c^5 - 2*c^4*d - 66*c^3
```

```
*d^2 + 25*c^2*d^3 + 30*c*d^4 - 15*d^5))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*
x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]
))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[
d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[
e + f*x]])/(15*c^4*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e +
 f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1
- Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 +
Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*
Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b
 + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^4*Sqrt[c + d]*f*Sqrt[b
+ a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*d^2*(b + a*Cos[e + f*x])*S
qrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(5*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x
])^2*Sqrt[c + d*Sec[e + f*x]]) - (2*d*(10*b*c^3 - 13*a*c^2*d - 2*b*c*d^2 +
5*a*d^3)*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(15*c^2*(c^2 - d^2)^2*f*(d
+ c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
```

```
 f*x]))))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
```

```
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{7/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\sqrt{b+a\cos(e+fx)}\left(-\frac{1}{2}d(5bc-3ad)+\frac{1}{2}\left(5bc^2-5acd-2bd^2\right)\cos(e+fx)\cdots}{(d+c\cos(e+fx))^{5/2}}}{5c\left(c^2-d^2\right)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} - \frac{2d\left(10bc^3-13ac^2d-2bcd^2+5ad^3\right)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(4\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\frac{1}{4}\left(a^2d^2\left(13c^2-5d^2\right)-8abcd\left(5c^2-d^2\right)+3b^2\left(5c^4+3c^2d^2\right)\right)-\frac{1}{2}\left(b^2cd(5\cdots\right.}{\sqrt{b+a\,c\cdots}}}{15c^2\left(c^2-d^2\right)^2\sqrt{b+a\cos(e+fx)}\sqrt{c+\cdots}}$$

$$= \frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} - \frac{2d\left(10bc^3-13ac^2d-2bcd^2+5ad^3\right)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(a^2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}\,dx}{c^4\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} + \frac{\left(4\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{-\frac{15}{4}a^2d^2\left(c^2-d^2\right)^2+\frac{1}{4}c^2\left(a^2d^2\left(13c^2-5d^2\right)-8abcd\left(5c^2-d^2\right)+3b^2(5\cdots\right.}{\cdots}}{15c^4\left(c^2-d^2\right)^2\sqrt{b+a\cos(e+f\cdots}}$$

$$= -\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticP}}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\,\text{s}}}$$
$$+\frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$
$$-\frac{2d(10bc^3-13ac^2d-2bcd^2+5ad^3)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2(c^2-d^2)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$
$$-\frac{\left((2abcd(35c^4-8c^2d^2+5d^4)-a^2d^2(58c^4-41c^2d^2+15d^4)-b^2(15c^6+19c^4d^2-2c^2d^4)\right)\sqrt{d+c}}{15c^3(c-d)(c^2-d^2)^2\sqrt{b+a\cos(e+fx)}\sqrt{c}}$$
$$-\frac{\left((b^2c^3(15c^3+10c^2d+9cd^2-2d^3)-2abc^2(15c^4+20c^3d-4c^2d^2-4cd^3+5d^4)+a^2d(60c^5-2c\right.}{15c^4(c-d)(c^2-d^2)^2\sqrt{}}$$

$$=\frac{2(a-b)\sqrt{a+b}(2abcd(35c^4-8c^2d^2+5d^4)-a^2d^2(58c^4-41c^2d^2+15d^4)-b^2(15c^6+19c^4d^2-2c^2}{15c^3(c-d)^3(c}$$
$$-\frac{2\sqrt{a+b}(b^2c^3(15c^3+10c^2d+9cd^2-2d^3)-2abc^2(15c^4+20c^3d-4c^2d^2-4cd^3+5d^4)+a^2d(60c}{}$$
$$-\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticP}}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\,\text{s}}}$$
$$+\frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$
$$-\frac{2d(10bc^3-13ac^2d-2bcd^2+5ad^3)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2(c^2-d^2)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2385 vs. 2(1122) = 2244.

Time = 7.98 (sec) , antiderivative size = 2385, normalized size of antiderivative = 2.13

$$\int\frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^4*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2)*((-2*(-(b
*c*d^2*Sin[e + f*x]) + a*d^3*Sin[e + f*x]))/(5*c^2*(c^2 - d^2)*(d + c*Cos[e
 + f*x])^3) - (4*(5*b*c^3*d*Sin[e + f*x] - 8*a*c^2*d^2*Sin[e + f*x] - b*c*d
^3*Sin[e + f*x] + 4*a*d^4*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^2*(d + c*Cos[e
```

```
 + f*x])^2) + (2*(15*b^2*c^6*Sin[e + f*x] - 70*a*b*c^5*d*Sin[e + f*x] + 58*
a^2*c^4*d^2*Sin[e + f*x] + 19*b^2*c^4*d^2*Sin[e + f*x] + 16*a*b*c^3*d^3*Sin
[e + f*x] - 41*a^2*c^2*d^4*Sin[e + f*x] - 2*b^2*c^2*d^4*Sin[e + f*x] - 10*a
*b*c*d^5*Sin[e + f*x] + 15*a^2*d^6*Sin[e + f*x]))/(15*c^2*(b*c - a*d)*(c^2
- d^2)^3*(d + c*Cos[e + f*x])))))/(f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x
])^(7/2)) + ((d + c*Cos[e + f*x])^(7/2)*Sec[e + f*x]^2*(a + b*Sec[e + f*x])
^(3/2)*((4*(b*c - a*d)*(-15*a*b^2*c^6 + 5*a^2*b*c^5*d + 25*b^3*c^5*d + 13*a
^3*c^4*d^2 - 38*a*b^2*c^4*d^2 + 25*a^2*b*c^3*d^3 + 7*b^3*c^3*d^3 - 18*a^3*c
^2*d^4 - 11*a*b^2*c^2*d^4 + 2*a^2*b*c*d^5 + 5*a^3*d^6)*Sqrt[((c + d)*Cot[(e
 + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])
*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d
))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d +
c*Cos[e + f*x]]) + 4*(b*c - a*d)*(-15*a^2*b*c^6 + 15*b^3*c^6 + 15*a^3*c^5*d
 - 55*a*b^2*c^5*d + 33*a^2*b*c^4*d^2 + 19*b^3*c^4*d^2 + 13*a^3*c^3*d^3 + 35
*a*b^2*c^3*d^3 - 70*a^2*b*c^2*d^4 - 2*b^3*c^2*d^4 + 4*a^3*c*d^5 - 12*a*b^2*
c*d^5 + 20*a^2*b*d^6)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c
 + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*
(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d
)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b
)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c +
d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e +
 f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/
2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[
Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2
]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[
b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(15*a*b^2*c^6 - 70*a^2*b
*c^5*d + 58*a^3*c^4*d^2 + 19*a*b^2*c^4*d^2 + 16*a^2*b*c^3*d^3 - 41*a^3*c^2*
d^4 - 2*a*b^2*c^2*d^4 - 10*a^2*b*c*d^5 + 15*a^3*d^6)*((Sqrt[(-a + b)/(a + b
)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt
[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (
2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/
(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a
 + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))])
 - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c
- d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*S
qrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e +
 f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)
/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
- ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b
+ a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Co
s[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c -
```

```
 a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)
/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f
*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(
a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]])
)))/(15*c^2*(c - d)^3*(c + d)^3*(-(b*c) + a*d)*f*(b + a*Cos[e + f*x])^(3/2)
*(c + d*Sec[e + f*x])^(7/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 54550 vs. 2(1037) = 2074.

Time = 21.53 (sec) , antiderivative size = 54551, normalized size of antiderivative = 48.62

method	result	size
default	Expression too large to display	54551

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Hanged}$$

```
[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(7/2),x)
[Out] \text{Hanged}
```

3.214 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal result 1504
Rubi [A] (verified) 1505
Mathematica [B] (warning: unable to verify) 1509
Maple [B] (warning: unable to verify) 1510
Fricas [F(-1)] 1510
Sympy [F(-1)] 1511
Maxima [F] 1511
Giac [F] 1511
Mupad [F(-1)] 1511

Optimal result

Integrand size = 29, antiderivative size = 891

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(7ac^2-4bcd-3ad^2)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(}{3c^2(c-d)^2(c+d)^{3/2}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d}}$$

$$+\frac{2\sqrt{a+b}(b^2c^2(c+3d)-abc(7c^2+4cd-3d^2)+a^2(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-}{(a-}}}{3c^3(c-d)^2(c+d)^{3/2}f\sqrt{b}}$$

$$-\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx}}$$

$$+\frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2/3*(-a*d+b*c)^2*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d+c*cos(f
*x+e))/(c+d*sec(f*x+e))^(1/2)-2/3*(a-b)*(7*a*c^2-3*a*d^2-4*b*c*d)*(d+c*cos(
f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)
^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*
(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+c
os(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^2/(c-d)^2
/(c+d)^(3/2)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)+2/3*(b^2*c^2*(
c+3*d)-a*b*c*(7*c^2+4*c*d-3*d^2)+a^2*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*(d+c*co
s(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+
b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2
)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1
+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/(c-d)
```

```
^2/(c+d)^(3/2)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2*a^2*(d+c*c
os(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(
a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/
2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e
))^(1/2)/c^3/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4027, 2871, 3132, 2890, 3077, 2897, 3075}

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},a\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$

$$+\frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

$$-\frac{2(a-b)\sqrt{a+b}(7ac^2-4bdc-3ad^2)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc}{3c^2(c-d)^2(c+d)^{3/2}f\sqrt{b+a\cos(e+fx)}\sqrt{c+}}$$

$$+\frac{2\sqrt{a+b}((9c^3-2dc^2-6d^2c+3d^3)a^2-bc(7c^2+4dc-3d^2)a+b^2c^2(c+3d))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(b}{(}}}{3c^3(c-d)^2(c+d)^{3/2}f\sqrt{}}$$

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(7*a*c^2 - 4*b*c*d - 3*a*d^2)*Sqrt[-(((b*c - a*d)*(
1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1
+ Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2
)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqr
t[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
qrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*f*Sqrt[b + a*Cos[e
+ f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^2*c^2*(c + 3*d) - a*b
*c*(7*c^2 + 4*c*d - 3*d^2) + a^2*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^3))*Sqrt[
-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(
((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos
[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3*(c - d)^2*(c + d)^(3/2)*f*S
qrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a^2*Sqrt[a + b]*Sqrt
[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-
```

```
(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*C
os[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[
(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]
])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^3*Sq
rt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c -
a*d)^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos
[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
```

```
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{5/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

$$+ \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\frac{1}{2}(7ab^2c^2-5a^2bcd-3b^3cd+a^3d^2)+\frac{1}{2}(b^3c^2-3a^3cd-5ab^2cd+a^2b(9c^2-}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+f}}{3c(c^2-d^2)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

$$+ \frac{\left(a^3\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^3\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$+ \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{-\frac{3}{2}a^3d^2(c^2-d^2)+\frac{1}{2}c^2(7ab^2c^2-5a^2bcd-3b^3cd+a^3d^2)+c(-3a^3d(c^2-}{\sqrt{b+a\cos(e+fx)}(d+c\,c}}{3c^3(c^2-d^2)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx}}$$

$$=$$

$$- \frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{Elliptic}}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d}}$$

$$+ \frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

$$+ \frac{\left((bc-ad)^2(7ac^2-4bcd-3ad^2)\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{1+\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(}}{3c^2(c-d)(c^2-d^2)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$+ \frac{\left((b^3c^3(c+3d)-ab^2c^3(7c+5d)+a^2bc^2(9c^2+5cd-2d^2)-a^3d(9c^3-2c^2d-6cd^2+3d^3)\right)\sqrt{d+}}{3c^3(c-d)(c^2-d^2)\sqrt{b+a\cos(e+fx)}\sqrt{c-}}$$

$$=$$

$$- \frac{2(a-b)\sqrt{a+b}(7ac^2-4bcd-3ad^2)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+f}{3c^2(c-d)^2(c+d)^{3/2}f\sqrt{b+a\cos(e+}}$$

$$+ \frac{2\sqrt{a+b}(b^2c^2(c+3d)-abc(7c^2+4cd-3d^2)+a^2(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+}{(a+b)(d+c\cos(e+f}}}{3c^3(c-d)^2(c+}$$

$$- \frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{Elliptic}}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d}}$$

$$+ \frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2026 vs. 2(891) = 1782.

Time = 7.03 (sec) , antiderivative size = 2026, normalized size of antiderivative = 2.27

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2*c^2*Sin[e + f*x
] - 2*a*b*c*d*Sin[e + f*x] + a^2*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c
*Cos[e + f*x])^2) + (2*(7*a*b*c^3*Sin[e + f*x] - 7*a^2*c^2*d*Sin[e + f*x] -
 4*b^2*c^2*d*Sin[e + f*x] + a*b*c*d^2*Sin[e + f*x] + 3*a^2*d^3*Sin[e + f*x]
))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])^2*(c
+ d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*(a + b*Sec[e + f*x])
^(5/2)*((4*(b*c - a*d)*(2*a^2*b*c^3 + b^3*c^3 + a^3*c^2*d - 8*a*b^2*c^2*d +
 2*a^2*b*c*d^2 + 3*b^3*c*d^2 - a^3*d^3)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(
c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e
 + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/
2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*
x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]
) + 4*(b*c - a*d)*(3*a^3*c^3 - 7*a*b^2*c^3 + 4*b^3*c^2*d + a^3*c*d^2 + 3*a*
b^2*c*d^2 - 4*a^2*b*d^3)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[
((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a -
b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*Ellip
ticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a
+ b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c
 + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(
e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*
x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcS
in[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqr
t[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sq
rt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-7*a^2*b*c^3 + 7*a^3
*c^2*d + 4*a*b^2*c^2*d - a^2*b*c*d^2 - 3*a^3*d^3)*((Sqrt[(-a + b)/(a + b)]*
(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-
a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(
b*c - a*d))/((-a + b)*(c + d))]])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b
+ a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a +
b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) -
(2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d
)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt
[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*
```

```
x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)
/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]
^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (
(b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a
*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e
 + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*
d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)
/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c
) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))
/(3*c*(c - d)^2*(c + d)^2*f*(b + a*Cos[e + f*x])^(5/2)*(c + d*Sec[e + f*x])
^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 18990 vs. $2(812) = 1624$.

Time = 16.32 (sec) , antiderivative size = 18991, normalized size of antiderivative = 21.31

method	result	size
default	Expression too large to display	18991

```
[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Hanged}$$

```
[In] int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(5/2),x)
[Out] \text{Hanged}
```

3.215 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

Optimal result . . . 1512
Rubi [A] (verified) . . . 1513
Mathematica [B] (warning: unable to verify) . . . 1518
Maple [B] (warning: unable to verify) . . . 1519
Fricas [F(-1)] . . . 1519
Sympy [F(-1)] . . . 1520
Maxima [F] . . . 1520
Giac [F] . . . 1520
Mupad [F(-1)] . . . 1520

Optimal result

Integrand size = 29, antiderivative size = 1150

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \frac{2(a-b)\sqrt{a+b}(b^2c^2d(29c^2+3d^2)-abc(35c^4+34c^2d^2-5d^4)+a^2(58c^4d-41\ldots}{\ldots}$$

$$+\frac{2\sqrt{a+b}(b^3c^4(5c^2+24cd+3d^2)-ab^2c^3(35c^3+42c^2d+21cd^2-2d^3)+a^2bc^2(45c^4+48c^3d+c^2d^2-8cd^3+\ldots}{\ldots}$$

$$-\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\ldots\right.}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}\ldots}$$

$$-\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2(bc-ad)(5bc^3-13ac^2d+3bcd^2+5ad^3)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2(c^2-d^2)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

```
[Out] -2/5*d*(-a*d+b*c)*(b+a*cos(f*x+e))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2
-d^2)/f/(d+c*cos(f*x+e))^2/(c+d*sec(f*x+e))^(1/2)+2/15*(-a*d+b*c)*(-13*a*c^
2*d+5*a*d^3+5*b*c^3+3*b*c*d^2)*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^2/(c^2-d
^2)^2/f/(d+c*cos(f*x+e))/(c+d*sec(f*x+e))^(1/2)+2/15*(a-b)*(b^2*c^2*d*(29*c
^2+3*d^2)-a*b*c*(35*c^4+34*c^2*d^2-5*d^4)+a^2*(58*c^4*d-41*c^2*d^3+15*d^5))
*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(
1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a
+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d
+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c
^3/(c-d)^3/(c+d)^(5/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2)+2/15*(b^3*c^4*(5*c^2+24*c*d+3*d^2)-a*b^2*c^3*(35*c^3+42*c^2*d+21*c*d
^2-2*d^3)+a^2*b*c^2*(45*c^4+48*c^3*d+c^2*d^2-8*c*d^3+10*d^4)-a^3*d*(60*c^5-
```

```
2*c^4*d-66*c^3*d^2+25*c^2*d^3+30*c*d^4-15*d^5))*(d+c*cos(f*x+e))^(3/2)*csc(
f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*
x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-co
s(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(
d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^4/(c-d)^3/(c+d)^(5/2)/(-a*d
+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2*a^2*(d+c*cos(f*x+e)
)^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2
)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(
a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*
d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/
c^4/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 4.36 (sec) , antiderivative size = 1150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {4027, 3068, 3126, 3132, 2890, 3077, 2897, 3075}

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx =$$

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},a\right.}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$

$$+\frac{2(bc-ad)\left(5bc^3-13adc^2+3bd^2c+5ad^3\right)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

$$-\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2(a-b)\sqrt{a+b}\left(\left(15d^5-41c^2d^3+58c^4d\right)a^2-bc\left(35c^4+34d^2c^2-5d^4\right)a+b^2c^2d\left(29c^2+3d^2\right)\right)\sqrt{-\frac{(bc-ad)(1-}{(a+b)(d+}}}{15c^3(c-d)^3(c+d)^{5/2}(bc}$$

$$+\frac{2\sqrt{a+b}\left(b^3\left(5c^2+24dc+3d^2\right)c^4-ab^2\left(35c^3+42dc^2+21d^2c-2d^3\right)c^3+a^2b\left(45c^4+48dc^3+d^2c^2-8d^3c-\right.\right.}{}$$

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(b^2*c^2*d*(29*c^2 + 3*d^2) - a*b*c*(35*c^4 + 34*c^2
*d^2 - 5*d^4) + a^2*(58*c^4*d - 41*c^2*d^3 + 15*d^5))*Sqrt[-(((b*c - a*d)*(
1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1
+ Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2
)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqr
t[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
qrt[a + b*Sec[e + f*x]])/(15*c^3*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f*Sqrt
[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^3*c^4*(5
```

```
*c^2 + 24*c*d + 3*d^2) - a*b^2*c^3*(35*c^3 + 42*c^2*d + 21*c*d^2 - 2*d^3) +
 a^2*b*c^2*(45*c^4 + 48*c^3*d + c^2*d^2 - 8*c*d^3 + 10*d^4) - a^3*d*(60*c^5
 - 2*c^4*d - 66*c^3*d^2 + 25*c^2*d^3 + 30*c*d^4 - 15*d^5))*Sqrt[-(((b*c - a
*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d
)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])
^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])
/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d
))]*Sqrt[a + b*Sec[e + f*x]])/(15*c^4*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f
*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a^2*Sqrt[a + b]*Sq
rt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt
[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c
*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSi
n[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*
x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^4*
Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*d*(b*
c - a*d)*(b + a*Cos[e + f*x])*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(5*c*(
c^2 - d^2)*f*(d + c*Cos[e + f*x])^2*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c - a
*d)*(5*b*c^3 - 13*a*c^2*d + 3*b*c*d^2 + 5*a*d^3)*Sqrt[a + b*Sec[e + f*x]]*S
in[e + f*x])/(15*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e
+ f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
```

```
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos(e+fx)(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{7/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\sqrt{b+a\cos(e+fx)}\left(\frac{1}{2}(5bc-3ad)(bc-ad)-\frac{1}{2}\left(5a^2cd+3b^2cd-2ab(5c^2-d\right.\right.}{(d+c\cos(e+fx))^{5/2}}}{5c\left(c^2-d^2\right)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{2(bc-ad)\left(5bc^3-13ac^2d+3bcd^2+5ad^3\right)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(4\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int \frac{\frac{1}{4}(bc-ad)\left(35abc^3-13a^2c^2d-24b^2c^2d-3abcd^2+5a^2d^3\right)-\frac{1}{4}\left(2ab^2cd(21\right.}{}}{15c^2\left(c^2-d^2\right)^2\sqrt{b+a\cos(e+fx)}}$$

$$= -\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{2(bc-ad)(5bc^3-13ac^2d+3bcd^2+5ad^3)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2(c^2-d^2)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{\left(a^3\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}\,dx}{c^4\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{\left(4\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{-\frac{15}{4}a^3d^2(c^2-d^2)^2+\frac{1}{4}c^2(bc-ad)(35abc^3-13a^2c^2d-24b^2c^2d-3abcd}{15c^4(c^2-d^2)^2\sqrt{b+a\cos}}$$

$$=$$
$$-\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{Ellipti}}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+}}$$
$$-\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{2(bc-ad)(5bc^3-13ac^2d+3bcd^2+5ad^3)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2(c^2-d^2)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{\left((b^3c^4(5c^2+24cd+3d^2)-ab^2c^3(35c^3+42c^2d+21cd^2-2d^3)+a^2bc^2(45c^4+48c^3d+c^2d^2-8\right.}{15c^4(c-}$$
$$-\frac{\left(4\left(c\left(-\frac{15}{4}a^3d^2(c^2-d^2)^2+\frac{1}{4}c^2(bc-ad)(35abc^3-13a^2c^2d-24b^2c^2d-3abcd^2+5a^2d^3)\right)-cd\right.\right.}{}$$

$$=\frac{2(a-b)\sqrt{a+b}(b^2c^2d(29c^2+3d^2)-abc(35c^4+34c^2d^2-5d^4)+a^2(58c^4d-41c^2d^3+15d^5))\sqrt{-}}{15c^3(c-d)^3(c+}$$
$$+\frac{2\sqrt{a+b}(b^3c^4(5c^2+24cd+3d^2)-ab^2c^3(35c^3+42c^2d+21cd^2-2d^3)+a^2bc^2(45c^4+48c^3d+c}{}$$
$$-\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{Ellipti}}{c^4\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+}}$$
$$-\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{2(bc-ad)(5bc^3-13ac^2d+3bcd^2+5ad^3)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{15c^2(c^2-d^2)^2f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2344 vs. $2(1150) = 2300$.

Time = 8.15 (sec) , antiderivative size = 2344, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^4*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((-2*(b^2*c
^2*d*Sin[e + f*x] - 2*a*b*c*d^2*Sin[e + f*x] + a^2*d^3*Sin[e + f*x]))/(5*c^
2*(c^2 - d^2)*(d + c*Cos[e + f*x])^3) + (2*(5*b^2*c^4*Sin[e + f*x] - 21*a*b
*c^3*d*Sin[e + f*x] + 16*a^2*c^2*d^2*Sin[e + f*x] + 3*b^2*c^2*d^2*Sin[e + f
*x] + 5*a*b*c*d^3*Sin[e + f*x] - 8*a^2*d^4*Sin[e + f*x]))/(15*c^2*(c^2 - d^
2)^2*(d + c*Cos[e + f*x])^2) + (2*(35*a*b*c^5*Sin[e + f*x] - 58*a^2*c^4*d*S
in[e + f*x] - 29*b^2*c^4*d*Sin[e + f*x] + 34*a*b*c^3*d^2*Sin[e + f*x] + 41*
a^2*c^2*d^3*Sin[e + f*x] - 3*b^2*c^2*d^3*Sin[e + f*x] - 5*a*b*c*d^4*Sin[e +
 f*x] - 15*a^2*d^5*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^3*(d + c*Cos[e + f*x]
))))/(f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(7/2)) + ((d + c*Cos[e
+ f*x])^(7/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c - a*d)*(10*a
^2*b*c^5 + 5*b^3*c^5 + 13*a^3*c^4*d - 48*a*b^2*c^4*d + 15*a^2*b*c^3*d^2 + 2
7*b^3*c^3*d^2 - 18*a^3*c^2*d^3 - 16*a*b^2*c^2*d^3 + 7*a^2*b*c*d^4 + 5*a^3*d
^5)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e +
 f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])
*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a -
 b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c
 - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a
*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(15*a^3*c^5 - 35*a
*b^2*c^5 + 23*a^2*b*c^4*d + 29*b^3*c^4*d + 13*a^3*c^3*d^2 - 5*a*b^2*c^3*d^2
 - 75*a^2*b*c^2*d^3 + 3*b^3*c^2*d^3 + 4*a^3*c*d^4 + 8*a*b^2*c*d^4 + 20*a^2*
b*d^5)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Co
s[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[(
(-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2
*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[
b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x
)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*
(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a
*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f
*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-35*a^2*b*c^5 + 58*a^3*c^4*d + 29*a*b^
2*c^4*d - 34*a^2*b*c^3*d^2 - 41*a^3*c^2*d^3 + 3*a*b^2*c^2*d^3 + 5*a^2*b*c*d
^4 + 15*a^3*d^5)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d +
```

```
c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])
/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d))]/
(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos
[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e +
f*x]))/((c + d)*(b + a*Cos[e + f*x]))]]) - (2*(b*c - a*d)*(((b*c + (a + b)*d
)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f
*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*C
sc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b
)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c -
a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*C
os[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e
+ f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2
)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
 - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a
- b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*
c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[
e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin
[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))/(15*c^2*(c - d)^3*(c + d)^3*f*(b
 + a*Cos[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^(7/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 39099 vs. $2(1065) = 2130$.

Time = 20.62 (sec) , antiderivative size = 39100, normalized size of antiderivative = 34.00

method	result	size
default	Expression too large to display	39100

```
[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a + b\sec(e + fx))^{5/2}}{(c + d\sec(e + fx))^{7/2}}\, dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Hanged}$$

```
[In] int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(7/2),x)
[Out] \text{Hanged}
```

3.216 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

Optimal result 1521
Rubi [A] (warning: unable to verify) 1522
Mathematica [B] (warning: unable to verify) 1528
Maple [B] (warning: unable to verify) 1530
Fricas [F(-1)] 1530
Sympy [F(-1)] 1530
Maxima [F] 1531
Giac [F] 1531
Mupad [F(-1)] 1531

Optimal result

Integrand size = 29, antiderivative size = 1428

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \frac{2(a-b)\sqrt{a+b}(2b^3c^3d(133c^4+62c^2d^2-3d^4)+2a^2bcd(406c^6+73c^4d^2+132}{}$$
$$+\frac{2\sqrt{a+b}(b^3c^4(35c^4+231c^3d+67c^2d^2+57cd^3-6d^4)-ab^2c^3(245c^5+413c^4d+439c^3d^2+53c^2d^3-12cd^4}{}$$
$$-\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}\left(\frac{(a+b)c}{a(c+d)}\right.}{c^5\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$
$$+\frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}}$$
$$-\frac{2d(14bc^3-19ac^2d-2bcd^2+7ad^3)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{35c^2(c^2-d^2)^2f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$
$$-\frac{2(2abcd(91c^4-2c^2d^2+7d^4)-a^2d^2(162c^4-101c^2d^2+35d^4)-b^2(35c^6+67c^4d^2-6c^2d^4))\sqrt{a+b\sec(e}}{105c^3(c^2-d^2)^3f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2/7*d^2*(b+a*cos(f*x+e))^2*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/
(d+c*cos(f*x+e))^3/(c+d*sec(f*x+e))^(1/2)-2/35*d*(-19*a*c^2*d+7*a*d^3+14*b*
c^3-2*b*c*d^2)*(b+a*cos(f*x+e))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^2/(c^2-
d^2)^2/f/(d+c*cos(f*x+e))^2/(c+d*sec(f*x+e))^(1/2)-2/105*(2*a*b*c*d*(91*c^4
-2*c^2*d^2+7*d^4)-a^2*d^2*(162*c^4-101*c^2*d^2+35*d^4)-b^2*(35*c^6+67*c^4*d
^2-6*c^2*d^4))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^3/(c^2-d^2)^3/f/(d+c*cos
(f*x+e))/(c+d*sec(f*x+e))^(1/2)+2/105*(a-b)*(2*b^3*c^3*d*(133*c^4+62*c^2*d^
2-3*d^4)+2*a^2*b*c*d*(406*c^6+73*c^4*d^2+132*c^2*d^4-35*d^6)-a*b^2*c^2*(245
*c^6+852*c^4*d^2+41*c^2*d^4+14*d^6)-a^3*(582*c^6*d^2-485*c^4*d^4+392*c^2*d^
```

```
6-105*d^8))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*co
s(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d)
)^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1
/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+
e))^(1/2)/c^4/(c-d)^4/(c+d)^(7/2)/(-a*d+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+
d*sec(f*x+e))^(1/2)+2/105*(b^3*c^4*(35*c^4+231*c^3*d+67*c^2*d^2+57*c*d^3-6*
d^4)-a*b^2*c^3*(245*c^5+413*c^4*d+439*c^3*d^2+53*c^2*d^3-12*c*d^4+14*d^5)+a
^2*b*c^2*(315*c^6+497*c^5*d+219*c^4*d^2-73*c^3*d^3+208*c^2*d^4+56*c*d^5-70*
d^6)-a^3*d*(525*c^7+57*c^6*d-699*c^5*d^2+214*c^4*d^3+672*c^3*d^4-280*c^2*d^
5-210*c*d^6+105*d^7))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/
2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(
a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f
*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+
b*sec(f*x+e))^(1/2)/c^5/(c-d)^4/(c+d)^(7/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(
1/2)/(c+d*sec(f*x+e))^(1/2)-2*a^2*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*Ellipti
cPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(
a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1
-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b
)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^5/f/(c+d)^(1/2)/(b+a*cos
(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 6.17 (sec) , antiderivative size = 1428, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used

$= \{4027, 3127, 3126, 3132, 2890, 3077, 2897, 3075\}$

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx =$$
$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},a\right.}{c^5\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+f}}$$
$$-\frac{2(-((35c^6+67d^2c^4-6d^4c^2)b^2)+2acd(91c^4-2d^2c^2+7d^4)b-a^2d^2(162c^4-101d^2c^2+35d^4))\sqrt{a+b\,\mathrm{se}}}{105c^3(c^2-d^2)^3f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$
$$-\frac{2d(14bc^3-19adc^2-2bd^2c+7ad^3)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{35c^2(c^2-d^2)^2f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{2(a-b)\sqrt{a+b}(-((-105d^8+392c^2d^6-485c^4d^4+582c^6d^2)a^3)+2bcd(406c^6+73d^2c^4+132d^4c^2-35d^6)}{}$$
$$+\frac{2\sqrt{a+b}(b^3(35c^4+231dc^3+67d^2c^2+57d^3c-6d^4)c^4-ab^2(245c^5+413dc^4+439d^2c^3+53d^3c^2-12d^4c+}{}$$

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*b^3*c^3*d*(133*c^4 + 62*c^2*d^2 - 3*d^4) + 2*a^2*
b*c*d*(406*c^6 + 73*c^4*d^2 + 132*c^2*d^4 - 35*d^6) - a*b^2*c^2*(245*c^6 +
852*c^4*d^2 + 41*c^2*d^4 + 14*d^6) - a^3*(582*c^6*d^2 - 485*c^4*d^4 + 392*c
^2*d^6 - 105*d^8))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*
Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Co
s[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sq
rt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(105*c^4*(
c - d)^4*(c + d)^(7/2)*(b*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^3*c^4*(35*c^4 + 231*c^3*d + 67*c^2*d^2 +
 57*c*d^3 - 6*d^4) - a*b^2*c^3*(245*c^5 + 413*c^4*d + 439*c^3*d^2 + 53*c^2*
d^3 - 12*c*d^4 + 14*d^5) + a^2*b*c^2*(315*c^6 + 497*c^5*d + 219*c^4*d^2 - 7
3*c^3*d^3 + 208*c^2*d^4 + 56*c*d^5 - 70*d^6) - a^3*d*(525*c^7 + 57*c^6*d -
699*c^5*d^2 + 214*c^4*d^3 + 672*c^3*d^4 - 280*c^2*d^5 - 210*c*d^6 + 105*d^7
))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]
*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(
d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b
 + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(105*c^5*(c - d)^4*(c + d)
^(7/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (
2*a^2*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*C
os[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos
```

```
[e + f*x]))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c
)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*S
qrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*
Sec[e + f*x]])/(c^5*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e
 + f*x]]) + (2*d^2*(b + a*Cos[e + f*x])^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e +
f*x])/(7*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3*Sqrt[c + d*Sec[e + f*x]]) -
 (2*d*(14*b*c^3 - 19*a*c^2*d - 2*b*c*d^2 + 7*a*d^3)*(b + a*Cos[e + f*x])*Sq
rt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(35*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e +
 f*x])^2*Sqrt[c + d*Sec[e + f*x]]) - (2*(2*a*b*c*d*(91*c^4 - 2*c^2*d^2 + 7*
d^4) - a^2*d^2*(162*c^4 - 101*c^2*d^2 + 35*d^4) - b^2*(35*c^6 + 67*c^4*d^2
- 6*c^2*d^4))*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(105*c^3*(c^2 - d^2)^3
*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\cos^2(e+fx)(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{9/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{(b+a\cos(e+fx))^{3/2}\left(-\frac{1}{2}d(7bc-5ad)+\frac{1}{2}\left(7bc^2-7acd-2bd^2\right)\cos(e+fx\cdots}{(d+c\cos(e+fx))^{7/2}}}{7c\left(c^2-d^2\right)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} - \frac{2d(14bc^3-19ac^2d-2bcd^2+7ad^3)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{35c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{\left(4\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\sqrt{b+a\cos(e+fx)}\left(\frac{1}{4}\left(3a^2d^2\left(19c^2-7d^2\right)-16abcd\left(7c^2-d^2\right)+5b^2\left(7c^4+\cdots\right.\right.\right.}{\cdots}}{35c^2\left(c^2-d^2\right)^2\sqrt{b+a\cos(e+fx\cdots}}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} - \frac{2d(14bc^3-19ac^2d-2bcd^2+7ad^3)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{35c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} - \frac{2(2abcd(91c^4-2c^2d^2+7d^4)-a^2d^2(162c^4-101c^2d^2+35d^4)-b^2(35c^6+67c^4d^2-6c^2d^4))\sqrt{a\cdots}}{105c^3\left(c^2-d^2\right)^3f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(8\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\frac{1}{8}\left(-3b^3c^3d\left(77c^2+19d^2\right)+ab^2c^2\left(245c^4+439c^2d^2-12d^4\right)-a^2bcd\left(497c\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}}$$
$$-\frac{2d(14bc^3-19ac^2d-2bcd^2+7ad^3)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{35c^2(c^2-d^2)^2f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$
$$-\frac{2(2abcd(91c^4-2c^2d^2+7d^4)-a^2d^2(162c^4-101c^2d^2+35d^4)-b^2(35c^6+67c^4d^2-6c^2d^4))\sqrt{a}}{105c^3(c^2-d^2)^3f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{\left(a^3\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^5\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{\left(8\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{-\frac{105}{8}a^3d^2(c^2-d^2)^3+\frac{1}{8}c^2(-3b^3c^3d(77c^2+19d^2)+ab^2c^2(245c^4+439}{}}{}$$

$$=$$
$$-\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{Ellipti}}{c^5\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+}}$$
$$+\frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}}$$
$$-\frac{2d(14bc^3-19ac^2d-2bcd^2+7ad^3)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{35c^2(c^2-d^2)^2f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$
$$-\frac{2(2abcd(91c^4-2c^2d^2+7d^4)-a^2d^2(162c^4-101c^2d^2+35d^4)-b^2(35c^6+67c^4d^2-6c^2d^4))\sqrt{a}}{105c^3(c^2-d^2)^3f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{\Big((b^3c^4(35c^4+231c^3d+67c^2d^2+57cd^3-6d^4)-ab^2c^3(245c^5+413c^4d+439c^3d^2+53c^2d^3-1}{}$$
$$-\frac{\Big((2b^3c^3d(133c^4+62c^2d^2-3d^4)+2a^2bcd(406c^6+73c^4d^2+132c^2d^4-35d^6)-ab^2c^2(245c^6+8}{105c^4(}$$

$$= \frac{2(a-b)\sqrt{a+b}(2b^3c^3d(133c^4+62c^2d^2-3d^4)+2a^2bcd(406c^6+73c^4d^2+132c^2d^4-35d^6)-ab^2c^2(}{}$$

$$+ \frac{2\sqrt{a+b}(b^3c^4(35c^4+231c^3d+67c^2d^2+57cd^3-6d^4)-ab^2c^3(245c^5+413c^4d+439c^3d^2+53c^2d}{}$$

$$- \frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{Elliptic}}{c^5\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d}}$$

$$+ \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}}$$

$$- \frac{2d(14bc^3-19ac^2d-2bcd^2+7ad^3)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{35c^2(c^2-d^2)^2f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}}$$

$$- \frac{2(2abcd(91c^4-2c^2d^2+7d^4)-a^2d^2(162c^4-101c^2d^2+35d^4)-b^2(35c^6+67c^4d^2-6c^2d^4))\sqrt{a-}}{105c^3(c^2-d^2)^3f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2979 vs. 2(1428) = 2856.

Time = 9.06 (sec) , antiderivative size = 2979, normalized size of antiderivative = 2.09

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2*
c^2*d^2*Sin[e + f*x] - 2*a*b*c*d^3*Sin[e + f*x] + a^2*d^4*Sin[e + f*x]))/(7
*c^3*(c^2 - d^2)*(d + c*Cos[e + f*x])^4) + (2*(-14*b^2*c^4*d*Sin[e + f*x] +
 43*a*b*c^3*d^2*Sin[e + f*x] - 29*a^2*c^2*d^3*Sin[e + f*x] + 2*b^2*c^2*d^3*
Sin[e + f*x] - 19*a*b*c*d^4*Sin[e + f*x] + 17*a^2*d^5*Sin[e + f*x]))/(35*c^
3*(c^2 - d^2)^2*(d + c*Cos[e + f*x])^3) + (2*(35*b^2*c^6*Sin[e + f*x] - 224
*a*b*c^5*d*Sin[e + f*x] + 234*a^2*c^4*d^2*Sin[e + f*x] + 67*b^2*c^4*d^2*Sin
[e + f*x] + 52*a*b*c^3*d^3*Sin[e + f*x] - 209*a^2*c^2*d^4*Sin[e + f*x] - 6*
b^2*c^2*d^4*Sin[e + f*x] - 20*a*b*c*d^5*Sin[e + f*x] + 71*a^2*d^6*Sin[e + f
*x]))/(105*c^3*(c^2 - d^2)^3*(d + c*Cos[e + f*x])^2) + (2*(245*a*b^2*c^8*Si
n[e + f*x] - 812*a^2*b*c^7*d*Sin[e + f*x] - 266*b^3*c^7*d*Sin[e + f*x] + 58
2*a^3*c^6*d^2*Sin[e + f*x] + 852*a*b^2*c^6*d^2*Sin[e + f*x] - 146*a^2*b*c^5
*d^3*Sin[e + f*x] - 124*b^3*c^5*d^3*Sin[e + f*x] - 485*a^3*c^4*d^4*Sin[e +
f*x] + 41*a*b^2*c^4*d^4*Sin[e + f*x] - 264*a^2*b*c^3*d^5*Sin[e + f*x] + 6*b
^3*c^3*d^5*Sin[e + f*x] + 392*a^3*c^2*d^6*Sin[e + f*x] + 14*a*b^2*c^2*d^6*S
in[e + f*x] + 70*a^2*b*c*d^7*Sin[e + f*x] - 105*a^3*d^8*Sin[e + f*x]))/(105
```

c^3(b*c - a*d)*(c^2 - d^2)^4*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(9/2)) + ((d + c*Cos[e + f*x])^(9/2)*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c - a*d)*(-70*a^2*b^2*c^8 - 35*b^4*c^8 - 77*a^3*b*c^7*d + 427*a*b^3*c^7*d + 162*a^4*c^6*d^2 - 522*a^2*b^2*c^6*d^2 - 298*b^4*c^6*d^2 + 348*a^3*b*c^5*d^3 + 666*a*b^3*c^5*d^3 - 263*a^4*c^4*d^4 - 586*a^2*b^2*c^4*d^4 - 51*b^4*c^4*d^4 + 127*a^3*b*c^3*d^5 + 59*a*b^3*c^3*d^5 + 136*a^4*c^2*d^6 + 26*a^2*b^2*c^2*d^6 - 14*a^3*b*c*d^7 - 35*a^4*d^8)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(-105*a^3*b*c^8 + 245*a*b^3*c^8 + 105*a^4*c^7*d - 567*a^2*b^2*c^7*d - 266*b^4*c^7*d + 190*a^3*b*c^6*d^2 + 586*a*b^3*c^6*d^2 + 162*a^4*c^5*d^3 + 706*a^2*b^2*c^5*d^3 - 124*b^4*c^5*d^3 - 1261*a^3*b*c^4*d^4 - 83*a*b^3*c^4*d^4 + 145*a^4*c^3*d^5 - 223*a^2*b^2*c^3*d^5 + 6*b^4*c^3*d^5 + 548*a^3*b*c^2*d^6 + 20*a*b^3*c^2*d^6 - 28*a^4*c*d^7 + 84*a^2*b^2*c*d^7 - 140*a^3*b*d^8)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(245*a^2*b^2*c^8 - 812*a^3*b*c^7*d - 266*a*b^3*c^7*d + 582*a^4*c^6*d^2 + 852*a^2*b^2*c^6*d^2 - 146*a^3*b*c^5*d^3 - 124*a*b^3*c^5*d^3 - 485*a^4*c^4*d^4 + 41*a^2*b^2*c^4*d^4 - 264*a^3*b*c^3*d^5 + 6*a*b^3*c^3*d^5 + 392*a^4*c^2*d^6 + 14*a^2*b^2*c^2*d^6 + 70*a^3*b*c*d^7 - 105*a^4*d^8)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)

```
*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d +
c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b
*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e +
f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e
 + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*
x]]))))/(105*c^3*(c - d)^4*(c + d)^4*(-(b*c) + a*d)*f*(b + a*Cos[e + f*x])^
(5/2)*(c + d*Sec[e + f*x])^(9/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 88655 vs. $2(1337) = 2674$.

Time = 23.70 (sec) , antiderivative size = 88656, normalized size of antiderivative = 62.08

method	result	size
default	Expression too large to display	88656

```
[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \text{Hanged}$$

```
[In] int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(9/2),x)
[Out] \text{Hanged}
```

3.217 $\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal result 1532
Rubi [F] 1533
Mathematica [C] (warning: unable to verify) 1533
Maple [A] (verified) 1533
Fricas [F(-1)] 1534
Sympy [F] 1534
Maxima [F] 1534
Giac [F] 1535
Mupad [F(-1)] 1535

Optimal result

Integrand size = 29, antiderivative size = 652

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx =$$

$$-\frac{2c(c+d)\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(1+\sec(e+fx))}{(c-d)(a+b\sec(e+fx))}}(a+b}{a(a+b)f\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2d(c+d)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(1+\sec(e+fx))}{(c-d)(a+b\sec(e+fx))}}(a+b}{b(a+b)f\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2(bc-ad)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(-1+\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(1+}{(c-d)(a+b}}}{abf\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}}$$

```
[Out] -2*c*(c+d)*cot(f*x+e)*EllipticPi(((a+b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x
+e)))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*x+e
))^(3/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*((a+b)*(-
a*d+b*c)*(-1+sec(f*x+e))*(c+d*sec(f*x+e))/(c+d)^2/(a+b*sec(f*x+e))^2)^(1/2)
/a/(a+b)/f/(c+d*sec(f*x+e))^(1/2)+2*d*(c+d)*cot(f*x+e)*EllipticPi(((a+b)*(c
+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(
a+b)/(c-d))^(1/2))*(a+b*sec(f*x+e))^(3/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/
(a+b*sec(f*x+e)))^(1/2)*(-(a+b)*(a*d-b*c)*(-1+sec(f*x+e))*(c+d*sec(f*x+e))/
(c+d)^2/(a+b*sec(f*x+e))^2)^(1/2)/b/(a+b)/f/(c+d*sec(f*x+e))^(1/2)+2*(-a*d+
b*c)*cot(f*x+e)*EllipticF(((a+b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(
1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*((-a*d+b*c)*(-1+sec(f*x+e))/(c+d)/(a+
b*sec(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/
2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/a/b/f/((a+b)*(c+d*sec(f*x+
e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)
```

Rubi [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$$

```
[In] Int[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]],x]
[Out] Defer[Int][(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi steps

$$\text{integral} = \int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 37.55 (sec) , antiderivative size = 50041, normalized size of antiderivative = 76.75

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]],x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 17.07 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.70

method	result
default	$-\frac{2\sqrt{c+d\sec(fx+e)}\,\sqrt{a+b\sec(fx+e)}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\left(\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\right.\right.}{}$

```
[In] int((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/f/((a-b)/(a+b))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(1/(
a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(
f*x+e)+1))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((
a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c^2-2*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*
x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c*d+EllipticF(((a-b)/(a+b
))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*d^2-2*El
lipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),(a+b)/(a-b),((c-d)/(c
```

```
+d))^(1/2)/((a-b)/(a+b))^(1/2))*d^2-2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(
f*x+e)+csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*c^
2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(c+d\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{a+b\sec{(e+fx)}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral((c + d*sec(e + f*x))**(3/2)/sqrt(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec{(fx+e)}+c)^{\frac{3}{2}}}{\sqrt{b\sec{(fx+e)}+a}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^(3/2)/(a + b/cos(e + f*x))^(1/2),x)
[Out] int((c + d/cos(e + f*x))^(3/2)/(a + b/cos(e + f*x))^(1/2), x)
```

3.218 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal result 1536
Rubi [A] (verified) 1536
Mathematica [A] (verified) 1537
Maple [A] (verified) 1538
Fricas [F(-1)] 1538
Sympy [F] 1538
Maxima [F] 1539
Giac [F] 1539
Mupad [F(-1)] 1539

Optimal result

Integrand size = 29, antiderivative size = 198

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(b}{(c}}}{a\sqrt{c+d}f}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+
d*sec(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*s
ec(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(
1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/a/f/(c+d)^(1
/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {4021}

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\text{arcs}\right.}{af\sqrt{c+d}}$$

```
[In] Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sq
rt[c + d]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x])
)/((a + b)*(c + d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/(
(a - b)*(c + d*Sec[e + f*x])))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*f)
```

Rule 4021

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

integral =

$$-\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-}}{a\sqrt{c+d}f}$$

Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx$$
$$=\frac{4\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(e+fx)\right)}{a-b}}\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{bc-ad}}\csc(e+fx)\left(a(c+d)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(c+d)(b+a\,\mathrm{c}}{}}\right.\right.\right.}{a(c+}$$

```
[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (4*Sqrt[((a + b)*Cot[(e + f*x)/2]^2)/(a - b)]*Sqrt[((c + d)*(b + a*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*(a*(c + d)*EllipticF[Ar
cSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(2*b*c - 2*a*d)
]], (2*(-(b*c) + a*d))/((a - b)*(c + d))] - (a + b)*c*EllipticPi[(-(b*c) +
a*d)/(a*(c + d)), ArcSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2
]^2)/(2*b*c - 2*a*d)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))])*Sqrt[c + d*S
ec[e + f*x]]*Sin[(e + f*x)/2]^2)/(a*(c + d)*f*Sqrt[((a + b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 11.56 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.63

method	result
default	$-\frac{2\sqrt{c+d\sec(fx+e)}\,\sqrt{a+b\sec(fx+e)}\left(\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)c-\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e\right)\right)}{f}$

```
[In] int((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/((a-b)/(a+b))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(Ell
ipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d
))^(1/2))*c-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(
c-d)/(a-b)/(c+d))^(1/2))*d-2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+cs
c(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*c)*(1/(a+b)
*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+
e)+1))^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\text{Timed out}$$

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{c+d\sec{(e+fx)}}}{\sqrt{a+b\sec{(e+fx)}}}\,dx$$

```
[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxim a")

[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)

Giac [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

[In] int((c + d/cos(e + f*x))^(1/2)/(a + b/cos(e + f*x))^(1/2),x)

[Out] int((c + d/cos(e + f*x))^(1/2)/(a + b/cos(e + f*x))^(1/2), x)

3.219 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result 1540
Rubi [A] (verified) 1541
Mathematica [C] (verified) 1542
Maple [A] (verified) 1543
Fricas [F(-1)] 1543
Sympy [F] 1543
Maxima [F] 1544
Giac [F] 1544
Mupad [F(-1)] 1544

Optimal result

Integrand size = 29, antiderivative size = 398

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\cdots}}{a\sqrt{a+b}cf} - \frac{2b\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(1\cdots}{(a-b)(c+\cdots}}}{a\sqrt{c+d}(bc-ad)f}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a+
b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*s
ec(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/a/c/f/(a+b)^
(1/2)-2*b*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/
2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))
*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a
*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/a/(-a*d+b*c)/f/(c+d)^(
1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4023, 4021, 4069}

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx =$$
$$-\frac{2b\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{af\sqrt{c+d}(bc-ad)}$$
$$-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\text{arc}\right.}{acf\sqrt{a+b}}$$

```
[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(a*Sqrt[a + b]*c*f) -
 (2*b*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e
+ f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f
*x])))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 4021

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4023

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x
_)]*(d_.) + (c_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqr
t[c + d*Csc[e + f*x]], x], x] - Dist[b/a, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0]
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx}{a}$$

$$= -\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c}, \arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right), \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+}{(c+d)(a+b\sec(e+}}}{a\sqrt{a+b}cf}$$
$$-\frac{2b\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right), \frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-}}{a\sqrt{c+d}(bc-ad)f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \frac{4i\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\sqrt{\frac{d+c\cos(e+fx)}{(c+d)(1+\cos(e+fx))}}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{-a+b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right), \frac{(a}{(a}\right.\right.}{\sqrt{\frac{-a+b}{a+b}}f\sqrt{a+b\sec(e+fx)}\sqrt{c}}$$

```
[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] ((4*I)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f
*x]))]*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))]*(EllipticF[I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a -
b)*(c + d))] - 2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
+ b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a - b)*(c + d))])*Sec[e + f*x]
)/(Sqrt[(-a + b)/(a + b)]*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x
]])
```

Maple [A] (verified)

Time = 13.40 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.66

method	result
default	$\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\left(\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\cot(fx+e)-\csc(fx+e)),\sqrt{\frac{(a+}{(a-}}\right.\right.}{f\sqrt{\frac{a-b}{a+b}}\,(d+c\cos(fx+e))(b+a\cos(f}$

[In] int(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)

[Out] 2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/(a
+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f
x+e)+1))^(1/2)(EllipticF(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),((a+
b)*(c-d)/(a-b)/(c+d))^(1/2))-2*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(f*x+e)-c
sc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2)))/(d+c*cos(
f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fri
cas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec{(e+fx)}}\sqrt{c+d\sec{(e+fx)}}}\,dx$$

[In] integrate(1/(c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="max
ima")
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="gia
c")
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),x)
[Out] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)), x)
```

3.220 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal result 1545
Rubi [A] (verified) 1546
Mathematica [B] (warning: unable to verify) 1549
Maple [B] (warning: unable to verify) 1550
Fricas [F] 1552
Sympy [F] 1552
Maxima [F] 1552
Giac [F] 1552
Mupad [F(-1)] 1553

Optimal result

Integrand size = 29, antiderivative size = 622

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}d^2\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(1+s}{(a-b)(c+d\,s}}}{c(c-d)\sqrt{c+d}(bc-ad)^2f}$$

$$-\frac{2\sqrt{a+b}(2c-d)d\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-}}{c^2(c-d)\sqrt{c+d}(bc-ad)f}$$

$$-\frac{2\sqrt{a+b}\cot(e+fx)\operatorname{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc}{(a-}}}{ac^2\sqrt{c+d}f}$$

```
[Out] -2*(a-b)*d^2*cot(f*x+e)*EllipticE((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^
(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+
e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-
(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/c/(c-d)/(-a*d+b*c)^
2/f/(c+d)^(1/2)-2*(2*c-d)*d*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e
))^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2)
)*(c+d*sec(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*
x+e)))^(1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/c^2/
(c-d)/(-a*d+b*c)/f/(c+d)^(1/2)-2*cot(f*x+e)*EllipticPi((c+d)^(1/2)*(a+b*sec
(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c
-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x
+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*s
ec(f*x+e)))^(1/2)/a/c^2/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4027, 3133, 2890, 3077, 2897, 3075}

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{2d\sqrt{a+b}(2c-d)\csc(e+fx)\sqrt{a+b\sec(e+fx)}(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos}{(a-b)(c\cos(}}}{c^2f(c-d)\sqrt{c+d}(bc-ad)\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec}}$$

$$-\frac{2\sqrt{a+b}\csc(e+fx)\sqrt{a+b\sec(e+fx)}(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}\,\mathrm{I}}{ac^2f\sqrt{c+d}\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$-\frac{2d^2(a-b)\sqrt{a+b}\csc(e+fx)\sqrt{a+b\sec(e+fx)}(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(}{(a-b)(c\cos(}}}{cf(c-d)\sqrt{c+d}(bc-ad)^2\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+}}$$

```
[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*d^2*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b
)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*
(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[A
rcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e
+ f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(
c*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*S
ec[e + f*x]]) - (2*Sqrt[a + b]*(2*c - d)*d*Sqrt[-(((b*c - a*d)*(1 - Cos[e +
 f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f
*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f
*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sq
rt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*S
ec[e + f*x]])/(c^2*(c - d)*Sqrt[c + d]*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x
]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[
e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e
+ f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e
+ f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*C
os[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(a*c^2*Sqrt[c + d]*f*Sqrt[b + a*
Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
```

```
 b*Sin[e + f*x]))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3133

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
```

```
C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\cos^2(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}\,dx}{c^2\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} + \frac{\left(\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{-d^2-2cd\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}\,dx}{c^2\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}}{ac^2\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\text{s}}} - \frac{\left((2c-d)d\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b+a\cos(e+fx)}\sqrt{d+c\cos(e+fx)}}\,dx}{c^2(c-d)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}} + \frac{\left(d^2\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\right)\int\frac{1+\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}\,dx}{c(c-d)\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2(a-b)\sqrt{a+b}d^2\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)E}{c(c-d)\sqrt{c+d}(bc-ad)^2f\sqrt{b+a\cos(e+fx)}\sqrt{c}} - \frac{2\sqrt{a+b}(2c-d)d\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{I}}{c^2(c-d)\sqrt{c+d}(bc-ad)f\sqrt{b+a\cos(e+fx)}} - \frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}}{ac^2\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\text{s}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1761 vs. $2(622) = 1244$.

Time = 19.32 (sec) , antiderivative size = 1761, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}\sec^2(e+fx)\Bigg(}{}$$

$$+\frac{2d^2(b+a\cos(e+fx))(d+c\cos(e+fx))\sec(e+fx)\tan(e+fx)}{(-bc+ad)\left(-c^2+d^2\right)f\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}$$

```
[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]]*(d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]^2*((-4*b*
c*d*(b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b
 + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*C
os[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin
[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[
2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d
)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(b*c^2
 - a*c*d - 2*b*d^2)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c +
 d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d
 + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[
ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*
(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)
*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f
*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sq
rt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]]
, (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b
+ a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) - 2*a*d^2*((Sqrt[(-a + b)/(a +
 b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sq
rt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]],
 (2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2
)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/
```

```
(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))
]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(
c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e
 + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/
2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*
x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]
) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(
b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c
 - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*
x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e +
 f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))
/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]
]))))/((c - d)*(c + d)*(b*c - a*d)*f*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e
+ f*x])^(3/2)) + (2*d^2*(b + a*Cos[e + f*x])*(d + c*Cos[e + f*x])*Sec[e + f
*x]*Tan[e + f*x])/((-(b*c) + a*d)*(-c^2 + d^2)*f*Sqrt[a + b*Sec[e + f*x]]*(
c + d*Sec[e + f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2866 vs. $2(577) = 1154$.

Time = 17.18 (sec) , antiderivative size = 2867, normalized size of antiderivative = 4.61

method	result	size
default	Expression too large to display	2867

```
[In] int(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/f/((a-b)/(a+b))^(1/2)/(a*d-b*c)/(c-d)/(c+d)/c*((a*(1-cos(f*x+e))^2*csc(f
*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2
-1))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*((c*(1-cos(f*x+e))^2*csc(f*x+e
)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))
^(1/2)*(((a-b)/(a+b))^(1/2)*a*c*d^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-((a-b)/(a
+b))^(1/2)*a*d^3*(1-cos(f*x+e))^3*csc(f*x+e)^3-((a-b)/(a+b))^(1/2)*b*c*d^2*
(1-cos(f*x+e))^3*csc(f*x+e)^3+((a-b)/(a+b))^(1/2)*b*d^3*(1-cos(f*x+e))^3*cs
c(f*x+e)^3+2*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+
e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^
2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e
)+csc(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*c^2*d
-2*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/
(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+
e)^2-c-d)/(c+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+
e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*d^3-2*(-(a*(1-c
```

```
os(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)
*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c
+d))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),-(a+b)/(
a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*c^3+2*(-(a*(1-cos(f*x+e))^2
*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos
(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*E
llipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),-(a+b)/(a-b),((c-d)/
(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*c*d^2-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^
2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticF(((a
-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*
a*c^2*d-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-
a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc
(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(
f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c*d^2+(-(a*(1-cos(f*x+e))^2*csc(
f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+
e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*Ellipt
icF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^
(1/2))*b*c^3+(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+
e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^
2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)
+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c^2*d+(-(a*(1-cos(f*x+e))^2
*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos
(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*E
llipticE(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))*a*c*d^2+(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f
*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot
(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*d^3-(-(a*(1-cos(f*x+
e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(
1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1
/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-
b)/(c+d))^(1/2))*b*c*d^2-(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e)
)^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*(-(c*(1-cos(f*x+e))^2*csc(f*x+e)^2-d*(1-
cos(f*x+e))^2*csc(f*x+e)^2-c-d)/(c+d))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*
(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*d^3-((a-b)/(a+b
))^(1/2)*a*c*d^2*(-cot(f*x+e)+csc(f*x+e))+((a-b)/(a+b))^(1/2)*a*d^3*(-cot(f
*x+e)+csc(f*x+e))-((a-b)/(a+b))^(1/2)*b*c*d^2*(-cot(f*x+e)+csc(f*x+e))+((a-
b)/(a+b))^(1/2)*b*d^3*(-cot(f*x+e)+csc(f*x+e)))/(a*(1-cos(f*x+e))^2*csc(f*x
+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(c*(1-cos(f*x+e))^2*csc(f*x+e)^2
-d*(1-cos(f*x+e))^2*csc(f*x+e)^2-c-d)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)/(b*d^2*sec(f*x +
 e)^3 + a*c^2 + (2*b*c*d + a*d^2)*sec(f*x + e)^2 + (b*c^2 + 2*a*c*d)*sec(f*
x + e)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a+b\sec\left(e+fx\right)}\left(c+d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(3/2)),x)
[Out] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(3/2)), x)
```

3.221 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$

Optimal result 1554
Rubi [N/A] 1554
Mathematica [N/A] 1555
Maple [N/A] (verified) 1555
Fricas [F(-1)] 1555
Sympy [N/A] 1556
Maxima [N/A] 1556
Giac [N/A] 1556
Mupad [N/A] 1557

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \frac{\sqrt[3]{d+c\cos(e+fx)}\,\sqrt[3]{a+b\sec(e+fx)}\,\text{Int}\left(\frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}},x\right)}{\sqrt[3]{b+a\cos(e+fx)}\,\sqrt[3]{c+d\sec(e+fx)}}$$

```
[Out] (d+c*cos(f*x+e))^(1/3)*(a+b*sec(f*x+e))^(1/3)*Unintegrable((b+a*cos(f*x+e))
^(1/3)/(d+c*cos(f*x+e))^(1/3),x)/(b+a*cos(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1
/3)
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$$

```
[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(1/3)*(a + b*Sec[e + f*x])^(1/3)*Defer[Int][(b + a*Co
s[e + f*x])^(1/3)/(d + c*Cos[e + f*x])^(1/3), x])/((b + a*Cos[e + f*x])^(1/
3)*(c + d*Sec[e + f*x])^(1/3))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt[3]{d+c\cos(e+fx)}\sqrt[3]{a+b\sec(e+fx)}\right)\int\frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}}\,dx}{\sqrt[3]{b+a\cos(e+fx)}\sqrt[3]{c+d\sec(e+fx)}}$$

Mathematica [N/A]

Not integrable

Time = 14.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int\frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int\frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int\frac{\left(a+b\sec\left(fx+e\right)\right)^{\frac{1}{3}}}{\left(c+d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}dx$$

```
[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int\frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt[3]{a+b\sec{(e+fx)}}}{\sqrt[3]{c+d\sec{(e+fx)}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(1/3),x)
[Out] Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{1}{3}}}{(d\sec{(fx+e)}+c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{1}{3}}}{(d\sec{(fx+e)}+c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 99.95 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{1/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(1/3),x)
[Out] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(1/3), x)
```

3.222 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$

Optimal result 1558
Rubi [N/A] 1558
Mathematica [N/A] 1559
Maple [N/A] (verified) 1559
Fricas [F(-1)] 1559
Sympy [N/A] 1559
Maxima [N/A] 1560
Giac [N/A] 1560
Mupad [N/A] 1560

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx=\text{Int}\left(\frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

```
[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3),x]
[Out] Defer[Int][(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Rubi steps

$$\text{integral}=\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 87.53 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{1}{3}}}{(c+d\sec(fx+e))^{\frac{4}{3}}}dx$$

```
[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [N/A]

Not integrable

Time = 10.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(4/3),x)
[Out] Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(4/3), x)
```

Maxima [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Giac [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Mupad [N/A]

Not integrable

Time = 105.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{1/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(4/3),x)
[Out] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(4/3), x)
```

3.223 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$

Optimal result 1561
Rubi [N/A] 1561
Mathematica [N/A] 1562
Maple [N/A] (verified) 1562
Fricas [F(-1)] 1562
Sympy [F(-1)] 1562
Maxima [N/A] 1563
Giac [N/A] 1563
Mupad [F(-1)] 1563

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Int}\left(\frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

```
[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3),x]
[Out] Defer[Int][(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 84.79 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{1}{3}}}{(c+d\sec(fx+e))^{\frac{7}{3}}}dx$$

```
[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(7/3),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Giac [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Hanged}$$

```
[In] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(7/3),x)
[Out] \text{Hanged}
```

3.224 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$

Optimal result 1564
Rubi [N/A] 1564
Mathematica [N/A] 1565
Maple [N/A] (verified) 1565
Fricas [F(-1)] 1565
Sympy [N/A] 1565
Maxima [N/A] 1566
Giac [N/A] 1566
Mupad [N/A] 1566

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\frac{(d+c\cos(e+fx))^{2/3}(a+b\sec(e+fx))^{2/3}\text{Int}\left(\frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}},x\right)}{(b+a\cos(e+fx))^{2/3}(c+d\sec(e+fx))^{2/3}}$$

```
[Out] (d+c*cos(f*x+e))^(2/3)*(a+b*sec(f*x+e))^(2/3)*Unintegrable((b+a*cos(f*x+e))
^(2/3)/(d+c*cos(f*x+e))^(2/3),x)/(b+a*cos(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2
/3)
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$$

```
[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(2/3)*(a + b*Sec[e + f*x])^(2/3)*Defer[Int][(b + a*Co
s[e + f*x])^(2/3)/(d + c*Cos[e + f*x])^(2/3), x])/((b + a*Cos[e + f*x])^(2/
3)*(c + d*Sec[e + f*x])^(2/3))
```

Rubi steps

$$\text{integral}=\frac{\left((d+c\cos(e+fx))^{2/3}(a+b\sec(e+fx))^{2/3}\right)\int \frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}}\,dx}{(b+a\cos(e+fx))^{2/3}(c+d\sec(e+fx))^{2/3}}$$

Mathematica [N/A]

Not integrable

Time = 14.91 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{2}{3}}}dx$$

```
[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{\frac{2}{3}}}{(c+d\sec(e+fx))^{\frac{2}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(2/3),x)
[Out] Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(2/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{2}{3}}}{(d\sec(fx+e)+c)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

Giac [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{2}{3}}}{(d\sec(fx+e)+c)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

Mupad [N/A]

Not integrable

Time = 107.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{2/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(2/3),x)
[Out] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(2/3), x)
```

3.225 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$

Optimal result 1567
Rubi [N/A] 1567
Mathematica [N/A] 1568
Maple [N/A] (verified) 1568
Fricas [F(-1)] 1568
Sympy [N/A] 1568
Maxima [N/A] 1569
Giac [N/A] 1569
Mupad [N/A] 1569

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \text{Int}\left(\frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

```
[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 89.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{5}{3}}}dx$$

```
[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [N/A]

Not integrable

Time = 61.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{\frac{2}{3}}}{(c+d\sec(e+fx))^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(5/3),x)
[Out] Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(5/3), x)
```

Maxima [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{2}{3}}}{(d\sec(fx+e)+c)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)
```

Giac [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{2}{3}}}{(d\sec(fx+e)+c)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)
```

Mupad [N/A]

Not integrable

Time = 107.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{2/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(5/3),x)
[Out] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(5/3), x)
```

3.226 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$

Optimal result 1570
Rubi [N/A] 1570
Mathematica [N/A] 1571
Maple [N/A] (verified) 1571
Fricas [F(-1)] 1571
Sympy [F(-1)] 1571
Maxima [N/A] 1572
Giac [N/A] 1572
Mupad [F(-1)] 1572

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \text{Int}\left(\frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

```
[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 87.72 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{8}{3}}}dx$$

```
[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(8/3),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{2}{3}}}{(d\sec{(fx+e)}+c)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)
```

Giac [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{2}{3}}}{(d\sec{(fx+e)}+c)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \text{Hanged}$$

```
[In] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(8/3),x)
```

```
[Out] \text{Hanged}
```

3.227 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$

Optimal result 1573
Rubi [N/A] 1573
Mathematica [N/A] 1574
Maple [N/A] (verified) 1574
Fricas [F(-1)] 1574
Sympy [N/A] 1574
Maxima [N/A] 1575
Giac [N/A] 1575
Mupad [N/A] 1575

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\frac{(d+c\cos(e+fx))^{4/3}(a+b\sec(e+fx))^{4/3}\text{Int}\left(\frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}},x\right)}{(b+a\cos(e+fx))^{4/3}(c+d\sec(e+fx))^{4/3}}$$

```
[Out] (d+c*cos(f*x+e))^(4/3)*(a+b*sec(f*x+e))^(4/3)*Unintegrable((b+a*cos(f*x+e))
^(4/3)/(d+c*cos(f*x+e))^(4/3),x)/(b+a*cos(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4
/3)
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

```
[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(4/3)*(a + b*Sec[e + f*x])^(4/3)*Defer[Int][(b + a*Co
s[e + f*x])^(4/3)/(d + c*Cos[e + f*x])^(4/3), x])/((b + a*Cos[e + f*x])^(4/
3)*(c + d*Sec[e + f*x])^(4/3))
```

Rubi steps

$$\text{integral}=\frac{\left((d+c\cos(e+fx))^{4/3}(a+b\sec(e+fx))^{4/3}\right)\int \frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}}\,dx}{(b+a\cos(e+fx))^{4/3}(c+d\sec(e+fx))^{4/3}}$$

Mathematica [N/A]

Not integrable

Time = 75.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{4}{3}}}dx$$

```
[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [N/A]

Not integrable

Time = 147.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{\frac{4}{3}}}{(c+d\sec(e+fx))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(4/3),x)
[Out] Integral((a + b*sec(e + f*x))**(4/3)/(c + d*sec(e + f*x))**(4/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Giac [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Mupad [N/A]

Not integrable

Time = 110.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{4/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(4/3),x)
[Out] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(4/3), x)
```

3.228 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$

Optimal result 1576
Rubi [N/A] 1576
Mathematica [N/A] 1577
Maple [N/A] (verified) 1577
Fricas [F(-1)] 1577
Sympy [F(-1)] 1577
Maxima [N/A] 1578
Giac [N/A] 1578
Mupad [F(-1)] 1578

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \operatorname{Int}\left(\frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

```
[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 98.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{7}{3}}}dx$$

```
[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(7/3),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxim
a")
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Giac [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Hanged}$$

```
[In] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(7/3),x)
[Out] \text{Hanged}
```

3.229 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$

Optimal result 1579
Rubi [N/A] 1579
Mathematica [N/A] 1580
Maple [N/A] (verified) 1580
Fricas [F(-1)] 1580
Sympy [F(-1)] 1580
Maxima [N/A] 1581
Giac [N/A] 1581
Mupad [F(-1)] 1581

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \text{Int}\left(\frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

```
[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3),x]
[Out] Defer[Int][(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 130.90 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{10}{3}}}dx$$

```
[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="fric
as")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(10/3),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{10}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="maxi
ma")
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)
```

Giac [N/A]

Not integrable

Time = 4.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{10}{3}}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="giac
")
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\text{Hanged}$$

```
[In] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(10/3),x)
[Out] \text{Hanged}
```

3.230 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx$

Optimal result . . . 1582
Rubi [A] (verified) . . . 1582
Mathematica [B] (warning: unable to verify) . . . 1584
Maple [F] . . . 1585
Fricas [F] . . . 1585
Sympy [F] . . . 1586
Maxima [F] . . . 1586
Giac [F] . . . 1586
Mupad [F(-1)] . . . 1586

Optimal result

Integrand size = 27, antiderivative size = 106

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx = -\frac{\text{AppellF1}\left(np,\frac{1}{2},\frac{1}{2}-m,1+np,\sec(e+fx),-\sec(e+fx)\right)(c(d\sec(e+fx))^p)^n(1+\sec(e+fx))^{-\frac{1}{2}-m}}{fnp\sqrt{1-\sec(e+fx)}}$$

```
[Out] -AppellF1(n*p,1/2-m,1/2,n*p+1,-sec(f*x+e),sec(f*x+e))*(c*(d*sec(f*x+e))^p)^
n*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/p/(1-sec(f*x+e)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4033, 3913, 3912, 138}

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx = -\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m \text{AppellF1}\left(np,\frac{1}{2},\frac{1}{2}-m,np+1,\sec(e+fx),-\right.}{fnp\sqrt{1-\sec(e+fx)}}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -((AppellF1[n*p, 1/2, 1/2 - m, 1 + n*p, Sec[e + f*x], -Sec[e + f*x]]*(c*(d*
Sec[e + f*x])^p)^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan
[e + f*x])/(f*n*p*Sqrt[1 - Sec[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+a\sec(e+fx))^m\,dx$$

$$= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n(1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (d\sec(e+fx))^{np}(1+\sec(e+fx))^m\,dx$$

$$= -\frac{\left(d(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+ \ldots}{f\sqrt{1-\sec(e+fx)}}$$

$$= -\frac{\text{AppellF1}\left(np,\frac{1}{2},\frac{1}{2}-m,1+np,\sec(e+fx),-\sec(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n(1+\sec(e+ \ldots}{fnp\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2425 vs. 2(106) = 212.

Time = 13.88 (sec) , antiderivative size = 2425, normalized size of antiderivative = 22.88

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx = \text{Result too large to show}$$

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n*p)*(Cos[(e + f*x)/2]^2*Sec[e +
 f*x])^(m + n*p)*(c*(d*Sec[e + f*x])^p)^n*(a*(1 + Sec[e + f*x]))^m*Tan[(e +
 f*x)/2])/(f*(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p
, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2)*((3*2^m*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^
(m + n*p))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p,
1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
 + (3*2^(1 + m)*(-1 + n*p)*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n*p)*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m + n*p
, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*Ap
pellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (m + n*p)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2
)^(-1 + n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]*(
-1/3*((1 - n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n*p)*Appell
F1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, m + n*p, 1 - n*p
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/
2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n
*p)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 + m)*AppellF1[1/2, m + n*p, 1 - n
*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1
+ n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]*(2*((-1
 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/
3*((1 - n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[
```

```
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n*p)*AppellF1[
3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n*p)*((
-3*(2 - n*p)*AppellF1[5/2, m + n*p, 3 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n*p)*Appel
lF1[5/2, 1 + m + n*p, 2 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (m + n*p)*((-3*(1 - n*p)*Appell
F1[5/2, 1 + m + n*p, 2 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + m + n*p)*AppellF1[5/2, 2
+ m + n*p, 1 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n
*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*Appe
llF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m + n*p)*AppellF1[1/2, m + n*p, 1
 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^
(-1 + n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n*p)*Tan[(e + f*x)/2
]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*S
ec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 -
 n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/
2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2)))
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+a\sec(fx+e))^m \, dx$$

[In] `int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)`

[Out] `int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)`

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx = \int ((d\sec(fx+e))^p c)^n (a\sec(fx+e)+a)^m \, dx$$

[In] `integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")`

[Out] `integral(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)`

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx$$
$$= \int (a(\sec(e+fx)+1))^m (c(d\sec(e+fx))^p)^n \, dx$$

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**m,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*(c*(d*sec(e + f*x))**p)**n, x)
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx$$
$$= \int ((d\sec(fx+e))^p c)^n (a\sec(fx+e)+a)^m \, dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx$$
$$= \int ((d\sec(fx+e))^p c)^n (a\sec(fx+e)+a)^m \, dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m \, dx$$
$$= \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{a}{\cos(e+f\,x)}\right)^m \, dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^m,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^m, x)
```

3.231 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx$

Optimal result 1587
Rubi [A] (verified) 1588
Mathematica [A] (verified) 1590
Maple [F] 1591
Fricas [F] 1591
Sympy [F] 1591
Maxima [F] 1592
Giac [F] 1592
Mupad [F(-1)] 1592

Optimal result

Integrand size = 27, antiderivative size = 275

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx$$
$$= \frac{a^3(7+4np) \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right) (c(d\sec(e+fx))^p)^n \sin(e+fx)}{fnp(2+np)\sqrt{\sin^2(e+fx)}}$$
$$- \frac{a^3(1+4np)\cos(e+fx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right) (c(d\sec(e+fx))^p}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}}$$
$$+ \frac{a^3(5+2np) (c(d\sec(e+fx))^p)^n \tan(e+fx)}{f(1+np)(2+np)}$$
$$+ \frac{(c(d\sec(e+fx))^p)^n (a^3+a^3\sec(e+fx))\tan(e+fx)}{f(2+np)}$$

```
[Out] a^3*(4*n*p+7)*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*se
c(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(n*p+2)/(sin(f*x+e)^2)^(1/2)-a^3*(4*n*p+1)*
cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d
*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+a^3*(2*n*p
+5)*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/(n*p+1)/(n*p+2)+(c*(d*sec(f*x+e))^p
)^n*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f/(n*p+2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4033, 3899, 4082, 3872, 3857, 2722}

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3\,dx =$$

$$-\frac{a^3(4np+1)\sin(e+fx)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec}{f\,(1-n^2p^2)\sqrt{\sin^2(e+fx)}}$$
$$+\frac{a^3(4np+7)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp(np+2)\sqrt{\sin^2(e+fx)}}$$
$$+\frac{a^3(2np+5)\tan(e+fx)\,(c(d\sec(e+fx))^p)^n}{f(np+1)(np+2)}$$
$$+\frac{\tan(e+fx)\,(a^3\sec(e+fx)+a^3)\,(c(d\sec(e+fx))^p)^n}{f(np+2)}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(7 + 4*n*p)*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*
x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*(2 + n*p)*Sqrt[Sin[e +
f*x]^2]) - (a^3*(1 + 4*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2
, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1
 - n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (a^3*(5 + 2*n*p)*(c*(d*Sec[e + f*x])^p)
^n*Tan[e + f*x])/(f*(1 + n*p)*(2 + n*p)) + ((c*(d*Sec[e + f*x])^p)^n*(a^3 +
 a^3*Sec[e + f*x])*Tan[e + f*x])/(f*(2 + n*p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\text{integral} = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+a\sec(e+fx))^3\,dx$$

$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2+np)} + \frac{\left(a(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+a\sec(e+fx))(a(2+2np)+}{2+np}$$

$$= \frac{a^3(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{\left(c(d\sec(e+fx))^p\right)^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2+np)} + \frac{\left(a(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}\left(a^2(2+np)(1+4np)+a^2(1+n\right.}{(1+np)(2+np)}$$

$$= \frac{a^3(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{\left(c(d\sec(e+fx))^p\right)^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2+np)} + \frac{\left(a^3(1+4np)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}\,dx}{1+np} + \frac{\left(a^3(7+4np)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{1+np}\,dx}{d(2+np)}$$

$$= \frac{a^3(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{\left(c(d\sec(e+fx))^p\right)^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2+np)} + \frac{\left(a^3(1+4np)\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx}{1+np} + \frac{\left(a^3(7+4np)\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-1-np}dx}{d(2+np)}$$

$$= \frac{a^3(7+4np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{fnp(2+np)\sqrt{\sin^2(e+fx)}} - \frac{a^3(1+4np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\left(c(d\sec(e+\right.}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}} + \frac{a^3(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{\left(c(d\sec(e+fx))^p\right)^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2+np)}$$

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.69

$$\int \left(c(d\sec(e+fx))^p\right)^n(a+a\sec(e+fx))^3\,dx$$

$$= \frac{a^3\cot(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(\left(2+9np+4n^2p^2\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)\sqrt{\right.\right.}{}$$

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*Cot[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*((2 + 9*n*p + 4*n^2*p^2)*Hyperge
ometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]
 + n*p*((6 + 3*n*p + (1 + n*p)*Sec[e + f*x])*Tan[e + f*x]^2 + (7 + 4*n*p)*H
ypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2]*Sec[e + f*x
]*Sqrt[-Tan[e + f*x]^2])))/(f*n*p*(1 + n*p)*(2 + n*p))
```

Maple [F]

$$\int (c(d\sec (fx+e))^p)^n (a+a\sec (fx+e))^3 dx$$

[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 dx = \int (a\sec (fx+e)+a)^3((d\sec (fx+e))^p c)^n dx$$

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")

[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3)*((d*sec(f*x + e))^p*c)^n, x)

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 dx = a^3\left(\int (c(d\sec (e+fx))^p)^n dx + \int 3(c(d\sec (e+fx))^p)^n \sec (e+fx) dx + \int 3(c(d\sec (e+fx))^p)^n \sec^2 (e+fx) dx + \int (c(d\sec (e+fx))^p)^n \sec^3 (e+fx) dx\right)$$

[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**3,x)

[Out] a**3*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(3*(c*(d*sec(e + f*x))**p)**n*sec(e + f*x), x) + Integral(3*(c*(d*sec(e + f*x))**p)**n*sec(e + f*x)**2, x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e + f*x)**3, x))

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx$$
$$= \int (a\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n \, dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx$$
$$= \int (a\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n \, dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx$$
$$= \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{a}{\cos(e+f\,x)}\right)^3 dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^3,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^3, x)
```

3.232 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx$

Optimal result . . . 1593
Rubi [A] (verified) . . . 1593
Mathematica [A] (verified) . . . 1596
Maple [F] . . . 1596
Fricas [F] . . . 1596
Sympy [F] . . . 1597
Maxima [F] . . . 1597
Giac [F] . . . 1597
Mupad [F(-1)] . . . 1598

Optimal result

Integrand size = 27, antiderivative size = 205

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx$$
$$= \frac{2a^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}$$
$$-\frac{a^2(1+2np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}}$$
$$+\frac{a^2(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)}$$

```
[Out] 2*a^2*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e)
)^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a^2*(2*n*p+1)*cos(f*x+e)*hyper
geom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^
n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+a^2*(c*(d*sec(f*x+e))^p)^n
*tan(f*x+e)/f/(n*p+1)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

= {4033, 3873, 3857, 2722, 4131}

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 \, dx =$$

$$-\frac{a^2(2np+1)\sin(e+fx)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec}{f\,(1-n^2p^2)\sqrt{\sin^2(e+fx)}}$$

$$+\frac{2a^2\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp\sqrt{\sin^2(e+fx)}}$$

$$+\frac{a^2\tan(e+fx)\,(c(d\sec(e+fx))^p)^n}{f(np+1)}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^2*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(c*(
d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a^2*(1 +
 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e
 + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*Sqrt[Sin
[e + f*x]^2]) + (a^2*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 + n*p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\text{integral} = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}(a+a\sec(e+fx))^2\,dx$$

$$= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}\left(a^2+a^2\sec^2(e+fx)\right)\,dx + \frac{\left(2a^2(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{1+np}\,dx}{d}$$

$$= \frac{a^2\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)} + \frac{\left(2a^2\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-1-np}\,dx}{d} + \frac{\left(a^2(1+2np)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}\,dx}{1+np}$$

$$= \frac{2a^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} + \frac{a^2\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)} + \frac{\left(a^2(1+2np)\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}\,dx}{1+np}$$

$$= \frac{2a^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a^2(1+2np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\left(c(d\sec(e\right.}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}} + \frac{a^2\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 \, dx$$
$$= \frac{a^2\cot(e+fx)\,(c(d\sec(e+fx))^p)^n \left((1+2np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)\sqrt{-\tan^2(e}{}$$

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]

[Out] (a^2*Cot[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*((1 + 2*n*p)*Hypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2] + n*p*(Tan[e + f*x]^2 + 2*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2])))/(f*n*p*(1 + n*p))

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+a\sec(fx+e))^2 \, dx$$

[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 \, dx = \int (a\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n \, dx$$

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*((d*sec(f*x + e))^p*c)^n, x)

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx$$
$$= a^2\left(\int (c(d\sec(e+fx))^p)^n\,dx + \int 2(c(d\sec(e+fx))^p)^n\sec(e+fx)\,dx + \int (c(d\sec(e+fx))^p)^n\sec^2(e+fx)\,dx\right)$$

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(2*(c*(d*sec(e + f*
x))**p)**n*sec(e + f*x), x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e + f
*x)**2, x))
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx$$
$$= \int (a\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx$$
$$= \int (a\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{a}{\cos(e+f\,x)}\right)^2 dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^2,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^2, x)
```

3.233 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx$

Optimal result 1599
Rubi [A] (verified) 1599
Mathematica [A] (verified) 1601
Maple [F] 1601
Fricas [F] 1601
Sympy [F] 1602
Maxima [F] 1602
Giac [F] 1602
Mupad [F(-1)] 1602

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \frac{a\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

```
[Out] a*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)
^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a*cos(f*x+e)*hypergeom([1/2, -1/2*
n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(
-n*p+1)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4033, 3872, 3857, 2722}

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \frac{a\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a\sin(e+fx)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Se
c[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*Cos[e + f*
x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*S
ec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}(a+a\sec(e+fx))\,dx\\
&= \left(a(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}\,dx\\
&\quad+\frac{\left(a(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{1+np}\,dx}{d}\\
&= \left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}\,dx\\
&\quad+\frac{\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-1-np}\,dx}{d}
\end{aligned}
$$

$$= \frac{a \,\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{np}{2}, \frac{1}{2}(2-np), \cos^2(e+fx)\right) (c(d\sec(e+fx))^p)^n \sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}$$
$$- \frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1-np), \frac{1}{2}(3-np), \cos^2(e+fx)\right) (c(d\sec(e+fx))^p)^n \text{s}}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx$$
$$= \frac{a\csc(e+fx)\left((1+np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{np}{2}, 1+\frac{np}{2}, \sec^2(e+fx)\right) + np\,\text{Hypergeometr}\right.}{fnp(1+np)}$$

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] (a*Csc[e + f*x]*((1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 +
 (n*p)/2, Sec[e + f*x]^2] + n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*
p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f*n
*p*(1 + n*p))
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+a\sec(fx+e))\,dx$$

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = a\left(\int (c(d\sec(e+fx))^p)^n\,dx + \int (c(d\sec(e+fx))^p)^n \sec(e+fx)\,dx\right)$$

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e)),x)
[Out] a*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral((c*(d*sec(e + f*x))**
p)**n*sec(e + f*x), x))
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a + \frac{a}{\cos(e+f\,x)}\right)dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x)),x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x)), x)
```

3.234 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$

Optimal result 1603
Rubi [A] (verified) 1604
Mathematica [A] (verified) 1606
Maple [F] 1606
Fricas [F] 1606
Sympy [F] 1606
Maxima [F] 1607
Giac [F] 1607
Mupad [F(-1)] 1607

Optimal result

Integrand size = 27, antiderivative size = 208

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \frac{(c(d\sec(e+fx))^p)^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{af\sqrt{\sin^2(e+fx)}} + \frac{(1-np)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2-np),\frac{1}{2}(4-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{af(2-np)\sqrt{\sin^2(e+fx)}}$$

```
[Out] (c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))-cos(f*x+e)*hypergeom([
1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(
f*x+e)/a/f/(sin(f*x+e)^2)^(1/2)+(-n*p+1)*cos(f*x+e)^2*hypergeom([1/2, -1/2*
n*p+1],[-1/2*n*p+2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/a/f/(-n
*p+2)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4033, 3905, 3872, 3857, 2722}

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx =$$

$$-\frac{\sin(e+fx)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{af\sqrt{\sin^2(e+fx)}}$$

$$+\frac{(1-np)\sin(e+fx)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2-np),\frac{1}{2}(4-np),\cos^2(e+fx)\right)(c(d\sec(e}{af(2-np)\sqrt{\sin^2(e+fx)}}$$

$$+\frac{\sin(e+fx)\,(c(d\sec(e+fx))^p)^n}{f(a\sec(e+fx)+a)}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] ((c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - (Cos[e +
 f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(
d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) + ((1 - n*p)*
Cos[e + f*x]^2*Hypergeometric2F1[1/2, (2 - n*p)/2, (4 - n*p)/2, Cos[e + f*x
]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(a*f*(2 - n*p)*Sqrt[Sin[e + f*x
]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
```

```
(a + b*Csc[e + f*x]))), x] + Dist[d*((n - 1)/(a*b)), Int[(d*Csc[e + f*x])^(
n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[
a^2 - b^2, 0]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\frac{(d\sec(e+fx))^{np}}{a+a\sec(e+fx)}\,dx$$

$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\left(d(1-np)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{-1+np}(a-a\sec(e+fx))\,dx}{a^2}$$

$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left((1-np)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}\,dx}{a} - \frac{\left(d(1-np)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{-1+np}\,dx}{a}$$

$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left((1-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx}{a} - \frac{\left(d(1-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{1-np}dx}{a}$$

$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin}{af\sqrt{\sin^2(e+fx)}} + \frac{(1-np)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2-np),\frac{1}{2}(4-np),\cos^2(e+fx)\right)\left(c(d\sec(e+\right.}{af(2-np)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.76

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \frac{\cot\left(\frac{1}{2}(e+fx)\right)(c(d\sec(e+fx))^p)^n\left(-\left((-1+np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)\sqrt{-\text{t}\cdots}\right.\right.}{\cdots}$$

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]

[Out] (Cot[(e + f*x)/2]*(c*(d*Sec[e + f*x])^p)^n*(-((-1 + n*p)*Hypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]) + n*p*(1 - Cos[e + f*x] + Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + n*p)/2, (1 + n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2])))/(a*f*n*p*(1 + Sec[e + f*x]))

Maple [F]

$$\int \frac{(c(d\sec(fx+e))^p)^n}{a+a\sec(fx+e)}dx$$

[In] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)

[Out] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)

Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \int \frac{((d\sec(fx+e))^p c)^n}{a\sec(fx+e)+a}\,dx$$

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")

[Out] integral(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)

Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{(c(d\sec(e+fx))^p)^n}{\sec(e+fx)+1}\,dx}{a}$$

[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e)),x)

[Out] Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x) + 1), x)/a

Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \int \frac{\left((d\sec\left(fx+e\right))^p c\right)^n}{a\sec\left(fx+e\right)+a}\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \int \frac{\left((d\sec\left(fx+e\right))^p c\right)^n}{a\sec\left(fx+e\right)+a}\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x)),x)
[Out] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x)), x)
```

3.235 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal result 1608
Rubi [A] (verified) 1608
Mathematica [A] (verified) 1611
Maple [F] 1611
Fricas [F] 1612
Sympy [F] 1612
Maxima [F] 1612
Giac [F] 1612
Mupad [F(-1)] 1613

Optimal result

Integrand size = 27, antiderivative size = 248

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{2(2-np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}$$
$$-\frac{(3-2np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\,\mathrm{s}}{3a^2f\sqrt{\sin^2(e+fx)}}$$
$$-\frac{2(2-np)\,(c(d\sec(e+fx))^p)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{(c(d\sec(e+fx))^p)^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] 2/3*(-n*p+2)*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec
(f*x+e))^p)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-1/3*(-2*n*p+3)*cos(f*x+
e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x
+e))^p)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-2/3*(-n*p+2)*(c*(d*sec(f*x+
e))^p)^n*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-1/3*(c*(d*sec(f*x+e))^p)^n*tan(f*x
+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

= {4033, 3902, 4105, 3872, 3857, 2722}

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{2(2-np)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{3a^2f\sqrt{\sin^2(e+fx)}}$$
$$-\frac{(3-2np)\sin(e+fx)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(}{3a^2f\sqrt{\sin^2(e+fx)}}$$
$$-\frac{2(2-np)\tan(e+fx)\,(c(d\sec(e+fx))^p)^n}{3a^2f(\sec(e+fx)+1)}-\frac{\tan(e+fx)\,(c(d\sec(e+fx))^p)^n}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*(2 - n*p)*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2
]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (
(3 - 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, C
os[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e +
 f*x]^2]) - (2*(2 - n*p)*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(3*a^2*f*(1
 + Sec[e + f*x])) - ((c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(3*f*(a + a*Sec
[e + f*x])^2)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
```

```
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^{p}\right)^{n}\right)\int\frac{(d\sec(e+fx))^{np}}{(a+a\sec(e+fx))^{2}}\,dx\\
&= -\frac{\left(c(d\sec(e+fx))^{p}\right)^{n}\tan(e+fx)}{3f(a+a\sec(e+fx))^{2}}-\frac{\left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^{p}\right)^{n}\right)\int\frac{(d\sec(e+fx))^{np}(a(-3+np)-a(-1+np)\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{3a^{2}}\\
&= -\frac{2(2-np)\left(c(d\sec(e+fx))^{p}\right)^{n}\tan(e+fx)}{3a^{2}f(1+\sec(e+fx))}-\frac{\left(c(d\sec(e+fx))^{p}\right)^{n}\tan(e+fx)}{3f(a+a\sec(e+fx))^{2}}-\frac{\left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^{p}\right)^{n}\right)\int(d\sec(e+fx))^{np}\left(-a^{2}(3-2np)(1-np)-2a^{2}np(2-\right.}{3a^{4}}\\
&= -\frac{2(2-np)\left(c(d\sec(e+fx))^{p}\right)^{n}\tan(e+fx)}{3a^{2}f(1+\sec(e+fx))}-\frac{\left(c(d\sec(e+fx))^{p}\right)^{n}\tan(e+fx)}{3f(a+a\sec(e+fx))^{2}}\\
&\quad+\frac{\left((3-2np)(1-np)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^{p}\right)^{n}\right)\int(d\sec(e+fx))^{np}\,dx}{3a^{2}}\\
&\quad+\frac{\left(2np(2-np)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^{p}\right)^{n}\right)\int(d\sec(e+fx))^{1+np}\,dx}{3a^{2}d}
\end{aligned}
$$

$$= -\frac{2(2-np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$
$$+ \frac{\left((3-2np)(1-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx}{3a^2}$$
$$+ \frac{\left(2np(2-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-1-np}dx}{3a^2d}$$
$$= \frac{2(2-np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}$$
$$- \frac{(3-2np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\left(c(d\sec(e+\right.}{3a^2f\sqrt{\sin^2(e+fx)}}$$
$$- \frac{2(2-np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.87

$$\int \frac{\left(c(d\sec(e+fx))^p\right)^n}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\left(c(d\sec(e+fx))^p\right)^n\left(-np(1+np)\tan(e+fx)+(1+\sec(e+fx))\left(2np(-2+np)(1+np)\tan(e+fx)\right.\right.}{}$$

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c*(d*Sec[e + f*x])^p)^n*(-(n*p*(1 + n*p)*Tan[e + f*x]) + (1 + Sec[e + f*x
])*(2*n*p*(-2 + n*p)*(1 + n*p)*Tan[e + f*x] + ((-1 + n*p)*(1 + n*p)*(-3 + 2
*n*p)*Cot[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x
]^2] - 2*n^2*p^2*(-2 + n*p)*Csc[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2
, (3 + n*p)/2, Sec[e + f*x]^2])*(1 + Sec[e + f*x])*Sqrt[-Tan[e + f*x]^2]))
/(3*a^2*f*n*p*(1 + n*p)*(1 + Sec[e + f*x])^2)
```

Maple [F]

$$\int \frac{\left(c(d\sec(fx+e))^p\right)^n}{(a+a\sec(fx+e))^2}dx$$

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2} \, dx = \int \frac{((d\sec(fx+e))^p c)^n}{(a\sec(fx+e)+a)^2} \, dx$$

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out] integral(((d*sec(f*x + e))^p*c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)

Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2} \, dx = \frac{\int \frac{(c(d\sec(e+fx))^p)^n}{\sec^2(e+fx)+2\sec(e+fx)+1} \, dx}{a^2}$$

[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e))**2,x)

[Out] Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**2

Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2} \, dx = \int \frac{((d\sec(fx+e))^p c)^n}{(a\sec(fx+e)+a)^2} \, dx$$

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")

[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a)^2, x)

Giac [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2} \, dx = \int \frac{((d\sec(fx+e))^p c)^n}{(a\sec(fx+e)+a)^2} \, dx$$

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")

[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x))^2,x)
[Out] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x))^2, x)
```

3.236 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx$

Optimal result 1614
Rubi [N/A] 1614
Mathematica [N/A] 1615
Maple [N/A] (verified) 1615
Fricas [N/A] 1615
Sympy [N/A] 1616
Maxima [N/A] 1616
Giac [N/A] 1616
Mupad [N/A] 1617

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx = (d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \operatorname{Int}((d\sec(e+fx))^{np}(a+b\sec(e+fx))^m, x)$$

```
[Out] (c*(d*sec(f*x+e))^p)^n*Unintegrable((d*sec(f*x+e))^(n*p)*(a+b*sec(f*x+e))^m
,x)/((d*sec(f*x+e))^(n*p))
```

Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] ((c*(d*Sec[e + f*x])^p)^n*Defer[Int][(d*Sec[e + f*x])^(n*p)*(a + b*Sec[e +
f*x])^m, x])/(d*Sec[e + f*x])^(n*p)
```

Rubi steps

$$\text{integral} = \left((d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n\right) \int (d\sec(e+fx))^{np}(a+b\sec(e+fx))^m \, dx$$

Mathematica [N/A]

Not integrable

Time = 6.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx$$

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]
[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (c(d\sec(fx+e))^p)^n (a+b\sec(fx+e))^m \, dx$$

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx = \int ((d\sec(fx+e))^p c)^n (b\sec(fx+e)+a)^m \, dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Sympy [N/A]

Not integrable

Time = 5.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx$$

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**m,x)
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**m, x)
```

Maxima [N/A]

Not integrable

Time = 4.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx = \int ((d\sec(fx+e))^p c)^n (b\sec(fx+e)+a)^m dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx = \int ((d\sec(fx+e))^p c)^n (b\sec(fx+e)+a)^m dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Mupad [N/A]

Not integrable

Time = 15.97 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+f\,x)}\right)^m \, dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^m,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^m, x)
```

3.237 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3\,dx$

Optimal result 1618
Rubi [A] (verified) 1619
Mathematica [A] (verified) 1621
Maple [F] 1622
Fricas [F] 1622
Sympy [F] 1622
Maxima [F] 1622
Giac [F] 1623
Mupad [F(-1)] 1623

Optimal result

Integrand size = 27, antiderivative size = 296

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3\,dx$$
$$= \frac{b(b^2(1+np)+3a^2(2+np))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n \sin}{fnp(2+np)\sqrt{\sin^2(e+fx)}}$$
$$- \frac{a(3b^2np+a^2(1+np))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}}$$
$$+ \frac{ab^2(5+2np)(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)(2+np)}$$
$$+ \frac{b^2(c(d\sec(e+fx))^p)^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)}$$

```
[Out] b*(b^2*(n*p+1)+3*a^2*(n*p+2))*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*
x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(n*p+2)/(sin(f*x+e)^2)^(1/2
)-a*(3*b^2*n*p+a^2*(n*p+1))*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*
n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(si
n(f*x+e)^2)^(1/2)+a*b^2*(2*n*p+5)*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/(n*p+
1)/(n*p+2)+b^2*(c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))*tan(f*x+e)/f/(n*p+2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4033, 3927, 4132, 3857, 2722, 4131}

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx =$$
$$-\frac{a(a^2(np+1)+3b^2np)\sin(e+fx)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\ldots}{f\,(1-n^2p^2)\sqrt{\sin^2(e+fx)}}$$
$$+\frac{b(3a^2(np+2)+b^2(np+1))\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec\ldots}{fnp(np+2)\sqrt{\sin^2(e+fx)}}$$
$$+\frac{ab^2(2np+5)\tan(e+fx)\,(c(d\sec(e+fx))^p)^n}{f(np+1)(np+2)}$$
$$+\frac{b^2\tan(e+fx)(a+b\sec(e+fx))\,(c(d\sec(e+fx))^p)^n}{f(np+2)}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] (b*(b^2*(1 + n*p) + 3*a^2*(2 + n*p))*Hypergeometric2F1[1/2, -1/2*(n*p), (2
- n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*(2
+ n*p)*Sqrt[Sin[e + f*x]^2]) - (a*(3*b^2*n*p + a^2*(1 + n*p))*Cos[e + f*x]*
Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[
e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (a*b^
2*(5 + 2*n*p)*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 + n*p)*(2 + n*p)
) + (b^2*(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])*Tan[e + f*x])/(f*(2
+ n*p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
```

```
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\text{integral} = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+b\sec(e+fx))^3\,dx$$

$$= \frac{b^2\left(c(d\sec(e+fx))^p\right)^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)} + \frac{\left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}\left(ad(b^2np+a^2(2+np))+bd(b^2(1+\right.}{d(2+np)}$$

$$= \frac{b^2\left(c(d\sec(e+fx))^p\right)^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)} + \frac{\left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}\left(ad(b^2np+a^2(2+np))+ab^2d(5+\right.}{d(2+np)} + \frac{\left(b(b^2(1+np)+3a^2(2+np))\right)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int (d\sec(e+fx))^{1+np}\,dx}{d(2+np)}$$

$$= \frac{ab^2(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{b^2\left(c(d\sec(e+fx))^p\right)^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)} + \frac{\left(b(b^2(1+np)+3a^2(2+np))\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-1-np}dx}{d(2+np)} + \frac{\left(a(3b^2np+a^2(1+np))\,(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}dx}{1+np}$$

$$= \frac{b(b^2(1+np)+3a^2(2+np))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+f\right.}{fnp(2+np)\sqrt{\sin^2(e+fx)}} + \frac{ab^2(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{b^2\left(c(d\sec(e+fx))^p\right)^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)} + \frac{\left(a(3b^2np+a^2(1+np))\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx}{1+np}$$

$$= \frac{b(b^2(1+np)+3a^2(2+np))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+f\right.}{fnp(2+np)\sqrt{\sin^2(e+fx)}} - \frac{a(3b^2np+a^2(1+np))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}} + \frac{ab^2(5+2np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{b^2\left(c(d\sec(e+fx))^p\right)^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.94

$$\int\left(c(d\sec(e+fx))^p\right)^n(a+b\sec(e+fx))^3\,dx = -\frac{\csc^3(e+fx)\left(a^3(6+11np+6n^2p^2+n^3p^3)\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+f\right.\right.}{}$$

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((Csc[e + f*x]^3*(a^3*(6 + 11*n*p + 6*n^2*p^2 + n^3*p^3)*Cos[e + f*x]^3*Hy
pergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(3*a*b*(
3 + 4*n*p + n^2*p^2)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + (n*p)/2, 2 + (
```

```
n*p)/2, Sec[e + f*x]^2] + (2 + n*p)*(3*a^2*(3 + n*p)*Cos[e + f*x]^2*Hyperge
ometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2] + b^2*(1 + n*p)*H
ypergeometric2F1[1/2, (3 + n*p)/2, (5 + n*p)/2, Sec[e + f*x]^2]))*(c*(d*Se
c[e + f*x])^p)^n*(-Tan[e + f*x]^2)^(3/2))/(f*n*p*(1 + n*p)*(2 + n*p)*(3 + n
*p)))
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+b\sec(fx+e))^3\,dx$$

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3\,dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**3,x)
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**3, x)
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx = \int (b\sec(fx+e)+a)^3 ((d\sec(fx+e))^p c)^n \, dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+f\,x)}\right)^3 dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^3,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^3, x)
```

3.238 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx$

Optimal result 1624
Rubi [A] (verified) 1624
Mathematica [A] (verified) 1627
Maple [F] 1627
Fricas [F] 1627
Sympy [F] 1628
Maxima [F] 1628
Giac [F] 1628
Mupad [F(-1)] 1628

Optimal result

Integrand size = 27, antiderivative size = 211

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx = \frac{2ab\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} - \frac{(b^2np+a^2(1+np))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}} + \frac{b^2(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)}$$

```
[Out] 2*a*b*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e)
)^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-(b^2*n*p+a^2*(n*p+1))*cos(f*x+
e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x
+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+b^2*(c*(d*sec(f*x+
e))^p)^n*tan(f*x+e)/f/(n*p+1)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used

$= \{4033, 3873, 3857, 2722, 4131\}$

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx = -\frac{(a^2(np+1)+b^2np)\sin(e+fx)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx\right.}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}} + \frac{2ab\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp\sqrt{\sin^2(e+fx)}} + \frac{b^2\tan(e+fx)\,(c(d\sec(e+fx))^p)^n}{f(np+1)}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (2*a*b*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(c*(
d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - ((b^2*n*p
 + a^2*(1 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p
)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2
)*Sqrt[Sin[e + f*x]^2]) + (b^2*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1
 + n*p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
```

```
] &&  !IntegerQ[n]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}(a+b\sec(e+fx))^2\,dx\\
&= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}\left(a^2+b^2\sec^2(e+fx)\right)\,dx\\
&\quad+\frac{\left(2ab(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{1+np}\,dx}{d}\\
&= \frac{b^2\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)}\\
&\quad+\frac{\left(2ab\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-1-np}\,dx}{d}\\
&\quad+\left(\left(a^2+\frac{b^2np}{1+np}\right)(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int(d\sec(e+fx))^{np}\,dx\\
&= \frac{2ab\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}\\
&\quad+\frac{b^2\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)}+\left(\left(a^2\right.\right.\\
&\quad\left.\left.+\frac{b^2np}{1+np}\right)\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}\,dx\\
&= \frac{2ab\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}\\
&\quad-\frac{\left(a^2+\frac{b^2np}{1+np}\right)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)\left(c(d\sec(e+\right.}{f(1-np)\sqrt{\sin^2(e+fx)}}\\
&\quad+\frac{b^2\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{f(1+np)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx$$
$$= \frac{\csc(e+fx)\left(a^2(2+3np+n^2p^2)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)+bnp\big(b(1$$

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Csc[e + f*x]*(a^2*(2 + 3*n*p + n^2*p^2)*Cos[e + f*x]^2*Hypergeometric2F1[1
/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(b*(1 + n*p)*Hypergeometr
ic2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sec[e + f*x]^2] + 2*a*(2 + n*p)*Cos[e
+ f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2]))*S
ec[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f*n*p*(1 + n*p
)*(2 + n*p))
```

Maple [F]

$$\int \left(c\left(d\sec\left(fx+e\right)\right)^{p}\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{2}dx$$

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx = \int (b\sec(fx+e)+a)^2\left((d\sec(fx+e))^p c\right)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*((d*sec(f*x + e))^
p*c)^n, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx$$

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**2,x)
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx = \int (b\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n \, dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx = \int (b\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n \, dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx = \int \left(c\left(\frac{d}{\cos(e+fx)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+fx)}\right)^2 dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^2,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^2, x)
```

3.239 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$

Optimal result 1629
Rubi [A] (verified) 1629
Mathematica [A] (verified) 1631
Maple [F] 1631
Fricas [F] 1631
Sympy [F] 1632
Maxima [F] 1632
Giac [F] 1632
Mupad [F(-1)] 1632

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

```
[Out] b*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)
^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a*cos(f*x+e)*hypergeom([1/2, -1/2*
n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(
-n*p+1)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4033, 3872, 3857, 2722}

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \frac{b\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a\sin(e+fx)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] (b*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Se
c[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*Cos[e + f*
x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*S
ec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}(a+b\sec(e+fx))\,dx \\
&= \left(a(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{np}\,dx \\
&\quad+\frac{\left(b(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx))^{1+np}\,dx}{d} \\
&= \left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}\,dx \\
&\quad+\frac{\left(b\left(\frac{\cos(e+fx)}{d}\right)^{np}\left(c(d\sec(e+fx))^p\right)^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-1-np}\,dx}{d}
\end{aligned}$$

$$= \frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\,\text{s}}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

$$\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))\,dx$$
$$= \frac{\csc(e+fx)\left(a(1+np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)+bnp\,\text{Hypergeomet}\right.}{fnp(1+np)}$$

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] (Csc[e + f*x]*(a*(1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 +
 (n*p)/2, Sec[e + f*x]^2] + b*n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 +
n*p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f
*n*p*(1 + n*p))
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n\,(a+b\sec(fx+e))\,dx$$

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)((d\sec(fx+e))^p\,c)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e)),x)
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a + \frac{b}{\cos(e+f\,x)}\right)\,dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x)),x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x)), x)
```

3.240 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$

Optimal result 1633
Rubi [A] (verified) 1633
Mathematica [B] (warning: unable to verify) 1635
Maple [F] 1636
Fricas [F] 1636
Sympy [F] 1636
Maxima [F] 1636
Giac [F] 1637
Mupad [F(-1)] 1637

Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = -\frac{b\,\text{AppellF1}\left(\frac{1}{2},\frac{np}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}(c(d\sec(e+fx))^p)^n\sin(e+fx)}{(a^2-b^2)\,f} + \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+np),1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}(c(d\sec(e+}{(a^2-b^2)\,f}$$

```
[Out] -b*AppellF1(1/2,1/2*n*p,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(cos
(f*x+e)^2)^(1/2*n*p)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)/f+a*Appell
F1(1/2,1/2*n*p-1/2,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*cos(f*x+e
)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)/
f
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4033, 3954, 2902, 3268, 440}

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}(c(d\sec(e+fx))^p)^n\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(np-1),1,\frac{3}{2},\sin^2(e+f\right.}{f\,(a^2-b^2)} - \frac{b\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}(c(d\sec(e+fx))^p)^n\,\text{AppellF1}\left(\frac{1}{2},\frac{np}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\,(a^2-b^2)}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] -((b*AppellF1[1/2, (n*p)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(Cos[e + f*x]^2)^((n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x]
)/((a^2 - b^2)*f)) + (a*AppellF1[1/2, (-1 + n*p)/2, 1, 3/2, Sin[e + f*x]^2,
 (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n*p
)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)*f)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))^{np}}{a+b\sec(e+fx)}\,dx$$

$$= \left(\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{1-np}(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= -\left(\left(a\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{2-np}(e+fx)}{b^2-a^2\cos^2(e+fx)}\,dx\right) + \left(b\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{1-np}(e+fx)}{b^2-a^2\cos^2(e+fx)}\,dx$$

$$= \frac{\left(b\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\right)\text{Subst}\left(\int \frac{(1-x^2)^{-\frac{np}{2}}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(e+fx)\right)}{f} - \frac{\left(a\cos^{np+2\left(\frac{1}{2}-\frac{np}{2}\right)}(e+fx)\cos^2(e+fx)^{-\frac{1}{2}+\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\right)\text{Subst}\left(\int \frac{(1-x^2)^{\frac{1}{2}(1-np)}}{-a^2+b^2+a^2x^2}\,dx, x, \ldots\right.}{f}$$

$$= -\frac{b\,\text{AppellF1}\left(\frac{1}{2},\frac{np}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\sin(e+f\ldots}{(a^2-b^2)f} + \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+np),1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}(c(\ldots}{(a^2-b^2)f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5411 vs. $2(206) = 412$.

Time = 32.62 (sec) , antiderivative size = 5411, normalized size of antiderivative = 26.27

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(c(d\sec(fx+e))^p)^n}{a+b\sec(fx+e)}dx$$

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{((d\sec(fx+e))^p c)^n}{b\sec(fx+e)+a}\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
[Out] integral(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e)),x)
[Out] Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{((d\sec(fx+e))^p c)^n}{b\sec(fx+e)+a}\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{((d\sec(fx+e))^p\,c)^n}{b\sec(fx+e)+a}\,dx$$

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x)),x)
[Out] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x)), x)
```

3.241 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal result 1638
Rubi [A] (verified) 1639
Mathematica [B] (warning: unable to verify) 1641
Maple [F] 1641
Fricas [F] 1642
Sympy [F] 1642
Maxima [F] 1642
Giac [F] 1642
Mupad [F(-1)] 1643

Optimal result

Integrand size = 27, antiderivative size = 322

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = -\frac{2ab\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-2+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}(c(d\sec(e+fx))^p)^n\sin(e+\ldots}{(a^2-b^2)^2 f} + \frac{a^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-3+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}(c(d\sec(e\ldots}{(a^2-b^2)^2 f} + \frac{b^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}(c(d\sec(e\ldots}{(a^2-b^2)^2 f}$$

```
[Out] -2*a*b*AppellF1(1/2,1/2*n*p-1,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2)
)*(cos(f*x+e)^2)^(1/2*n*p)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)^2/f+
a^2*AppellF1(1/2,1/2*n*p-3/2,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))
*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/
(a^2-b^2)^2/f+b^2*AppellF1(1/2,1/2*n*p-1/2,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e
)^2/(a^2-b^2))*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)
^n*sin(f*x+e)/(a^2-b^2)^2/f
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4033, 3954, 2903, 3268, 440}

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = -\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,(c(d\sec(e+fx))^p)^n\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(np-2),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+}{a^2-b^2}\right.}{f\,(a^2-b^2)^2} + \frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\,(c(d\sec(e+fx))^p)^n\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(np-3),2,\frac{3}{2},\sin^2(e\right.}{f\,(a^2-b^2)^2} + \frac{b^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\,(c(d\sec(e+fx))^p)^n\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(np-1),2,\frac{3}{2},\sin^2(e\right.}{f\,(a^2-b^2)^2}$$

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a*b*AppellF1[1/2, (-2 + n*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*
x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^((n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[
e + f*x])/((a^2 - b^2)^2*f) + (a^2*AppellF1[1/2, (-3 + n*p)/2, 2, 3/2, Sin[
e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)
^((-1 + n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)^2*f) +
(b^2*AppellF1[1/2, (-1 + n*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^
2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n*p)/2)*(c*(d*Sec[e +
f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)^2*f)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2903

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
```

```
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))^{np}}{(a+b\sec(e+fx))^2}\,dx \\
&= \left(\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{2-np}(e+fx)}{(b+a\cos(e+fx))^2}\,dx \\
&= \left(\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \left(\frac{b^2\cos^{2-np}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2} - \frac{2ab\cos^{3-np}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2} + \frac{a^2\cos^{4-np}(e+fx)}{\left(-b^2+a^2\cos^2(e+fx)\right)^2}\right)dx \\
&= \left(a^2\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{4-np}(e+fx)}{\left(-b^2+a^2\cos^2(e+fx)\right)^2}\,dx \\
&\quad - \left(2ab\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{3-np}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2}\,dx \\
&\quad + \left(b^2\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{2-np}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2}\,dx
\end{aligned}
$$

$$= -\frac{\left(2ab\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\right)\text{Subst}\left(\int\frac{(1-x^2)^{\frac{1}{2}(2-np)}}{(-a^2+b^2+a^2x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$+\frac{\left(a^2\cos^{np+2\left(\frac{1}{2}-\frac{np}{2}\right)}(e+fx)\cos^2(e+fx)^{-\frac{1}{2}+\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\right)\text{Subst}\left(\int\frac{(1-x^2)^{\frac{1}{2}(3-np)}}{(a^2-b^2-a^2x^2)^2}\,dx, x\right.}{f}$$
$$+\frac{\left(b^2\cos^{np+2\left(\frac{1}{2}-\frac{np}{2}\right)}(e+fx)\cos^2(e+fx)^{-\frac{1}{2}+\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\right)\text{Subst}\left(\int\frac{(1-x^2)^{\frac{1}{2}(1-np)}}{(-a^2+b^2+a^2x^2)^2}\,dx,\right.}{f}$$
$$= -\frac{2ab\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-2+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right.}{(a^2-b^2)^2 f}$$
$$+\frac{a^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-3+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}\,(c}{(a^2-b^2)^2 f}$$
$$+\frac{b^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}\,(c}{(a^2-b^2)^2 f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 14108 vs. 2(322) = 644.

Time = 46.81 (sec) , antiderivative size = 14108, normalized size of antiderivative = 43.81

$$\int\frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]
[Out] Result too large to show
```

Maple [F]

$$\int\frac{(c(d\sec(fx+e))^p)^n}{(a+b\sec(fx+e))^2}dx$$

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{((d\sec(fx+e))^p c)^n}{(b\sec(fx+e)+a)^2}\,dx$$

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")

[Out] integral(((d*sec(f*x + e))^p*c)^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2), x)

Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx$$

[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e))**2,x)

[Out] Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x))**2, x)

Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{((d\sec(fx+e))^p c)^n}{(b\sec(fx+e)+a)^2}\,dx$$

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")

[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a)^2, x)

Giac [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{((d\sec(fx+e))^p c)^n}{(b\sec(fx+e)+a)^2}\,dx$$

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")

[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x))^2,x)
[Out] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x))^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 1645

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```